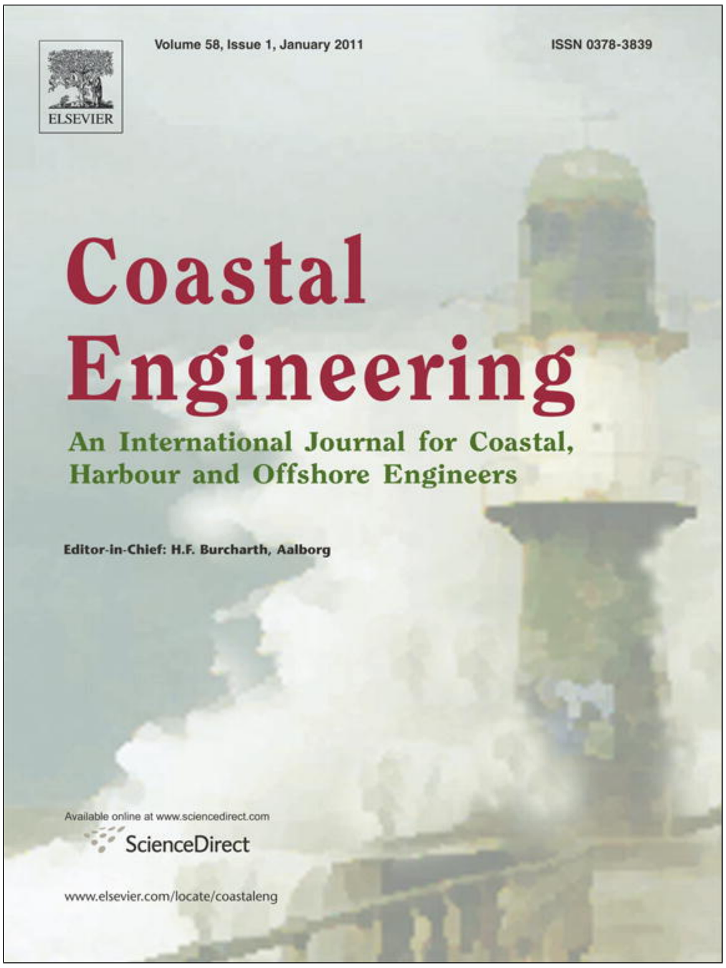
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Statistical characteristics of long waves nearshore

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ABSTRACT

The random long wave runup on a beach of constant slope is studied in the framework of the rigorous solutions of the nonlinear shallow water theory. These solutions are used for calculation of the statistical characteristics of the vertical displacement of the moving shoreline and its horizontal velocity. It is shown that probability characteristics of the runup heights and extreme values of the shoreline velocity coincide in the linear and nonlinear theory. If the incident wave is represented by a narrow-band Gaussian process, the runup height is described by a Rayleigh distribution. The significant runup height can also be found within the linear theory of long wave shoaling and runup. Wave nonlinearity nearshore does not affect the Gaussian probability distribution of the velocity of the moving shoreline. However the vertical displacement of the moving shoreline becomes non-Gaussian due to the wave nonlinearity. Its statistical moments are calculated analytically. It is shown that the mean water level increases (setup), the skewness is always positive and kurtosis is positive for weak amplitude waves and negative for strongly nonlinear waves. The probability of the wave breaking is also calculated and conditions of validity of the analytical theory are discussed. The spectral and statistical characteristics of the moving shoreline are studied in detail. It is shown that the probability of coastal floods grows with an increase in the nonlinearity. Randomness of the wave field nearshore leads to an increase in the wave spectrum width.

1. Introduction

Giant surface waves approaching the coast frequently cause extensive coastal flooding, destruction of coastal constructions and loss of lives. Such waves can be generated by various phenomena: strong storms and cyclones, underwater earthquakes, high-speed ferries, aerial and submarine landslides. The most recent examples of such events are the catastrophic tsunami in the Indian Ocean, which occurred on 26 December 2004 (Lay et al., 2005; Rabinovich and Thomson, 2007) and hurricane Katrina (28 August 2005) in the Atlantic Ocean (Kim et al., 2008). The rogue waves frequently reported in the World Ocean including the North Sea may induce the unusual and short-lived catastrophic flooding on the coast (Didenkulova et al., 2006a; Kharif et al., 2009). The huge storm in the Baltic Sea on 9 January 2005, which produced unexpected long waves in many areas of the Baltic Sea (Tõnisson et al., 2008), and the influence of unusually high surge created by long waves from high-speed ferries in Tallinn Bay (Soomere, 2007; Torsvik et al., 2009), should also be mentioned as examples of regional marine natural hazards connected with extensive runup of certain types of waves.

The prediction of possible flooding and properties of the water flow on the coast is an important practical task for physical oceanography and coastal engineering. That explains the multitude of empirical formulas describing runup characteristics available in the engineering literature (see, for instance, Le Mehaute et al., 1968; Stockdon et al., 2006). For the most part these formulas are specific for different geographic areas due to local characteristics of wave regimes (wind direction, coastal effects of wave refraction and diffraction). Numerical simulation of these processes should be carried out within fully-nonlinear Euler or Navier–Stokes equations including effects of wave breaking and dissipation in the near-bottom boundary layer (Liu et al., 1995; Kennedy et al., 2000; Choi et al., 2007, 2008; Fuhrman and Madsen, 2008). In the case of an irregular incoming wave field such simulations are costly and complicated and have only been undertaken with the use of empirical assumptions (Massel, 1989).

This situation improves in the case of long non-breaking waves, when the basic hydrodynamical model is based on nonlinear shallow water theory. Analytical rigorous solutions of the nonlinear shallow water system for wave runup are available for a beach of constant slope in the vicinity of the shoreline (Carrier and Greenspan, 1958). Using the Carrier–Greenspan approach various solutions of the nonlinear problem have been obtained and actively used in the study of runup of different incident wave shapes: sine wave (Carrier and Greenspan, 1958; Pelinovsky and Mazova, 1992; Massel and Pelinovsky, 2001; Madsen and

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Fuhrman, 2008), cnoidal wave (Synolakis et al., 1988), soliton (Pedersen and Gjevik, 1983; Synolakis, 1987; Kânoğlu, 2004), sine pulse (Mazova et al., 1991), Lorentz pulse (Pelinovsky and Mazova, 1992), Gaussian pulse (Carrier et al., 2003; Kânoğlu and Synolakis, 2006), *N*-waves (Tadepalli and Synolakis, 1994), nonlinear deformed waves (Didenkulova et al., 2006b), symmetrical bell-shaped waves (Didenkulova et al., 2008b) and "characterized tsunami waves" (Tinti and Tonini, 2005). Nevertheless, all these solutions are deterministic.

