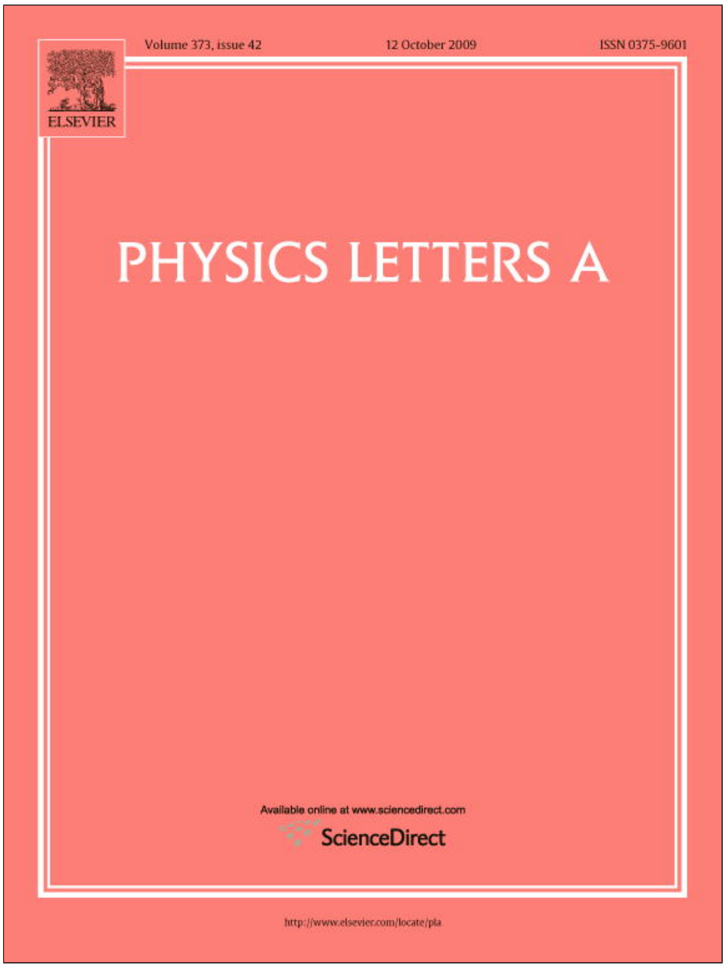
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Non-dispersive traveling waves in inclined shallow water channels

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ABSTRACT

Existence of traveling waves propagating without internal reflection in inclined water channels of arbitrary slope is demonstrated. It is shown that traveling non-monochromatic waves exist in both linear and nonlinear shallow water theories in the case of a uniformly inclined channel with a parabolic cross-section. The properties of these waves are studied. It is shown that linear traveling waves should have a sign-variable shape. The amplitude of linear traveling waves in a channel satisfies the same Green's law, which is usually derived from the energy flux conservation for smoothly inhomogeneous media. Amplitudes of nonlinear traveling waves deviate from the linear Green's law, and the behavior of positive and negative amplitudes are different. Negative amplitude grows faster than positive amplitude in shallow water. The phase of nonlinear waves (travel time) is described well by the linear WKB approach. It is shown that nonlinear traveling waves of any amplitude always break near the shoreline if the boundary condition of the full absorption is applied.

1. Introduction

Traveling waves are usually studied for media with constant parameters along the wave path. Well-known solutions exist for most integrable or near-integrable equations within nonlinear wave physics, including large-amplitude water waves. If parameters of the medium vary slowly along the wave path, solutions in the traveling wave form with variable amplitude and phase can be obtained with the use of asymptotic methods (WKB approach for linear waves, perturbation soliton theory, etc.). The amplitude of both linear and nonlinear waves satisfy the conservation of energy flux. It has been pointed out in [1,2] that for certain conditions of parameters of the medium, an asymptotic WKB solution for a linear monochromatic wave coincides with an exact solution even for rapidly varying media. It has been shown that traveling waves in general form can be found within the framework of the variable-coefficient linear wave equation using direct methods and Lie algebra [3–9]. Concerning nonlinear waves in strongly inhomogeneous media, the number of such exact traveling wave solutions is very limited [10–12]. One of the examples here is the solution of the variable-coefficient nonlinear Schrödinger equation, which has been found in the form of a soliton with constant amplitude due to a balance between the wave focusing in inhomogeneous media on caustics and wave defocusing, related to the phase chirp [10]. The interest to the problem of traveling waves is related to the possibility of reflectionless transfer of wave energy over long distances.

