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This paper explores how adjustment costs affect equilibrium outcomes in the labor market. It does so using two equilibrium models: the classical framework of Burdett – Mortensen (1998) and its latest extension — the model by Burdett et al. (2009), which includes human capital accumulation. In both cases, firm hiring costs are introduced into the model. The paper presents an analysis of the impact of these costs on the equilibrium distribution of wage offers, the distribution of observed wages, firms' profits, and in the extended model, the distribution of experiences and the variance decomposition of wages. Finally, using the framework of the extended model, the paper analyzes in detail the individual wage process, decomposing it into the effects of experience and of tenure. The main results are as follows: hiring costs make the distribution of wage offers and that of observed wages less dispersed and workers are typically paid less. In the extended model, the differences in the quality of workers between high- and low-paying firms are enhanced and the variance of observed wages within a cohort of given experience declines. Moreover, the differences in initial abilities and in accumulated experiences acquire bigger weight in explaining the variance of observed wages, whereas firm effects become less important. Analysis of the individual life-time wage profile shows that more qualified workers usually end up in high-paying firms because longer experience is associated with longer duration of uninterrupted search on-the-job.

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1. Introduction

Nowadays the model of Burdett-Mortensen (1998, henceforth BM) is widely accepted as a baseline way to think about the labor market. This model was the first one to show that when there are search frictions and agents behave strategically, assuming on-the-job search is enough to obtain persistent wage differentials even when all workers and firms are respectively identical. This paper gave rise to a voluminous literature that has in many ways enriched the initial environment (for the overview see Rogerson, Shimer, Wright (2005)). One of the most up-to-date extensions of the original model is the work by Burdett, Coles and Carrillo-Tudela (2009, henceforth BCC), that makes the first step to connect between the nature of equilibrium in search markets and the study of human capital accumulation through learning-by-doing. The introduction of the human capital accumulation allows to improve on the original model in many respects, in particular, to obtain a Pareto-like wage distribution with a "fat" right tail.

In our work we are extending the baseline model of BM, and then the model of BCC by assuming that there is a cost that a firm pays upon each hire. This assumption is a realistic one — indeed, in most industries a firm bears expenses in order to adjust between worker's skills and the job. One example of these expenses are training costs, another — the costs of providing equipment for the new-hired. Labor adjustment costs have been studied by many authors (s. for example, Abowd, Kramarz (2003) and Cooper, Willis (2009)), but mostly with respect to the problem of explaining the dynamics of employment on micro and macro level. The novelty of our work is that we use the baseline framework, and then the framework of BCC to explore the influence of adjustment costs on the equilibrium outcomes — the distribution of offers, the distribution of wages, firm's profits, the properties of equilibrium distribution of experiences.

We model hiring costs differently in the baseline and the extended environments. For the BM model we assume equal fixed hiring cost for all workers, which is natural as all workers are identical. In the BCC model we assume that hiring costs are proportional to worker's productivity — that is, hiring a qualified worker is more costly than a worker without experience.

In both models the introduction of adjustment costs does not directly influence the optimal behavior of workers, only indirectly, through the change in the offers distribution. In general we can say that higher costs mean lower equlibrium profits, lower equilibrium wage dispersion, the contraction in the range of equilibrium offers. In the extended model adjustment costs have an impact on the joint distribution of piece rates and experiences and on the variance decomposition of wages. Namely, the higher are the costs, the higher is the correlation between piece rates and experiences. More than that, for each cohort of given experience the variance of observed piece rates declines; the quality of workforce in low-paying firms deteriorates, whereas the quality of workforce in high-paying firms improves. Finally, as costs grow, the differentials in initial abilities and accumulated experiences become more important in explaining the observed variance of wages, whereas the input of the variance in observed piece rates goes down.

There are a few papers which have investigated learning-by-doing effects within a similar turnover framework as in BCC (2009). Bunzel et al. (1999) analyze the BM model with human capital accumulation. Unlike the approach of BCC, they assume that the entire human capital is lost when changing a job. This leads to quite different results — for example, Bunzel et al. always have one point in wage distribution that is not differentiable, more than that, the tail is not Pareto-like, as obtained in BCC. Rubinstein and Weiss (2007) analyze human capital accumulation and on-the-job search but do not consider equilibrium. Barlevy (2008) uses record statistics approach to estimate the wage-offers distribution, and shows, using NLSY data, that we reject the lognormal distribution in favor of Pareto. He also shows that for the workers in his sample (young workers) the wage growth is mostly due to the accumulation of general human capital and on-the-job search — the model of BCC is in line with these findings, they do not assume firm-specific capital at all.

Bagger et al. (2006) use the outside offer-matching framework developed in Postel-Vinay and Robin (2002a) to incorporate learning-on-the-job with individual productivity shocks. Unlike BCC, they are interested in the individual wage process rather than in the cross-sectional distributions. The main analytical result of their paper is that they decompose the individual wage growth (conditional on experience) into a term reflecting the contribution of human capital accumulation and a term, reflecting the impact of job search. They find that the job-search-related wage growth is

similar across education groups, whereas human capital's role differs markedly. For low-educated workers, human capital accumulation is found to contribute slightly negatively (if anything) to total wage growth. Among high-educated workers human capital accumulation is the primary source for early career wage growth, and this contribution of human capital accumulation declines sharply with experience. This is different from the analytical implications of BCC that the early wage growth is mostly due to on-the-job search, whereas at long experiences it is mostly due to human capital accumulation.

There are also papers that focus on the decomposition of wage dispersion. Abowd, Kramarz and Margolis (1999) study a longitudinal sample of over one million French workers.They find that at the individual level, person effects tend to be more important than firm effects in explaining compensation variability. The theoretical findings of BCC are consistent with these facts — the share of firm fixed effects in explaining the cross-section variance of wages is always much lower than that of the person effects. In addition, Abowd, Kramarz and Margolis find that enterprises paying higher wages are also more high-skilled employee intensive. The latter is a fundamental insight of the BCC model.

Postel-Vinay and Robin (2002b) build an equilibrium model with counter-offer matching and ex ante heterogeneous workers and firms. The structural model is estimated on French panel data, providing a decomposition of the cross-employee wage variance. The authors find that the share of cross-sectional wage variance that is explained by person effects varies across skill groups — it is the highest (almost 40%) for high-skilled white-collars and goes down to 0 as the observed skill level declines. The analysis of BCC does not cover the wage variance decomposition by skill groups; in general, the overall share of experience in explaining the wage dispersion varies in BCC from 18% to 65%.

The paper is organized as follows: in sections 2 and 3 we present and analyze the basic BM model with adjustment costs, in section 4 we describe the setting of the BCC and derive the equilibrium, in section 5 we analyze the properties of the equilibrium with adjustment costs, section 6 contains simulations and the analysis of variance decomposition. Section 7 is devoted to the analysis of the career development in BCC over time. Section 8 concludes.

2. Baseline Burdett-Mortensen: the model with adjustment costs

We keep the basic assumptions of the model unchanged — there is a continuum of measure m of identical workers and measure 1 of identical firms participating in a labor market. At a moment in time, each worker is either unemployed (state 0) or employed (state 1). The expected discounted lifetime income when a worker is unemployed V_0 is:

$$rV_0 = b + \lambda_0 \cdot \left[\int \max\left\{V_0, V_1(w')\right\} dF(w') - V_0\right] \quad (1)$$

where r is the discount rate, b is the unemployment benefit, λ_0 is the Poisson arrival rate of offers, F(·) is the cumulative distribution function of offers. In other words, the flow value of being unemployed equals to the income while unemployed plus the expected capital gain attributable to searching for a job where acceptance occurs only if the value of employment $V_1(w')$ exceeds that of continued search. Similarly, the expected discounted lifetime income when a worker is employed at wage rate w solves:

$$rV_1(w) = w + \lambda_1 \cdot \left[\int \max\left\{V_1(w), V_1(w')\right\} - V_1(w)\right] dF(w') + \\ + \delta \cdot \left[V_0 - V_1(w)\right] \quad (2)$$

where δ is an exogenous Poisson rate of job destruction, and $\lambda_1 < \lambda_0$ is the offers arrival rate for the employed. As $V_1(\cdot)$ is increasing in w whereas V_0 is independent of w, there exists a reservation wage R such that:

$$V_1(w) >< V_0 \qquad as \qquad w >< R$$

where $V_1(R) = V_0$. Thus, the optimal strategy of a worker is to accept an offer if and only if it is above R, and reject it otherwise. Evaluating (2) at w=R, subtracting (1) from (2) and assuming $r / \lambda_0 \to 0$ gives the following expression for R:

$$R - b = \left[\frac{\lambda_0 - \lambda_1}{\delta}\right] \int_R^\infty \frac{1 - F(x)}{1 + \frac{\lambda_1}{\delta}\left(1 - F(x)\right)} dx \quad (3)$$

Given the reservation wage, the flows of workers into and out of unemployment can be easily specified. Let u denote the steady-state number of workers unemployed. In steady state the flow of workers into employment

is $\lambda_0 \cdot \left(1 - F(R)\right) \cdot u$ equals the outflow into unemployment due to destruction shocks $\delta \cdot \left(m - u\right)$, therefore:

$$u = \frac{m}{1 + \frac{\lambda_0}{\delta}\left[1 - F(R)\right]}$$

is the steady-state unemployment rate. In the same manner we can characterize the inflows and outflows from the pool of the workers currently employed at a wage below w, $(m-u)\cdot G(w)$: the inflow is $u\lambda_0\left[F(w) - F(R)\right]$, that is, those unemployed workers who receive and accept an offer below w. The outflow is: $(m-u)G(w)\cdot\left[\delta + \lambda_1 \cdot (1 - F(R))\right]$— workers are exogenously laid-off, and quit to better-paid jobs. In steady state the pool is constant, the flows are equal:

$$u\lambda_0\left[F(w) - F(R)\right] = (m-u)G(w)\cdot\left[\delta + \lambda_1\left(1 - F(R)\right)\right]$$

$$G(w) = \frac{F(w) - F(R)}{\left[1 - F(R)\right]\cdot\left[1 + \frac{\lambda_1}{\delta}\left(1 - F(w)\right)\right]} \quad (4)$$

Now we can express the measure of workers per firm earning a wage w in steady-state as:

$$l(w \mid R, F) = \lim_{\varepsilon \to 0} \frac{G(w) - G(w - \varepsilon)}{F(w) - F(w - \varepsilon)} \cdot \left(m - u\right)$$

$$l(w \mid R, F) = \frac{G'(w)}{F'(w)} \cdot (m - u)$$

$$l(w \mid R, F) = \frac{m\frac{\lambda_0}{\delta}\left[1 + \frac{\lambda_1}{\delta}(1 - F(R))\right]}{\left[1 + \frac{\lambda_1}{\delta}(1 - F(w))\right]^2 \cdot \left[1 + \frac{\lambda_0}{\delta}(1 - F(R))\right]} \qquad if \quad w \geq R \quad (5)$$

$$l(w \mid R, F) \qquad if \qquad w < R$$

Thus, (5) specifies the steady state number of workers available to a firm offering any particular wage, given the wages offered by other firms, represented by $F(w)$, and the workers reservation wage R. From (5) it follows immediately that $l(\cdot \mid R,F)$ is (i) increasing in w, (ii) continuous except where F has a mass point. Till equation (5) inclusive we have been presenting the original BM model; in what follows our assumption about the hiring cost comes into play.

Now we consider the behavior of a firm. Let p denote the flow of revenue generated per employed worker (p is common for all workers, they are ex ante identical). Let c denote the adjustment cost per worker. Conditional on R and F, each employer is assumed to post a wage that maximizes its steady-state flow profit, that is, an optimal wage offer solves the following problem:

$$\pi = \max_{w}\left(p - w - c\right)\cdot l(w \mid R,F)$$

The equilibrium solution to the search and wage-posting game described above can be described by a triple (R, F, π) such that R is compatible with workers' optimal behavior, π is the solution to the maximization problem of the firm, and F is such that:

$$\begin{aligned}&(p - w - c)\cdot l(w \mid R,F) = \pi \ \textit{for all } w \textit{ on the } \sup \textit{port of } F\\ &(p - w - c)\cdot l(w \mid R,F) \le \pi \ \textit{otherwise}\end{aligned} \tag{6}$$

Let w and w denote the infimum and the supremum of the support of equilibrium F (given one exists). Note that no employer will offer a wage below R because then an employer would have no employees. Hence, without loss of generality, we consider only those distribution functions which have w⩾R.

