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Games with Connected Player Strategies for Analyzing the Competitiveness of a Railroad Company in a Cargo Transportation Marketplace

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Abstract—**The competitiveness of a railroad company in a marketplace of cargo transportation in a region of a country is analyzed with the use of two mathematical models in the form of two-person games in which all the feasible player strategies are connected, i.e., cannot be chosen by the players independently, and the set of these connected strategies is a polyhedron described by a system of compatible linear inequalities. The first model is used to analyze the case in which the railroad company competes with all the other cargo carriers that offer their services in the region, for instance, with tracking companies, whereas the second model helps to find potential profitable coalitions of the company with some of these carriers in an attempt to enlarge the company's fair share of the market as much as possible on account of providing a "door-to-door" service for clients that need to move high volumes of cargo over long distances.**

Keywords—coalitions; competitiveness; equilibrium; games; polyhedron; quadratic optimization problems; railroad company; transportation tariffs.

I. INTRODUCTION

The competitiveness of a legal entity in a marketplace implies that its fair share of the market a) is "captured" by this market participant, and b) is large enough to justify its investment in the activities that constitute the subject of the participant's presence there while earning the legal entity an acceptable profit. In conformity to cargo transportation this particularly means that the volume of cargo operations under the participant's control is acceptable and stable in the sense that other market participants cannot decrease this volume (the market share) by acting individually or collectively taking into account the existing and expected tendencies in the market growth and expected technological innovations. The competitive equilibrium concept underlies the market behavior to which all the participants are interested to adhere, and none of them is interested in changing this behavior. Though calculating this equilibrium presents a challenge under any structure of the market [1] and under any status of business relations among the market participants [2], one should bear in mind that the participants may form coalitions (allowable by law and by market regulations), and every time such coalitions emerge, generally, the competitive equilibrium is to be recalculated.

It is clear that any quantitative approaches to market analysis, including the calculation of the above competitive equilibrium, require developing appropriate mathematical models to describe the functioning of all the market participants, their goals, and their intents. In conformity to the market of cargo services these models should particularly allow the participants to analyze the competitiveness of the tariffs for the services that they and their competitors can offer acting both alone and within any legally allowable coalitions, and cooperative game theory has long been recognized as the most suitable tool for such an analysis [3]. In analyzing the potential coalitions with other market participants that a particular market participant may be interested in forming, one should find the set of those potential partners a cooperation with which would increase the market share of the coalition. This increase should be such that a fair imputation of the additional profit among the coalition members would let each coalition member earn at least not less than it could earn by acting individually. Each coalition that can potentially be formed can pursue two goals, that is, to maximize its share of the market or to minimize the share of the coalition's surrounding. As it is known, there exist games in which strategies of the corresponding (to these two goals) coalition lead to different results (different coalition shares), whereas in some games the share of the coalition remains the same in both games [4].

Let a particular market participant consider the expediency of forming a coalition with the other market participants, and let

$T = \overline{1,n}$ be a set of participants acting in a marketplace of cargo transportation services,

$K \subseteq T$ be a set of potential partners for the participant to form a coalition,

S_i be a set of feasible strategies of market participant (player) i, $i \in \overline{1,n}$, and

H_i, $i \in T$ be the goal (utility) function of market participant (player) i, $i \in \overline{1,n}$ on $\prod_{i=1}^{n} S_i$.

Then the utility function of the coalition K is $\sum_{i \in K} H_i\ s_i$,

where $H_i : \prod_{i=1}^{n} S_i \to R^1$, and R^1 is the set of real numbers.

