A Technique to Simulate Melting Probes Movement and to Estimate Penetration Velocities Range

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***Abstract*—This paper considers some problems related to penetration through thick ice layer and to study effectiveness of a melting probe like instrument for penetration through thick Jovians moon Europa icy surface.**

A melting probes movement model creation, verification and validation, as well as studying a melting process in general is discussed. To create the simulation model of the melting probes movement a mathematical model based on theory of solid water was developed. Model verification was based on experiments carried out by the Austrian Academy of Sciences.

I. Introduction

Last decades researches showed that there are several space objects covered by thick icy layer. There are only three of them (except the Earth) where water presence was proved. Such objects where ice thickness estimation varies from several and up to several dozen kilometers are Jovian's moon Europa, Saturn's moon Enceladus and the dwarf planet Ceres. For Europa it is assumed that there should be a deep ocean (with a depth up to 100 km) under Europas icy shell (e.g. [1], [2]). To study this possible ocean and to look for lifes traces, it is necessary to penetrate the icy sheet. This means that special equipment should be designed in order of meeting the space mission requirements:

- small size and weight;
- relatively small power consumption;
- a device should provide a relatively clean ice penetration.

In terrestrial conditions such kind of device to study subsurface area under thick icy layer is successfully used. The idea of a special probe creation appered in the mid 1960s [3]–[5]. Based on techniques for studying terrestrial ices it was decided to consider some of them for extraterrestrial application.

There are several ways to penetrate through ice. One of them is to use a melting probe that operates by melting surrounding ice and move down through the ice by gravitational force. Other one is a thermo drill that operates by melting and drilling simultaneously. And cryobot that uses water jets to create a hole. For extraterrestrial application the most effective is the melting probe. The more detailed information about different techniques of ice penetration could be found in [6].

Other unsolved problems are in the areas of analyzing how the probe will move in low gravity and low atmospheric pressure; whether the hole formed in the ice will be closed when the probe penetrates far enough or not; what is the influence of the probes characteristics on the melting process; what would be the order of magnitude of the penetration velocity. This study explores the method based on elasto-plastic theory and so-called solid water theory to estimate the melting velocity and to study the melting process. Based on this method, several cases of melting probe motion was considered, the velocity of the melting probe was estimated, the influence of different factors was studied and discussed, and an easy way to optimize the parameters of the probe was proposed.

II. Previous research

Last decades several experiments were held to study characteristics of probes that are suitable to penetrate through deep ice layer for space mission. These experiments results showed that it is necessary to optimize the probes working parameters. Such parameters are probes shape, probes surface temperature, and probes penetration velocity [7]–[10].

The main task is to choose optimal probes characteristics so that it could be possible to achieve the optimal ice penetration velocity that will require minimal energy consumption, dimensions and weight. Experiments to study melting process and probes optimization are laborious and costly task, since in addition to creating the probe itself, experiments should be conducted at sufficiently low temperatures under vacuum conditions.

Nowadays almost all theoretical estimations of melting probes parameters influence on a melting process are based on theory that was proposed by Aamot and Sherve [3], [4]. This theory provides an estimation of the melting probes velocity taking into account probes geometry, power consumption and properties of surrounding ice.

The disadvantage of this model is the fact that it takes into account only the radius and length of the melting probe but does not consider a form of the tip; the second problem is that resulting value does not show the dynamics of melting process, and observed velocity is just a mean value.

Thus, there is a need for a melting probes movement model creation that allow to take into account the full geometry of the probe and to observe data for characterizing the melting process in general.

III. Problem formulation

A mathematical model for estimation of the melting probe penetration velocity through ice should be based on reliable information on probes geometry and the surrounding ice, the thermodynamic properties of the materials, temperatures of the probe and surrounding ice, the temperature at which phase change occurs under these conditions.

Simulation of the melting process taking into account the phase changes as well as moving object is an extremely difficult task. To simplify this task it is possible to use ”solid water theory” proposed by Nikolaev [11] that allows considering both water and ice as a solid medium. To take into account melting probe movement it is proposed to do multi-step simulation for each task, as it is accepted for solving quasi-static problems [12].