However real long wave records are irregular, and even tide-gauge records of tsunami waves, which have deterministic origin, show an irregular structure of the wave field, determined by processes of wave diffraction, refraction and scattering: see Fig. 1, which demonstrates the tide-gauge record of the 1883 Krakatau tsunami in South Georgia Island in the Atlantic (Pelinovsky et al., 2005). These effects are even more apparent for infragravity and edge waves and storm surges (Rabinovich, 1993). The numerous experimental studies of wave runup on a beach in natural conditions also demonstrate an irregular character (Bowen et al., 1968; Huntley et al., 1977; Guza and Thornton, 1980; Holman and Sallenger, 1985; Holman, 1986; Raubenneimer and Guza, 1996; Raubenneimer et al., 2001).

The first attempt to study the runup of irregular waves on a beach of constant slope in the context of nonlinear shallow water theory has been made in Didenkulova et al. (2008a), exploring the statement that extreme runup characteristics (maximum runup and rundown heights and maximum runup and rundown velocities of the shoreline) in linear and nonlinear theories coincide (Carrier and Greenspan, 1958; Synolakis, 1991; Didenkulova et al., 2007, 2008b). It follows from this statement that statistical distributions of extreme runup characteristics can be found from the linear theory. It has been shown in Didenkulova et al. (2008a) that for an incident wave field represented by narrow-band Gaussian process, extreme runup characteristics can be described by the Rayleigh distribution even in the nonlinear problem.

In this manuscript we study detailed statistical characteristics of the moving shoreline (vertical displacement and horizontal velocity). The main result here is that the statistical moments of the vertical displacement of the moving shoreline are modified by the wave nonlinearity in the nearshore zone. The paper is organized as follows. The rigorous solutions describing the nonlinear dynamics of the moving shoreline in the context of the nonlinear-shallow water theory are given in Section 2. It is shown that extreme values of the runup height in linear and nonlinear theories coincide, and therefore, the statistical distribution of runup heights does not change in consequence of wave nonlinearity. In the case of the narrow-band Gaussian processes the runup height distribution is described by the Rayleigh law. Statistical moments of runup characteristics (vertical displacement and velocity) are calculated in Section 3. The wave nonlinearity perturbs the moments of the displacement of the moving shoreline being away of a Gaussian distribution: the mean water level on the coast increases, the skewness grows monotonically with an increase in wave amplitude. The kurtosis is positive for weak amplitude waves and negative for large amplitude waves. In Section 4 statistical characteristics of the narrow-band dynamics of the moving shoreline are studied. The probability characteristics of the wave breaking are also calculated here and conditions for validity of the analytical theory are discussed. The probability of the narrow-band runup process resulting in coastal floods is studied in Section 5. It is shown that the probability grows with an increase in the nonlinearity. In particular, the probability of unusual and short-lived catastrophic flooding (freak wave appearance) is three times larger for nonlinear than for linear waves. Spectral characteristics of the runup of the monochromatic wave with random amplitudes and phases are analyzed in Section 6. It is shown that nonlinearity intensifies the generation of high harmonics and leads to the increase in the runup spectrum width. The main results are summarized in Section 7.

2. Theoretical model and distributions of extreme runup characteristics

We use the 1D nonlinear shallow water theory for describing runup of irregular waves on a beach. The basic equations are

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}[(h+\eta)u] = 0, \quad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial \eta}{\partial x} = 0 \tag{1}$$

where $\eta(x,t)$ is water displacement, $u(x,t)$ is depth-averaged velocity, $h(x)$ is unperturbed water depth, g is gravity acceleration, x is a coordinate, directed onshore, and t is time. The geometry of the problem is presented in Fig. 2. Using the hodograph transformation Carrier and Greenspan (1958) found rigorous analytical solutions to the problem of wave runup on a beach of constant slope $h(x) = -\alpha x$. These solutions describe the wave field nearshore, including the moving shoreline which determines the destructive power of the wave impact on the beach. For the moving shoreline, the Carrier and Greenspan method can be reduced to a two-step approach, described in detail in Pelinovsky and Mazova (1992) and Didenkulova et al. (2007, 2008b).

At the first step the linearized Eq. (1) is solved for given characteristics of the incident wave, and the wave field (vertical water displacement $R(t)$ and horizontal velocity $U(t)$) at the

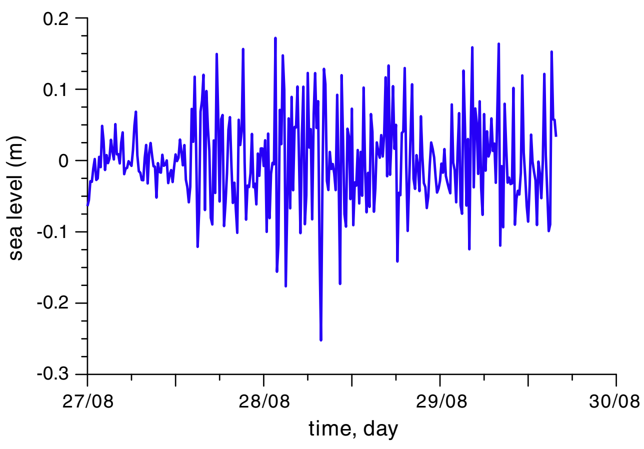

Fig. 1. The tide-gauge record of the 1883 Krakatau tsunami in South Georgia Island.

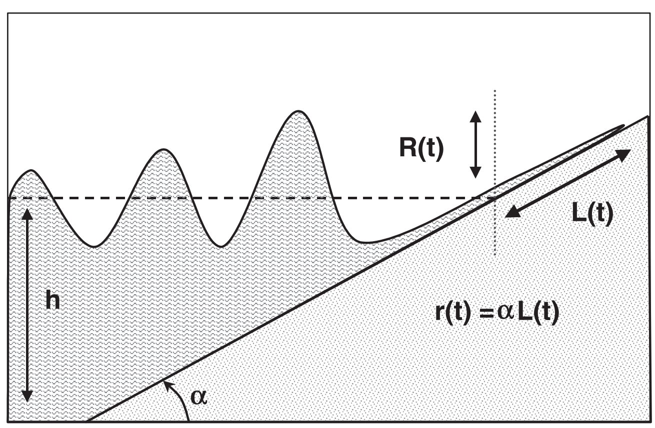

Fig. 2. The geometry of the problem.

unperturbed shoreline ($x = 0$) is defined. These characteristics are related by Didenkulova et al. (2007)

$$U = \frac{1}{\alpha}\frac{dR}{dt} \tag{2}$$

At the second step we find characteristics of the moving shoreline (vertical water displacement $r(t)$ and horizontal velocity $u(t)$) in the nonlinear problem with the use of a simple Riemann transformation of time (Pelinovsky and Mazova, 1992; Didenkulova et al., 2007, 2008b)

$$u(t) = U\left(t + \frac{u}{\alpha g}\right) \tag{3}$$

$$r(t) = R\left(t + \frac{u}{\alpha g}\right) - \frac{u^2}{2g} \tag{4}$$

It is evident that functions $r(t)$ and $u(t)$ satisfy the same kinematical relation (Eq. (2)). Solutions (3)–(4) are valid for both deterministic and random functions $R(t)$ and $U(t)$, which are determined by characteristics of the incident wave in the framework of the linear theory (it reduces to the variable-coefficient wave equation). The theory of linear differential equations with deterministic coefficients and random initial and boundary conditions is well developed. That is why we do not analyze the influence of linear shoaling and runup on the probability characteristics of the random water waves here assuming the statistical properties of functions $R(t)$ and $U(t)$ to be known.