Each coalition K that can be formed by a particular market participant may compete with its surrounding $T \setminus K$, where the surrounding is understood as a set of all the market participants that are not members of the coalition K, so the following two games between the coalition and its surrounding should be considered:

Game I

$$\min_{\eta_{T\setminus K} \in I_{T\setminus K}} \sum_{i \in K} H_i(\xi_K, \eta_{T\setminus K}) \to \max_{\xi_K \in I_K}$$

$$\max_{\xi_K \in I_K} \sum_{i \in K} H_i(\xi_K, \eta_{T\setminus K}) \to \min_{\eta_{T\setminus K} \in I_{T\setminus K}},$$

which is an antagonistic game between the coalition K and its surrounding $T \setminus K$ and

Game 2

$$\sum_{i \in K} H_i(\xi_K, \eta_{T\setminus K}) \to \max_{\xi_K \in I_K}$$

$$\sum_{i \in T\setminus K} H_i(\xi_K, \eta_{T\setminus K}) \to \max_{\eta_{T\setminus K} \in I_{T\setminus K}},$$

which is a non-cooperative two-person game between the coalition K and its surrounding $T \setminus K$.

Here, ξ_K and $\eta_{T\setminus K}$ are a strategy of the coalition K and a strategy of its surrounding $T \setminus K$, respectively, and both strategies are certain probabilistic measures over the sets $I_K = \prod_{i \in K} S_i$ and $I_{T\setminus K} = \prod_{i \in T\setminus K} S_i$. It is natural to assume that both games are solvable, i.e., both games possess Nash equilibrium points at least in mixed strategies [4], [5].

Certainly, to develop both games under particular market conditions and for particular market participants one should choose the particular functions H_i and sets S_i in the direct product of which these functions are defined, where $i \in \overline{1,n}$, and n is the number of the market participants.

II. The Problem Statement and Mathematical Formulation

Let us first consider a railroad company that competes with other transportation modes in a particular segment of a cargo transportation market, for instance, in a segment of container transportation. The interaction of the railroad company with its surrounding (that is formed by the other transportation modes functioning in the marketplace) can be analyzed in the framework of a two-person game with a constant sum, where this sum is the total volume of cargoes available in the marketplace, and each piece of this cargo is to be moved between certain geographic locations. Let the goal function of each carrier (player) be a difference between a linear function of cargo volumes moved by this carrier (which has the form of the scalar product of a vector of these volumes and a vector of particular transportation tariffs per unit of the volume, offered by the carrier) and a linear function describing the expenses of this carrier associated with moving a unit volume of each type of cargo, and let the available cargo volumes be bounded from above and from below. Then finding a competitive equilibrium for all the market participants can be achieved by solving a two-person game on a polyhedron of connected player strategies, and these strategies are volumes of cargo of each type that each game player is to move [6], [7].

Let

m be the number of types of cargo that is offered for transportation within a particular market segment,

n be the number of the market participants acting within this market segment besides the railroad company,

x_{ji} be the volume of cargo of type j that will be moved by market participant (carrier) i from the surrounding,

u_{ji} be the tariff of carrier i for moving a unit volume of cargo of type j, $i \in \overline{1,n}$, $j \in \overline{1,m}$,

p_{ji} be the expenses of carrier i from the surrounding associated with moving a unit volume of cargo of type j, $i \in \overline{1,n}$, $j \in \overline{1,m}$,

y_j be the volume of cargo of type j, that will be moved by the railroad company, $j \in \overline{1,m}$,

z_j be the tariff of the railroad company for moving a unit volume of cargo of type j, $j \in \overline{1,m}$,

q_j be the expenses of the railroad company associated with moving a unit volume of cargo of type j, $j \in \overline{1,m}$,

h_j be the total volume of cargo of type j available for transportation within a particular period of time (the market volume), $j \in \overline{1,m}$.

In the framework of this game, the railroad company considers its surrounding as a unified player which aspires to capture as large share of the market (i.e., as large volumes of cargoes of all the types) as possible.

Let $A \in \Omega_1$, $B \in \Omega_2$, and let Ω_1, Ω_2 be sets of railway stations between each pair of which the cargoes of all the types can be moved, and let A and B be two particular railroad stations. If the geography of cargo transportation within the segment of the market under consideration is limited by the set of pairs of railway stations, then the railroad company can compete with the other carriers directly, and its competitive edge is associated with the ability to move sizable volumes of cargo contemporaneously. However, if there are geographic locations, where a particular piece of cargo is originated and to where this piece of cargo should be moved, which are not railway stations, then the railroad company can extend its services to the customers from these locations by forming a partnership, for instance, with a tracking company (or with several tracking companies) that is not (are not) part of the set of carriers forming the surrounding of the railroad company in the marketplace segment.

