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# STATE EFFICIENCY IN PUBLIC SECTOR PRODUCTION: COMBINING DEA AND MATHEMATICAL SIMULATION

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A common situation that state efficiency research has to deal with is total lack of information about actual efficiency parameters: the only data available for analysis is the amount of inputs (in most cases, monetary, corresponding to budget expenditure) and the amount of outputs (in most cases, quantifiable public goods). A non-parametric method such as DEA is well-suited for estimating DMU efficiency in such cases, although different input and output specifications might yield contradictory estimates. In this paper we provide an example of how a dynamic model of the state redistribution based on common assumptions about production technology allows to benchmark efficiency estimates. The model becomes a systematic generator of inputs and outputs to be estimated via DEA; model (pre-set) parameters are to be compared to DEA efficiency scores. We also propose a way to combine both "static" and "dynamic" approaches to measuring public sector efficiency by using aggregated DEA inputs.

Key words: State efficiency, public capital, dynamical formal model, data envelopment analysis

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## 1. Introduction

The development of methods aimed at estimating public sector performance is a thriving research field. Despite a few complications inherent to the connection between public sector and politics (see e.g. Arndt 2008), new methods appear every year, while existing ones are perfected and fine-tuned to specific tasks. This multitude of options not only brings the researcher face-to-face with the daunting task of choosing the proper methodology, but also derails academic discourse from testing the theories on actual empirical data towards methodological discussion that often boils down to purely mathematical arguments: which regression model to use, what functional form should the production technology take, what is the most feasible way to allocate weights to inputs and outputs etc.

In this context, assessing the quality of various efficiency estimates becomes a priority: while theoretical boundaries of a given method are important, a researcher should bear in mind that applied political science and economics should be used to further the understanding of actual political processes. However, benchmarking efficiency estimates faces the same problems as producing efficiency estimates: it is a difficult task without deeper understanding of the underlying process (in this case, the production technology used to create public goods). A few qualitative benchmarking techniques have been suggested (e.g. Hood et al. 2008), but in most of the literature the actual public good production process remains in the background.

Mathematical simulation is a tool that might help provide insights into efficiency estimation techniques, bridging the gap between purely methodological and empirical research. In this paper we provide an example of how a dynamic model of public resource redistribution based on common assumptions about production technology allows to benchmark efficiency estimates. We use a public good production model with a Cobb-Douglas technology at its core to provide a test dataset; we then use Data Envelopment Analysis to obtain efficiency estimates that can be compared to the multifactor productivity parameters[1] set in the model. The model allows us to address the issue of production time lags within the public sector by benchmarking DEA efficiency scores obtained for various input combinations.

A further methodological issue arises from the fact that most empirical data available for public sector research is a “snapshot” of the system’s state, whereas public good production is a constant process. While using system inputs and outputs as data for efficiency estimation seems logical, a more feasible estimation method would take into account the dynamic nature of a process

[1] Our methodology also provides an opportunity to compare empirical estimations to other pre-set efficiency parameters like efficiency losses in public capital accumulation (see section III)

in question. In this paper we propose a way to combine both "static" and "dynamic" approaches to measuring public sector efficiency by using aggregated DEA inputs.

The rest of the paper is organized as follows: in Section II we provide a brief literature review; Section III describes the system dynamics model used in our work; Section IV provides details of the numerical experiments conducted. Section V concludes.

## 2. Literature review

Non-parametric envelopment research, of which DEA methodology is an important part, has come a long way since the pioneering work of Farrell (1957). Seminal papers published by Charnes, Cooper and Rhodes (1978) and by Banker, Charnes and Cooper (1984), providing a formal linear-programming method of frontier estimation for constant and variable returns to scale, respectively, have given researchers the tools necessary to conduct multiple empirical studies concerning DMU[2] efficiency in both public and private sector. To name a few, DEA efficiency studies have been carried out for public health (Afonso and Aubyn 2006), public library systems (Hammond 2002), state universities (Agasisti 2009), municipalities (Balaguerr-Coll et al. 2012, Benito et al. 2014), bank sector (Fukuyama 2014) and many other fields.

Aside from empirical measurement, considerable effort has been put by researchers into perfecting the efficiency estimation methodology. Key problems raised by researchers include assessing the accuracy of DEA scores and estimating how observed DEA inefficiencies are affected by external factors. Various benchmarking techniques for DEA estimators have been used by e.g. Huang et al. (2012) and Cherchye et al. (2013).