Next, we can rule out the noncontinuous wage offer distributions. Suppose, F has a mass point in some w, R⩽w<p. Then any employer offering a wage slightly greater than w would have a significantly larger steady-state labor force (see (5)) and only a slightly smaller profit per worker (p-w-c is continuous in w). Therefore, a wage just above w yields a greater steady-state profit. Thus offering a wage equal to a mass point can not be profit maximizing in the sense of (6). Note that this conclusion rules out a single market wage as an equilibrium possibility.

Now, when we know that the distribution F is continuous, we can calculate the labor force of a firm offering the lowest possible wage w :

$$l(\underline{w} \mid R,F) = \frac{m\frac{\lambda_0}{\delta}\left[1 + \frac{\lambda_1}{\delta}(1 - F(R))\right]}{\left[1 + \frac{\lambda_1}{\delta}\right]^2 \cdot \left[1 + \frac{\lambda_0}{\delta}(1 - F(R))\right]} \qquad \textit{if} \quad w \ge R \tag{7}$$

$l(\underline{w} \mid R,F)$ is independent of the value of w, whereas the profit of the lowest-wage firm is positive as long as $\underline{w} \ge R$ and declines with $\underline{w}$. This implies that the employer offering the lowest wage in the market will maximize its profit flow if and only if

$$\underline{w} = R \tag{8}$$

In equilibrium any offer must yield the same steady-state profit, which equals

$$\pi = \left(p - R - c\right)\cdot\frac{m\frac{\lambda_0}{\delta}}{\left[1 + \frac{\lambda_1}{\delta}\right]\cdot\left[1 + \frac{\lambda_0}{\delta}\right]} = \left(p - w - c\right)\cdot l(w \mid R,F)$$

for all w in the support of F, by equation (7). Using (5), the above implies that the unique candidate for equilibrium F is:

$$F(w) = \left(1 + \frac{\delta}{\lambda_1}\right)\cdot\left(1 - \sqrt{\frac{p - w - c}{p - R - c}}\right) \tag{9}$$

Equations (3) and (8) imply that:

$$R = \frac{b(1 + \lambda_1)^2 + \lambda_1\left(\lambda_0 - \lambda_1\right)\left(p - c\right)}{(1 + \lambda_1)^2 + \lambda_1\left(\lambda_0 - \lambda_1\right)} \tag{10}$$

The highest wage on the market can be found from $F(\overline{w}) = 1$:

$$\overline{w} = p - c - \left(\frac{\delta}{\delta + \lambda_1}\right)^2\cdot\left(p - R - c\right) \tag{11}$$

Equations (8) and (11) imply that the support of the equlibrium F, $\left[\underline{w},\overline{w}\right]$ is non-degenerate and lies strictly below p, as long as hiring cost c is lower than p-R. Therefore, profit on the support, π, is strictly positive.

Following Burdett-Mortensen, we have to complete the proof that (8), (9), (10) and (11) characterize the unique equilibrium by showing that no

wage off the support of the candidate F yields higher profits. Wages below the support of F attract no workers and therefore yield zero profits, whereas wages above $\overline{w}$ attract no more workers than $l(\overline{w} \mid R, F)$ and hence yield lower profits. Thus the claim is established.

Finally, from (4) and (9) it follows that the distribution of earned wages is:

$$G(w) = \frac{F(w)}{1 + \frac{\lambda_1}{\delta}(1 - F(w))} \qquad (12)$$

for all wages on the support of F.

3. Baseline Burdett-Mortensen: the effects of adjustment costs

We will build our analysis as a set of propositions about the properties of equilibrium in the presence and without the adjustment costs.

Proposition 1: In equilibrium with adjustment costs the highest offered wage is always lower than in equilibrium without costs.

Proof: See Appendix[1].

This result looks intuitive — firms have to pay for each hire and therefore there will be no such high wages as if there were no costs. More than that, if a cost is too high — above p-R — then there will be no equlibrium at all, the profit of any employer will turn negative.

Proposition 2: In equilibrium with adjustment costs the reservation wage of the unemployed is lower than in equilibrium without costs.

Proof: See Appendix.

The intuition for this result is not straightforward. As shown earlier, in equilibrium with costs the highest offered wage on the market goes down. The higher is the maximal wage, the higher is the value of search option, that is, the expected value of employment that can be gained due to job search, where the expectation is taken with respect to the offers distribution function. The wages that are regarded as acceptable are $\left[R, \overline{w}\right]$ for the

[1] Предоставляется автором по запросу.

unemployed and $(\underline{w}, \overline{w}]$ for the employed. When the value of $\overline{w}$ goes down, it means an equal, for both states, reduction in the span of wages that can be attained. This obviously lowers the search option value in both states. The weight of the search option in the value of a state is defined by the offers arrival rate — the higher the rate, the higher the weight of an option. We have assumed that the offers arrive to unemployed more frequently in a unit of time than to employed, and therefore the value of the search option will add more to the value of unemployment, than to the value of employment, and, conversely, the unemployed will be more «hurt» by a reduction in the search option value. Therefore, the decrease in $\overline{w}$ following from hiring costs lowers the value of unemployment more than the value of employment, the latter state becomes relatively more attractive, and the reservation rate of the unemployed goes down, they are more eager to leave to employment.

The assumption of different offers arrival rates is crucial here, for example, if we assume equal rates, then both with and without costs the reservation wage of the unemployed would simply equal b. Note also, from the two propositions above, that though the support of F in general case is shifting to the left, the degree of the shift is not trivial, it is not a parallel shift by c, as one may intuitively assume in such a model. Proposition 3 below describes it in detail.

Proposition 3: The decrease in the highest offered wage as a result of adjustment costs is larger than the decrease in the reservation wage of the unemployed. Therefore, the equilibrium range of offers contracts.

Proof: See Appendix.

Adjustment costs influence directly the firm's optimization problem, and indirectly — the optimization of the workers. Therefore, it is rather intuitive that the impact of costs on the reservation wage will be weaker, than on the highest wage offer — the former is more linked to the worker's side, whereas the latter — to the employer's.

Proposition 4: The offers distribution function in equilibrium without costs FOSD the offers distribution function in equilibrium with costs.

Proof: See Appendix.

Proposition 5: The wages distribution function in equilibrium without costs FOSD the wages distribution function in equilibrium with costs and is less dispersed.

Proof: See Appendix.

Thus, adjustment costs, borne by employers, result in lower wages for the workers.

Proposition 6: The steady-state profit of the firms in equilibrium with costs is strictly lower than in equilibrium without costs.

Proof: See Appendix.

Propositions 3 to 6 summarize how the distribution of offers, the distribution of observed wages and the equilibrium profits of the firms change if we assume hiring costs. What they state is that for the firms additional costs mean lower profits. For the workers hiring costs mean that the observed wages will be lower than earlier, however there will be less wage dispersion — the range of wages (and offers) contracts. That is, if hiring costs could be exogenously set by a policymaker, then it would be impossible to simultaneously obtain higher incomes and less inequality. Obviously, the higher are the costs, the stronger would be the aforementioned effects.

Finally, we perform a simulation exercise, in order to get a better notion of the distributions described above. We use the same parameter values as in BCC. We set $\lambda 1 = 0.15$, $\lambda 0 = 0.3$, $\delta = 0.055$, $b = 0.71$, $p = 1$, $c = 0.15$. The value of hiring costs is rather arbitrary here, however, it roughly corresponds to the marginal hiring costs in terms of output per worker as estimated in Yashiv (2009). The following graphs depict the distributions of offered and observed wages:

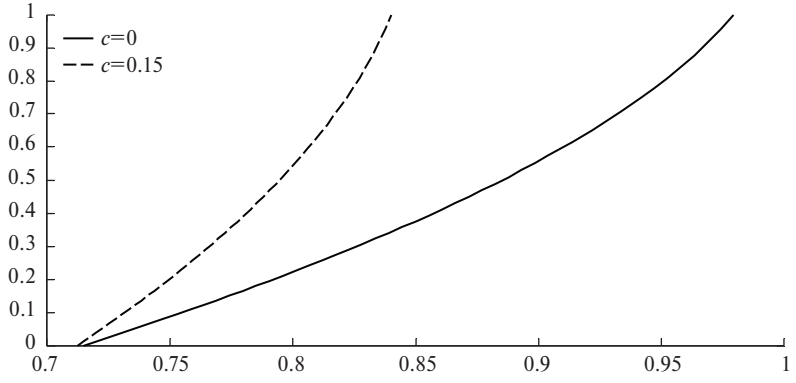

Figure 1: The CDF of offers in a baseline BM model with adjustment costs, $F(w)$

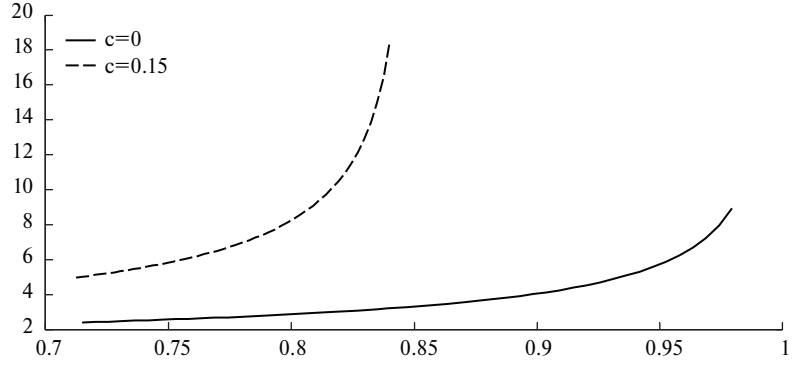

Figure 2: The PDF of offers in a baseline BM model with adjustment costs, $F'(w)$

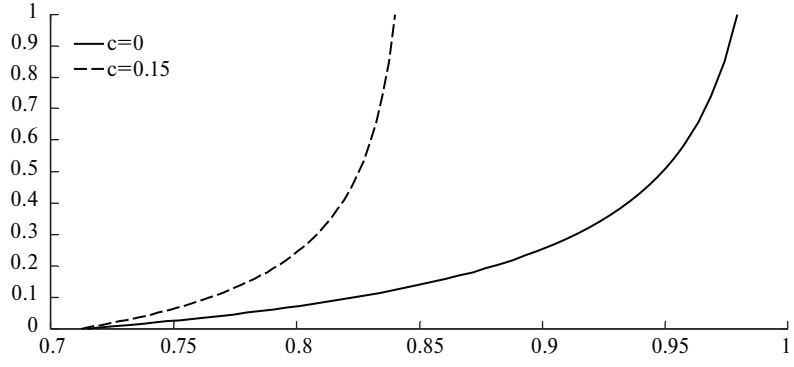

Figure 3: The CDF of observed wages in a baseline BM model with adjustment costs, $G(w)$

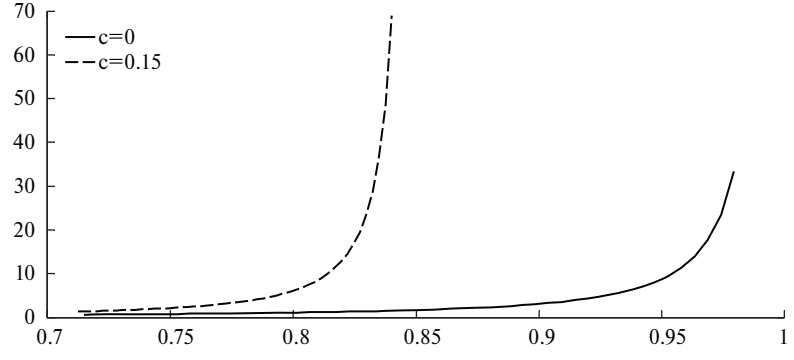

Figure 4: The PDF of observed wages in a baseline BM model with adjustment costs, $G'(w)$

The density of wages in the BM model with adjustment costs, as well as in the baseline model, is rising — the fact which is not observed in reality. The model of BCC, presented in the following sections, avoids this problem by enriching the basic framework with human capital accumulation.

To sum up, we have found that in the baseline model of BM 1998 adjustment costs result in the contraction of the range of offered wages, where the new offers distribution is FOSD by the offers distribution without costs. The observed wages with costs are characterized by a lower dispersion, and are also FOSD by the distribution of observed wages in equilibrium without costs.