Here we consider the water wave dynamics in a narrow channel with variable depth along both cross-section and longitudinal channel directions (Fig. 1) and discuss traveling wave solutions in both linear and nonlinear cases. It will be shown later, that traveling waves exist in inclined shallow water channels of arbitrary slope when the WKB approach is not valid.

Governing equations for describing long waves in narrow channels consist of the hyperbolic 1D nonlinear-shallow water system

$$\frac{\partial S}{\partial t} + \frac{\partial}{\partial x}(Su) = 0, \qquad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial \eta}{\partial x} = 0, \tag{1}$$

where $\eta(x,t)$ is the water displacement, $u(x,t)$ is the depth-averaged flow, $S(x,t)$ is the variable water cross-section of the channel, and g is gravity acceleration.

In the case of a parabolic channel cross-section the water cross-section is $S \sim H^{3/2}$, and Eqs. (1) can be re-written for variables H and u:

$$\frac{\partial H}{\partial t} + u\frac{\partial H}{\partial x} + \frac{2H}{3}\frac{\partial u}{\partial x} = 0, \qquad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial H}{\partial x} = g\frac{\partial h}{\partial x}, \tag{2}$$

where $H(x,t) = h(x) + \eta(x,t)$ is the total depth along the longitudinal channel axis and $h(x)$ is unperturbed water depth. These equations differ from the "classic" 1D shallow-water equations for a plane beach by the 2/3 coefficient in the continuity equation. In the case of a beach of constant slope waves always reflect from the variable bottom [13–20]. At the same time, as it is shown below, in the case of an inclined channel of a parabolic shape the wave does not have internal reflection and propagates as a traveling wave.

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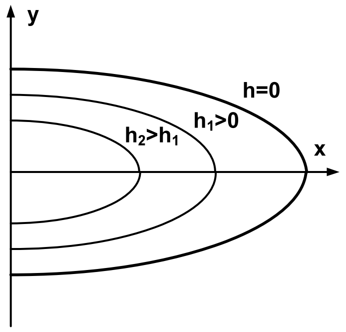

Fig. 1. Bathymetry contours of a channel of a parabolic cross-section.

The linear approximation of Eqs. (2) leads to the variable-coefficient wave equation for the water displacement

$$\frac{\partial^2 \eta}{\partial t^2} - g\frac{dh}{dx}\frac{\partial \eta}{\partial x} - \frac{2gh}{3}\frac{\partial^2 \eta}{\partial x^2} = 0. \tag{3}$$

Below we find solutions of the nonlinear system equations (2) and its linear approximation equation (3) describing traveling waves in inhomogeneous channels.

2. Linear traveling waves in inclined channels: Rigorous and approximated solutions

Let us briefly discuss the prediction of the WKB approach for linear waves in channels with slowly varying depth. In this case the solution is sought in the form

$$\eta(x,t) = A(x)\exp\bigl(i\bigl[\omega t - \Psi(x)\bigr]\bigr), \tag{4}$$

where the amplitude $A(x)$ and wave number $k(x) = d\Psi(x)/dx$ are slowly varying functions of x. After substituting Eq. (4) into Eq. (3) we obtain equations for functions $A(x)$ and $k(x)$

$$\frac{d^2A}{dx^2} + \frac{3}{2h}\frac{dh}{dx}\frac{dA}{dx} + \left[\frac{3\omega^2}{2gh} - k^2\right]A = 0, \tag{5}$$