A number of papers have been dedicated to the so-called two-stage DEA, where efficiency estimators obtained during the first stage of analysis are then regressed against external variables likely to contribute to DMU inefficiencies. Due to the truncated nature of DEA estimators, developing feasible models allowing for statistical inference proved challenging and was addressed, among others, by Simar et al. (Daraio and Simar 2005, Simar and Wilson 2007), who used a bootstrap regression to obtain feasible results. Attempts to account for statistical noise despite the deterministic nature of DEA were made as well (i.e., Gstach 1998), ultimately leading to the development of a hybrid efficiency estimation technique that builds on both non-parametric and stochastic frontier analysis (Kuosmanen and Johnson 2010, Kuosmanen and Kortelainen 2012).

Yet another direction of research is dedicated to using Malmquist productivity indices to separate purely technological inefficiencies from DEA estimators obtained over several time

[2] Decision-making unit.

periods (most notably Fare et al. 1997), allowing for an insight into DMU efficiency dynamics which then developed into metafrontier studies by Battese et al. (2002, 2004).

Most of the abovementioned studies uphold the basic DEA model assumption that there is no notable time lag for a DMU between receiving the input and producing the output: all inputs are converted into outputs within the same time period, none are carried over to the next time period and no past inputs influence the output production in the future. Several attempts to account for time lag have been made over the years, most notably in econometric research: Fallah-Fini and Triantis (2014) provide an extensive review and name the most common causes of time interdependence between different production periods presented in the literature and describe lagged variable models used by e.g. Giriliches (1967) and Chen and Dalen (2010) for economic studies. Another popular approach to tackling the time lag problem is to construct a complex multi-stage production model that uses DEA estimators at each stage, with some of the inputs carrying over from previous stages (a good example is provided for the banking sector in Fukuyama 2014). However, as Fallah-Fini and Triantis correctly state, “creating realistic models that map back to the actual production systems is an open research challenge” (Fallah-Fini and Triantis 2014). Few attempts have been made to account for time lag in public sector since it presents a challenging task of modeling the production process properly. A notable exception are Ozpeynirci and Koksalan (2007), who used Monte Carlo simulations to generate data and assess the quality of DEA estimators in the presence of time lags of 1 to 3 years and then proceeded to test their findings on a real data set to evaluate state research institutions. Still, the problem of public sector efficiency in the presence of time lags between investment and the provision of public goods remains largely unexplored.

In this paper we attempt to build on early efforts to estimating DEA frontier production functions made by Banker et al. (1984) by constructing a dynamic public good production model with a Cobb-Douglas production function at its core. In the sequel, the model becomes a systematic generator of inputs and outputs to be estimated via DEA; model (pre-set) parameters are to be compared to DEA efficiency scores. This strategy gives us a practical opportunity to test various techniques aimed at capturing the lag effects.

## 3. Model

The model core is a Cobb-Douglas production technology with aggregate social output[3] $Y(t)$. It is produced using two input factors: private capital $K(t)$ and public capital $G(t)$:

[3] It could be thought of as the sum of education rate, life expectancy, infant survival rate and other values characterizing the achieved level of social development.

$$Y(t) = AK^{\alpha}(t)G^{1-\alpha}(t) \quad (1)$$

Public capital includes material assets created and maintained by the government, on the one hand, and participating in social output production, on the other: schools, hospitals, sewer systems etc[4]. It is considered to be non-rivalrous and non-excludable – in other words, a "pure" public good. The latter means that it can be produced only at the expense of state or municipal budget funds.

The sum of elasticities with respect to all inputs is equal to unity, so that the function in general exhibits constant returns to scale (CRS). It means that a 1% increase in both factors gives an equal increase in output. A return to each input is diminishing.

$A$ is a time-invariant multifactor productivity parameter (or total factor productivity, TFP). Students of political economy, especially those being closer to political science than to economics, tend to understand it straightforwardly as a measure of technological development (see, e.g. Przeworski 2004). It is one possible interpretation, but the whole picture is more complicated. Basically, $A$ is a residual component: it is "responsible" for changes in output not explained by changes in inputs; that is why TFP is sometimes described as a "measure of our ignorance" (Hulten 2001: 9). Such an ambiguity makes various interpretations possible, and many of them are much more theoretically rich than just "level of technology" in a narrow R&D sense. Thus, some scholars view TFP at an institutional angle, – as an indicator of how well rules of the game regulate social interactions. Hall and Jones, for example, introduce a concept of social infrastructure, meaning "the institutions and government policies that determine the economic environment within which individuals accumulate skills, and firms accumulate capital and produce output"(Hall, Jones 1999: 84). Some of the others link TFP with economic openness (Keller, Yeaple 2003), or, even more generally, the system's flexibility and adoptability[5].

For our purposes and in our formal framework the most important point is that TFP is an *efficiency* parameter that reflects the connection (or even the coordination) between two basic inputs – private and public capital. One of the possibilities (but not the only one) is to consider it as a private accessibility to public infrastructural assets, i.e. (which in turn reflects) the ability of public capital to serve as a real public good. This position is obviously closer to institutional view on TFP than to technological one. Anyway, at this stage of analysis it should be enough to define TFP as a *system efficiency* parameter representing the "complementarity" of all input factors. In the sequel we call it *A-efficiency*.