The proposed method considers only thermal component to study the melting process.

IV. Mathematical model

Study the ice melting process, as well as taking into account the melting probe movement is a complex task, which setting up includes taking into account the properties of ice, the possible inclusions in the ice sheet, the energy spent on the phase transition, etc. For the presented problem several assumption that allow setting up a mathematical model of the melting process problem were accepted:

- a slot in the ice in accordance with probes shape with a given temperature on it is considered;
- it is assumed that the melting probes power propulsion system can maintain a constant temperature on probes surface;
- Ice and water are considered as solid, isotropic, homogeneous medium;
- the ice melting process is supposed to be a multi-step process: the temperature problem is solved with a modified geometry and boundary condition by quasi-static method.

Under these assumptions, mathematical formulation for each simulation step is a common heat equation, which is:

$$\rho c \frac{\partial T}{\partial t} = \lambda \sum_i \left[\frac{\partial^2 T}{\partial {x_i}^2} \right] \tag{1}$$

where, $T = T(x_i, t)$ temperature in the point x_i at moment t; $c = c(T)$ specific heat, (J/(kgK)); $\rho = \rho(T)$ density (kg/m3); $\lambda = \lambda(T)$ thermal conductivity (W/ (m K)).

Initial temperature distribution in the volume V is:

$$T(x_i, 0) = T_0(x_i) \tag{2}$$

Boundary conditions are as follows:

$$T|_L = T_{probe} \tag{3}$$

That on the boundary L is the temperature T_{probe} (Fig. 1).

A boundary position on which the temperature is kept constant is constantly changing from iteration to iteration. Its position is determined by the current melting probe position and the physical condition of the surrounding environment. An approximate solution of the problem is found by using the finite element method (FEM).

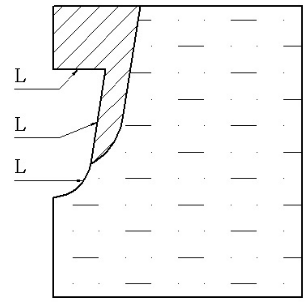


Fig. 1. Boundary conditions

The heat equation can be reduced to the following system of first order linear differential equations:

$$[C] = \frac{\partial \{T\}}{\partial \tau} + [K]\{T\} \tag{4}$$

where $[C] = \sum_{e=1}^{E} [C^{(e)}]$ - damping matrix;

$[C] = \sum_{e=1}^{E} [K^{(e)}]$ - heat conduction matrix;

$E-$number of finite elements.

In order to obtain the $\{T\}$ value at each time interval is necessary to solve a system of linear differential equations. To do this, it is necessary to replace the partial derivative with respect to time of its finite-difference analogue using the central difference scheme

$$\frac{\partial T}{\partial \tau} = \frac{T_j - T_i}{\tau_j - \tau_i} = \frac{T_j - T_i}{\Delta \tau} \tag{5}$$

where $T_i = T_{\tau = \tau_i}$

Considering the nodal values as a function of time it is possible to write

$$\frac{d\{T\}}{d\tau} = \left\{ \dot{T} \right\} = \frac{1}{\Delta \tau} (\{T\}_j - \{T\}_i) \tag{6}$$

Since $\{\dot{T}\}$ calculated at the midpoint of the time slot it is necessary to calculate$\{T\}$ at this point. This can be done as follows:

$$\{T\}^* = \frac{1}{2} (\{T\}_j + \{T\}_i) \tag{7}$$

that could be written as follow

$$\left([K] + \frac{2}{\Delta t} [C] \right) \{T\}_j =$$
$$= \left(\frac{2}{\Delta t} [C] - [K] \right) \{T\}_i - \left(\{F\}_j + \{F\}_i \right) \tag{8}$$

Considering the nodal values at time as known, the nodal values at time $\tau + \Delta\tau$ is possible to calculate. On the first step the initial temperature distribution $T(x, y, 0) = T_0(x, y)$ is used. Since some of the components $\{T\}$ are known $T_{L_i} = T^{const}$, then they should remain unchanged over time. The values of T are necessary to recover after each iteration.