A schematic example of linear and nonlinear motions of the shoreline for modulated wave packet is presented in Fig. 3 (the envelope frequency is 5 times less than the carrier frequency) in non-dimensional variables. The nonlinear velocity wave has the shape of a Riemann wave and differs strongly from the linear solution. The nonlinear vertical displacement $r(t)$ has a longer duration of runup and a shorter duration of the rundown stages than a linear one. Also the difference is larger for waves of large amplitude.

It follows from Eqs. (3) and (4) that extreme runup characteristics in linear ($R(t)$ and $U(t)$) and nonlinear ($r(t)$ and $u(t)$) theories coincide (Carrier and Greenspan, 1958; Synolakis, 1991; Didenkulova et al., 2007). This fact has been used by Didenkulova et al. (2008a) where the random long wave runup on a beach within linear shallow water theory has been studied. Since this problem is linear, statistical properties of the wave field (type of distribution) do not change. For example, if initial wave field is represented by a Gaussian stationary random process, then the functions $R(t)$ and $U(t)$ are also represented by Gaussian random processes, which should also be stationary from the physical point of view. When the incident wave field has a narrow-band spectrum, the height distributions of the incident wave and functions $R(t)$ and $U(t)$ are described by Rayleigh distributions (Massel, 1996). At the same time the processes of wave shoaling and runup affect the parameters of the distribution. For example, the significant runup height at the beach, R_s, for a narrow-band process is the same as for a deterministic sine wave (Carrier and Greenspan, 1958; Synolakis, 1991; Didenkulova et al., 2007)

$$R_s = \sqrt{\frac{4\pi\omega_0 L}{c_0}}A_s, \quad c_0 = \sqrt{gh_0}, \quad h_0 = L_0/\alpha \tag{5}$$

where A_s is a significant amplitude, defined by the standard deviation of the incident wave field $A_s \approx 2\sigma$, L_0 is a distance from the point where the incident wave field is defined (measured) to the shoreline, and ω_0 is a central frequency of the incident wave spectrum.

Thus, the Rayleigh probability distribution function $P(R)$ and probability density function $f(R)$ for runup heights R_{up} have a form

$$P\left(R_{up}\right) = \exp\left(-2\left[\frac{R_{up}}{R_s}\right]^2\right), \quad f\left(R_{up}\right) = 4\frac{R_{up}}{R_s^2}\exp\left(-2\left[\frac{R_{up}}{R_s}\right]^2\right) \tag{6}$$

It is obvious, that the same distribution (Eq. (6)) can be obtained for rundown heights R_{down} for the narrow-band process.

It follows from the coincidence of the runup heights in linear and nonlinear theories, that the distribution of the real (nonlinear) runup heights (extremes of $r(t)$) are described by the same Eqs. (5) and (6). This was the main result of Didenkulova et al. (2008a).

3. Statistical moments of the moving shoreline

As it was shown above distribution of the extreme runup values can be found from the linear theory. At the same time statistical characteristics of the moving shoreline (vertical displacement and horizontal velocity) require nonlinear theory.

Here we will calculate statistical moments of the velocity of the moving shoreline, assuming the process to be ergodic and using time averaging

$$\langle u^n \rangle = \frac{1}{T}\int_0^T u^n(t)dt = \frac{1}{T}\int_0^T U^n\left(t + \frac{u}{\alpha g}\right)dt, \quad n = 1, 2, 3... \tag{7}$$

where T is the time of the record (the length of the realization). It is convenient to introduce new variable

$$\tau = t + \frac{u(t)}{\alpha g} \tag{8}$$

In this case

$$d\tau = dt\left[1 + \frac{1}{\alpha g}\frac{du}{dt}\right] \tag{9}$$

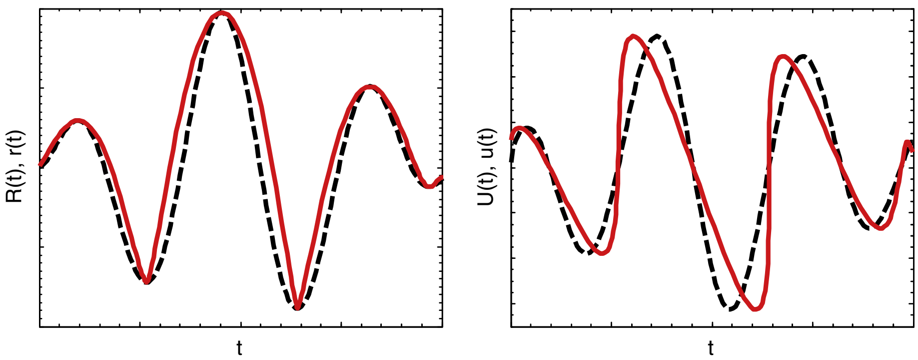

Fig. 3. Water displacement (left) and horizontal velocity (right) in nonlinear (solid lines) and linear (dashed lines) problems for modulated wave packet.

and du/dt in Eq. (9) can be found exactly from Eq. (3)

$$\frac{du}{dt} = \frac{dU/d\tau}{1-(g\alpha)^{-1}dU/d\tau} \quad (10)$$

Substituting Eq. (10) into Eq. (9) gives us the final expression for dt

$$dt = \left[1-(g\alpha)^{-1}dU/d\tau\right]d\tau \quad (11)$$

and integral (7) becomes explicit

$$<u^n> = \frac{1}{T}\int_0^T U^n(\tau)\left[1-\frac{1}{g\alpha}\frac{dU}{d\tau}\right]d\tau \quad (12)$$

In the case of the stationary random process the second term in Eq. (12) does not contribute into the integral, and

$$<u^n> = <U^n> \quad (13)$$

as a result, the nonlinearity of the wave field nearshore does not change the statistical moments of the velocity of the moving shoreline, and distribution of $u(t)$ is Gaussian if the incident wave, described by Gaussian distribution. It is important to point out that our results for velocity of the moving shoreline are similar to known results for Riemann waves in nonlinear acoustics, which are described by Eq. (3) in time and space (Gurbatov et al., 1991).