For the latter case, the interaction between the railroad company and the surrounding can be described by a two-person game in which the payoff function of the railroad company is

$$\sum_{j=1}^{m} y_j z_j - \sum_{j=1}^{m} y_j q_j \to \max_{(y_1 \dots, y_m)}$$

whereas the payoff function of the surrounding is

$$\sum_{j=1}^{m}\sum_{i=1}^{n} x_{ji} u_{ji} - \sum_{j=1}^{m}\sum_{i=1}^{n} p_{ji} x_{ji} \to \max_{(x_{11} \dots, x_{nm})}$$

and the system of linear constraints for the numbers $y_1 \dots, y_m$ and $x_{11} \dots, x_{mn}$

$$\sum_{i=1}^{n} x_{ji} + y_j = h_j$$

$$x_{ji} \geq 0,\ i \in \overline{1,n},\ j \in \overline{1,m}$$

$$y_j \geq 0,\ j \in \overline{1,m}$$

holds, along with other linear constraints that may be imposed on the vector variables x and y separately.

Let

$y \in R_+^m$, $y = (y_1, \dots, y_m)$ be an m-dimensional vector whose components are volumes of cargoes determining the market share of cargoes for the railroad company, and y_j is the volume of cargo of type j, to be moved by the railroad company, $j \in \overline{1,m}$,

$x \in R_+^{mn}$, $x = (x_{11}, \dots, x_{mn})$ be an mn-dimensional vector whose components are volumes of cargoes determining the market share of cargoes for the surrounding, and x_{ji} is the volume of cargo of type j, to be moved by carrier i, $j \in \overline{1,m}$, $i \in \overline{1,n}$,

$z \in R_+^m$, $z = (z_1, \dots, z_m)$ be an m-dimensional vector whose components are transportation tariffs offered by the railroad company for moving cargoes of all the types, and z_j is a tariff for moving a unit volume of cargo of type j, $j \in \overline{1,m}$,

$u \in R_+^{mn}$, $u = (u_{11}, \dots, u_{mn})$ be an mn-dimensional vector whose components are transportation tariffs offered by the carriers from the surrounding, where u_{ji} is a tariff offered by carrier i for moving a unit volume of cargo of type j, $i \in \overline{1,n}$, $j \in \overline{1,m}$,

$p \in R_+^{mn}$, $p = (p_{11}, \dots, p_{mn})$ be an mn-dimensional vector whose components reflect the transportation expenses associated with moving cargoes of all the types by carriers from the surrounding, where p_{ji} reflects the expenses associated with moving a unit volume of cargo of type j, $j \in \overline{1,m}$ by carrier i, $i \in \overline{1,n}$ and

$q \in R_+^m$, $q = (q_1, \dots, q_m)$ be an m-dimensional vector whose components reflect the expenses of the railroad company associated with moving cargoes of all the types, and q_j reflects the expenses associated with moving a unit volume of cargo of type j, $j \in \overline{1,m}$ by the railroad company.

Game (1) – (3) can be rewritten in the vector-matrix form as follows [7]:

$$< y, z > - < y, q > \to \max_{y \geq 0}$$
$$< x, u > - < x, p > \to \max_{x \geq 0}$$

where

a) $Ax + E_m y = h$, and E_m is a unit $m \times m$ matrix (i.e., the $m \times m$ matrix in which each component located on the main diagonal equals 1, and each of all the other components equals 0),

b) some above-mentioned additional balance constraints, imposed on the vectors x and y separately, hold, for instance, $Dx \leq d$ and $Cy \leq c$, where D and C are matrices, and d and c are vectors of correspondent dimensions,

c) $h \in R_+^m$, $h = (h_1, \dots, h_m)$ is an m-dimensional vector whose j-component equals the total volume of cargo of type j to be moved (the market volume with respect to the cargo of type j) j, $j \in \overline{1,m}$, and

d) A is an $m \times mn$ matrix

$$A = \begin{pmatrix} \overbrace{11\dots1}^{n} & 00\dots0 & \dots\dots\dots & 00\dots0 \\ 00\dots0 & \overbrace{11\dots1}^{n} & \dots\dots\dots & 00\dots0 \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \\ 00\dots0 & 00\dots0 & \dots\dots\dots & \underbrace{11\dots1}_{n} \end{pmatrix},$$

and this game is a non-cooperative game on a polyhedral set of connected player strategies determined by the constrains $Ax + E_m y = h$, $Dx \leq d$, $Cy \leq c$, $x \geq 0$, $y \geq 0$.

