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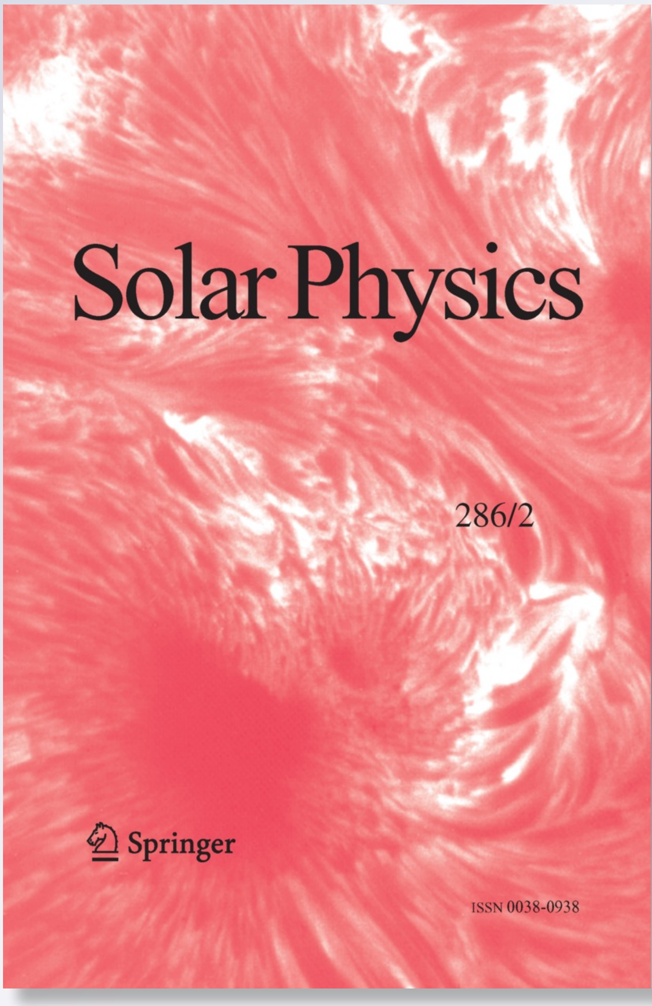

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Abstract Propagating kink waves are ubiquitously observed in solar magnetic wave guides. We consider the possibility that these waves propagate without reflection although there is some inhomogeneity. We briefly describe the general theory of non-reflective, one-dimensional wave propagation in inhomogeneous media. This theory is then applied to kink-wave propagation in coronal loops. We consider a coronal loop of half-circle shape embedded in an isothermal atmosphere, and assume that the plasma temperature is the same inside and outside the loop. We show that non-reflective kink-wave propagation is possible for a particular dependence of the loop radius on the distance along the loop. A viable assumption that the loop radius increases from the loop footpoint to the apex imposes a lower limit on the loop expansion factor, which is the ratio of the loop radii at the apex and footpoints. This lower limit increases with the loop height; however, even for a loop that is twice as high as the atmospheric scale height, it is small enough to satisfy observational constraints. Hence, we conclude that non-reflective propagation of kink waves is possible in a fairly realistic model of coronal loops.

Keywords Corona · Coronal magnetic loops · Waves · Wave reflection

1. Introduction

Standing transverse oscillations of coronal magnetic loops were one of the first wave phenomena observed in the solar corona with the new generation of spacecraft (Aschwanden

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et al., 1999; Nakariakov *et al.*, 1999). These oscillations were interpreted as fast kink oscillations of magnetic tubes (for a review see Ruderman and Erdélyi, 2009; Terradas, 2009). Recently, observations of transverse perturbations propagating along the magnetic field were reported. These propagating perturbations were detected with the *Solar Optical Telescope* (SOT) onboard *Hinode* in prominence fibrils (Okamoto *et al.*, 2007) and chromospheric spicules (De Pontieu *et al.*, 2007; He *et al.* 2009a, 2009b), and with the *X-ray Telescope* (XRT) onboard *Hinode* in soft X-ray jets (Cirtain *et al.*, 2007). Similar perturbations were observed in coronal loops by the *Coronal Multi-Channel Polarimeter* (COMP: Tomczyk *et al.*, 2007).

An important property of the observed propagating transverse perturbations is that they mainly propagate upwards, with the amplitude of the upward-propagating perturbations much larger than those of the downward-propagating perturbations. In the application to open wave guides, such as spicules, this property is considered as evidence that perturbations are generated in the lower parts of the solar atmosphere and then propagate in the upper part of the chromosphere and corona. For closed wave guides, such as coronal magnetic loops, it is considered as evidence that there is sufficiently strong damping, so that there is no wave reflected from the end of the wave guide opposite to the one where the propagating wave is driven.

Both of these statements describe definitely necessary conditions for the absence of downward-propagating waves. However, they are not sufficient. When the wavelength of a wave propagating in an inhomogeneous wave guide is of the order of the characteristic length of inhomogeneity, as is the case in all of the observations mentioned above, then, in general, there is sufficiently strong wave reflection from the inhomogeneity. However, there are some exceptions. For particular forms of the propagation-speed variation along the wave guide, waves can propagate without reflection even in strongly inhomogeneous wave guides. Such non-reflective wave propagation along inhomogeneous wave guides has been studied in applications to plasma physics (Ginzburg, 1970), oceanography (Brekhovskih, 1980; Vlasenko, 1987; Vlasenko *et al.*, 2003; Didenkulova, Pelinovsky, and Soomere, 2008; Grimshaw, Pelinovsky, and Talipova, 2010), acoustics (Ibragimov and Rudenko, 2004), and atmospheric science (Petrukhin, Pelinovsky, and Batsyna, 2011). Recently, the theory of non-reflective wave propagation has been applied to solar physics. Petrukhin, Pelinovsky, and Talipova (2012) studied non-reflective vertical propagation of acoustic waves in the solar atmosphere, while Cally (2012) investigated non-reflective propagation of Alfvén waves.