Let us return to the model economy. Aggregate output $Y(t)$ is taxed at a fixed rate $\tau$, tax revenue forms the state budget $T(t)$:

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[4] The major developments in public capital studies have been made by Aschauer (1989, 2000), Arslanalp et al. (2010), Crowder and Himarios (1997), Dabla-Norris et al. (2011), Gupta et al. (2014), Hulten (1996), Pritchett (2000) and many others.

[5] See (Isaksson 2007) for a detailed review.

$$T(t) = \tau Y(t),\ \tau \in (0,\ 1] \tag{2}$$

The budget generates two monetary flows, directed towards public and private sectors. Coefficient $\mu$ defines the fraction of the budget to be spent for public capital investment, *I*(*t*):

$$I(t) = \mu T(t),\ \mu \in [0,\ 1] \tag{3}$$

We should stress here that state expenditures is the only source of public capital accumulation.

The rest of the budget $(1-\mu)$ returns to private sector. Thus, the dynamics of private capital $K(t)$ is determined by the tax burden and the public investment share. What private producers get is a composition of a part of the output left after taxation and a part of the budget left after reservation of money for public projects:

$$K(t+1) = (1-\tau)Y(t) + (1-\mu)T(t) \tag{4}$$

The dynamics of public capital is very different. It is governed by two processes: depreciation of capital assets overtime, on the one hand, and budget investment inflow, on the other:

$$G_{t+1} = G_t - \delta G_t + \gamma I_t \tag{5}$$

Each of the processes is characterized by its own efficiency parameter, namely depreciation rate $\delta$ and investment efficiency $\gamma$. Let us consider it in more detail.

In the literature (e.g. Arestoff and Hurlin 2006; Arslanalp et al. 2010) depreciation rates of capital assets are typically considered to be determined by physical or technological reasons. For example, traditional materials like cement deteriorate many times slower in comparison to IT-based assets. In our approach we emphasize another class of depreciation factors, namely the behavioral one (Pritchett 2000). Variance in depreciation rates is affected not only by the nature of assets in question, but also by the quality of operation (maintenance) and / or corruption practices. Poor quality of materials and labor used in the construction of public infrastructure, resulting in speedy deterioration, often has corruptive origins. The reduction in quality creates a difference between actualand declared costs of the project and makes bribery possible. Bad maintenance may also be a consequence of a bureaucrat's effort to receive corruption rent.

In this study the concrete mechanisms of efficiency losses are not our focus; much more important is the general interpretation of the depreciation rate as a behaviorally motivated efficiency parameter. For brevity we will call it $\delta$*-efficiency*.

The inclusion of investment efficiency parameter $\gamma \in [0,\ 1]$ into the model (5) requires less explanation, especially since this innovation has been acknowledged in the recent literature. It means that in the general case only some share of budget spending transforms into productive public capital (Pritchett 2000). Investment losses occur due to transaction costs, misallocation of recourses and – once again – corruption. Current empirical studies (Gupta et al. 2014) demonstrate that in developing countries the average gamma value may not exceed 0.5.

Hereafter efficiency related to a transformation of budget money into useful public assets is referred to as $\gamma$*-efficiency*. Equation (5) for public capital dynamics is called efficiency adjusted perpetual inventory equation.

Thus, the model takes the form:

$$Y(t) = AK^{\alpha}(t)G^{1-\alpha}(t)\,,\ \alpha \in (0,\ 1) \tag{6}$$

$$T(t) = \tau Y(t),\ \tau \in (0,\ 1] \tag{7}$$

$$I(t) = \mu T(t),\ \mu \in [0, 1] \tag{8}$$

$$K(t+1) = (1-\tau)Y(t) + (1-\mu)T(t) \tag{9}$$

$$G(t+1) = G_t - \delta G_t + \gamma I_t\ \gamma \in [0,\ 1], \tag{10}$$

where $A$, $\gamma$ and $\delta$ are efficiency parameters of various kinds, responsible for the general performance of the system. To simulate a set of different DMUs, we just have to perform a number of model runs with different values of efficiency parameters.

However, there are two other important variables affecting the system's economy in the model (6–10). They are policy settings $\tau$ and $\mu$, regulating tax rate and budget investment share correspondingly. A detailed description of their impact upon the dynamics of public and private capital stocks, along with the complete formal analysis of the system (6–10) could be found in (Akhremenko, Petrov 2014). Here we limit ourselves to a few key points.