$$A\frac{dk}{dx} + 2k\frac{dA}{dx} + \frac{3}{2h}\frac{dh}{dx}Ak = 0. \tag{6}$$

Taking into account that in a slowly inhomogeneous medium the first two terms in Eq. (5) are relatively small; they can therefore be neglected and thus

$$k(x) = \frac{d\Psi}{dx} = \frac{\omega}{\sqrt{2gh(x)/3}}. \tag{7}$$

A negative sign of $k(x)$ in Eq. (7) describes the wave propagating in the opposite direction. Eq. (6) represents the energy flux conservation, and after using Eq. (7) it determines the variation of wave amplitude with depth

$$A(x) \sim h^{-1/2}(x). \tag{8}$$

Expression (8) can also be obtained from the generalized Green's law $A \sim h^{-1/4}B^{-1/2}$ for a wave in a rectangular channel of variable depth $h(x)$ and width $B(x)$ [21–24], taking into account the characteristic width of a parabolic channel $B \sim S/H \sim h^{1/2}$. Thus, the wave amplitude in a parabolic channel grows faster with a decrease in depth ($h^{-1/2}$), than for a plane beach ($h^{-1/4}$). The WKB approach for a linear wave transformation in a channel of slowly varying depth gives simple but important results, which are useful for finding traveling waves solutions in cases where the depth is not a slow function of the distance.

The WKB solution obtained above neglects the two first terms in Eq. (5). Meanwhile there is a condition, when their sum gives zero. The corresponding differential equation can be obtained by inserting Eq. (7) into Eq. (5)

$$\frac{d^2A}{dx^2} + \frac{3}{2h}\frac{dh}{dx}\frac{dA}{dx} = 0. \tag{9}$$

Together with the Green's law (8) Eq. (9) gives us the exact solution

$$h(x) = -\alpha x, \tag{10}$$

where α is a bottom slope. It follows from Eq. (10) that the WKB approach, applied to the linear wave equation for a uniformly inclined parabolic channel, gives an exact solution valid for any values of the bottom slope α. The existence of similar exact solutions in the variable-coefficient linear wave equation has been pointed out in [1,2], and these solutions have been applied for various physical problems in relation to acoustical, surface and internal waves [3,5–8,25,26].

Let us study the structure of the traveling wave along the uniformly inclined parabolic channel. The general solution for the traveling wave can be expressed through Fourier superposition of elementary solutions (4)

$$\eta(x,t) = A(x)f\bigl[t - \tau(x)\bigr], \tag{11}$$

$$A(x) = A_0\left(\frac{h(x)}{h_0}\right)^{-1/2},$$

$$\tau(x) = \int_{x_0}^{x}\frac{dy}{\sqrt{2gh(y)/3}} = \sqrt{\frac{6}{g\alpha^2}}\bigl[\sqrt{h_0} - \sqrt{h(x)}\bigr]. \tag{12}$$

The traveling wave (11) propagates onshore without reflection with an increase in its amplitude. A similar solution, which describes the wave propagating offshore with a decrease in amplitude, can be found by the change of the sign of $\tau(x)$ in Eq. (11). The linear superposition of these solutions gives us the general solution of the Cauchy problem for the wave equation (3). Both solutions should be considered in the "liquid" domain ($x < 0$), where the point of the shoreline $x = 0$ is a singular point of the linear wave equation. Wave dynamics in the vicinity of the coastal line depends on the boundary condition in the singular point. Waves approaching the shoreline point can be removed from the domain by applying the condition of full absorption of wave energy at $x = 0$. This approach has been used in [27] for study a 3D wave motion in a basin of arbitrary depth. In the case of a fully reflecting boundary condition the superposition of two traveling waves of the same amplitudes and phases shifted by π should be used. In this case, even though the wave field at the shoreline is bounded, nevertheless the nonlinear effects are important in its vicinity and we consider this effect in the next section.