Game (4) is reducible to an auxiliary antagonistic game on the same polyhedral set of connected player strategies, and solving this auxiliary game (if it is solvable) is reducible to solving a pair of quadratic optimization problems so that the pair of the vectors (x^*, y^*) that form a solution to these optimization problems form a solution to Game (4) [8]. Here, the vector y^* in the pair of vectors (x^*, y^*) is the vector whose components are the volumes of cargoes of all the types (that represent the railroad company's fair share of the market segment under consideration) that this company can capture under its particular tariffs (being components of the vector z) and under particular tariffs offered by the company's competitors (components of the vector u). By changing the transportation tariffs that are components of the vector z , the railroad company can estimate how these changes affect the size of its fair share of the market segment under consideration by solving problem (4) under any particular tariffs (components of the vector u) that can be offered by the competing companies forming the railroad company's surrounding.

Though Game (4) is the simplest among those that can be used for analyzing the potential of the railroad company in the segment of the market under consideration, one can use this game to analyze the expediency of forming possible coalitions between the railroad company and a particular transportation company representing the other transportation modes competing with the railroad company in this segment. That is, solving this game can help find which potential coalition participants (if any) can enlarge the market share of the railroad company in the framework of the above coalition.

Now, let i be a transportation company competing with the railroad company in a particular segment of the market under consideration, and let the railroad company consider company i as a potential partner with which to form a coalition for the purpose of increasing the profit of each of the two companies.

Logistically, there are two ways to form such a coalition between company i and the railroad company.

1. Company i either continues to move all the types of cargoes (which this company moved prior to forming a coalition with the railroad company) or focuses on moving only a part of these types of cargoes, and it does this from the points of cargo origination to the destination points. However, a part of the cargo volume of every type that can be moved by the railroad company in the framework of the coalition more effectively (in the sense of the profit that can be earned by this coalition compared with the profit that could have been earned by the coalition if company i continued to move this part of the cargo volume) will be moved by the railroad company. The effectiveness of a new scheme of assigning the cargoes to be moved by each of the coalition partners is attained on account of moving the above part of the cargoes by the railroad company either up to the cargo destination points or up to some intermediate railroad stations from which either company i or any other transportation company that works together with the railroad company (and, consequently, with the coalition) delivers the cargo from these intermediate railroad stations to the destination points under a "door-to-door" scheme.

2. Company i moves cargoes only from the points of their origination to a certain set of railways stations, as well as from these railways stations to the destination points (to the cargo recipients).

It is clear that in both cases company i is interested in forming a coalition with the railroad company only if the level of its profit will increase compared with the one that company i can secure by acting on its own, i.e., by remaining an independent carrier from the surrounding of the railroad company (provided no other coalition between the railroad company and one of the other carriers competing with the railroad company is formed).

Let

$\widetilde{y} = (y_1,\dots,y_m;x_{1i},x_{2i},\dots,x_{mi}) \in R_+^{2m}$ be a vector whose first m components form the vector y , and the remaining m components are volumes of cargoes of types $\overline{1,m}$ that are moved by company i so that x_{ki} is the volume of cargo of type k that is moved by company i as a member of the coalition, $k \in \overline{1,m}$,

$\widetilde{z} = (z_1,\dots,z_m;u_{1i},u_{2i},\dots,u_{mi}) \in R_+^{2m}$ be a vector whose first m components form the vector z , and the remaining m components are transportation tariffs offered by the coalition (consisting of the railroad company and company i) for moving a unit volume of each type of cargo (out of types $\overline{1,m}$) so that u_{ki} is the tariff for moving a unit volume of cargo of type k by company i , $k \in \overline{1,m}$.