First, all possible trajectories of economic development could be reduced to two basic scenarios. One is long-run growth: both private and public capital dynamics evolve into a stationary regime described by positive exponential function. Another is a long-run decline: both public and private capital (and, correspondingly, social output) converges asymptotically to zero. The choice between the former and the latter is determined by the policy vector $(\tau \quad \mu)$ in the following way. If the policy vector $(\tau \quad \mu)$ falls within the specific area on the policy plane (the grey one in Fig. 1), the long-run growth scenario is realized. Otherwise we face long-run decline (recession).

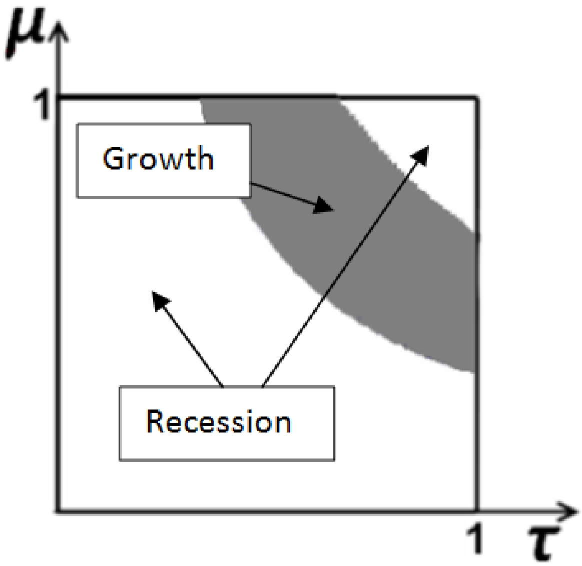


*Fig. 1.* The area of long-run growth policies

The size of the long-run growth area is determined by efficiency parameters $A$, $\gamma$ and $\delta$. A decrease in $A$ and/or $\gamma$, as well as an increase in $\delta$, squeezes the gamut of the "successful" policies, and vice versa (see Fig. 2).

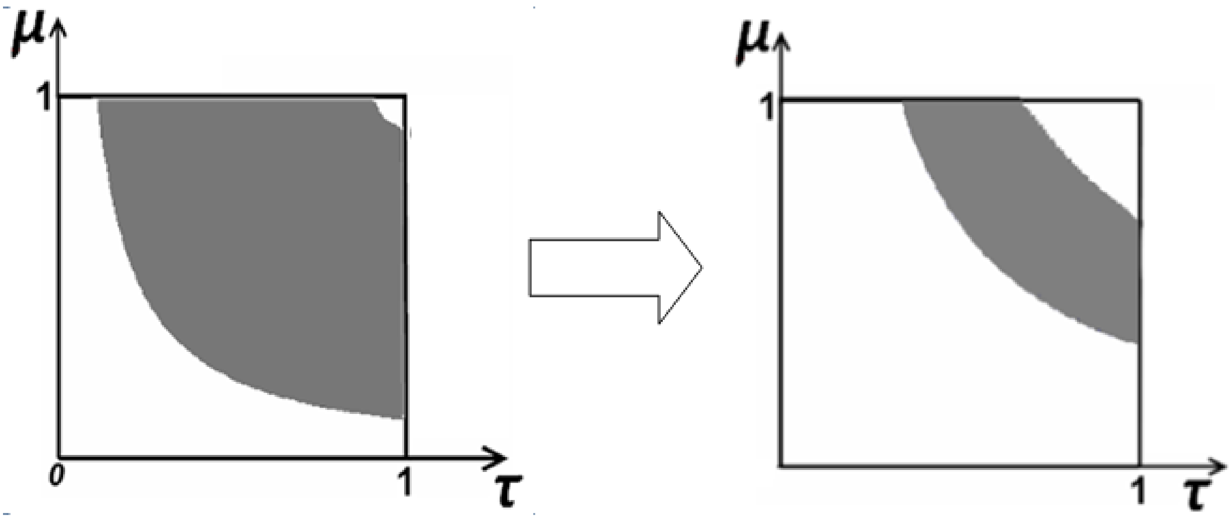


*Fig. 2.* A decrease in any type of efficiency leads to the reduction of the long-run growth policy area

The reasoning above leads to one important practical choice in the research strategy. One may consider DMUs in the model as fully autonomous, i.e. characterized by individual policies *and* individual rates of efficiency. Empirically, it corresponds either to independent states, which are able to implement their own policies "by definition", or to the regions within federated states with high levels of decentralization. Another strategy is to model DMUs as the regions of a unitary state (or a bogus federation) which do not have enough autonomy to differ strongly in their tax and redistribution policies. In the latter case we model the variance in efficiency, changing the values of $A$, $\gamma$ and $\delta$ from one DMU to another, keeping the policy vector constant.