4. Burdett, Coles, Carrillo-Tudela (2009): the model with adjustment costs

In this section we will describe the setting of the BCC model with an assumption that there is a cost for the firm associated with hiring a new worker. The paper of BCC (2009), a direct extension of BM (1998), is the first attempt to integrate the "two pillars" of modern labor economics: the theory of human capital accumulation and equilibrium turnover in labor markets where workers search for better paid employment. The authors build a model where workers accumulate human capital when employed and where they search on-the-job and leave the firm whenever they get a better offer. In this way the model is able to explain simultaneously how the wages grow as the workers become more experienced (the central topic of the human capital theory) and why and when workers change jobs (the focus of the search approach to labor market). As a result, BCC are able to fully characterize the cross-section distribution of experiences and earned piece rates, to analyze the equilibrium sorting effect — there is a payoff to experience in the model, the old typically earn more than the young, — and to build the distribution of wages that, unlike the distribution obtained in BM (1998) has an inside mode and a Pareto-like "fat" right tail — the properties of the empirical wage distributions which simple search models fail to replicate, they are in general "plagued" by a rising wage density.

4.1 The setting

There is a continuum of workers and firms, each of measure one. There is a turnover of workers: each worker permanently exits the labor market at rate $\phi > 0$, while ϕ also describes the inflow of new labor market entrants. Assume that each labor market entrant comes to the market with some initial productivity y_i, where γ_i is the proportion of the new entrants who are of type i. As all have the same exit rate ϕ, steady-state turnover implies that γ_i is also the measure of type i workers in the market.

Learning-by-doing in this model means that a worker's productivity increases at rate ρ when working. Thus, after x years of work experience, a type i worker's productivity is $y = y_i e^{\rho x}$. An unemployed worker's productivity y remains constant through time.

A worker with productivity y generates flow output y while employed. The price of the production good is normalized to one, so y aslo describes the flow revenue. Each firm pays each of its employees the same piece rate θ. Thus for an employee with productivity y the flow wage is $w = \theta y$. The firms' revenue flow is the total flow output from its employees multiplied by $1 - \theta$. Here enters our assumption about the adjustment cost. Namely, we assume that at the moment when a new worker enters a firm, this firm pays a one-time adjustment cost, that expresses all the expenses on hiring — such as training costs, the costs of equipment, etc.. The costs of hiring a worker of productivity y are $c \cdot y$. We make an assumption of proportional costs for two reasons. First, for simplicity — in the model of BCC everything is proportional to productivity. Second, by making such an assumption we actually claim that the more experienced a worker — the more costly it will be for a firm to hire her. This does not sound unnatural — indeed, the common sense is that the costs of hiring a builder are much lower than the costs of hiring a computer engineer, where the process of training may take several months before an employee starts in fact producing. Apart from common sense, empirical studies find that hiring costs are "largest for highly skilled employees" (see Abowd, Kramarz 2003).

As different firms may offer different piece rates, let $F(\theta)$ denote the proportion of firms offering a piece rate no greater than θ, where $[\underline{\theta}, \theta]$ denote the infimum and the supremum of the support of $F(\cdot)$.

There are destruction shocks in that each employed worker is displaced into unemployment according to the Poisson rate $\delta > 0$. There is a job offers arrival rate $\lambda > 0$, common for employed and unemployed workers. The search is random, any job offer θ is a random draw from F, there is no recall. The

standard tie-breaking assumptions are: an unemployed worker accepts an offer if indifferent to accepting it or remaining unemployed, while an employed worker quits only if the job offer is strictly preferred. A worker of productivity y in the state of unemployment enjoys flow income by, where $0<b<1$. All agents are risk neutral, the rate of time preference is zero for simplicity, but BCC assume $\phi > \rho$ to ensure that total expected lifetime payoffs are finite. Each worker maximizes expected lifetime income, each firm chooses piece rate θ to maximize steady-state flow profit, taking into account the search strategies of workers.

4.2 The workers

In this section the distribution of piece rate offers F is considered as given, and the optimal worker behavior is characterized.

Consider first an unemployed worker with productivity y. As there is no learning-by-doing, and no skills depreciation when unemployed, the expected lifetime payoff of such a worker $W^U(y)$ is described be the following Bellman equation:

$$\phi W^U(y) = by + \lambda \int_{\underline{\theta}}^{\bar{\theta}} \max\left\{W^E(y,\theta') - W^U(y), 0\right\} dF(\theta') \qquad (13)$$

While unemployed the worker enjoys flow income by. Job offers arrive at rate λ and, conditional on the realized draw θ', a worker either accepts an offer and gets a welfare gain $W^E(y,\theta') - W^U(y)$, or remains unemployed with productivity y.

Consider now a worker of productivity y employed at a firm paying piece rate θ. As it is always better to be employed at a firm paying a higher piece rate, it is immediate that the expected lifetime payoff of such a worker $W^E(y,\theta)$ is increasing in θ. Thus an employed worker quits to an outside offer θ' if and only if $\theta'>\theta$. Assuming that an employed worker never voluntarily quits to unemployment, the Bellman equation describing $W^E(y,\theta)$ is:

$$\phi W^E(y,\theta) = \theta y + \frac{\partial W^E(y,\theta)}{\partial t} +$$

$$+\lambda \int_{\theta}^{\bar{\theta}} (W^E(y,\theta') - W^E(y,\theta)) dF(\theta') + \delta\left(W^U(y) - W^E(y,\theta)\right)$$

The first term on the right hand side describes the flow earnings, the second describes increased value through learning-by-doing, the third describes the capital gain upon receiving a preferred outside offer $\theta'>\theta$, while the last corresponds to the welfare loss through being laid-off.

It can easily be seen that as all payoffs are linear in y, the value functions are linear as well. Therefore, we can re-write

$$\frac{\partial W^E(y,\theta)}{\partial t} = \frac{\partial W^E(y,\theta)}{\partial y} \cdot \frac{\partial y}{\partial t} = \frac{\partial W^E(y,\theta)}{\partial y} \cdot y\rho = W^E(y,\theta) \cdot \rho =$$

And:

$$\phi W^E(y,\theta) = \theta y + \rho W^E(y,\theta) + \lambda \int_{\theta}^{\bar{\theta}} (W^E(y,\theta') -$$

$$-W^E(y,\theta)) dF(\theta') + \delta\left(W^U(y) - W^E(y,\theta)\right)$$

As a useful shorthand, define:

$$q(\theta) = \phi + \delta + \lambda \cdot \left(1 - F(\theta)\right)$$

which is the rate at which any employee exits a firm offering piece rate θ (due to death, layoff or attractive outside offer). Note that $q(\theta) > \rho$ for all θ (as $\phi > \rho$)

Proposition 7: Optimal job search implies:

(1) all unemployed workers have the same reservation piece rate θ^R; that is, any unemployed worker acepts job offer θ if and only if $\theta \geq \theta^R$;

(2) the reservation piece rate θ^R is determined by the following equation:

$$\theta^R = \frac{b(\phi - \rho)}{\phi} - \frac{\rho}{\phi} \cdot \int_{\theta^R}^{\bar{\theta}} \frac{\lambda(1 - F(\theta))}{q(\theta) - \rho} d\theta \qquad (15)$$

Further, for any F a solution exists, is unique, implies $\theta^R < b$, and θ^R is strictly decreasing in ρ.

Proof: see Appendix.

Equation (15) implies that with no learning-by-doing $\theta^R = b$: when experience has no value, unemployed workers reject all offers below b. But with learning-by-doing, when $\rho > 0$, experience is valuable, because it increases future productivity. Therefore, unemployed workers then have a

reservation piece rate below b. Proposition 1 shows that a higher rate of learning-by-doing implies a strictly lower reservation piece rate of the unemployed for a given F. Obviously, the change in reservation rate of the unemployed affects the wage posting incentives of firms. The next step is to characterize the equilibrium wage competition and so determine equilibrium F. Till now there was no change in the presentation comparing to BCC, in the next part we introduce adjustment costs.

4.3 The firms

It must be noted first that offering a piece rate $\theta < \theta^R$ means that the firm makes zero profit, because it attracts no workers. If the hiring costs are not too high, then offering $\theta = b$ generates strictly positive profit (as b<1). Thus in any market equilibrium we must have $\theta \geq \theta^R$ and each unemployed worker, regardless of type, always accepts the first job offer received. As will be shown this implies that each type *i* will have in Market Equilibrium (a) the same unemployment rate U (b) the same distribution of experiences across the unemployed $N(x)$ and (c) the same joint distribution of experiences and piece rates across the employed workers $H(x,\theta)$. Proposition 8 below fully characterizes these distribution functions.

As there is no discounting, the steady-state flow profit equals the hiring rate of the firm, multiplied by the expected profit per hire. Given offer , steady state flow profit is :

$$\pi(\theta) = \sum_i \begin{bmatrix} \lambda\gamma_i U \int_{x=0}^{\infty} \left[\int_{\tau=0}^{\infty} e^{-q(\theta)\tau}(1-\theta) y_i e^{\rho(\tau+x)} d\tau - c y_i e^{\rho x} \right] dN(x) + \\ \lambda\gamma_i (1-U) \int_{\underline{\theta}}^{\theta} \int_{x=0}^{\infty} \left[\int_{\tau=0}^{\infty} e^{-q(\theta)\tau}(1-\theta) y_i e^{\rho(\tau+x)} d\tau - c y_i e^{\rho x} \right] dH(x,\theta') \end{bmatrix}$$

The flow profit consists of two parts — the profit from hiring an unemployed worker and the profit from hiring from the job. For each *i* the first term of the above is the steady state flow profit due to attracting type *i* unemployed workers with experiences $x \in [0,\infty]$. The hiring inflow of such workers is $\lambda\gamma_i U dN(x)$ and the inside bracketed integral is the expected profit per hired unemployed worker with experience x. The productivity increases on the job at rate ρ — it implies that the worker will have productivity $y_i e^{\rho(\tau+x)}$ at each tenure $\tau \geq 0$. The integrand also takes into account that the worker leaves the firm at rate $q(\theta)$. Finally, and this is where our

central assumption comes into light, upon hiring the firm incurs the cost $cy_i e^{\rho x}$.

The second term of the above is the flow profit due to attracting type *i* employed workers. Here the structure is similar, but in addition we take into account that an offer θ attracts only those workers who are currently employed at lower piece rates — hence the integrand $\int_{\underline{\theta}}^{\theta}$. Integrating over and simplifying yields:

$$\pi(\theta) = \left(\frac{\lambda(1-\theta)}{q(\theta)-\rho} - \lambda c \right) \cdot$$

$$\cdot \left[U \cdot \int_{x=0}^{\infty} e^{\rho x} dN(x) + (1-U) \cdot \int_{\theta'=\underline{\theta}}^{\theta} \int_{x=0}^{\infty} e^{\rho x} dH(x,\theta') \right] \cdot \bar{y}$$

where $\bar{y} = \sum_i \gamma_i y_i$ is the mean ability of labor market entrants. Note that the expression for profit that we get here is very much alike the respective equation in the BCC paper, but for the element -λc inside the brackets. This already means that the equilibrium in our model with adjustment costs will be different from that in BCC. From now on, for the ease of comparison, we will denote the equilibrium variables in our model with index c. We now formally define an equilibrium.

A Market Equilibrium is a set $\left\{\theta^R{}_c, U^c, N^c(\cdot), H^c(\cdot), F^c(\cdot)\right\}$, such that:

(i) $\theta^R{}_c$ is the optimal reservation piece rate of any unemployed worker;

(ii) $U^c, N^c(\cdot), H^c(\cdot)$ are consistent with steady-state turnover given piece rate offers $F^c(\cdot)$ and optimal worker search strategies;

(iii) The constant profit condition is satisfied:

$$\pi(\theta) = \bar{\pi}^c \quad for \quad all \quad \theta \quad where \quad dF^c(\cdot) > 0$$

$$\pi(\theta) < \bar{\pi}^c \quad for \quad all \quad \theta \quad where \quad dF^c(\cdot) = 0$$

The constant profit condition requires that all equilibrium offers, those with $dF^c(\cdot) > 0$, enjoy the same profit $\bar{\pi}^c > 0$, while others make no greater profit. Lemma 1 below presents a useful result.

Lemma 1: A Market Equilibrium implies

)i) $F^c(\cdot)$ contains no mass points

)ii) $F^c(\cdot)$ has a connected support

)iii) $\underline{\theta}^c = \theta^R_c$

Proof: see Appendix.

In order to construct $\pi(\theta)$ we have to solve for $U^c, N^c(\cdot), H^c(\cdot)$ in a Market Equilibrium. We consider each of these objects in turn.