Similar transformations can be done for statistical moments of the vertical displacement of the moving shoreline $r(t)$. The first moment can be found from Eq. (4) with the use of Eq. (11)

$$<r> = \frac{1}{T}\int_0^T R\left(t+\frac{u}{g\alpha}\right)dt - \frac{<u^2>}{2g} = \frac{1}{T}\int_0^T R(\tau)d\tau - \frac{1}{g\alpha T}\int_0^T R(\tau)\frac{dU}{d\tau}d\tau - \frac{<U^2>}{2g} \quad (14)$$

Since $R(t)$ is defined with respect to the mean sea level and the mean sea level is constant in the linear theory, we can assume it to be zero. In this case the first term in Eq. (14) is zero. The second term in Eq. (14) can be integrated by parts, and the final expression for the mean displacement of the moving shoreline is

$$<r> = \frac{<U^2>}{2g} \quad (15)$$

It follows from Eq. (15) that the nonlinearity leads to an increase in the mean sea level at the coast (setup) for any distribution of the wave field. This effect has been noticed in experimental studies of long wave runup on a beach (Bowen et al., 1968; Huntley et al., 1977; Raubenneimer and Guza, 1996; Dean and Walton, 2009). Wave periods in these studies varied from 10 s to 20 min. The wave setup on a beach has been pointed out for all wave periods.

Using the same procedure the second moment of the displacement of the moving shoreline can be found. After several mathematical manipulations the expression it reduces to the following form

$$<r^2> = <R^2> - \frac{<U^4>}{12g^2} + \frac{<RU^2>}{g} \quad (16)$$

Since in the Gaussian case the functions $R(t)$ and $U(t)$ do not correlate, the third term in Eq. (16) splits into $<RU^2>=<R><U^2>=0$ and the second term can be expressed through the standard deviation $<U^4>=3<U^2>^2$. Eq. (16) transforms to

$$<r^2> = <R^2> - \frac{<U^2>^2}{4g^2} = \sigma_R^2 - <r>^2 \quad (17)$$

where σ_R is a standard deviation of water level oscillations at the shoreline in the linear theory. Eq. (17) demonstrates that the nonlinearity reduces the second moment of the vertical displacement of the moving shoreline. The second central moment (variance) of the vertical displacement of the moving shoreline is defined as

$$\sigma_r^2 = <r^2> - <r>^2 = \sigma_R^2 - 2<r>^2 \quad (18)$$

Omitting mathematical manipulations and again using assumptions of the Gaussian stationary process we can write expressions for skewness and kurtosis in the final form

$$s = \frac{<(r-<r>)^3>}{\sigma_r^3} = \frac{8<r>^3}{(\sigma_R^2-2<r>^2)^{3/2}} \quad (19)$$

$$k = \frac{<(r-<r>)^4>}{\sigma_r^4} - 3 = \frac{<r>^2\left(4\sigma_R^2-23<r>^2\right)}{(\sigma_R^2-2<r>^2)^2} \quad (20)$$

It follows from Eq. (19) that the skewness is always positive. The kurtosis can be positive or negative depending on the wave field parameters. It demonstrates the non-Gaussianity of the vertical displacement of the moving shoreline.

The statistical characteristics of the moving shoreline have been measured by Huntley et al. (1977) at two Canadian beaches. Beach 1 was located 50 km northeast of Halifax and had a slope of 0.07. Beach 2 was on the Gulf of Saint Lawrence coast of Cape Breton Island and a slope of 0.12. For both beaches they have found that the skewness is positive (equal to 0.2) and kurtosis is negative (equal to −0.6) demonstrating that the runup process is not Gaussian.

It follows from Eqs. (17)–(20) that the first four moments of $r(t)$ can be expressed through standard deviations σ_R and σ_U of the random Gaussian functions $R(t)$ and $U(t)$, which do not correlate with each other. In practice, the displacement of the moving shoreline $R(t)$ is usually measured and the shoreline velocity $U(t)$ is computed using Eq. (2). Thus, standard deviations σ_R and $\sigma_{R'}$ ($U=dR/dt$) are assumed to be known. As a result, Eqs. (17)–(20) can be expressed in a non-dimensional form with the use of single parameter

$$Br_\sigma = \frac{\sigma_{R'}^2}{g\alpha^2\sigma_R}, \quad \frac{<r>}{\sigma_R} = \frac{Br_\sigma}{2}, \quad \frac{\sigma_r}{\sigma_R} = \sqrt{1-\frac{Br_\sigma^2}{2}} \quad (21)$$

$$s = \frac{Br_\sigma^3}{\left[1-\frac{Br_\sigma^2}{2}\right]^{3/2}}, \quad k = \frac{Br_\sigma^2\left[1-\frac{23}{16}Br_\sigma^2\right]}{\left[1-\frac{Br_\sigma^2}{2}\right]^2} \quad (22)$$

The physical interpretation of the parameter Br_σ will be discussed in the next section.

It should be noted here that Eqs. (21) and (22) do not use any assumptions on spectral characteristics of the water waves.

4. Statistical characteristics of the narrow-band Gaussian process

Further simplifications can be achieved for narrow-band processes. In this case the rigorous expression for standard deviation of $dR(t)/dt$ through the correlation function $K(t)$ (Kendall and Stuart, 1969)

$$\left\langle\left(\frac{dR}{dt}\right)^2\right\rangle = <R^2> \left.\frac{d^2K}{d\tau^2}\right|_{\tau=0} \quad (23)$$

can be reduced to

$$\sigma_{R'} \approx \omega_0\sigma_R \quad (24)$$

As a result, all statistical moments can be expressed through the significant runup height ($R_s=2\,\sigma_R$) which is widely used in physical oceanography and ocean engineering [we have used it in the Rayleigh