In this article we hold to the "unitary" strategy in order to keep the dimensionality of parameter space more compact. Making the first step injoining the dynamic models and DEA, a simpler strategy seems to be a better one. Thus, all DEA estimations of relative DMU efficiency are made under the assumption that all units conduct the same redistribution policy. Policy parameters change only between experiments to check the robustness of the results obtained. It should be noticed, however, that nothing in our methodology restricts a more complicated experiment involving fully autonomous DMUs.

## 4. Computational Experiment

A common situation that efficiency research has to deal with is total lack of information about actual efficiency parameters ($A$, $\gamma$ and $\delta$): the only data available for analysis is the amount of inputs (in most cases, monetary, corresponding to budget expenditure) and the amount of outputs (in most cases, quantifiable public goods). A non-parametric method such as DEA is well-suited for estimating DMU efficiency in such cases, although different input and output specifications might yield contradictory estimates. In the experiments presented below we demonstrate how accounting for public good production time lags can influence the precision of DEA estimates. Since we use model data, we can directly compare DEA estimates to the "true" (controlled by the researcher) efficiency parameter ($A$) for a given DMU. The experiments with other efficiency parameters, namely $\gamma$ and $\delta$, give basically the same results; thus their description is skipped in this section.

We conduct several types of quantitative experiments with the model using various DEA estimates in comparison with "true" efficiency in order to assess how different time lags influence DEA performance. First, we use simultaneous inputs and outputs to demonstrate that DEA fails to provide correct efficiency estimators for DMUs within the model. Second, we use inputs with various time lags to show that, while maintaining correct efficiency ranks between DMUs, DEA estimates fail to capture proportions between true efficiencies. Third, we apply the same DEA model to inputs accumulated over several time periods and receive efficiency estimates that are much closer to "true" DMU efficiencies.

a. Simultaneous inputs and outputs

The first case to consider is the following: inputs are converted into outputs within the same time period (this is the basic, non-lagged DEA assumption). We arbitrarily assign efficiency parameters to 11 DMUs ($i$ = 1, 2,.., 11), i.e. the first DMU has been assigned $A$ = 1,5; $\delta$ =0,01; $\gamma$ = 0,95, the second one $A$ = 0,7; $\delta$ =0,2; $\gamma$ = 0,6 and so on.

Let us first assume all the DMUs are equal public good providers within a unitary state, therefore they have no means of changing the established tax and investment rates: $\tau_1 = \tau_2 = \ldots = \tau_{12}$; $\mu_1 = \mu_2 = \ldots = \mu_{12}$. For each DMU we calculate the values of budget investment $I_i(t)$, $(i = 1, 2, \ldots 11)$ at each point in time for 100 time periods ($t$ = 1, 2, …,100), then run DEA algorithms with various returns to scale to determine efficiency estimates. However, despite the differences in inner efficiency parameters, all DMUs at all points in time form a straight line within the two-dimensional input-output space, therefore producing DEA efficiency scores of 1 for all DMUs under all returns to scale settings (CRS, VRS, NIRS[6]). This is due to the fact that, under our model's assumptions, the output/input ratio $\frac{Y(t)}{I(t)}$ that determines a DMU's position within the output/input space remains constant and independent of any efficiency parameters. Investment is a percentage of the budget: $I(t) = \mu T(t)$; which, in turn, is a percentage of public capital: $T(t) = \tau Y(t)$. Therefore,

$$\frac{Y(t)}{I(t)} = \frac{Y(t)}{\mu \tau Y(t)} = \frac{1}{\mu \tau} \tag{11}$$

See Fig. 3 for an example at $t$ = 95; we use the logarithmic scale to compensate for enormous differences in input and output levels that develop by $t$ = 95 due to initial gaps in efficiency set for DMUs. Therefore, DEA scores calculated using simultaneous inputs and outputs do not correspond to actual efficiency parameters in our model.

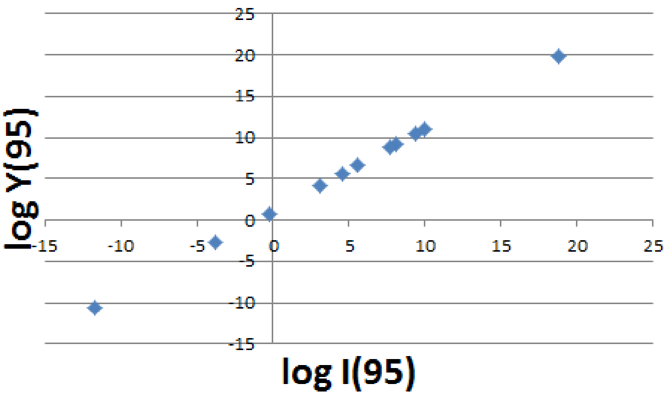


*Fig. 3.* DMUs within the two-dimensional "log investment – log public good production" space

Furthermore, using simultaneous inputs and outputs wouldn't provide a good setting for DEA even if DMU inefficiencies were attributed to differences in policy $(\tau, \mu)$: if we look once again at Fig. 1, economic growth is non-linearly, non-monotonically dependent on policy, which the ratio

[6] The acronyms stand for constant, variable and non-increasing returns to scale correspondingly.