The shape of the water displacement in the framework of the wave equation (3) can be arbitrary, but the wave field contains two components: water displacement and water flow, which are connected. The depth averaged flow can be calculated exactly from the second equation in Eqs. (2) in linear approximation

$$u(x,t) = U(x)f\bigl[t - \tau(x)\bigr] + U_1(x)\Phi\bigl[t - \tau(x)\bigr], \tag{13}$$

where

$$\Phi(\varsigma) = \int f(\varsigma)\,d\varsigma, \qquad U(x) = \sqrt{\frac{3g}{2h(x)}}A(x),$$

$$U_1(x) = \frac{\alpha g}{2h(x)}A(x). \tag{14}$$

It follows from Eqs. (13) and (14) that generally the shape of the water flow does not coincide with the shape of the water displacement, except in the case of a slowly varying bottom when the last term in Eq. (13) is weak. Moreover, if the water displacement is a sign-constant disturbance, the flow behind the wave is not zero, and this does not result in boundedness of the wave energy. It means that the shape of the traveling wave should be sign-variable

$$\int_{-\infty}^{+\infty} f(t)\,dt = 0. \tag{15}$$

Thus, in the framework of linear theory, traveling waves propagate along a uniformly inclined parabolic channel in both offshore and onshore directions without any limitations with respect to the bottom slope. The shape of the water displacement should be sign-variable, which is appropriate for the description of the leading tsunami wave [28].

3. Nonlinear traveling waves in inclined channels

Eqs. (2) for a uniformly inclined channel is a hyperbolic system with constant coefficients. Its solution can be obtained with the use of the Legendre (hodograph) transformation. This approach has been very popular for long wave runup on a plane beach [13–20, 28–31]; and has been generalized for the case of narrow bays [32, 33]. With the use of the Legendre transformation Eqs. (2) can be reduced to (Appendix A)

$$\frac{\partial^2 \Phi}{\partial \lambda^2} - \frac{\partial^2 \Phi}{\partial \sigma^2} - \frac{2}{\sigma}\frac{\partial \Phi}{\partial \sigma} = 0, \tag{16}$$

and all the physical variables can be expressed through Φ, λ and σ

$$\eta = \frac{1}{2g}\left[\frac{2}{3}\frac{\partial \Phi}{\partial \lambda} - u^2\right], \qquad u = \frac{1}{\sigma}\frac{\partial \Phi}{\partial \sigma}, \tag{17}$$

$$x = \frac{\eta}{\alpha} - \frac{\sigma^2}{6g\alpha}, \qquad t = \frac{\lambda - u}{g\alpha}. \tag{18}$$

Thus, the nonlinear system (2) is reduced to the linear variable-coefficient wave Eq. (16). As the variable σ has a meaning of the total depth according to Eq. (A.6), Eq. (16) should be solved on the semi-axis $\sigma \geqslant 0$. It can be shown that the wave equation (16) has a solution in the form of a traveling wave propagating onshore

$$\Phi(\sigma, \lambda) = \frac{\Theta(\lambda + \sigma)}{\sigma}, \tag{19}$$

where Θ is an arbitrary function describing the shape of the wave. The "nonlinear" solution (17)–(19) is transformed to the "linear" traveling wave solution (11)–(12) with $f \sim \Theta'$ far from the shoreline. Therefore, the traveling wave of the water elevation $\eta(t)$ in nonlinear theory has a sign-variable shape as in the linear theory, even in the case when the function $\Theta(\lambda)$ is a sign-constant disturbance of a finite duration.