Further, let $\widetilde{x} \in R_+^{m(n-1)}$, $\widetilde{u} \in R_+^{m(n-1)}$, $\widetilde{p} \in R_+^{m(n-1)}$, $\widetilde{q} \in R_+^{2m}$ be the vectors with the following components:

$$\widetilde{x} = \big(x_{11},x_{12},\dots,x_{1(i-1)},x_{1(i+1)},\dots,x_{1n},x_{21},x_{22},\dots,x_{2(i-1)}, x_{2(i+1)},\dots,x_{2n},\dots,x_{m1},x_{m2},\dots,x_{m(i-1)},x_{m(i+1)},\dots,x_{mn}\big),$$

$$\widetilde{u} = \big(u_{11},u_{12},\dots,u_{1(i-1)},u_{1(i+1)},\dots,u_{1n},u_{21},u_{22},\dots,u_{2(i-1)}, u_{2(i+1)},\dots,u_{2n},\dots,u_{m1},u_{m2},\dots,u_{m(i-1)},u_{m(i+1)},\dots,u_{mn}\big),$$

$$\widetilde{p} = \big(p_{11},p_{12},\dots,p_{1(i-1)},p_{1(i+1)},\dots,p_{1n},p_{21},p_{22},\dots,p_{2(i-1)}, p_{2(i+1)},\dots,p_{2n},\dots,p_{m1},p_{m2},\dots,p_{m(i-1)},p_{m(i+1)},\dots,p_{mn}\big),$$

$$\widetilde{q} = (\widetilde{q}_1,\dots,\widetilde{q}_m;\widetilde{p}_{1i},\widetilde{p}_{2i},\dots,\widetilde{p}_{mi}) .$$

Here,

$\widetilde{x}$ is an $m(n-1)$ -dimensional vector whose components are volumes of cargoes of all the types that are to be moved by all the carriers from the surrounding (of the coalition formed by the railroad company and company i) in the game under consideration, and x_{jl} is the volume of cargo of type j that is to be moved by carrier l , $l \in \overline{1,n} \setminus \{i\}$, $j \in \overline{1,m}$,

$\widetilde{u}$ is an $m(n-1)$ -dimensional vector whose components are transportation tariffs for moving a unit of cargo volume for each type of cargoes offered by the carriers from the surrounding (of the coalition formed by the railroad company and company i) in the game under consideration, and u_{jl} is the tariff for moving a unit volume of cargo of type j , offered by carrier l , $l \in \overline{1,n} \setminus \{i\}$, $j \in \overline{1,m}$,

$\widetilde{p}$ is an $m(n-1)$ -dimensional vector whose components reflect the expenses associated with moving a unit of cargo volume for each type of cargoes by the surrounding (of the coalition formed by the railroad company and company i) in the game under consideration, and p_{ji} reflect the expenses of carrier l, $l \in \overline{1,n} \setminus \{i\}$, associated with moving a unit of cargo of type j, $j \in \overline{1,m}$, and

$\widetilde{q}$ is a $2m$ -dimensional vector whose first m components reflect the expenses of the railroad company in the framework of the coalition (formed by the railroad company and company i), and these expenses are associated with moving a unit volume for each type of cargoes, whereas the other m components reflect the same expenses for company i.

Since, generally, the expenses $\widetilde{q}_j$ and $\widetilde{p}_{ji}$ may differ from (usually are smaller than) the expenses q_j and p_{ji}, respectively, one should assume that either the inequalities $\widetilde{q}_j \le q_j$, $\widetilde{p}_{ji} \le p_{ji}$, $j \in \overline{1,m}$ or the inequalities

$$\sum_{j=1}^{m} \widetilde{q}_j \le \sum_{j=1}^{m} q_j, \quad \sum_{j=1}^{m} \widetilde{p}_{ji} \le \sum_{j=1}^{m} p_{ji}$$

hold though the inequality

$$\sum_{j=1}^{m} (\widetilde{q}_j + \widetilde{p}_{ji}) \le \sum_{j=1}^{m} (q_j + p_{ji})$$

or the inequalities being a combination of the above three types of the inequalities may also hold.