$\frac{1}{\mu\tau}$ cannot fully describe. A further DEA experiment supports this conclusion: let us assume 11 DMU's have equal efficiency parameters (for instance, $A$ = 1, $\delta$ = 0.1, $\gamma$ = 1), but different policy parameters. Having previously analyzed the model, we can rank policies by their "successfulness" (the higher the number, the higher its chance to succeed) and compare the results with DEA efficiency ranks. The results are presented in Table 1. The "worst" policy ($\tau$ = 0.01, $\mu$ = 0.01) that leads to public goods being drastically underfinanced is ranked best in all DEA models. Moreover, all variable returns to scale models make no distinction between DMUs 1 to 8 despite the differences in their actual performance.

The experiments presented above illustrate that it is incorrect to view inputs and their conversion to outputs as a simultaneous process: in public capital production, investment always precedes actual results.

*Table 1*. "True" and estimated ranks of polity "successfulness"

| DMU | τ | μ | True "successfulness" rank | CRS | input – NIRS | output – NIRS | input – VRS | output – VRS |
|---|---|---|---|---|---|---|---|---|
| 1 | 0,01 | 0,01 | 2 | 11 | 4 | 4 | 4 | 4 |
| 2 | 0,1 | 0,1 | 3 | 10 | 4 | 4 | 4 | 4 |
| 3 | 0,2 | 0,2 | 4 | 9 | 4 | 4 | 4 | 4 |
| 4 | 0,3 | 0,3 | 5 | 8 | 4 | 4 | 4 | 4 |
| 5 | 0,4 | 0,4 | 6 | 7 | 4 | 4 | 4 | 4 |
| 6 | 0,5 | 0,5 | 8 | 6 | 4 | 4 | 4 | 4 |
| 7 | 0,6 | 0,6 | 10 | 5 | 4 | 4 | 4 | 4 |
| 8 | 0,7 | 0,7 | 11 | 4 | 4 | 4 | 4 | 4 |
| 9 | 0,8 | 0,8 | 9 | 3 | 3 | 3 | 3 | 3 |
| 10 | 0,9 | 0,9 | 7 | 2 | 2 | 2 | 2 | 2 |
| 11 | 0,99 | 0,99 | 1 | 1 | 1 | 1 | 1 | 1 |

b. Lagged inputs

In this section we consider the ratio $\frac{Y(t)}{I(t-l)}$, where $l$ is the time lag between investment and public capital production. The experiment is organized as follows. Let us assume 11 DMUs have different $A$-efficiency rates increasing with DMU number $i$: the first DMU ($i$ = 1) has $A$ = 0.5, the second one $A$ = 0.6, up to DMU #11 with $A$ = 1.5. The policy for all DMUs remains the same $(\tau = 0.6,\ \mu = 0.6)$. We then run DEA models for several values of $l$ using various returns to scale.

In this case, DEA estimates actually capture efficiency ranks for the DMUs regardless of the value of $l$ (see Table2).

*Table 2.* DEA efficiency estimates using lagged inputs

| DMU # | Efficiency ($A$) | Time lag ($l$) | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 10 | 30 |
| 1 | 0,5 | 0,739 | 0,546 | 0,404 | 0,298 | 0,220 | 0,049 | 0,000 |
| 2 | 0,6 | 0,758 | 0,574 | 0,435 | 0,330 | 0,250 | 0,062 | 0,000 |
| 3 | 0,7 | 0,779 | 0,606 | 0,472 | 0,368 | 0,286 | 0,082 | 0,001 |
| 4 | 0,8 | 0,802 | 0,643 | 0,515 | 0,413 | 0,331 | 0,110 | 0,002 |
| 5 | 0,9 | 0,827 | 0,683 | 0,565 | 0,467 | 0,386 | 0,149 | 0,004 |
| 6 | 1 | 0,853 | 0,727 | 0,620 | 0,529 | 0,451 | 0,204 | 0,010 |
| 7 | 1,1 | 0,880 | 0,775 | 0,683 | 0,601 | 0,529 | 0,280 | 0,025 |
| 8 | 1,2 | 0,909 | 0,826 | 0,751 | 0,683 | 0,621 | 0,385 | 0,063 |
| 9 | 1,3 | 0,939 | 0,881 | 0,827 | 0,776 | 0,729 | 0,531 | 0,159 |
| 10 | 1,4 | 0,969 | 0,939 | 0,910 | 0,882 | 0,854 | 0,730 | 0,401 |
| 11 | 1,5 | 1,000 | 1,000 | 1,000 | 1,000 | 1,000 | 1,000 | 1,000 |

However, $A$ and DEA estimates are scaled differently, so in order to compare the results we use the ratio of efficiency estimates between the most efficient DMU in the set (in this case, DMU #11, corresponding to $A$ = 1.5)and a given DMU. The comparison is presented in Table 3.