First, consider the steady state pool of type *i* unemployed workers. The balance of outflow $\left((\phi+\lambda)\gamma_i U^c\right.$ and inflow $\left.\left(\phi\gamma_i + \delta\gamma_i(1-U^c)\right)\right.$ defines the steady-state unemployment rate:

$$U^c = \frac{\phi+\delta}{\phi+\delta+\lambda}$$

which is indeed the same across all types. The unemployment rate in our model coincides with the unemployment rate in the BCC paper, the adjustment costs do not affect the equilibrium share of the unemployed in the market. This is natural, because unemployment is essentially exogenous in the model, it is independent of the workers' and firms' behavior, therefore, it is insensitive to any changes in the agents' optimization problems.

Next, consider the pool of type i unemployed workers who have experience no greater than x (x≥0). Balancing the outflow $\left((\phi+\lambda)\gamma_i U^c N^c(x)\right)$ and the inflow $\left(\phi\gamma_i + \delta\gamma_i(1-U^c)H^c(x,\bar{\theta})\right)$, and using U^c from above, we get:

$$N^c(x) = \frac{\phi(\phi+\delta+\lambda)+\lambda\delta H^c(x,\bar{\theta})}{(\phi+\delta)(\phi+\lambda)} \qquad (16)$$

for all x≥0.

Finally, consider the pool of type i employed workers with experience no greater than x working for a piece rate no greater than θ. Consider the flows over any instant of time dt>0. The steady state requires the equality of outflow

$\left(\gamma_i(1-U^c)q^c(\theta)H^c(x,\theta)dt + \gamma_i(1-U^c)\left[H^c(x,\theta) - H^c(x-dt,\theta\right] + O(dt^2)\right)$

and inflow $\left(\lambda F^c(\theta)\gamma_i U^c N^c(x)dt\right)$, which yields, when we divide by dt and let dt→0 under the assumption $O(dt^2)/dt \to 0$ as dt→0:

$$F^c(\theta)(\phi+\delta)N^c(x) = \frac{\partial H^c(x,\theta)}{\partial x} + q^c(\theta)H^c(x,\theta) \qquad (17)$$

The (17) above is a partial differential equation on $H^c(x,\theta)$. Proposition 8 now presents closed form solutions for $H^c(x,\theta)$ and $N^c(x)$. Its proof is relegated to the Appendix.

Proposition 8: A Market Equilibrium implies distribution functions

$$N^c(x) = 1 - \frac{\delta\lambda}{(\phi+\delta)(\phi+\lambda)}e^{-\frac{\phi(\phi+\delta+\lambda)}{\phi+\lambda}x} \quad \textit{for} \quad \textit{all} \quad x \geq 0$$

$$H^c(x,\theta) = \frac{F^c(\theta)(\phi+\delta)}{q^c(\theta)}\cdot\left(1-e^{-q^c(\theta)x}\right) -$$

$$-\frac{\delta F^c(\theta)}{q^c(\theta)-\phi F^c(\theta)}\cdot\left(e^{-\frac{\phi(\phi+\delta+\lambda)}{\phi+\lambda}x} - e^{-q^c(\theta)x}\right)$$

$$\textit{for} \quad \textit{all} \quad \theta \in \left[\underline{\theta}_c, \bar{\theta}^c\right] \quad \textit{and} \quad x \geq 0$$

It is noteworthy that the distribution of experiences among the unemployed $N^c(x)$ is the same as in BCC, and is in fact independent of the cost parameter. It happens by the same reason as the independence of the unemployment rate from the assumption of costs. The thing is, that the outflow from unemployment is completely exogenous, and is defined by the death and the job-offer rate. The inflow is due to new-borns (exogenous) and layoffs. As far as experiences of the unemployed are concerned, the only flow that could be different in our model is the flow of laid off, but as all other flows are without change, it turns out that in equilibrium this flow also stays the same. Therefore, the pool $N^c(x)$ is the same as in the model of BCC. However, this does not hold for $H^c(x,\theta)$, which depends on the offers distribution $F^c(\theta)$ and will therefore be different in our model.

4.4 Market Equilibrium with adjustment costs

In this section we will solve for equilibrium with adjustment costs. As was shown in Lemma 1, the offers distribution function $F^c(\theta)$ has a connected support, so all we need to do is to solve for the constant profit con-

dition $\pi(\theta)=\bar{\pi}^c$ for all $\theta\in\left[\underline{\theta}_c,\bar{\theta}^c\right]$ for equilibrium $F^c(\theta)$. First, we had an expression for profit:

$$\pi(\theta)=\left(\frac{\lambda(1-\theta)}{q^c(\theta)-\rho}-\lambda c\right)\cdot$$

$$\cdot\left[U^c\cdot\int_{x=0}^{\infty}e^{\rho x}dN^c(x)+(1-U^c)\cdot\int_{\theta'=\underline{\theta}}^{\theta}\int_{x=0}^{\infty}e^{\rho x}dH^c(x,\theta')\right]\cdot\bar{y} \qquad (18)$$

In the previous section we found U^c, and solved explicitly for $N^c(x)$ and $H^c(x,\theta)$ as functions of $F^c(\theta)$. Using them, we can calculate the integrands inside the profit expression above. Substituting these expressions into (18), and imposing the constant profit condition we get:

$$\bar{\pi}^c=\left(\frac{\lambda(1-\theta)}{q^c(\theta)-\rho}-\lambda c\right)\cdot\frac{\phi(\phi+\delta-\rho)}{\phi(\phi+\delta+\lambda)-\rho(\phi+\lambda)}\left[\frac{\phi+\delta+\lambda-\rho}{q^c(\theta)-\rho}\right]\cdot\bar{y}$$

The above is a quadratic equation on $F^c(\theta)$, which is hidden inside $q^c(\theta)=\phi+\delta+\lambda\cdot\left(1-F^c(\theta)\right)$. Solving it for $F^c(\theta)$ yields the following:

$$F^c(\theta)=\frac{\phi+\delta+\lambda-\rho}{\lambda}-$$

$$-\frac{2(1-\theta)}{\lambda c+\lambda\sqrt{c^2+4(1-\theta)\frac{\bar{\pi}^c(\phi(\phi+\delta+\lambda)-\rho(\phi+\lambda))}{y\lambda\phi(\phi+\delta-\rho)(\phi+\delta+\lambda-\rho)}}} \qquad (19)$$

Now we have $F^c(\theta)$ as a function of θ, parameters and the steady-state profit $\bar{\pi}^c$. All that remains now is to determine $\bar{\pi}^c$. We do it by evaluating $F^c(\theta)$ at $\theta=\underline{\theta}^c$. From the absence of mass points in $F^c(\theta)$ we know that $F^c(\underline{\theta}^c)=0$. Using it in (19) above we get that $\bar{\pi}^c$ and $\underline{\theta}^c$ are related as:

$$\frac{\bar{\pi}^c(\phi(\phi+\delta+\lambda)-\rho(\phi+\lambda))}{y\lambda\phi(\phi+\delta-\rho)(\phi+\delta+\lambda-\rho)}=\left[\frac{(1-\underline{\theta}^c)}{(\phi+\delta+\lambda-\rho)^2}-\frac{c}{(\phi+\delta+\lambda-\rho)}\right]$$

Inserting into (19) yields:

$$F^c(\theta)=\frac{\phi+\delta+\lambda-\rho}{\lambda}\cdot\left[1-\frac{2(1-\theta)}{D+\sqrt{D^2+4(1-\theta)\cdot(1-\underline{\theta}^c-D)}}\right]$$

$$\textit{for all}\quad\theta\in\left[\underline{\theta}^c,\bar{\theta}^c\right] \qquad (20)$$

$$\textit{where}\quad D=\left(\phi+\delta+\lambda-\rho\right)\cdot c$$

Note that this expression is rather similar to the equilibrium distribution in BCC, and can be regarded as its generalization: indeed, for c=0 (no hiring costs), the $F^c(\theta)$ reduces to $F(\theta)$. In the presence of hiring costs the term D, which depends on the hiring costs and the parameters of the model, is different from zero and drives the wedge between the benchmark distribution of BCC and our general case here, with adjustment costs. The detailed analysis of the relation between $F^c(\theta)$ and $F(\theta)$ is relegated to the following section of the paper.

The final step is to determine the bounds $\left[\underline{\theta}_c,\bar{\theta}^c\right]$. What we have found above is actually $F^c(\theta\,|\,\underline{\theta}^c)$ — conditional on the minimal θ. From the workers' optimal behavior we know that the reservation piece rate of the unemployed must satisfy the following equation, given $F^c(\theta\,|\,\underline{\theta}^c)$:

$$\theta_c{}^R=\frac{b(\phi-\rho)}{\phi}-\frac{\rho}{\phi}\cdot\int_{\theta_c{}^R}^{\bar{\theta}(\underline{\theta})}\frac{\lambda(1-F^c(\theta\,|\,\underline{\theta}^c))}{q^c(\theta)-\rho}d\theta$$

So, given some $\underline{\theta}^c$, the above defines the reservation rate as a function of the lower bound $\theta_c{}^R(\underline{\theta}^c)$. But in Lemma 1 we showed that the minimal θ equals the reservation piece rate of the unemployed, $\underline{\theta}^c=\theta_c{}^R$. So all we need to do is to find the fixed point $\theta_c{}^R(\underline{\theta}^c)=\underline{\theta}^c$. To do it, we should make one last calculation, namely, to note that we still do not know $\bar{\theta}^c$, but can express it also as a function of $\underline{\theta}^c$. In particular we evaluate (20) at $\bar{\theta}^c$ and use $F^c(\bar{\theta}^c\,|\,\underline{\theta}^c)=1$. This gives us the following equation connecting the two bounds:

$$1 = \frac{\phi + \delta + \lambda - \rho}{\lambda} \cdot \left[1 - \frac{2(1 - \bar{\theta}^c)}{D + \sqrt{D^2 + 4(1 - \bar{\theta}^c) \cdot \left(1 - \underline{\theta}^c - D\right)}} \right]$$

where $D = \left(\phi + \delta + \lambda - \rho\right)c$

Thus the problem of finding $\underline{\theta}^c$ is reduced to the problem of solving the following equation:

$$\underline{\theta}^c = \frac{b(\phi - \rho)}{\phi} - \frac{\rho}{\phi} \cdot \int_{\underline{\theta}^c}^{\bar{\theta}(\underline{\theta})} \frac{\lambda(1 - F^c(\theta \mid \underline{\theta}^c))}{q^c(\theta) - \rho} d\theta \qquad (21)$$

In the model of BCC it could be done analytically, in our setting only the numerical solution is available, due to the complexity of the function $F^c(\theta)$, which enters the within-integrand expression itself and also defines $\bar{\theta}^c$ as a function of $\underline{\theta}^c$ (and this is the upper bound of the integral in the last equation above).

Theorem 1: For any $\rho < \phi$ a Market equilibrium exists, is unique and implies that:

(1) $\underline{\theta}^c = \theta_c{}^R$ is the solution to (21)

(2) The equilibrium $F^c(\theta)$ is then given by (20)

(3) The steady-state distribution functions $N^c(x)$, $H^c(x,\theta)$ are as described in Proposition 8.

Proof: See Appendix.

This completes the presentation of the BCC model with adjustment costs. In their paper BCC proceed by analyzing the properties of the obtained equilibrium and by simulating the model in order to get the density of wages. We will follow essentially the same scheme, in each case we will try to highlight the differences that the introduction of adjustment costs brings into the components of the equilibrium.

5. Burdett, Coles, Carrillo-Tudela (2009): the effects of adjustment costs

5.1 The distribution of offers and the profit of the firm

Lemma 2: For a given cost c there is a unique $\theta^{crit}(c)$ such that

$$F^c(\theta) < F(\theta) \quad for \quad \theta < \theta^{crit}(c)$$

$$F^c(\theta) > F(\theta) \quad for \quad \theta > \theta^{crit}(c)$$

$$F^c(\theta) = F(\theta) \quad for \quad \theta = \theta^{crit}(c)$$

where $F(\theta)$ is the cumulative distribution of offers in equilibrium without costs.

Proof: See Appendix.

Lemma 2 above states that the CDF's of offers in equilibrium with costs and without them have a single intersection, so that for relatively small piece rates the new CDF lies below the old, whereas for relatively high piece rates the opposite holds. This means that in equilibrium with costs there are less "extreme" offers, that is, the probability that an offer is very low or very high is lower in our setting than in BCC.