As in the linear theory, the solution (19) is unbounded at the singular point ($\sigma = 0$) if the boundary condition of full absorption is applied (there is no reflection from the shoreline). The new effect here is the wave breaking, caused by a strong nonlinear deformation of the wave profile. From the mathematical point of view the wave breaks when the Jacobian of the Legendre transformation becomes zero. It is easy to show that Jacobian can be expressed in the following form

$$J = \frac{\partial(t, x)}{\partial(I_+, I_-)} = \frac{2\sigma}{3}\frac{\partial t}{\partial I_+}\frac{\partial t}{\partial I_-} = \frac{\sigma}{6g^2\alpha^2}\left[\left(1 - \frac{\partial u}{\partial \lambda}\right)^2 - \left(\frac{\partial u}{\partial \sigma}\right)^2\right]. \tag{20}$$

It follows from Eq. (20) that $J = 0$ at the point $\sigma = 0$. Since the point $\sigma = 0$ is singular for the hyperbolic system (2) and does not characterize the wave breaking, it should be excluded from the analysis. For waves of weak amplitude the Jacobian $J \sim \sigma/6g^2\alpha^2$ is nonzero. Its magnitude decreases with an increase in the wave amplitude. At the same time velocity (in the nonlinear traveling wave) grows significantly in the vicinity of the singular point $u \sim \Theta/\sigma^2$, and it follows that the Jacobian must be zero at some point near the shore. This means that the traveling wave always breaks when it approaches the coast.

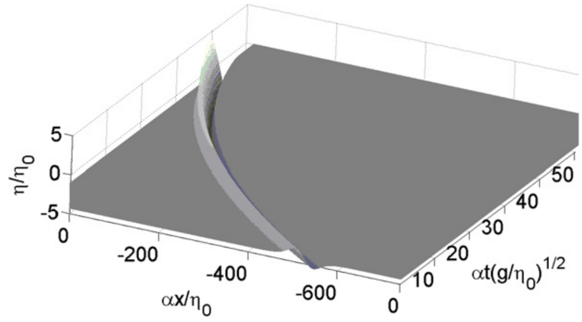

Fig. 2. Nonlinear traveling wave of water displacement in the uniformly inclined parabolic channel.

Besides the wave (19) Eq. (16) has a solution in the form of a traveling wave propagating offshore. Amplitude of this wave decreases with a distance and the wave never breaks.

If we apply the boundary condition of full reflection at the singular point, the bounded wave field at the shoreline will be described by the superposition of two waves traveling in opposite directions

$$\Phi(\lambda, \sigma) = \frac{\Theta(\lambda + \sigma) - \Theta(\lambda - \sigma)}{\sigma}. \tag{21}$$

In this case the wave approaching the coast will not break under special conditions for the wave amplitude, duration and bottom slope. This problem is analyzed in [32,33] for study of runup characteristics of water waves in channels.

Properties of the wave propagating onshore in the case, where the function $\Theta(\lambda)$ is in the shape of the Korteweg–de Vries soliton (sech^2) are discussed below. Figs. 2–7 show nonlinear traveling waves of the water displacement and particle velocity, computed from Eqs. (17)–(18) with the use of dimensionless variables (η/η_0, $u/(g\eta_0)^{1/2}$, $\alpha x/\eta_0$, $\alpha t(g/\eta_0)^{1/2}$). As it has been pointed out above, the water displacement has a sign-variable shape. Fig. 2 clearly demonstrates a wave amplification when the wave approaches the coast (shoaling effect), and the wave speed in shallow water decreases.

Due to the nonlinearity the wave shape deforms with a distance (Fig. 3), and its front becomes steeper. The variation of the wave amplitude with distance is shown in Fig. 4 for both positive (dashed line) and negative (dash-dotted line) amplitudes of the water wave displacement, and the linear Green's law (8) is marked with a solid line.

The negative wave amplitude grows faster than positive amplitude in shallow water. The difference between amplitude curves in Fig. 4, which represent the "nonlinear" Green's law, and the linear Green's law increases with a decrease in water depth. The difference in positive and negative amplitudes follows from the linear Green's law if the depth h is replaced by the total depth $H = h + \eta$. Since the total depth under the wave trough is smaller than under the wave crest, the negative amplitude is greater than the positive one.