*Table 3.* Efficiency estimate ratios between a given DMU and the most efficient DMU

| DMU # | $A$ ratio | Time lag ($l$) | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 10 | 30 |
| 1 | 3,00 | 1,35 | 1,83 | 2,48 | 3,35 | 4,54 | 20,59 | 6447,11 |
| 2 | 2,50 | 1,32 | 1,74 | 2,30 | 3,03 | 4,00 | 16,04 | 3125,52 |
| 3 | 2,14 | 1,28 | 1,65 | 2,12 | 2,72 | 3,49 | 12,20 | 1412,40 |
| 4 | 1,88 | 1,25 | 1,56 | 1,94 | 2,42 | 3,02 | 9,11 | 605,70 |
| 5 | 1,67 | 1,21 | 1,46 | 1,77 | 2,14 | 2,59 | 6,72 | 250,31 |
| 6 | 1,50 | 1,17 | 1,37 | 1,61 | 1,89 | 2,22 | 4,91 | 100,96 |
| 7 | 1,36 | 1,14 | 1,29 | 1,47 | 1,66 | 1,89 | 3,57 | 40,15 |
| 8 | 1,25 | 1,10 | 1,21 | 1,33 | 1,46 | 1,61 | 2,59 | 15,87 |
| 9 | 1,15 | 1,07 | 1,14 | 1,21 | 1,29 | 1,37 | 1,88 | 6,28 |
| 10 | 1,07 | 1,03 | 1,07 | 1,10 | 1,13 | 1,17 | 1,37 | 2,49 |
| 11 | 1,00 | 1,00 | 1,00 | 1,00 | 1,00 | 1,00 | 1,00 | 1,00 |

As Table 3 shows, DEA comes closer to estimating actual efficiency ratios when used with $l$ ranging from 2 to 4 time periods, after which the difference between true efficiency and DEA scores grows exponentially. However, using lagged inputs is still an improvement on the original DEA estimation procedure.

c. Accumulated inputs

One further argument can be made against using both simultaneous and lagged inputs while estimating efficiency. In both macroeconomics and system dynamics, stocks and flows are key concepts used to differentiate between achieved results and the process used to achieve them. In this sense, public investment is a flow and public capital is a stock. Therefore, using both values in order to estimate efficiency would produce a qualitatively heterogeneous model. However, it is entirely possible to transform a flow into a stock by using accumulated values: in this experiment, instead of $I(t-l)$ we consider $\sum_{\tau=t-l}^{t-1} I_\tau$ as an input for DEA, while the output $Y(t)$remains the same. Note that we do not account for investment in the current time period when building our accumulated input: the assumption that current year's investment does not contribute to efficiency has been tested above in Section IVa. The results of the DEA estimation are presented in Table 4.

*Table 4.* DEA estimates using accumulated inputs

| DMU # | Efficiency ($A$) | Time lag ($l$) | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 10 | 30 |
| 1 | 0,5 | N/A | 0,641 | 0,558 | 0,489 | 0,431 | 0,246 | 0,059 |
| 2 | 0,6 | N/A | 0,665 | 0,587 | 0,521 | 0,465 | 0,284 | 0,090 |
| 3 | 0,7 | N/A | 0,693 | 0,620 | 0,558 | 0,505 | 0,330 | 0,136 |
| 4 | 0,8 | N/A | 0,724 | 0,657 | 0,600 | 0,551 | 0,385 | 0,200 |
| 5 | 0,9 | N/A | 0,757 | 0,698 | 0,646 | 0,601 | 0,449 | 0,282 |
| 6 | 1 | N/A | 0,793 | 0,742 | 0,696 | 0,657 | 0,522 | 0,380 |
| 7 | 1,1 | N/A | 0,831 | 0,788 | 0,750 | 0,717 | 0,603 | 0,490 |
| 8 | 1,2 | N/A | 0,871 | 0,838 | 0,808 | 0,782 | 0,692 | 0,610 |
| 9 | 1,3 | N/A | 0,913 | 0,890 | 0,869 | 0,851 | 0,788 | 0,735 |
| 10 | 1,4 | N/A | 0,956 | 0,944 | 0,933 | 0,924 | 0,891 | 0,866 |
| 11 | 1,5 | N/A | 1,000 | 1,000 | 1,000 | 1,000 | 1,000 | 1,000 |

Table 4 shows that the disproportions between estimated made with different time lags are significantly reduced when using accumulated inputs.