Proposition 9: Hiring costs lower the highest and raise the lowest equilibrium piece rate offer.That is, the range of offers $\bar{\theta}^c - \underline{\theta}_c$ contracts, there is less price dispersion.

Proof: see Appendix.

It is interesting to compare the influence of the hiring costs on equilibrium offers in this model, with learning-by-doing, and in the simple model of BM. In both cases additional costs lower the highest offer, because obviously the firms are less eager to pay high wages (piece rates) in the situation when their costs go up. However, the influence of the hiring costs on the reservation wage (piece rate) is different in two models. In the baseline BM setting the reservation wage went down, whereas in the BCC model the reservation piece rate (and the minimum wage) goes up. The explanation here is in the assumptions of the models and in the influence of human capital accumulation. In the BM model we assumed that the offers arrival rate is higher for the unemployed, therefore the unemployed are more "hurt" by the reduction in the value of the search option due to the reduction in $\bar{w}$. This led to the decrease in the reservation wage of the un-

employed. In the BCC settings we assumed equal offers arrival rates for both states, therefore the value of the search option declines for all workers to the same extent. However, on the job workers accumulate human capital at a rate ρ, which adds to the value of employment. Namely, for each value of employment W we know that the input of learning-by-doing into this value is ρW (see equation (14)). It means that if the value of search option falls, then W falls twice — once directly (as the value of the search option is an additive part of W) and second time — indirectly — as W falls, the increment of learning-by-doing ρW also falls. Therefore, when workers accumulate human capital on-the-job, the value of employment falls more when there appear adjustment costs, than the value of unemployment. The reservation piece rate of the unemployed goes up, they are comparatively less willing to leave to employment. The logic described here may be illustrated by a simple picture:

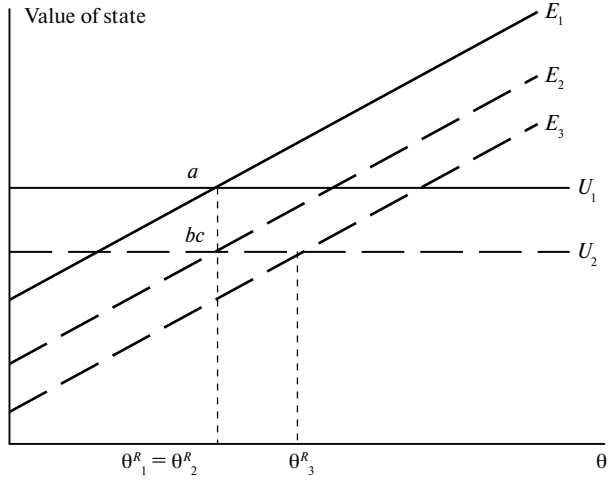

Figure 5: Determination of the reservation piece rate:

The value of employment (E) and unemployment (U) as functions of piece rate θ

The reservation piece rate is by definition the rate at which the values of employment and unemployment coincide — the intersection of the horizontal and upper-sloping curve. The initial intersection (of E_1 and U_1) is in point **a**. When there are costs, the value of both states goes down, the curves move. If there were no learning-by-doing, the value of the search

option would go down similarly for the employed and the unemployed, so that the value of the states would decline to the same extent (point **b**, the intersection of E_2 and U_2, the reservation piece rate without change $\theta_1^R = \theta_2^R$). However, due to learning-by-doing, there is an additional fall in the value of employment (to E_3), so that now the reservation piece rate is defined in the intersection point **c** and equals θ_3^R

Proposition 9 above is essential. Lower dispersion of offers will later imply lower dispersion of observed wages. Table 1 below illustrates how the support of the offers distribution contracts as the costs go up.

Table 1. The influence of adjustment costs on the support of equilibrium offer distribution

c	$\underline{\theta}^c$	$\overline{\theta}^{-c}$	$\overline{\theta}^{-c} - \underline{\theta}^c$
0	0.3459	0.9325	0.5866
0.5	0.3529	0.9091	0.5562
1	0.3599	0.8857	0.5258
1.5	0.3669	0.8624	0.4955
2	0.3739	0.839	0.4651
2.5	0.3809	0.8156	0.4347

We see how the range of offers contracts from 0.59 to 0.43 as costs grow from 0 to 2.5.

Lemma 3: For each set of parameters there is some critical value of costs, $c^{\max}$ such that there is no equilibrium with positive profit if $c > c^{\max}$. When $c = c^{\max}$, there is a non-degenerate uniform distribution of offers, i.e. the density of $F^c(\theta)$ is linear.

Proof: See Appendix.

The highest equilibrium-compatible value of costs in our calibration (the calibration of BCC) is $c^{\max} = 2.7833$. That is, if the hiring costs are more than 2.78 times the productivity with which a worker enters a firm, there will be no incentive for a firm to enter this market, the profit would turn negative. From Lemma 2 it can easily be seen that the higher the rate of human capital accumulation, the higher are the costs that a firm is ready to absorb. The opposite is true for the high rates of all kinds of separations between a firm and its employee $-\phi, \delta, \lambda$. When $c = c^{\max}$, the firms work with zero profit, there is still a non-degenerate distribution of offers, but

no offer is more frequent than the other, the distribution is uniform. Recalling equation (18) above helps to understand why it is so:

$$\pi(\theta)=\left(\frac{\lambda(1-\theta)}{q^c(\theta)-\rho}-\lambda c\right)\cdot$$

$$\cdot\left[U^c\cdot\int_{x=0}^{\infty}e^{\rho x}dN^c(x)+(1-U^c)\cdot\int_{\theta'=\underline{\theta}}^{\theta}\int_{x=0}^{\infty}e^{\rho x}dH^c(x,\theta')\right]\cdot\bar{y}$$

We are discussing the equilibrium in which the profit equals to zero for any piece rate offered and we want to understand why it means that any piece rate will be offered with the same frequency. Zero profit, as can easily be seen from the expression above, arises solely due to the expression in the first parentheses, because the expression in second parentheses is always positive. In other words, the profit is zero, no matter what is the composition of workforce in the firm. When the composition of workforce matters for the profit (the case when costs are below $c^{\max}$), the choice of the frequencies is meaningful for the firm, because by choosing a piece rate it chooses what will be the productivity of the new-hired. In case of maximal costs, the quality of workforce has no influence on firm's profits, therefore the firm is indifferent between various piece rates. It is noteworthy that the range of offers is still non-degenerate, by the reasoning above about the absence of mass points in the equilibrium distribution, when there is on-the-job search.

Proposition 10. In equilibrium with adjustment costs the steady-state profit of the firms is strictly lower than in equilibrium without costs.

Proof: See Appendix.

Figure 6 below illustrates, for our calibration, how the profit declines with costs, and reaches zero at the maximal possible cost.

Finally, to get a better notion of the influence of adjustment costs on the offer distributions, we present here two graphs, depicting the cumulative function $F^c(\theta)$, and its density $F^c{}'(\theta)$, in equilibrium without costs, and with costs growing to 2.78:

The figures above summarize the main findings so far. The figure with the cumulative distribution of offers illustrates how the reservation piece rate goes up and the highest piece rate goes down as costs grow. There is an intersection point, $\theta^{crit}(c)$, of each $F^c(\theta)$ with the original $F(\theta)$, and these points are very close for all c, but not identical. When c equals its highest

possible value, the cumulative distribution is linear, and the density is constant, as stated in Lemma 3.

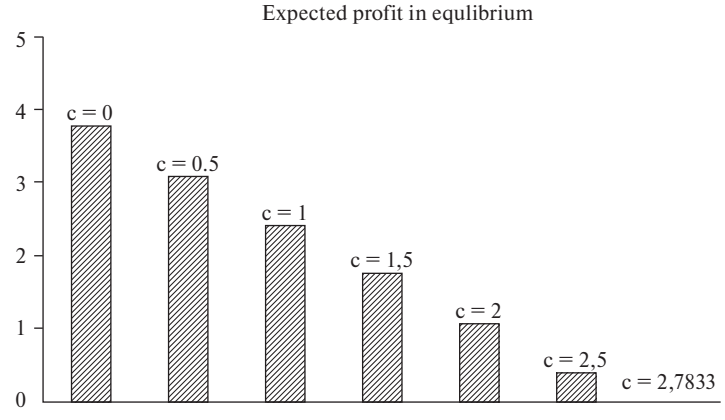

Figure 6: Steady-state profit in equilibrium with adjustment costs

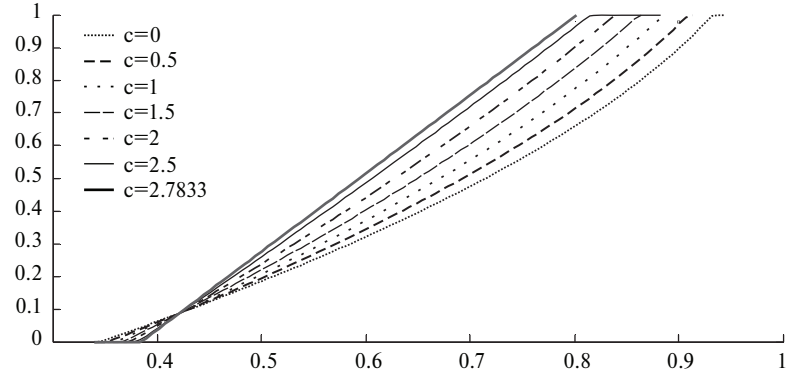

Figure 7: The CDF of offered piece rates $F^c(\theta)$ in the BCC model with adjustment costs:

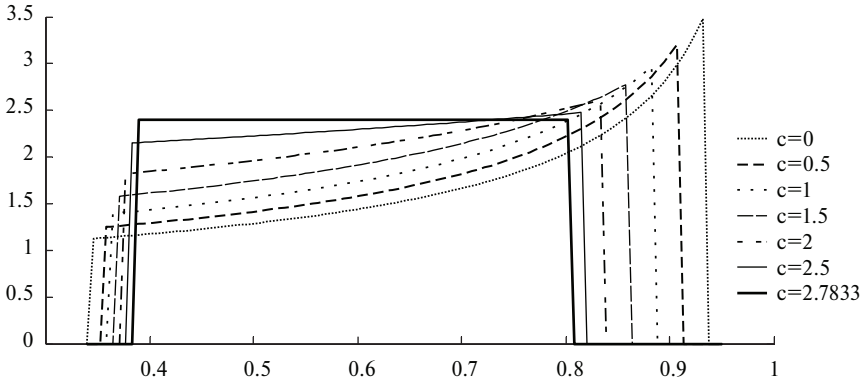

Figure 8: The PDF of offered piece rates $F^c(\theta)$ in the BCC model with adjustment costs:

5.2 Equilibrium composition and equilibrium sorting

One of the fundamental insights of the BCC paper is that there is equilibrium composition effect; in other words, the firms which offer a higher piece rate enjoy, in Market Equilibrium, a more experienced and thus more productive workforce. The distribution of experiences inside the firms paying θ is defined by $H(x \mid \theta)$:

$$H(x \mid \theta) = (1 - e^{-qx}) - \frac{\delta q^2}{(\phi + \delta)\left[q - \phi F\right]^2} \cdot$$

$$\cdot \left[e^{-\frac{\phi(\phi+\delta+\lambda)}{\phi+\lambda}x} - e^{-qx} \right] - \frac{\lambda F \phi (1 - F) qx}{(\phi + \delta)\left[q - \phi F\right]} e^{-qx}$$

Tedious, but simple algebra establishes that if $\delta > \phi$ (which actually means that each worker expects to be laid off at least once over a working lifetime), then:

$$\frac{\partial H(x \mid \theta)}{\partial \theta} < 0$$

That is, in equilibrium the quality of workforce in low-paying firms is worse than in high-paying firms. Proposition 11 below states that adjustment costs augment this differentiation.

Proposition 11: When there are hiring costs, the quality of workforce in low-paying firms deteriorates, whereas the quality of workforce in high-paying firms improves, compared to the equilibrium without costs.