Figs. 2–4 are presented in dimensionless variables. Let us make some physical estimates for the initial wave, given at the 7 km distance far from the shoreline at the 700 m depth with a bottom slope ~0.1. In this case the initial wave with an amplitude

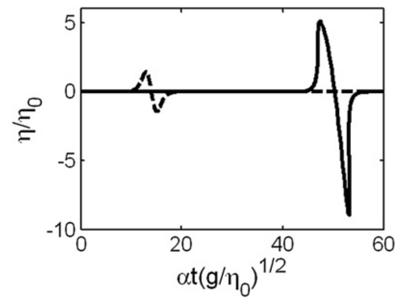

Fig. 3. Deformation of the wave shape in approaching wave: dashed and solid lines correspond to an incident wave ($\alpha x/\eta_0 = -353$) and the wave near the shoreline ($\alpha x/\eta_0 = -17$), respectively.

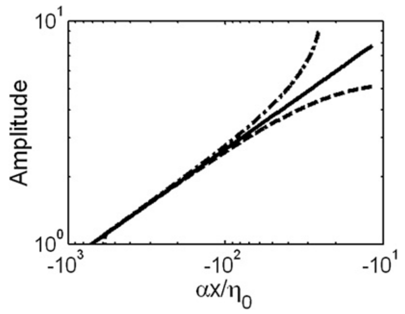

Fig. 4. Variation of positive (dashed line) and negative (dash-dotted line) amplitudes of the traveling wave with distance; solid line corresponds to the linear Green's law (8).

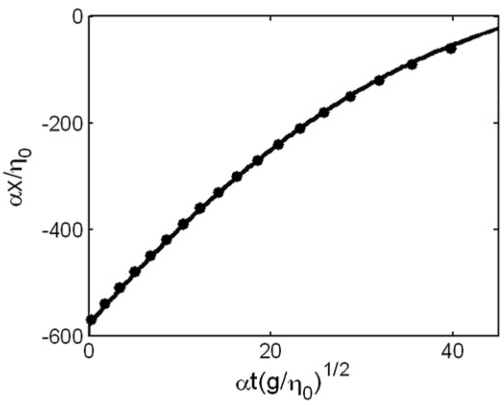

Fig. 5. Trajectory of the maximum wave amplitude in a plane (x, t); circles correspond to the linear function (12).

of 1 m and characteristic wavelength of about 2 km increases its positive and negative amplitude in 5 and 9 times nearshore (at the 300 m distance from the shoreline at the 30 m depth), respectively. This wave is strongly nonlinear, the ratio of the wave height to the depth nearshore is about 0.5.

The trajectory of the maximum wave amplitude in the (x, t) plane is shown in Fig. 5 by the solid line, where circles correspond to the linear function (12). It follows from Fig. 5 that the nonlinearity weakly influences on the phase (kinematic) characteristics.

The evolution of the traveling wave of velocity looks similar to Fig. 2 and it is not reproduced here. The shapes of both wave components (water displacement and velocity) are the same far offshore and differ near the shoreline (Fig. 6).

The variation of the maximum positive (dashed line) and minimum negative (dash-dotted line) amplitudes of the wave of velocity with distance is shown in Fig. 7. According to Eq. (13) in the linear theory, the amplitude of the wave of velocity is not a simple power function of the water depth. In the case of slowly varying

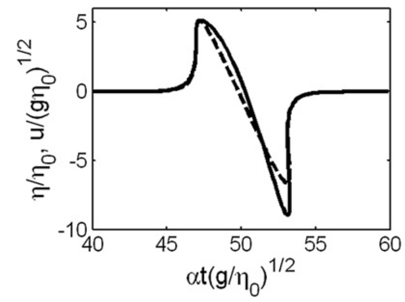

Fig. 6. Shapes of waves of water displacement (solid line) and velocity (dashed line) near the shoreline ($\alpha x/\eta_0 = -17$).