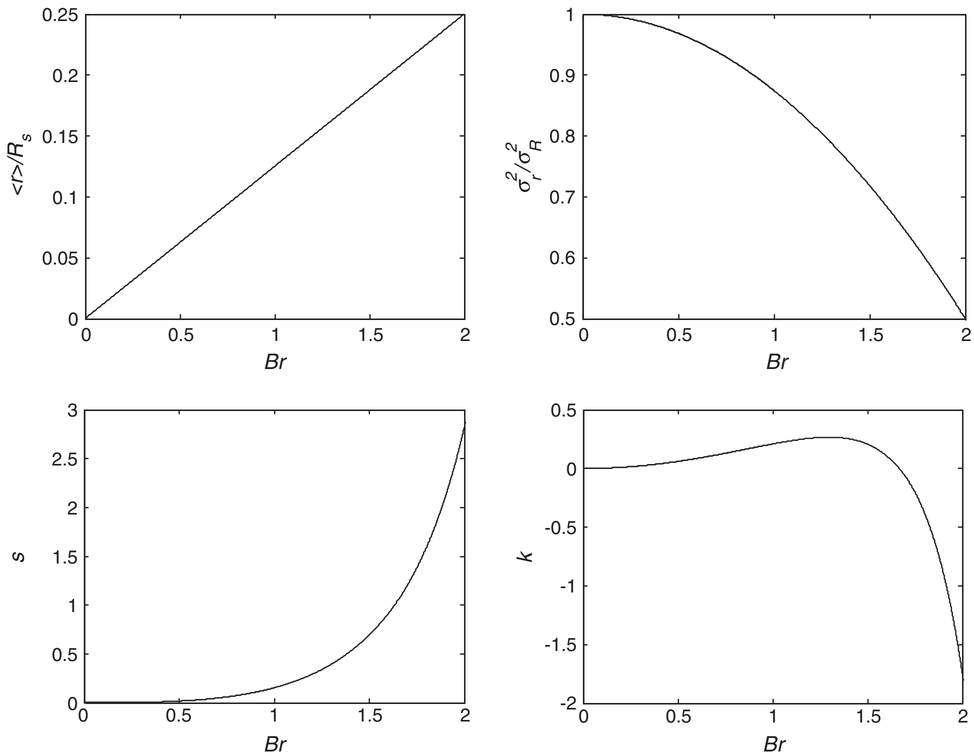

Fig. 4. Statistical moments of the runup characteristics of narrow-band Gaussian irregular waves.

distribution (Eq. (6))]. It is also convenient to re-express the parameter Br_σ

$$Br = \frac{\omega^2 R_s}{g\alpha^2} \tag{25}$$

which is close to the theoretical breaking parameter used in the theory of long wave runup of regular waves on a beach $Br_{CG} = \omega^2 R/g\alpha^2$, where ω and R is a frequency and runup height of an monochromatic wave (Carrier and Greenspan, 1958). It reflects the nonlinearity of waves: $Br_{CG} << 1$ corresponds to linear waves and increase in Br_{CG} corresponds to increase in the nonlinearity (wave amplitude) until $Br_{CG} = 1$, which is the case of the first breaking of a sine wave. However, for the random wave field the situation is principally different. We do not have information about individual waves and know only general statistics of the entire wave field. Moreover in this field there is always a nonzero probability to get a high-amplitude wave with $Br_{CG} > 1$. It is logical that if the probability of wave breaking is low or in other words if there are only a few such breaking waves we still can apply the shallow water model to describe the total wave field. So, here the breaking parameter Br [Eq. (25)] is a statistical parameter only and the main information on wave breaking is contained in the probability function of wave breaking that is discussed below.

It is evident that

$$Br = 2Br_\sigma \tag{26}$$

Final expressions for statistical moments of runup characteristics in the case of narrow-band process are:

$$<r> = \frac{\omega_0^2 R_s^2}{8g\alpha^2} = \frac{BrR_s}{8}, \quad \sigma_r^2 = \frac{R_s^2}{4}\left(1 - \frac{Br^2}{8}\right) \tag{27}$$

$$s = \left(\frac{Br}{2\sqrt{1 - Br^2/8}}\right)^3, \quad k = \frac{Br^2\left(1 - 23(Br/8)^2\right)}{4(1 - Br^2/8)^2} \tag{28}$$

Plots of the mean value, standard deviation, skewness and kurtosis Eqs. (27) and (28) of the displacement of the moving shoreline for the runup of narrow-band Gaussian irregular waves on a beach are presented in Fig. 4 as functions of Br.

As it has been pointed out in the previous section the mean value and skewness are always positive and increase with an increase in breaking parameter, while the standard deviation decreases. The kurtosis is positive for waves of weak amplitude and grows until $k \approx 0.27$ ($Br = 4\sqrt{2/19} \approx 1.3$), then it decreases to zero ($Br = 8/\sqrt{23} \approx 1.7$), starts to be negative and continue its rapid decrease down to minus infinity. These conclusions are in agreement with experimental results by Huntley et al. (1977), who obtained values 0.2 for skewness and -0.6 for kurtosis. As it follows from Fig. 4 strongly nonlinear waves (with large values of Br) should have positive skewness and negative kurtosis. The experimental value of kurtosis corresponds to $Br = 1.9$ in our theory, that is also close to the value of the breaking parameter $Br = 2.5$, estimated in (Huntley et al., 1977). It should be noted that the theoretical number $Br_{CG} = 1$ indicating the first wave breaking, which for a regular monochromatic wave occurs at the wave trough (Fig. 3), is not the best characteristics for characterizing wave breaking in natural conditions. Concentrating in a small area of the wave body usually this kind of wave breaking is not visible in the natural conditions. That is also why Huntley et al. (1977) considered these waves as non-breaking.

It should be mentioned, that the solutions obtained are valid only for non-breaking waves. Eqs. (3) and (4) for large values of amplitude give an ambiguous solution that physically means wave breaking. Since waves of large amplitude always exist in the irregular wave field, the obtained results cannot be valid for large values of Br. Some restrictions on Br follow from Eqs. (27) and (28), where the dispersion becomes zero for $Br = \sqrt{8} \approx 2.8$ and skewness and kurtosis tend to infinity, which does not have physical interpretation.

Necessary conditions for the validity of the shallow water theory for irregular wave field can be obtained estimating the probability of wave breaking. One of the conditions comes from Eq. (10), when the denominator in Eq. (10) is zero, in other words when the random process dU/dt reaches the level of $g\alpha$. In the case of a narrow-band Gaussian incident wave field, the function dU/dt also is Gaussian with a standard deviation $\omega_0\sigma_U = \omega_0^2\sigma_R/\alpha$. In this case the probability that $dU/dt > g\alpha$ (let us call it a notional probability of the wave breaking) is simply

$$P_{Br} = \frac{1}{\sqrt{2\pi}} \int_{1/Br}^{\infty} \exp\left(-\frac{\xi^2}{2}\right) d\xi = 1 - \Phi(1/Br) \quad (29)$$

where Φ is the standard Normal distribution function. The dependence of the notional probability of the wave breaking (Eq. (29)) on the parameter Br is shown in Fig. 5.

It follows from Fig. 5 that probability of wave breaking increases with an increase in parameter Br and tends to the limiting value 0.5, which means that only waves with positive values of dU/dt can break. It is important to mention that probability of wave breaking is small (below 5%) for $Br < 0.6$, therefore effects of wave breaking can be neglected in calculations of the statistical moments for these values of Br.