Proof: Taking the distribution of experiences inside the firms paying given theta $H(x \mid \theta)$:

$$H(x \mid \theta) = (1 - e^{-qx}) - \frac{\delta q^2}{(\phi + \delta)\left[q - \phi F\right]^2} \cdot$$

$$\cdot \left[e^{-\frac{\phi(\phi+\delta+\lambda)}{\phi+\lambda}x} - e^{-qx} \right] - \frac{\lambda F \phi (1 - F) qx}{(\phi + \delta)\left[q - \phi F\right]} e^{-qx}$$

Differentiating it (see Appendix), for a given θ, wrt F yields, under $\delta > \phi$:

$$\frac{\partial H(x \mid \theta)}{\partial F} < 0$$

That is, the higher is F for given θ, the lower is the share of relatively inexperienced workers in the θ-paying firms. Using Lemma 2 we can therefore deduce that:

$$H^c(x \mid \theta) > H(x \mid \theta) \quad for \quad \theta < \theta^{crit}(c)$$

$$H^c(x \mid \theta) < H(x \mid \theta) \quad for \quad \theta > \theta^{crit}(c)$$

$$or, equivalently,$$

$$1 - H^c(x \mid \theta) > 1 - H(x \mid \theta) \quad for \quad \theta > \theta^{crit}(c)$$

Verbally, the above states that in low-paying firms ($\theta < \theta^{crit}(c)$) the share of relatively inexperienced workers is higher in the presence of adjustment costs. Conversely, in high-paying firms ($\theta > \theta^{crit}(c)$) the share of workers with experience above given x ($1 - H^c(x \mid \theta)$) goes up compared to the equilibrium without costs. This completes the proof.

The structure of the model allows us to analyze the other side of the link between piece rates and experiences, namely, what is the distribution of piece rates among the cohort of workers of experience x? The answer to this question is given by a conditional distribution $H(\theta \mid x)$:

$$H(\theta \mid x) = F \cdot \left[\frac{\delta + (\phi + \lambda)(1 - F) e^{-\frac{\lambda(q - \phi F)}{\phi + \lambda} x}}{\delta + (\phi + \lambda)(1 - F)} \right]$$

It turns out that adjustment costs make this intra-cohort distribution of piece rates less dispersed.

Proposition 12: In the presence of adjustment costs the distribution of piece rates among workers of given experience is less dispersed than without costs.

Proof: The influence of F on the distribution of piece rates within a cohort can be measured by $\frac{\partial H(\theta \mid x)}{\partial F}$. Simple calculations show (see Appendix) that:

$$\frac{\partial H(\theta \mid x)}{\partial F} > 0$$

Thus, the higher is $F(\theta)$ for a given θ, the higher is the share of people of experience x who are employed at a rate below θ. In Lemma 2 above we have proved that in the presence of adjustment costs $F^c < F$ for the lower range of piece rates, and $F^c > F$ for the higher range of piece rates. This means that:

$$H^c(\theta \mid x) < H(\theta \mid x) \quad for \quad \theta < \theta^{crit}(c)$$

$$H^c(\theta \mid x) > H(\theta \mid x) \quad for \quad \theta > \theta^{crit}(c)$$

$$or, equivalently,$$

$$1 - H^c(\theta \mid x) < 1 - H(\theta \mid x) \quad for \quad \theta > \theta^{crit}(c)$$

That is, the share of workers with especially low and especially high piece rates within a cohort of a given experience is lower when there is adjustment cost, than without it. In the presence of hiring costs "extreme" values of piece rates within a cohort become less frequent, the distribution is less dispersed. This completes the proof.

To sum up, this section was devoted to the impact of adjustment costs on the properties of the joint cross-section distribution of piece rates and experiences among employed workers. We found that:

(1) Adjustment costs reduce the inequality of piece rates earned within a cohort of workers of given experience

(2) Adjustment costs augment the composition effect — the quality of workforce in low-paying firms deteriorates, whereas the quality of workforce in high-paying firms improves.

The intuition behind the aforementioned facts is the following. First, we have previously found that in the presence of costs, the equilibrium range of offers contracts. Therefore it is quite natural that the piece rates are more compact if we make cross-sections for each given experience.

The second property follows from the constant profit condition: the new distribution of offers is such that for relatively low piece rates the new F^c is below the old one, that is, $1 - F^c > 1 - F$, and it means that in the economy with adjustment costs the outflow of workers from relatively low piece rates will be higher, due to on-the-job search. The hiring costs are proportional to the productivity of a worker at the time she enters a firm, and these costs are paid in any case, regardless how long the worker will stay. In this situation, the firm with the higher outflow will have on average less time to cover the initial costs; therefore, it can not "afford" high costs, that is, high-skilled workers. Conversely, the outflow from high-paying firms is lower in the economy with adjustment costs, and these firms enjoy even more qualified workers, because they have on average more time to cover comparatively high hiring costs.

If an empirical experiment were available, where we raise adjustment costs and look how the equilibrium changes, we would observe less wage dispersion among the workers of given experience, and we would see how the average experience of a worker in a low-paying firm goes down, whereas the average experience of a worker in a high-paying firm goes up.

In fact the two properties mentioned above imply that we will see a higher correlation between experience and piece rates in the extended model. To illustrate this, and numerically explore the wage distributions, implied by the model, we perform simulation exercises in the next section.

6. Simulations

We now perform some numerical simulations to illustrate the model's implications for wage dispersion. The calibration is taken unchanged from the BCC, that is, we set:

Parameter	Value
ϕ	0.025
δ	0.055
λ	0.15
b	0.71
ρ	0.009

Suppose first that the initial distribution of productivities y_i is degenerate, and each labor market entrant has initial productivity $y_i = 1$. The surprising result of BCC is that even in this case the distribution of wages has an inside mode and a long right tail.

Figure 3 below describes the resulting equilibrium wage density (see Appendix for derivation) for the cases when c=0, c=0.5, c=1, c=1.5. It can be seen that the general form of the distribution remains the same, however, the lowest wage in the market goes up with c, reflecting the growth in the lowest offered θ.

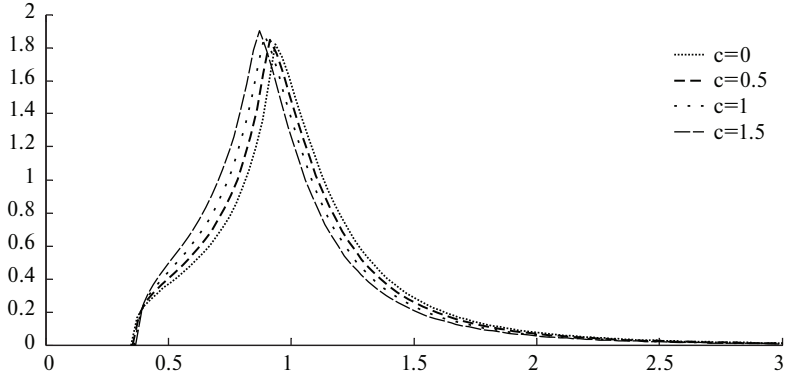

Figure 9: The PDF of observed wages g(w), in the BCC model with adjustment costs when workers are homogeneous:

The rising left tail of the wage distribution resembles the density of wages in the BM model, and reflects the fact that labor market entrants start their careers at low-paying jobs and move to better-paid jobs through on-the-job search. The right tail of the distribution reflects high productivities, and in that area the main engine of wage growth is the accumulation of experience, the on-the-job search opportunities are for the most part exhausted. It can be shown that at high productivities the density of wages paid

(asymptotically) mirrors the density of worker productivities, which is Pareto (this arises as the distribution of experiences $H(x \mid \theta)$ is exponential, while each worker's productivity is $y = y_i e^{\rho x}$). Hence the "fat" right tail of the wage density. This of course is a well-known property of the empirical wage distributions; see for example Weizsacker (1993) and Neal and Rosen (2000).

Smoothing out this wage distribution clearly requires some worker heterogeneity. For the ease of exposition the theory section assumed a finite number of types. But the analysis extends straightforwardly if there is a continuum of underlying abilities, described by the distribution function $A(\cdot)$. Suppose that $A(\cdot)$ is a Gamma-distribution where the density is:

$$\frac{dA(y_i \mid k_0.k_1)}{dy_i} = \frac{\left(\frac{y_i}{k_0}\right)^{k_1-1} e^{-\frac{y_i}{k_0}}}{k_0\Gamma(k_1)}$$

where $k_0, k_1 > 0$ and $\Gamma(\cdot)$ is a gamma-function. The mean and the variance of $A(\cdot)$ are $\mu = k_0 k_1$ and $\sigma^2 = k_0^2 k_1$. Figure 10 describes the three ability distributions with the same mean but different spreads:

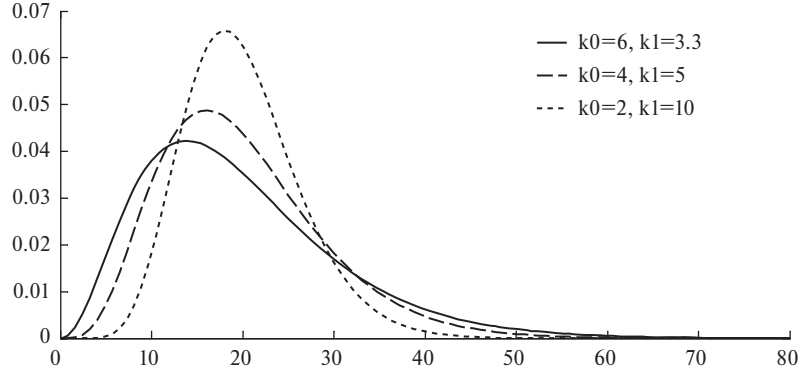

Figure 10: The PDF of initial productivities $A'(y_i)$ with the same mean but different spreads

To illustrate the impact of costs on the distribution of wages when there are many types of underlying abilities we take one distribution $A(\cdot)$ with $k_0 = 4, k_1 = 5$ and simulate the model for several values of costs: c=0, c=0.5,

c=1, c=1.5, c=2, c=2.5 . We see, from Figure 11 that the evolution of the wage distribution is essentially the same as in case of one type.

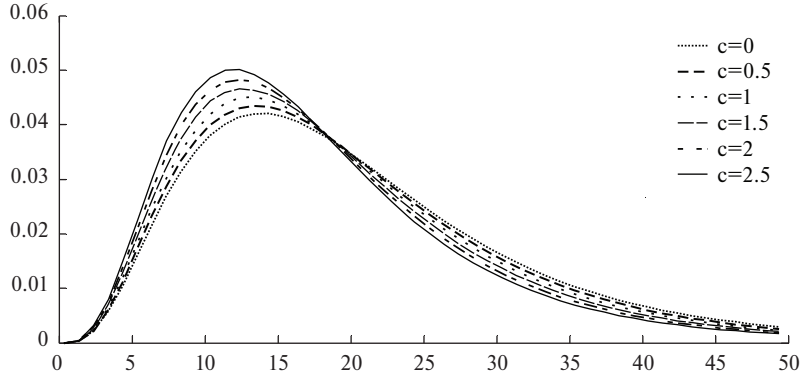

Figure 11: The PDF of observed wages g(w), in the BCC model with adjustment costs when workers are heterogeneous ($A(\cdot)$ with $k_0 = 4, k_1 = 5$)

The model of BCC does not allow to analytically explore the properties of the distribution of the observed wages. However, the properties of the distribution of piece rates and experiences, which underlies the resulting wage distribution, are clear and were explored in the previous sections. They are also reflected in the figures above. For example, in the figure with one type of workers we see that the range of observed wages contracts as costs go up — this is a consequence of the rise in the reservation piece rate of the unemployed (higher reservation piece rate implies higher minimal observed wage). The mode of the observed wages distribution goes down, that is, the most frequently observed wage declines when hiring costs grow. This is also a consequence of the changes in the piece rates distribution, and follows from the fact that the highest observed piece rate (as well as the highest offered piece rate) goes down and becomes less frequent, whereas the lowest observed piece rate goes up and becomes more frequent. Combined with the tighter link between piece rate and experience this yields a lower most frequent observed wage.