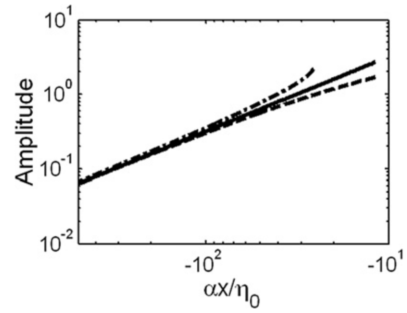

Fig. 7. Variation of positive (dashed line) and negative (dash-dotted line) amplitudes of the wave of velocity with a distance; the solid line corresponds to the linear Green's law.

depth the "velocity" amplitude is described by the function $U(x)$ (14), and $U(x) \sim h^{-1}(x)$ represents the linear Green's law for a wave of velocity; it is shown in Fig. 7 by a solid line. As it has been shown for a water displacement, the difference between positive and negative amplitudes can be explained by the difference in the total depth under the wave crest and trough.

4. Conclusion

It is shown that traveling waves may exist in inclined shallow water channels (along the wave path) within linear and nonlinear theory. These waves propagate over large distances without reflection. Such solutions are obtained for long water waves propagating in a uniformly inclined narrow channel with a parabolic cross-section. In the linear case they are found directly, and in the nonlinear case the hodograph transformation is used. It is shown that both linear and nonlinear traveling waves of water displacement in the channel should have a sign-variable shape in order to satisfy boundedness of the wave energy.

Traveling waves of water displacement conserve their shape in linear theory. Their amplitudes vary according to the Green's law, which follows from the energy flux conservation, without any limitations of the bottom slope. Traveling waves of water velocity do not conserve their shape even in linear theory and can be described by the superposition of two traveling waves of different amplitudes.

In the nonlinear case traveling waves propagating onshore become steeper with distance and always breaks near the shore if the boundary condition of the full absorption is applied. At the same time waves propagating offshore do not break. The variation of the wave amplitude with distance, which represents the "nonlinear" Green's law, differs from the prediction of the linear theory. The negative wave amplitude grows faster than positive amplitude in shallow water, demonstrating the difference in the total depth, which is less under the wave trough than under the wave crest. The travel time of a nonlinear wave is described well by the linear theory.

The obtained solutions can be useful for detailed analyses of the wave dynamics in channels including processes of wave shoaling, runup and wave breaking.

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Appendix A

In order to use the Legendre transformation for solving the nonlinear system (2) it is necessary to introduce Riemann invariants

$$I_\pm = u \pm 2\sqrt{\frac{3}{2}gH} + \alpha gt, \tag{A.1}$$

and re-write the system (2) in the form

$$\frac{\partial I_\pm}{\partial t} + c_\pm \frac{\partial I_\pm}{\partial x} = 0, \tag{A.2}$$

where the characteristic speeds are

$$c_\pm = \frac{2}{3}I_\pm + \frac{1}{3}I_\mp - \alpha gt. \tag{A.3}$$

After multiplying Eq. (A.2) by the nonzero Jacobian $\partial(t,x)/\partial(I_+, I_-)$, it transforms to the system

$$\frac{\partial x}{\partial I_\mp} - c_\pm \frac{\partial t}{\partial I_\mp} = 0, \tag{A.4}$$

which can easily be reduced to the linear equation by eliminating the coordinate x

$$\frac{\partial^2 t}{\partial I_+ \partial I_-} + \frac{2}{I_+ - I_-}\left(\frac{\partial t}{\partial I_-} - \frac{\partial t}{\partial I_+}\right) = 0. \tag{A.5}$$

Let us introduce new variables

$$\lambda = \frac{I_+ + I_-}{2} = u + \alpha gt, \qquad \sigma = \frac{I_+ - I_-}{2} = \sqrt{6gH}, \tag{A.6}$$

after which Eq. (A.5) transforms to

$$\frac{\partial^2 t}{\partial \lambda^2} - \frac{\partial^2 t}{\partial \sigma^2} - \frac{4}{\sigma}\frac{\partial t}{\partial \sigma} = 0. \tag{A.7}$$

After substituting $u = \frac{1}{\sigma}\frac{\partial \Phi}{\partial \sigma}$, Eq. (A.7) can be re-written in its final form to the wave equation (16).

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