(a) (b) (c)

Fig. 2. Experiment with: (a) spherical tip; (b) flat tip; (c) pyramidal tip.

V. Imitational simulation and verification techniques on known experiments

For verification of the proposed method a number of calculations were done based on known experiments. For this goal three experiments are considered: for melting probes with different body geometry. All experiments were carried out by the Austrian Academy of Sciences in Graz [8].

Two circular and one square cross-section melting probes were considered. First two melting probes had spherical and flat tips. The last one had pyramidal tip.

For all three melting probes a velocity analysis was performed by description of first parts of experiments that can be a subject of imitational simulation. This decision was accepted due to the fact that in all three cases experiments were stopped due to various reasons. The melting probe with spherical tip was stucked in ice; during experiment with the flat tip probe the triple water point was achieved; the problem with electronics was a reason for termination of experiment with the pyramidal tip.

A. Experiments description

Experiment with a spherical tip (Fig. 2(a), Table I) was held in Graz in a vacuum chamber. The probe was a cylinder with hemispherical brass tip. The motion was carried only by the melting.

Experiment with a flat tip (Fig. 2(b), Table I) was conducted in a vacuum chamber. In the experiment, smooth channel formed in a vicinity of the probe. Maximum penetration velocity was observed in the initial phase of the experiment.

Experiment with pyramidal tip (Fig. 2(c), Table I) was carried out in a vacuum. The melting probe with a room temperature was placed in ice. The maximal penetration velocity was achieved at the beginning of the experiment. After the tip totally got into ice the velocity started decrease slowly.

B. Numerical simulation

A numerical simulation of the melting probes penetration through ice was made for the case when thermal conductivity, heat capacity and density of ice were considered constant in a melting probes surrounding and for the case when the dependence of these parameters from the temperature was taken into account. For both cases ice thermal properties from [10] were used:

- Thermal conductivity, $[W/(m \times K)]$
  $\lambda = \dfrac{619.2}{T} + \dfrac{58646}{T^3} + 3.237 \times 10^{-3} \times T - 1.38210^{-5} \times T^2$
- Heat capacity, $[J/(kg \times K)]$
  $$C_p = x^3 \frac{c_1 + c_2 x^2 + c_3 x^6}{1 + c_4 x^2 + c_5 x^4 + c_6 x^8}$$
  $$x = \frac{T}{T_t}\ ,\ T_t = 273.16\ ,\ c_1 = 1.843 \times 10^5\ ,$$
  $$c_2 = 1.6357 \times 10^8\ ,\ c_3 = 3.5519 \times 10^9\ ,$$
  $$c_4 = 1.667 \times 10^2, c_5 = 6.465 \times 10^4\ ,\ c_6 = 1.6935 \times 10^6$$
- Density, $[kg/m^3]$
  $$\rho = 933.31 + 0.037978 \times T - 3.6274 \times 10^{-4} \times T^2$$

The model includes ice block geometry with the melting probe that is immersed in a half into ice. Materials properties, temperature boundary conditions were set up. Based on empirical criteria the penetration depth and elapsed time were calculated. Using temperature field corresponding to the calculated time water filled area in ice around the melting probe was determined. On the basis of these data geometry was reconstructed for the next iteration.

Numerical simulations were done using the finite element system MSC Patran / Nastran by MSC Software Corporation.

C. Simulation's results

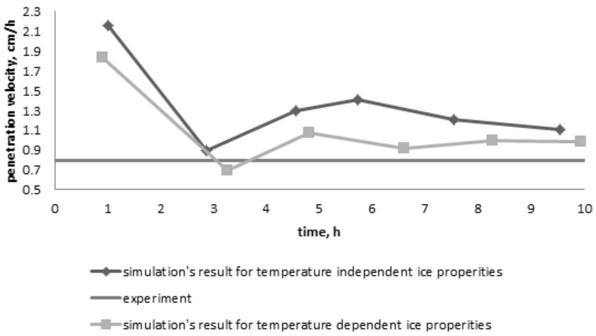
The observed velocities at different iterations of the melting process are presented in Fig. 3.