In order to test which efficiency estimates are more precise we use the method described above in Section IVb to transform raw values into ratios for both lagged and accumulated inputs, as well as true efficiency values. We then use sums of squared differences between true efficiency ratios and estimated efficiency ratios for all DMUs to determine the best approach.

*Table 5.* Sums of squared differences between true efficiency ratios and estimate ratios for different time lags

| | Time lag ($l$) | | | | | |
|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 10 | 30 |
| Lagged input | 2,351 | 0,362 | 1,575 | 9,586 | 690,332 | 53761582,801 |
| Accumulated input | 4,015 | 2,594 | 1,439 | 0,618 | 4,120 | 310,895 |

As Table 5 demonstrates, at low time lag values both methods produce comparable results. However, starting at $l = 5$ the accumulated input model yields considerably better results.

The experiments presented above have been carried out for other efficiency parameters ($\gamma$, $\delta$). The general conclusion about accumulated inputs yielding better efficiency estimates remains true in all cases.

## 5. Conclusions

In this paper we make two main contributions to the existing literature that deals with DEA estimation of public good provision. First, we propose the general methodology of testing the adequacy of DEA efficiency evaluations in the presence of time lag effects. The research algorithm can be briefly outlined as follows:

a) Construction of a dynamical formal model capable of simulating the process of public capital production and containing parameters determining the efficiency of this process. We use Cobb-Douglas production technology and corresponding mathematical tools such as (efficiency adjusted) perpetual inventory equations.

b) Setting the values of efficiency parameters in the way that each particular combination would represent a uniquely identified decision making unit. These values are the "true" efficiency attributes since they were explicitly pre-defined by the researcher.

c) A set of model runs (with different efficiency settings in each) provides data for the envelopment analysis – DMU's inputs and outputs at various periods of time. In our experimental environment it serves as a substitution for the empirical data. The basic distinction between experimental and "real life" data is that in the former case we know, as it was already stressed before, the true efficiency characters of DMUs under investigation.

d) DEA estimations of the experimentally generated data via different technical options (referred to input calculations in the study in question).

e) Comparison of DEA scores and pre-defined efficiency values. It enables to make a well-grounded choice in favor of one or another DEA estimation design.

This paper pictures the application of this methodology to the comparison of several techniques of taking into account time delay between budget investment and the resulting output in public sector. We argue that the best DEA estimates are made when monetary input is accumulated over time.

The dynamical model presented in the article is obviously not the only formal environment to test and fine-tune DEA estimations. One promising direction for the future improvement of our methodology is to try different model specifications based upon most recent advancements in public capital studies. In general, we hope that combining dynamical formal models and empirical research techniques in one analytical framework has a potential to become an expanding and fruitful research area in contemporary political economics.

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**Ахременко, А. С., Юрескул, Е. А.**
Оценивание государственной эффективности в общественном секторе: сочетание «оболочечного» анализа данных и математического моделирования [Электронный ресурс] : препринт WP14/2015/01 / А. Ахременко, Е. Юрескул ; Нац. исслед. ун-т «Высшая школа экономики». – Электрон. текст. дан. (500 Кб). – М. : Изд. дом Высшей школы экономики, 2015. – (Серия WP14 «Политическая теория и политический анализ»). – 20 с. (на англ. яз.)

В количественных исследованиях социальной эффективности стран и регионов в публичном секторе ученые обычно располагают лишь информацией о «входах» системы (как правило, бюджетных расходах) и ее «выходах» (в основном данных об объеме произведенных общественных благ). Хотя непараметрические методы оценивания, такие как «оболочечный» анализ данных (Data Envelopment Analysis, DEA), приспособлены к работе именно с такой информацией, различия в спецификации моделей ведут к противоречиям в результатах. В этой работе авторы демонстрируют, каким образом динамическая модель государственного перераспределения ресурсов и накопления общественного капитала позволяет «настроить» дизайн оболочечных техник анализа. Динамическая модель становится систематическим генератором входных и выходных данных для оценивания с помощью DEA; далее исследователь сопоставляет значения заложенных в модель (и в этом смысле истинных) параметров эффективности с полученными DEA-оценками. Именно это позволяет осмысленно выбирать оптимальную спецификацию оценочного инструментария. Основываясь на этом подходе, мы также предлагаем сочетание «статического» и «динамического» подходов в измерении эффективности публичного сектора с использованием накопленных входов в DEA-моделях.

Ключевые слова: эффективность государства, общественный капитал, динамическая математическая модель, оболочечный анализ данных

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