5. Probability of coastal floods for the narrow-band Gaussian process

As has been shown above, the velocity of the moving shoreline can still be described by the Gaussian distribution if the wave field offshore is also described by the normal distribution. At the same time the displacement of the moving shoreline differs from the Gaussian distribution. If the deviation is weak (small values of the parameter Br), its probability density function can be found by a perturbation technique based on the Gram–Charlier series of Type A (Kendall and Stuart, 1969; Massel, 1996). A Gram–Charlier expansion of the probability density function of displacement $f(r)$ is of the form

$$f(r) = \frac{1}{\sqrt{2\pi}\sigma_r} \exp\left(-\frac{(r - \langle r \rangle)^2}{2\sigma_r^2}\right) \left\{1 + \frac{s}{3!} H_3\left(\frac{r - \langle r \rangle}{\sigma_r}\right) + \frac{k}{4!} H_4\left(\frac{r - \langle r \rangle}{\sigma_r}\right) + \ldots\right\} \quad (30)$$

where $H(\rho)$ are the Hermite polynomials

$$H_3(\rho) = \rho^3 - 3\rho, \quad H_4(\rho) = \rho^4 - 6\rho^2 + 3 \quad (31)$$

This representation is used in oceanography for water waves, when the wave field is close to Gaussian (Massel, 1996). We note that there are several ways to represent perturbations of the Gaussian distribution, for example the Gram–Charlier series based on direct expansion in orthogonal polynomials or Edgeworth's series based on a similar expansion resting on a Fourier transformation (Kendall and Stuart, 1969). Both approaches lead to Eq. (30) if only two terms of the series expansion are used.

Introducing non-dimensional displacement ξ and its probability density function w

$$\xi = \frac{r}{R_s}, \quad w(\xi, Br) = f(r) R_s \quad (32)$$

and using Eqs. (27) and (28) the density w can be represented as

$$w(\xi, Br) = \sqrt{\frac{2}{\pi\Sigma}} \exp\left[-\frac{\Psi^2}{2\Sigma}\right] \left\{1 + \frac{s(Br)}{3!} H_3\left(\frac{\Psi}{\sqrt{\Sigma}}\right) + \frac{k(Br)}{4!} H_4\left(\frac{\Psi}{\sqrt{\Sigma}}\right) + \ldots\right\} \quad (33)$$

where

$$\Psi = 2\xi - \frac{Br}{4}, \quad \Sigma = 1 - \frac{Br^2}{8} \quad (34)$$

The probability density function w is shown in Fig. 6 for several values of the parameter Br. It is evident from Fig. 6 that w becomes asymmetric and shifts towards large values of shoreline displacement ξ with an increase in parameter Br. This again demonstrates that the process of wave runup prevails over the wave rundown even in the case when the incident wave is symmetrical with respect to the horizontal axis. The amplitude of the central peak also grows when the breaking parameter increases, suggesting that nonlinear waves will cause higher flooding at the coast.

Given an approximation to the probability density function, an approximation to the survivor function (the probability of exceeding a specific level) can be found

$$W(\xi, Br) = \int_{\xi}^{\infty} w(z, Br) dz \quad (35)$$

Results are shown on Fig. 7. It can be seen from Fig. 7 that the probability of appearance of high waves at the coast increases with an increase in breaking parameter. Some quantitative information on the exceedance probability of the displacement of the moving shoreline for different values of breaking parameter is presented in Table 1. There is a three-fold increase in the probability of exceedance of the level $r = 2\,R_s$, when Br reaches 0.6. We note here that the case $r > 2\,R_s$ corresponds to the case of unexpected short-lived high waves (so-called freak or rogue waves), which can appear both in the open sea and in the coastal zone (Kharif et al., 2009). In this connection an important result here is that the probability of freak wave appearance grows with an increase in nonlinearity, being for example three times larger for waves with $Br = 0.6$ than for linear waves.

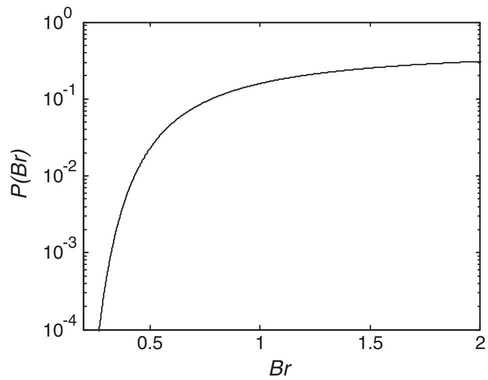

Fig. 5. Probability of wave breaking as a function of wave breaking parameter Br.

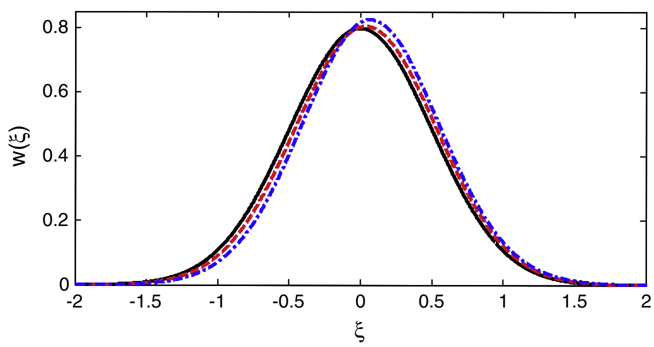

Fig. 6. Probability density function of the displacement of the moving shoreline for $Br = 0$ (solid line), $Br = 0.3$ (dashed line) and $Br = 0.6$ (dash-dotted line).

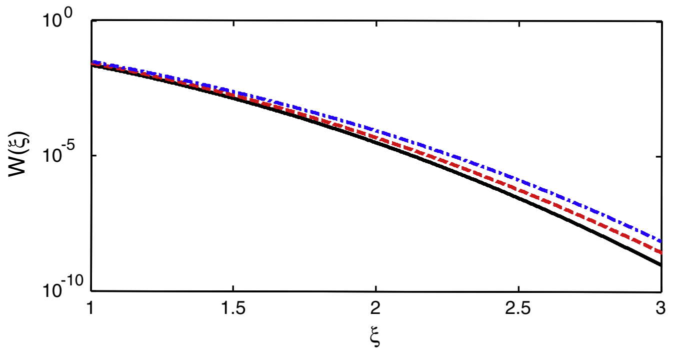

Fig. 7. Survivor function for the displacement of the moving shoreline for $Br=0$ (solid line), $Br=0.3$ (dashed line) and $Br=0.6$ (dash-dotted line).