The deviation of mean penetration velocity is:

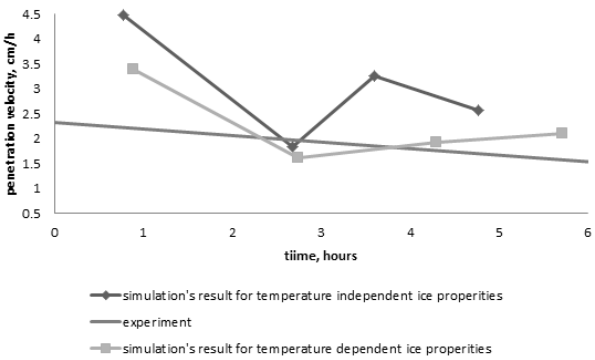
- for probe with spherical tip 31.10% (temperature dependent ice properties) and 5.95% (temperature dependent ice properties);
- for probe with flat tip 48.36% (temperature dependent ice properties) and 8.84% (temperature dependent ice properties);
- for probe with pyramidal tip -34.21% (temperature dependent ice properties) and -40.53% (temperature dependent ice properties);

TABLE I
Melting probes parameters

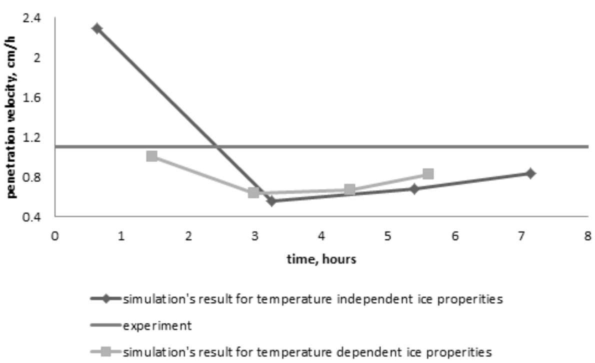
	Probe with spherical tip	Probe with flat tip	Probe with pyramidal tip
Probes length, cm	24.6	20.0	19.5
Probes radius, cm	3.175	3.175	5.66 (diagonal)
Power, W	80	60	35
Ice temperature, $^{\circ}C$	-30	-56.7	-53.2
Reached depth, cm	6.0	5.0	7.0
Mean velocity, cm/h	0.8	1.5	1.1



(a)



(b)



(c)

Fig. 3. Penetration velocity for a probe with: (a) spherical tip; (b) flat tip; (c) pyramidal tip.

VI. Discussion and Conclusions

The absolute deviation value of the melting probes penetration velocity does not exceed 50 %. Taking into account that exact ice properties are unknown those results could be considered as first approximations.

Using temperature dependent ice properties in the calculation leads to a reduction of melting probes estimated velocity that in the case of a cylindrical melting probe shape allows obtaining a more accurate estimation.

Results observed for probe with cylindrical body shape exceed experimental velocities. For the square cross-section probe observed velocity is less than experimental. In all cases, there is a trend of convergence of the data obtained from simulation, the experimental data. Such a result can be justified by the fact that the melting probe with a pyramidal tip had more complex equipment and a complete model describing its movement should take into account not only the temperature component.

A comparison of experimental results with the obtained numerical data by using the proposed method showed that this method allows estimating the melting probe velocity; to predict the overall dynamics of the melting process. Thus, knowing only the ice temperature in which the melting probe should operate as well as restrictions on its technical parameters, the proposed imitational simulation method allows optimizing the melting probes parameters to achieve maximal penetration velocity even before the melting probe creation. Thus, the proposed method can be used for preliminary analysis of various probes before the melting probe creation:

- Estimation of penetration velocity through ice under certain parameters (temperature of the probe, its geometry, etc.);
- Optimization of the melting probes parameters to increase the penetration velocity;
- Selection of the optimum temperature to achieve this velocity;
- Study of the changes that occurs in the surrounding ice.

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