The interaction of the coalition (the railroad company and company i) with its surrounding still can be considered as a two-person game (the coalition playing against its surrounding); the only difference is associated with the different form of the vectors $\widetilde{y}$, $\widetilde{z}$, $\widetilde{x}$, $\widetilde{u}$, $\widetilde{p}$, $\widetilde{q}$, and that of the matrices $\widetilde{A}$ and $\widetilde{E}$ in the mathematical formulation of the new game compared with the matrices A and E_m in game (4), respectively. This new game takes the following form:

$$< \widetilde{y}, \widetilde{z} > - < \widetilde{y}, \widetilde{q} > \to \max_{\widetilde{y} \ge 0}$$
$$< \widetilde{x}, \widetilde{u} > - < \widetilde{x}, \widetilde{p} > \to \max_{\widetilde{x} \ge 0}$$

where $\widetilde{A}\widetilde{x} + \widetilde{B}\widetilde{y} = h$, and, as in Game (4), some additional constraints of the balance type in the form of the inequalities $\widetilde{D}\widetilde{x} \le \widetilde{d}$ and $\widetilde{C}\widetilde{y} \le \widetilde{c}$ for the vectors $\widetilde{x}$ and $\widetilde{y}$ hold, whereas the $m \times 2m$ matrix $\widetilde{B}$ and the $m \times m(n-1)$ matrix $\widetilde{A}$ are

$$\widetilde{B} = (E_m | E_m) = \left(\begin{array}{c|c} 10...0 & 10...0 \\ 01...0 & 01...0 \\ .... & .... \\ 00...1 & 00...1 \end{array} \right),$$

and

$$\widetilde{A} = \begin{pmatrix} \overbrace{11...1}^{n-1} & \overbrace{00...0}^{n-1} & 00...0 \\ 00...0 & 11...1 & 00...0 \\ .... & .... & .... \\ 00...0 & 00...0 & \underbrace{11...1}_{n-1} \end{pmatrix}.$$

In just the same manner in which Game (4) is reducible to an auxiliary antagonistic game on a set of connected player strategies, Game (5) is reducible to an auxiliary antagonistic game on a set of connected player strategies, which is described by the system of linear inequalities $A_0\widetilde{x} + B_0\widetilde{y} = h_0$, in which the matrices from the above inequalities $\widetilde{D}\widetilde{x} \le \widetilde{d}$ and $\widetilde{C}\widetilde{y} \le \widetilde{c}$ are parts of the matrices A_0 and B_0 respectively. However, in analyzing the possibility of the coalition (between the railroad company and company i) to form, the additional linear inequality:

$$< \widetilde{y}, \widetilde{z} - \widetilde{q} > \ \ge \ < y^*, z > - < y^*, q > + \\ + < x^*, \widehat{u} > - < x^*, \widehat{p} > + \mu$$

where (x^*, y^*) is a solution to (an equilibrium in) Game (4), $\mu > 0$, and $\hat{u} \in R^{mn}$, $\hat{p} \in R^{mn}$ are vectors whose all the components equal zero, except for those occupying the numbers $(j-1)n + i$, $j \in \overline{1,m}$, which coincide with the corresponding components of the vectors $u \in R_+^{mn}$, $u = (u_{11}, \dots, u_{mn})$ and $p \in R_+^{mn}$, $p = (p_{11}, \dots, p_{mn})$, should be added. This inequality requires that the profit of both members of the coalition combined should exceed the sum of their profits that would be earned if both of them acted independently (i.e., if company i competed with the railroad company as did the other carriers), at least by $\mu > 0$.