An increase in the probability of the large values of $r(t)$ together with invariance of extreme values (runup and rundown heights) can be interpreted as an increase in the total time during which wave runups exceed a specific level. For example, if we talk about a one-day storm, the duration of wave exceedance of the level R_s is 30 min for linear waves and about 45 min for nonlinear waves with $Br=0.6$. At the same time exceedance of the level 2 R_s ("rogue wave" level) is 3 s only for linear waves and 8 s for nonlinear waves. The exceedance of the level 2 R_s corresponds to appearance of rogue waves, which usually happen suddenly and last for a very short time. Nearshore such waves are often observed as large amplitude water splashes. Their descriptions can be found in (Didenkulova et al., 2006a,b; Kharif et al., 2009).

A general conclusion is that the probability of coastal floods or wave overtopping grows with the nonlinearity.

6. Spectral analysis of runup characteristics

If the wave approaching the coast is a monochromatic wave with random amplitudes and phases, functions $R(t)$ and $U(t)$ can be represented in the form

$$R(t) = R\sin\left(\omega_0 t + \phi - \frac{\pi}{2}\right), U(t) = V\sin(\omega_0 t + \phi) \quad (36)$$

where amplitudes R, $V=\omega_0 R/\alpha$ and phase ϕ are random. For the Gaussian process, the phase ϕ is distributed uniformly in the range $[0, 2\pi]$

$$W(\phi) = \frac{1}{2\pi} \quad (37)$$

and amplitudes R and V have Rayleigh distributions (Eq. (6)).

The spectral representation of Eq. (3) is extensively used in nonlinear acoustics (Gurbatov et al., 1991) and also for the long sea surface waves (Zahibo et al., 2008); $u(t)$ is represented as the Bessel–Fubini series

$$u(t) = \sum V_n \sin[n\omega_0 t + \phi_n], \quad \text{where} \quad (38)$$

$$Vn\frac{2g\alpha}{\omega_0 n}J_n\left(\frac{\omega_0 nV}{g\alpha}\right) = \frac{2\omega_0 R_s}{n\alpha Br}Jn\left(nBr\frac{R}{R_s}\right)$$

Table 1
Exceedance probability for the displacement of the moving shoreline.

	$Br=0$	$Br=0.3$	$Br=0.6$
$r=R_s$	$2.28\cdot10^{-2}$	$2.66\cdot10^{-2}$	$3.05\cdot10^{-2}$
$r=2\,R_s$	$3.17\cdot10^{-5}$	$4.87\cdot10^{-5}$	$8.73\cdot10^{-5}$
$r=3\,R_s$	$9.87\cdot10^{-10}$	$2.86\cdot10^{-9}$	$7.39\cdot10^{-9}$

J_n being a Bessel function of the n-th order. The vertical displacement of the moving shoreline can be found by integration of Eq. (38):

$$r(t) = R_0 + \sum R_n \sin\left[n\omega_0 t + \phi_n - \frac{\pi}{2}\right], \quad R_n = \frac{2R_s}{n^2 Br}J_n\left(nBr\frac{R}{R_s}\right) \quad (39)$$

In the series (39) again the phase ϕ is distributed uniformly and the amplitude of the linear displacement R is distributed by the Rayleigh law (Eq. (6)). The zero harmonic R_0 in Eq. (39) represents the mean water level and can be calculated from Eq. (15):

$$R_0 = \frac{V^2}{4g} = \frac{\omega_0^2 R^2}{4g\alpha^2} = \frac{BrR^2}{4R_s} \quad (40)$$

If the breaking parameter Br is small, the spectral amplitudes R_n can be expressed by the asymptotic expression

$$\frac{R_n}{R_{n0}} = \left(\frac{R}{R_s}\right)^n \quad (41)$$

where

$$R_{n0} = \frac{R_s}{nn!}\left(\frac{nBr}{2}\right)^{n-1} \quad (42)$$

Using asymptotic Eqs. (41) and (42), we can find the approximated distributional properties and moments of the spectral amplitudes analytically. Since from Eq. (39) the spectral amplitude R_n is a function of R, its probability density function and distribution function follow from those of R by

$$f(R_n) = f(R)\frac{dR}{dR_n}, \quad P(R_n) = P(R) \quad (43)$$

where in the right-side of Eq. (43) R is expressed in terms of R_n with the use of Eq. (39). As has been indicated above, $P(R)$ and $f(R)$ are the Rayleigh distribution function and probability density function respectively [Eq. (6)].

In the limit of weakly nonlinear waves (Eq. (41)) the probability density function (Eq. (43)) simplifies to

$$f(R_n) = \frac{4}{nR_{n0}}\left[\frac{R_n}{R_{n0}}\right]^{\frac{2-n}{n}} \exp\left(-2\left[\frac{R_n}{R_{n0}}\right]^{2/n}\right) \quad (44)$$

If $n=1$ Eq. (44) represents a classical Rayleigh distribution; for the second harmonic ($n=2$) it is the exponential distribution. If $n>2$ the probability density function decreases monotonically with the wave amplitude and has a singularity in $R_n=0$. This asymptotic analysis is illustrated in Fig. 8 for $Br=0.1$ for $n=1, \ldots, 4$. Here R_{n0} is determined by

$$R_{n0} = \frac{2R_s}{n^2 Br}J_n(nBr) \quad (45)$$

The same features of the probability density function can be also observed for all values of the parameter $Br\leq0.6$.

A similar analysis can be carried out for the distribution function of R_n. In the limit of waves of weak nonlinearity it is described by

$$P(R_n) = \exp\left[-2\left(\frac{R_n}{R_{n0}}\right)^{2/n}\right] \quad (46)$$

The distribution function decreases with the spectral amplitude slower for higher numbers of harmonics (Fig. 9, left). This is also true for waves of moderate amplitude (Fig. 9, right), when Eqs. (39) and (45) are used. With an increase in the breaking parameter the

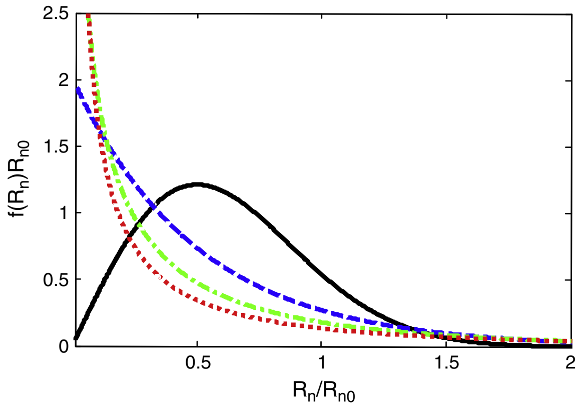

Fig. 8. Probability density function of overtone amplitudes of the displacement of the moving shoreline for $Br=0.1$, $n=1$ (solid line), $n=2$ (dashed line), $n=3$ (dash-dotted line), $n=4$ (dotted line).

amplitude of the first harmonics slightly differs from the Rayleigh distribution. It can be seen in Fig. 9 (right) for waves with $Br=0.5$. Thus, we may conclude that the probability of appearance of spectral harmonics decay with amplitude slower for higher harmonics.