Finally, we perform the decomposition of the variance of wages. The model of BCC is built so that it allows a very simple and insightful decomposition of the wage dispersion. The wage of a worker is, by definition: $w = \theta y_i e^{\rho x}$. Therefore, as there is no sorting by underlying types ($\mathrm{cov}(\theta, y_i) = 0$) the following holds:

$$\mathrm{var}(\ln w) = \mathrm{var}(\ln y_i) + \mathrm{var}(\ln \theta) + \rho^2 \mathrm{var}(x) + 2\rho \mathrm{cov}(x, \ln \theta)$$

This equation in fact states that in a cross section of workers we observe different wages because the workers were born with different abilities, because workers are employed at firms offering different piece rates, because workers have different experiences, and, finally, the wage dispersion is augmented by the covariance of piece rate and experience. This covariance arises because among the workers with long experience the high piece rates are more frequent, the workers with long experience had more time to search on-the-job and to find a better opportunity. Having simulated the model, we can calculate the relative weight of each component of wage variance. Table 2 below presents the results for the model with one type ($A(\cdot)$ is degenerate):

Table 2. Variance decomposition of ln wages under different costs when A is degenerate

Degenerate A	**Total variation**	**Relative contribution (%)**			
	var(ln$w$)	var(lny_i)	var(lnθ)	ρ^2var(x)	2ρcov(x, lnθ)
c=0	0.1388	0	35.73	52.50	11.77
c=0.5	0.1351	0	34.45	53.69	11.86
c=1	0.1315	0	33.23	54.90	11.87
c=1.5	0.1280	0	31.89	56.16	11.94
c=2	0.1247	0	30.58	57.45	11.97
c=2.5	0.1215	0	29.22	58.78	12.00

The results in Table 2 above are completely in line with our analysis. First, we see that as the costs grow, the variance of wages decreases. The observed piece rates become more compact, and their weight in explaining the wage dispersion contracts. Conversely, the difference in the observed experiences becomes more important in explaining the wage differentials, and, finally, the weight of the interaction of piece rate and experiences goes up as well, but the rise is minor. A somewhat more distinct way to see how piece rate and experience become more tightly linked with the growth of costs is the correlation coefficient: in the table below we see how the correlation between piece rates and experiences grows as costs become high:

Costs	corr(x, lnθ)
c=0	0.1361
c=0.5	0.1377
c=1	0.1393
c=1.5	0.141
c=2	0.1428
c=2.5	0.1446

It is rather clear, that the regularities described in Table 2 will hold for the case when the distribution of underlying abilities is non-degenerate. We present the results of the simulations for the non-degenerate cases in two following tables. The first one, Table 3, is built for the equilibria without costs, and contains the decomposition results for several forms of A.

Table 3. Variance decomposition of ln wages when there are no costs, for different initial A

Parameters of A	**Total variation**	**Relative contribution (%)**			
	var(ln w)	var(ln y_i)	var(ln θ)	ρ^2 var(x)	2ρcov(x, ln θ)
$k_0 = 6, k_1 = 3.3$	0.491	71.88	10.05	14.77	3.31
$k_0 = 4, k_1 = 5$	0.361	61.72	13.68	20.1	4.51
$k_0 = 2, k_1 = 10$	0.244	43.26	20.27	29.79	6.68
Degenerate A	0.139	0	35.73	52.5	11.77

In all cases, when there is a distribution of initial abilities, their variance explains the lion's part of the resulting wage dispersion. The higher the variance of initial abilities — the lower is the weight of all other components of the wage dispersion — as the variance of $\ln y_i$ goes up from 0 to 0.352, the weight of var(ln θ) decreases from 35.73% to 10.05%, the weight of the equilibrium dispersion of experiences decreases from 52.50% to 14.77% and finally, the weight of the interaction term goes down from 11.77% to 3.31%.

The second table, Table 4, takes $k_0 = 4, k_1 = 5$ and analyzes the decomposition for various levels of costs:

*Table 4. Va*riance decomposition of ln wages under different costs when A is non-degenerate with parameters $k_0 = 4, k_1 = 5$

Non-Degenerate A	**Total variation**	**Relative contribution (%)**			
$k_0 = 4, k_1 = 5$	var(ln w)	var(ln y_i)	var(ln θ)	ρ^2 var(x)	2ρcov(x, ln θ)
c=0	0.361	61.72	13.68	20.10	4.51
c=0.5	0.356	62.34	12.97	20.22	4.47
c=1	0.354	62.96	12.31	20.33	4.40
c=1.5	0.350	63.56	11.62	20.47	4.35
c=2	0.347	64.18	10.95	20.58	4.28
c=2.5	0.344	64.76	10.30	20.71	4.23

Here the findings are consistent with the results in Table 2, which does the same for the degenerate case: as costs grow, the variance of wages goes down, the weight of the dispersion of observed piece rates goes down, the weight of the dispersion of observed experiences goes up. The only difference between the two cases is that when there was one type of workers, the interaction between piece rate and experience became more important in explaining wage differentials as costs went up, whereas in case of non-degenerate *A*, this interaction term becomes less important. However, in both cases the change is negligibly small.

7. The development of a career in Burdett-Coles-Carrillo-Tudela (2009)

7.1 Equilibrium sorting and payoff to experience

The equilibrium cross-section distributions in BCC (2009), though analytically clear, are obtained from an exogenously imposed condition of equal flows in- and out of different pools of workers. The resulting distributions imply, for example, that in equilibrium there is a payoff to experience — higher experience means higher probability of earning a high piece rate. However, the analysis of BCC does not shed light on the mechanism that allocates, in equilibrium, more experienced workers to more high-paying firms. They rather vaguely claim that workers with long experience had more time, than new-comers, to search for a better-paid job. But in

effect, once a person is laid-off (which happens at the same rate for all), she starts looking for a new job from the beginning, no matter how much she got on her last job, and this is true both for experienced workers and those who are just entering the market. That is, during each uninterrupted employment spell a worker has a chance to reach higher piece rates doing on-the-job search until she is thrown back into unemployment and again has to wait for the start of employment cycle. In this setting past experience has no explicit influence on one's current piece rate, experienced workers and new entrants behave identically in searching for the job. However, in equilibrium the positive correlation between piece rate and experience emerges, and we aim to explain where it stems from. To see this, it is necessary to analyze individual career path, that is, to abandon the cross-section analysis of BCC in favor of the time-series framework, where we will follow the development of a representative career. Indeed, in the presence of ergodicity, both approaches should yield the same results. Therefore, the results of BCC will serve a benchmark for our work.

The population of workers in BCC is a peculiar one — once a worker dies, she is immediately replaced by a new entrant, who is "born" into unemployment and has zero experience, so that the size of the population is always constant. BCC regard this population as a cross-section at a point in time, and focus on analyzing the joint distribution of piece rates and experiences in this cross-section. An equivalent approach is to regard this population as a single individual, who is from time to time hit by a specific shock that brings her experience to zero and makes her unemployed. That is, the life of the population is equivalent to the life of one worker, which may look as follows (time is on x-axis, experience — on y-axis:

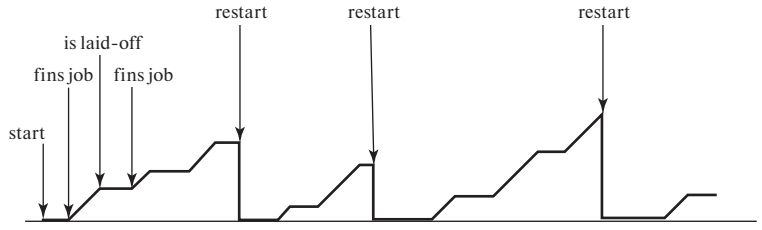

Figure 12: A representative career as an ergodic equivalent of the cross-section population in BCC:

Above we see the path of experience over time, which is cyclical, where the end of each cycle happens because an individual is hit by the shock ϕ which resets her experience to zero. The infinite life of this individual, that is, the infinite sequence of such cycles is an ergodic equivalent of the cross-section population in BCC. The shock ϕ may be realized both when an individual is off the job (as in the first two cycles above) or when employed (as in the last case above). The horizontal sections correspond to the periods of unemployment, and the upward sloping sections are the periods of employment (maybe at several firms consequently), during which an individual accumulates experience. Keeping this in mind, it is easy to characterize the distribution of experiences *S(x)* over a representative life, that is, the share of cycles in which an individual has reached at least experience *x*. Namely, *S(x)* and *S(x+Δx)* are linked as:

$$S(x+\Delta x) = S(x)\cdot\left(1-(\phi+\delta)\Delta x\right) + S(x)\cdot\left(\Delta x\delta\cdot\int_0^\infty \lambda e^{-\lambda u}e^{-\phi u}du\right)$$

The expression above states that the cycles in which one reaches experience $x+\Delta x$ are the cycles in which one had experience x and continued to be employed and did not die during Δx (the probability $\left(1-(\phi+\delta)\Delta x\right)$), or if one had x, got unemployed during Δx (probability $\delta\Delta x$) but managed to get an offer and come back to job before the cycle is terminated (probability $\int_0^\infty \lambda e^{-\lambda u}e^{-\phi u}du$). This gives us:

$$\frac{S(x+\Delta x)-S(x)}{\Delta x} = S(x)\cdot\left(-\underbrace{(\phi+\delta)}_{A1} + \underbrace{\frac{\delta\lambda}{\phi+\lambda}}_{A2}\right)$$

Solving this differential equation yields:

$$S(x) = e^{-\frac{\phi(\phi+\delta+\lambda)}{\phi+\lambda}x}$$

Thus we have computed the share of cycles, in which experience x has been reached, in an infinite representative life. In terms of cross-section it is the probability that a given worker (employed or unemployed) has experience higher than x. When an individual is in a cycle and has reached experience x, her piece rate, if employed, is defined by the time that she has spent climbing up to better jobs due to on-the-job search. Denote this time

by t — it is the length of the current period of uninterrupted employment. Then $q=x-t$ is the part of experience that was accumulated in a current cycle up to the start of the last uninterrupted employment period. How is q distributed, given x? Denote this distribution by $P(q|x)$. When x grows to $x+\Delta x$ (and we have seen that the share of cycles where we reach $x+\Delta x$ $S(x+\Delta x)$ is $(1-A1\Delta x+A2\Delta x)$ times the share $S(x)$ of cycles where we reach x), the time spent before the last uninterrupted employment will stay the same if and only if an individual does not "die" (rate ϕ) or is not laid-off (rate δ) during Δx (the probability $1-A1\Delta x$). That is,

$$P(q \mid x+\Delta x) = P(q \mid x) \cdot \frac{1-A1\Delta x}{1-A1\Delta x+A2\Delta x}$$

$$P(q \mid x+\Delta x) = P(q \mid x) \cdot \left(1-\frac{A2\Delta x}{1-A1\Delta x+A2\Delta x}\right)$$

$$\frac{P(q \mid x+\Delta x)-P(q \mid x)}{\Delta x} = -P(q \mid x) \cdot \frac{A2}{1-A1\Delta x+A2\Delta x}$$

$$P(q \mid x) = C(q) \cdot e^{-A2x}$$

$$P(x \mid x) = 1 \Rightarrow$$

$$C(q)\big|_{q=x} = e^{A2x}$$

$$C(q) = e^{A2q}$$

$$P(q \mid x) = e^{A2q} \cdot e^{-A2x} = e^{-A2(x-q)} = e^{-A2t} = e^{-\frac{\delta\lambda}{\phi+\lambda}t}$$

Above we have found the probability that the experience gained up to the start of the last uninterrupted employment spell is less than q, given that the overall experience is x. Therefore, the probability that the length of the last uninterrupted employment spell is less than t, given x, is actually the complementary probability to $P(x-t \mid x)$:

$$P(t \mid x) = 1 - e^{-\frac{\delta\lambda}{\phi+\lambda}t}$$

$$P(t = x \mid x) = 1 \quad (\textit{there is a mass at } t = x)$$

Here is the clue to the link between experience x and piece rate θ, that remained unclear in the cross-section analysis of BCC. When an individual has gained experience x in her current cycle, what can we tell about her piece rate? The answer is: it depends on how long has she been in the last period of uninterrupted employment. If this period is long (high t), then the chances of climbing high up the ladder of piece rates are high. If the individual has just been hired from unemployment (small t), no matter the experience, the odds are that her θ is low. The probability describing the time spent in the last uninterrupted employment given x, is $P(t|x)$ above. We now understand that the higher is t, the higher will be θ (the exact influence of t upon θ will be derived below). Why will t be typically higher for high x? Simply because for low x it is impossible to reach high t — when a person is in the cycle in which she has up to now accumulated experience 2, it can not be, that she has been employed for, say, 3. In other words, long periods of uninterrupted employment are simply impossible in the cycles where the overall experience is relatively low. Therefore, relatively high piece rates are less frequent in such cycles.