III. On Solving Game (4) and Game (5), (6)

Both Game (4) and Game (5), (6) are those on polyhedral sets of connected player strategies in the form

$$S = \{(x, y) \ge 0 : Ax + By \ge h\}$$

with the payoff function

$$\phi(x, y) = \langle p, x \rangle + \langle q, y \rangle .$$

A pair of vectors (x^*, y^*) is called an equilibrium point in this game, if the inequalities

$$\phi(x^*, y) \le \phi(x^*, y^*) \le \phi(x, y^*), \ \forall \ (x^*, y) \in S, \ \forall \ (x, y^*) \in S$$

hold [6]. This game can also be rewritten in the form

$$(x^*, y^*) \in Ep_{(x,y) \in S} \{\langle p, x \rangle + \langle q, y \rangle\},$$
$$S = \{(x, y) \ge 0 : Ax + By \ge h\},$$

where $Ep_{(x,y) \in S} \{\langle p, x \rangle + \langle q, y \rangle\}$ is a set of equilibrium points of the game, and it can be solved on the basis of the approach to solving antagonistic games on polyhedral sets of connected player strategies in which the payoff function is a sum of a bilinear and two linear functions of vector variables, which is proposed in [6]. Particularly, in conformity to Game (5), (6), the following assertion holds:

Assertion[8]. A pair of vectors $(\tilde{x}^{**}, \tilde{y}^{**})$ is an equilibrium point in Game (5), (6), if and only if $(\tilde{x}^{**}, \tilde{y}^{**})$ is an equilibrium in the game on the polyhedral set

$$A_0\tilde{x} + B_0\tilde{y} \ge h_0,$$
$$< \tilde{y}, \tilde{z} - \tilde{q} > \ \ge \ < y^*, z > - < y^*, q > +$$
$$+ < \tilde{x}^*, \tilde{u} > - < \tilde{x}^*, \tilde{p} > + \mu$$

with the payoff function $\phi(\tilde{x}, \tilde{y}) = \langle \tilde{y}, \tilde{z} - \tilde{q} \rangle - \langle \tilde{x}, \tilde{u} - \tilde{p} \rangle$.

This Assertion allows one to reduce finding equilibria in each of the two games to finding solutions to the following non-linear system of two quadratic equations and linear inequalities [6]:

$$Th \ge \delta,$$
$$\langle h, H_1 h \rangle + \langle \rho, h \rangle = 0,$$
$$\langle h, H_2 h \rangle + \langle \rho, h \rangle = 0,$$

where T is a $\tau \times \sigma$ matrix, H_1, H_2 are $\sigma \times \sigma$ symmetric quadratic matrices, $\rho \in R^{\sigma}$, and $h \in R_+^{\sigma}$ (if the games are solvable). Necessary and sufficient conditions for equilibria in Game (5), (6), presented in [6], allow one to reduce finding equilibria in solvable games to finding extrema in the so-called quadratic optimization problems [9]

$$K_0 \to \inf_{x \in M \subset R^{\sigma}},$$
$$K_i \le 0, i \in I, K_j = 0, j \in J, I \cap J = \emptyset,$$
$$K_\nu = \langle A_\nu x, x \rangle + \langle l_\nu, x \rangle + c_\nu,$$

where A_ν are symmetric $\sigma \times \sigma$ quadratic matrices, l_ν are vectors from R^{σ}, c_ν are constants, ν in $\{0\} \cup I \cup J$, $I, J \subseteq N$, N is the set of all natural numbers, and M is either a polyhedral set or coincides with R^{σ}. Effective techniques that are based on non-differential optimization ones were proposed in [9] for solving these quadratic optimization problems.

If $(\tilde{x}^{**}, \tilde{y}^{**})$ is a solution to (an equilibrium in) Game (5), (6), then the vector $\tilde{y}^{**}$ determines the volumes of cargoes of types $\overline{1,m}$ to be moved by the coalition, which is its fair share of the market under the transportation tariffs comprising the vectors $\tilde{z}$ and $\tilde{u}$. In just the same manner as in Game (4), by changing the tariffs, the coalition can evaluate how these changes affect the size of the fair share of the market segment under consideration by solving Game (5), (6) under any particular tariffs offered by the carriers comprising the surrounding of the coalition. Moreover, by solving Game (5), (6) for all possible coalitions that the railroad company can form with any of the other carriers functioning in the segment of the market, the railroad company can evaluate which coalition (if any) would be the best to form from the viewpoint of the profit that the coalition can receive by acting against its surrounding. The same types of the games can be formulated for evaluating the expediency to form a coalition between the railroad company and more than one of the other carriers competing with it in the market segment.