The probability distribution of the zero harmonics R_0 coincides with the distribution of the second harmonics R_2 in the limiting case of weakly nonlinear waves (exponential distribution) and is not shown in Figs. 8 and 9.

The randomness of the wave field nearshore influences the mean values of the spectral amplitudes. In the case of weekly nonlinear waves they can be calculated explicitly from Eq. (41)

$$\left\langle \frac{R_n}{R_{n0}} \right\rangle = 4\int_0^\infty \zeta^{n+1}\exp\left(-2\zeta^2\right)d\zeta = \begin{cases} \dfrac{m!}{2^m} & n=2m \\ \sqrt{\dfrac{\pi}{2}}\dfrac{(m+1)!!}{2^{3m}} & n=2m+1 \end{cases} \tag{47}$$

The mean values of spectral amplitudes increase with n. It follows that on average nonlinearity increases spectral amplitudes and leads to an increase in the wave spectrum width.

The same effects should also take place for narrow-band (or more precisely, for any one-peak spectrum) Gaussian processes: the nonlinearity leads to the generation of high and low-frequency components and, therefore, increases the spectrum width. For example, spectral broadening of the Riemann waves has been studied in nonlinear acoustics (Pelinovskii, 1976).

7. Conclusion

The runup of random long non-breaking waves on a beach of constant slope is studied in the context of nonlinear shallow water theory. The exact analytical solutions obtained with the use of the hodograph transformation allow study of the processes of shoaling and runup of random (irregular) waves assuming that the incident wave field is represented by a Gaussian stationary process.

Attention is concentrated on the nonlinear dynamics of the moving shoreline. It is shown that the probability characteristics of the runup heights and extreme values of the shoreline velocity coincide in the linear and nonlinear theory, and therefore nonlinearity does not influence the amplitude characteristics of the runup process. If the incident wave field is represented by a narrow-band Gaussian process, the runup height (amplitude) is described by a Rayleigh distribution. Thus the significant runup height can also be found within the linear theory of long wave shoaling in the coastal zone.

The moments of the runup distribution are calculated analytically. Wave nonlinearity nearshore does not influence the probability distribution of the horizontal velocity of the moving shoreline and its moments, and it can be described by a Gaussian distribution. However the vertical displacement of the moving shoreline becomes non-Gaussian. In the case of a narrow-band process its statistical moments are studied in relation to the breaking parameter Br. It is shown that the mean water level increases with an increase in Br (setup), while the standard deviation decreases. The skewness is always positive and grows with Br. The kurtosis is positive for waves of weak amplitude and grows until $k\approx0.27$ ($Br\approx1.3$), then it decreases to zero ($Br\approx1.7$), starts to be negative. These conclusions are in agreement with experimental results by Huntley et al. (1977), who obtained values 0.2 for skewness and -0.6 for kurtosis. The experimental value of kurtosis corresponds to $Br=1.9$ in our theory, which is also close to the value of the breaking parameter $Br=2.5$ estimated in Huntley et al. (1977).

The probability of wave breaking as a function of the breaking parameter Br is also calculated. It does not exceed 5% for $Br<0.6$.

The probability density function of the vertical displacement of the moving shoreline is found with the use of the Gram-Charlier series. It is shown that the probability of coastal floods grows with an increase in the nonlinearity. An important result here is that the exceedance of the level $r=2\ R_s$, which corresponds to the case of freak or rogue waves (Kharif et al., 2009), increases by a factor of 3 in comparison to the linear theory, when Br reaches 0.6.

An increase in the probability of large values of $r(t)$ together with an insensitivity of extreme values (runup and rundown heights) to nonlinearity can be interpreted as an increase in the duration of wave runup stage. For example, if we talk about one-day storm, the duration of wave exceedance of the level R_s is 30 min for linear waves and about 45 min for nonlinear waves with $Br=0.6$. At the same time

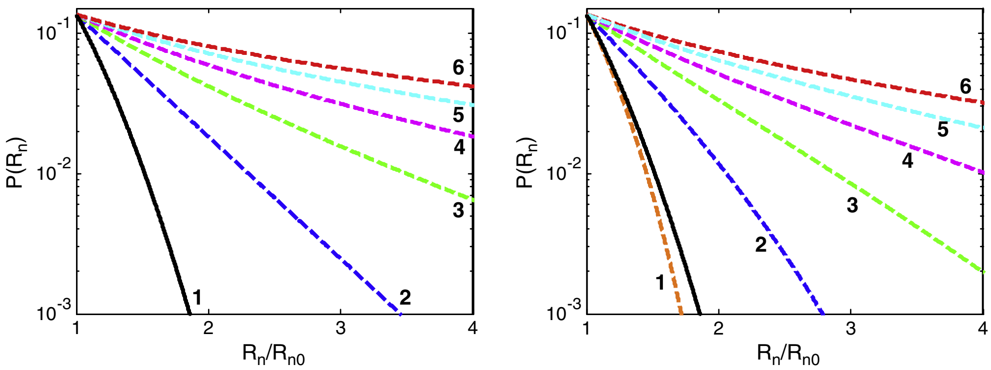

Fig. 9. Probability distribution of overtone amplitudes ($n=1$–6) of the displacement of the moving shoreline for $Br=0.01$ (left) and $Br=0.5$ (right), solid line corresponds to the Rayleigh distribution.

exceedance of the level 2 R_s ("rogue wave" level) is 3 s only for linear waves and 8 s for nonlinear waves. The exceedance of the level 2 R_s corresponds to appearance of rogue waves, which usually happen suddenly and last for a very short time. Nearshore such waves are often observed as large amplitude water splashes. Their descriptions can be found in Didenkulova et al. (2006a,b) and Kharif et al. (2009).

The spectral characteristics of the moving shoreline when the approaching wave is a monochromatic wave with random amplitudes and phases are also studied. In particular, the probabilities of spectral amplitudes are calculated. In the limiting case of weakly nonlinear waves the first harmonic is described by the Rayleigh distribution, the second harmonic by an exponential distribution, and higher harmonics by probability density functions with a singularity at $R=0$. It is shown that the distribution functions decrease with the spectral amplitude slower for higher harmonics. Randomness of the wave field nearshore leads to an increase in the wave spectrum width.

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