To illustrate the point, we plot $1-P(t|x)$ (the probability that an employed worker of experience x is employed longer than some given t (t is on the horizontal axis)) for different x (on the graph below we use x=12, x=25, x=38, x=50:

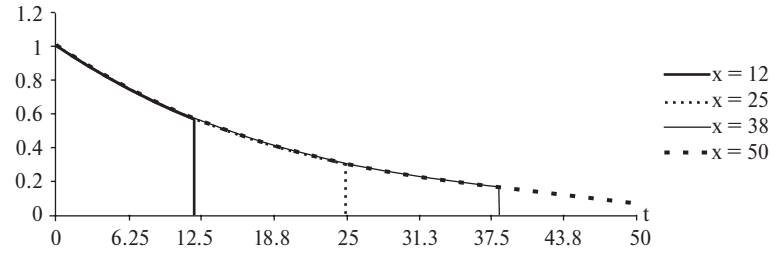

Figure 13: The probability to be uninterruptedly employed for longer than t, given overall experience x:

As can be easily seen, the distribution of t for big x FOSD that for the smaller x. For example, an employee with experience 25 has the same chance to be in relatively short employment spells as an employee with experience 12, but the former has also a positive chance to be uninterruptedly employed for 15 or even 20 units of time. And, of course, a chance of being in a long spell is a chance to reach high piece rate due to on-the-job search. This is the heart of the mechanism that allocates more experienced workers to jobs

with higher piece rates — those with higher x simply have a chance to be in a long spell now, and that's why they reach higher piece rates. This is essentially the only reason why in the model of BCC there is a payoff to experience in the form of piece rates. This payoff, however, emerges solely by the reasons of probability and not because more experienced workers behave differently in their search for θ than the labor market entrants.

7.2 Payoff to tenure

In the previous section we have shown why an experienced worker has a good chance to be a long-tenured in her last employment. In this section we will show analytically why it is good to have a long tenure. Intuitively, it is clear that being employed for a long time means having time to sample better and better piece rates that are offered with constant intensity by outside firms. We can measure this payoff by building distributions of piece rates earned at each tenure – $P(\theta \mid t)$.

First of all, for an individual hired from unemployment *(t=0),* the distribution of piece rates is simply the unconditional distribution of offers:

$$P(\theta \mid 0) = F(\theta)$$

Second, in the extreme case of infinite tenure a worker will finally arrive at the highest possible piece rate:

$$P(\theta \mid t) = \begin{Bmatrix} 0 \text{ if } \theta < \bar{\theta} \\ 1 \text{ if } \theta = \bar{\theta} \end{Bmatrix}$$

In between, for positive tenures, we find $P(\theta \mid t)$ recursively. Suppose we know $P(\theta \mid t)$. Then, the probability that a worker is still below θ at $t+dt$ (this is $P(\theta \mid t+dt)$) is the probability that he was in the group below θ at the moment t and during dt did not find a better job. That is (see Appendix for detailed derivation):

$$P_{t+dt}(\theta) = P_t(\theta) \cdot \left(1 - \lambda dt \cdot (1 - F(\theta)\right)$$

$$\Rightarrow P(\theta \mid t) = F(\theta) \cdot e^{-\lambda(1-F(\theta))t}$$

It can easily be seen that $P(\theta \mid t)$ converges from full *F(θ)* to a mass point at θ as t goes and approaches infinity. The next figure illustrates this:

On the horizontal axis above we have piece rate θ and on the vertical axes — the conditional cumulative distribution function $P(\theta \mid t)$, where each

line corresponds to a different t. For example, when *t=0*, which in fact means zero tenure, a just-started job, the distribution of piece rates is actually the distribution of offers. As tenure goes up, that is, a worker stays employed and samples better and better offers, the distribution evolves, low piece rates become less and less frequent, and in the extreme, if infinite tenure were possible, a worker would for certain achieve highest possible theta, so that the distribution would be a mass point in θ. The line for t=50 approximates this case.

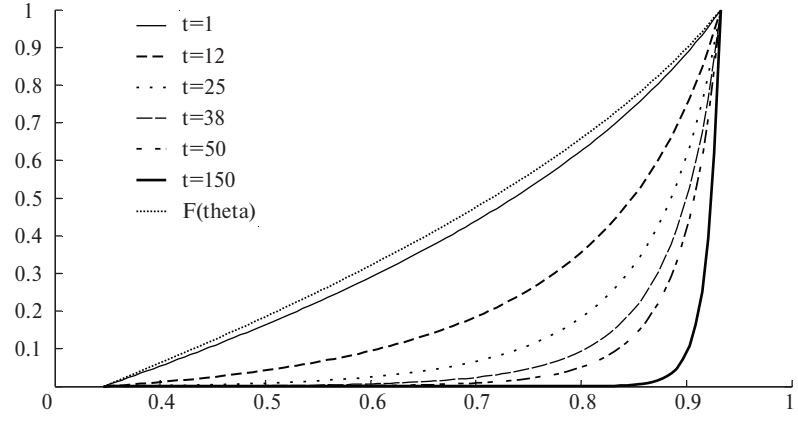

Figure 14: The CDF of earned piece rates conditional on tenure:

The payoff to tenure can be expressed as FOSD of this distribution for high t over that for low t:

$$P_{t1}(\theta) < P_{t2}(\theta) \text{ iff } \quad t1 > t2$$

Having calculated $P(t \mid x)$ and $P(\theta \mid t)$ we can compute the distribution of piece rates among the employed given experience (we show here the initial equation and the result). The detailed derivation is relegated to the Appendix:

$$P(\theta \mid x) = \int_{t=0}^{x} \frac{dP(t \mid x)}{dt} \cdot P(\theta \mid t)dt + e^{-\frac{\delta\lambda}{\phi+\lambda}x} \cdot P(\theta \mid t = x)$$

$$\Rightarrow P(\theta \mid x) = F(\theta) \cdot \left[\frac{\delta + (\phi + \lambda)(1 - F(\theta))e^{-\frac{\lambda(q(\theta)-\phi F(\theta))}{\phi+\lambda}x}}{\delta + (\phi + \lambda)(1 - F(\theta))} \right]$$

It is noteworthy that we get an expression identical to $H(\theta \mid x)$ in BCC (2009), thus our $P(\theta \mid x)$ has the same properties as $H(\theta \mid x)$ in BCC, for example, there is equilibrium sorting — higher experiences are typically associated with higher piece rates. However, our approach, unlike BCC, provides an elegant and explicit structural explanation of this property. Namely, we have shown that higher experience means higher probability of being uninterruptedly employed for a long time (see the discussion of $P(t \mid x)$ above), and we have directly computed how the time spent in uninterrupted employment is translated into piece rate growth (see $P(\theta \mid t)$ above). These two distributions constitute the mechanism governing the development of one's career, they are intuitive and clear, and they directly show that essentially the influence of experience is very mechanical in the model of BCC, the workers do not in fact "learn" to sample better offers and they are not offered better jobs because of their high qualification, it is just that an inexperienced worker could not be employed for a long time, and therefore did not have a chance to find herself a better job.

Essentially what we have done is the decomposition of the wage process of an individual into two components — the duration of the uninterrupted employment spells and the growth of piece rate within each such spell. We think that this structure is a convenient framework for analyzing various modifications of the original model of BCC. Consider, for example, the case of adjustment costs. As was shown in the previous section, hiring costs influence the equilibrium distribution of offers, $F(\theta)$. In the terms of individual career it is clear that this change has no impact on the lengths of the uninterrupted employment spells ($P(t \mid x)$ stays the same), however, the process of accumulating higher piece rates within employment is affected. The probability that a worker employed for t periods earns now a piece rate less than θ (see above):

$$P(\theta \mid t) = F(\theta) \cdot e^{-\lambda(1-F(\theta))t}$$

The higher is $F(\theta)$ for a given θ, the higher is this probability, that is, relatively low piece rates become more frequent. We know, from the analysis of the model with adjustment costs, that for small piece rates the new $F^c(\theta)$ is lower than the old $F(\theta)$, whereas for high piece rates the opposite holds. It means that in the economy with hiring costs the probability of earning extremely high or extremely low piece rates, conditioned on being uninterruptedly employed for t periods, goes down, the new distribution of piece rates earned at each t has less mass in the tails. An individual, uninterruptedly employed for t periods, is more likely to earn a piece rate in the middle of the offers range, and not in its ends. Hence our aforementioned result that for each given experience the distribution of piece rates becomes less dispersed. We believe that there are additional extensions of the original framework of BCC that can be intuitively and comprehensively analyzed with the use of the career structure derived here.

8. Conclusion

We have analyzed the influence of adjustment costs on equilibrium outcomes in two models of labor market. In the baseline model of 1998 there is no human capital, all workers are equally productive at each point in time. In this model we assumed that a firm pays a fixed given sum for each new-hired worker. In the new model of 2009 workers' productivities differ, because they accumulate experience while working and they have different initial abilities. In such a model our assumption was that the hiring cost is proportional to the productivity of a worker — that is, hiring a qualified engineer is more costly than hiring a plumber and these costs are paid no matter how long a worker stays with the firm. We find that in both models hiring costs make equilibrium profits of the firm go down, and reduce the dispersion of observed wages. However, there are several differences. In the simple model the reservation benchmark of the unemployed goes down, whereas in the model with human capital accumulation it goes up. Finally, a rich environment of BCC allows to analyze additional properties of the equilibrium. In the economy where firms expend a lot on hiring a qualified worker, the differentiation of workforce quality between high and low paying firms will increase — that is, the workers in low-paying firms will be less qualified in the new equilibrium, whereas high-paying firms will now enjoy even more qualified workforce. In a cross-section of workers with the same experience there will be less wage inequality than earlier. In sum, all these properties are linked to the rising correlation between piece rate and experience when hiring costs go up.

Hiring costs influence the variance decomposition of wages in a rich model. When there is human capital accumulation, there are several sources of the dispersion of wages: variance in initial abilities, variance in accumulated experience, variance in the observed piece rates, and the interac-

tion between piece rates and experiences, which augment each other. Both for single type, and for non-degenerate distribution of initial abilities, higher costs mean that the greater part of wage differentials is due to the dispersion in initial abilities and accumulated experiences, and lesser part is due to the differences in the observed piece rates. The part of the interaction term changes insignificantly.

Finally, we have decomposed the development of a career in the model with human capital accumulation into two processes — the process of gaining tenure with experience and the process of accumulating higher piece rates as tenure grows. This structure allows to see why there is a positive correlation between piece rates and experiences — it turns out that longer experiences are associated with long tenures, which in turn are translated into high piece rates due to on-the-job search. The introduction of adjustment costs has no impact on the link between experience and tenure, however, it modifies on-the-job search process through the equilibrium distribution of offers. We get, that for all employment tenures the extreme values of piece rates become less frequent, which explains why for different experiences the distribution of earned piece rates becomes less dispersed. We believe that additional extensions can be comprehensively analyzed within the time-series framework that we have derived. The main innovation would be to model non-mechanical effects of experience.

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Равновесие на рынке труда, человеческий капитал и издержки найма: Препринт WP12/2009/08. — М.: Издательский дом Государственного университета – Высшей школы экономики, 2009. — 52 с. (на англ. яз.).

Данная работа посвящена анализу влияния предпосылки об издержках приспособления на равновесие на рынке труда. Исследование проводится в рамках двух моделей равновесия: классическая модель Бардетта – Мортенсена (1998) и её новейшая модификация – модель Бардетта и др. (2009), включающая накопление человеческого капитала. В обоих случаях мы добавляем в модель предположение о том, что наем работника связан для фирмы с издержками, и анализируем влияние данной предпосылки на равновесное распределение предлагаемых зарплат, распределение наблюдаемых зарплат, прибыли фирм, а также, в более новой модели, – распределение стажей и разложение дисперсии зарплат на компоненты. В последней части статьи в рамках расширенной модели подробно изучается процесс изменения зарплаты агента в течение жизни с точки зрения эффектов общего стажа и последнего периода непрерывной работы. Основные результаты работы: издержки приспособления снижают разброс предлагаемых и наблюдаемых зарплат, при этом средний уровень доходов агентов снижается. В расширенной модели мы получили, что различия в квалификации работников между фирмами с разными уровнями зарплат усиливаются, в то время как распределение доходов внутри когорты агентов с данным стажем становится более равномерным. Различия во врожденных способностях и накопленном стаже приобретают, при наличии издержек приспособления, больший вес в объяснении дисперсии наблюдаемых зарплат, в то время как значение различий между фирмами снижается. Анализ индивидуального профиля дохода в течение жизни показал, что более квалифицированные работники, как правило, заняты в фирмах, платящих высокую зарплату, потому что больший стаж означает более продолжительный последний период непрерывной занятости, что дает агенту возможность найти лучшую работу.

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