One should notice that though in both Game (4) and Game (5), (6), the railroad company attempts to maximize its own gain or that of the coalition that it forms with another carrier or with a group of other carriers (from among the carriers competing with the railroad company), this strategy leads to the same result that would be obtained if the railroad company attempted to reduce the gain of the surrounding, i.e., either a) the gain of all the other carriers combined (if the railroad company competes with all of them), or b) the gain of the surrounding of a coalition that the railroad company may decide to form with any of the other carriers or with any group of these carriers. This follows from the fact that both games are those with constant sum, and the coincidence of the gains in both such games with constant sum was established in [5].

Finally, one should bear in mind that though game-theoretic approaches to analyzing transportation systems are well known [10]-[16], their applications to railroad transportation systems have so far been limited, and the complexity of the mathematical problems to be solved in finding the equilibria in the corresponding games seems to be one of the reasons for this state of affairs in the field. In particular, as shown in this article, in studying railroad transportations systems, games on sets of connected player strategies that are more complicated then games on sets of disjoin players strategies (which are mostly studied, in particular in transportation systems [10], [11]), i.e., games in which the players cannot choose their strategies independently of each other, emerge [3], [6], [15]. Solving these games, describing the functioning of railroad companies in any particular cargo marketplace, is not an easy task.

Nevertheless, detecting classes of games describing the functioning of railroad companies in a cargo transportation marketplace for which either known effective optimization techniques can be used, or new such techniques can be developed may help game-theoretic approaches become parts of the tools to analyze the market of both railroad and multimodal transportation services and make the market of cargo transportation more understandable to both the shippers and the carriers [10], [11].

IV. Conclusion

To be competitive in a segment of a particular cargo transportation market, the transportation company should attempt to capture its fair share of the market by offering competitive tariffs for moving cargoes of all the types available there and by forming coalitions with the other market participants acting there. In so doing, the railroad company may face the following four situations:

a) there exists a set of tariffs that make the railroad company acting alone competitive in the market segment by allowing the company to attain a desirable level of profit,

b) there exists a set of tariffs that make the railroad company acting alone competitive in the market segment;

however, its profit is unacceptably low, and the railroad company cannot increase the level of the profit by changing its tariffs,

c) under any tariffs, the railroad company cannot be profitable by acting alone, while by forming a coalition with another carrier or with a group of other carriers competing in a particular market segment, the railroad company can make an acceptable profit as part of the coalition and thus can be competitive, and

d) the railroad company cannot compete with the other carriers alone, and no coalition with another carrier or with a group of any other carriers competing in a particular market segment allows the railroad company to be profitable.

While analyzing all these situations, generally, it requires developing more complicated game models (in which both the cargo volumes and the tariffs for their moving are the variables), solving Game (4) and Game (5), (6) allows the railroad company to do the competitiveness analysis for any particular set of tariffs that all the other carriers can offer in a particular market segment of cargo transportation, which can help the railroad company develop both its long term and its short term business strategies.

In fact, a strategic analysis of a particular market segment implies developing a characteristic function of a cooperative game, since generally, any group of carriers can do the same analysis of the collective market share of a potential coalition that these carriers may decide to form as does the railroad company in an attempt to increase its fair share of the market segment.

Another option to increase a fair share of the market segment for the railroad company is to form a partnership with some of its clients, first of all with those with whom stable cargo flows have existed for a certain period of time, and where the loading and unloading the cargo can be done with the use of the client's railway sidings. Also, one should understand that while a railroad company may not be competitive in moving cargoes within a particular geographic region, moving cargoes between several regions can provide a competitive edge to the railroad company, especially when the volumes of cargoes to move are sizable. Certainly, in this case, the market segment to be analyzed changes, and so does the set of the competitors (players), which requires a recalculation of the profit that the railroad company may attaint with (possibly new) different partners.

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