The aim of this article is to extend the theory of non-reflective wave propagation to fast kink oscillations in magnetic tubes that are inhomogeneous in the longitudinal direction. The article is organised as follows: In the next section we formulate the problem and write down the governing equations. In Section 3 we briefly describe the general theory of one-dimensional non-reflective wave propagation in inhomogeneous media. In Section 4 we apply the general theory to propagating kink waves in coronal loops. Section 5 contains the summary of the results obtained and our conclusions.

2. Problem Formulation

We consider the propagation of fast kink waves along a straight thin magnetic tube in the cold-plasma approximation. It is assumed that the plasma density varies along the tube, but does not vary in the radial direction. Hence, in cylindrical coordinates $[r, \varphi, z]$ with the z-axis coinciding with the tube axis, the density is given by

$$\rho = \begin{cases} \rho_{\mathrm{i}}(z), & r < R(z), \\ \rho_{\mathrm{e}}(z), & r > R(z). \end{cases} \tag{1}$$

Here $R(z)$ is the tube radius, which can also vary along the tube. In the thin-tube approximation, plane polarised kink waves are described by

$$\frac{\partial^2(\eta/R)}{\partial t^2} - c_{\mathrm{k}}^2 \frac{\partial^2(\eta/R)}{\partial z^2} = 0, \tag{2}$$

where η is the tube-axis displacement,

$$c_{\mathrm{k}}^2 = \frac{2B^2}{\mu_0(\rho_{\mathrm{i}} + \rho_{\mathrm{e}})}, \tag{3}$$

B the magnetic-field magnitude, and μ_0 the magnetic permeability of free space. Due to the magnetic-flux conservation, B and R are related by

$$B(z)R^2(z) = \text{const.} \tag{4}$$

Equation (2) was first derived by Dymova and Ruderman (2005) for a particular case where $R = \text{const}$, and was then generalised for the case with variable R by Ruderman, Verth, and Erdélyi (2008). It is used here to study the non-reflective propagation of kink waves.

3. General Theory

In this section we briefly describe the general theory of non-reflective wave propagation. Since it is relevant for any wave equation with a variable phase speed, we drop the subscript k. We seek the solution to Equation (2) in the form

$$u(z,t) \equiv \eta(z,t)/R(z) = A(z)\Phi\big(\tau(z), t\big), \tag{5}$$

where, at present, $A(z)$, $\tau(z)$, and $\Phi(\tau, t)$ are unknown functions. Obviously, any function $u(z,t)$ can be written in this form, so this representation does not impose any restriction on the solution. Substituting Equation (5) in Equation (2), we obtain

$$A\frac{\partial^2\Phi}{\partial t^2} - c^2 A\left(\frac{\mathrm{d}\tau}{\mathrm{d}z}\right)^2 \frac{\partial^2\Phi}{\partial\tau^2} = c^2\left(2\frac{\mathrm{d}A}{\mathrm{d}z}\frac{\mathrm{d}\tau}{\mathrm{d}z} + A\frac{\mathrm{d}^2\tau}{\mathrm{d}z^2}\right)\frac{\partial\Phi}{\partial\tau} + c^2\frac{\mathrm{d}^2 A}{\mathrm{d}z^2}\Phi. \tag{6}$$

Now we consider this equation as an equation for Φ and define the unknown functions $A(z)$ and $\tau(z)$ in such a way that this equation reduces to the Klein–Gordon equation. First of all, we take

$$\frac{\mathrm{d}\tau}{\mathrm{d}z} = \frac{1}{c}, \qquad \tau(z) = \int\frac{\mathrm{d}z}{c(z)}, \tag{7}$$

where, for definiteness, we have chosen the plus sign corresponding to the wave propagation in the positive z-direction. To obtain the Klein–Gordon equation we also need to eliminate the first term on the right-hand side of Equation (6). For this we put

$$2\frac{\mathrm{d}A}{\mathrm{d}x}\frac{\mathrm{d}\tau}{\mathrm{d}z} + A\frac{\mathrm{d}^2\tau}{\mathrm{d}z^2} = 0. \tag{8}$$

Integrating this equation we obtain

$$A(z) = A_0 c^{1/2}(z), \tag{9}$$

where A_0 is a constant. Since u is defined as the product of A and Φ, we can take $A_0 = 1$ without loss of generality. As a result, we reduce Equation (6) to the variable-coefficient Klein–Gordon equation

$$\frac{\partial^2\Phi}{\partial t^2} - \frac{\partial^2\Phi}{\partial\tau^2} = \frac{c^2}{A}\frac{\mathrm{d}^2 A}{\mathrm{d}z^2}\Phi. \tag{10}$$

So far, we have not made any assumptions about $c(z)$, so the derivation of Equation (10) is absolutely general. We see that the wave equation with variable phase speed can always be reduced to the Klein–Gordon equation with one variable coefficient.

Now we would like to have the Klein–Gordon equation with constant coefficients. Hence, we impose the condition

$$c^2\frac{\mathrm{d}^2 A}{\mathrm{d}z^2} = \beta A. \tag{11}$$

Then Equation (10) reduces to

$$\frac{\partial^2 \Phi}{\partial t^2} - \frac{\partial^2 \Phi}{\partial \tau^2} = \beta \Phi. \tag{12}$$

Substituting Equation (9) in this equation, we arrive at the equation for c:

$$2c\frac{\mathrm{d}^2 c}{\mathrm{d}z^2} - \left(\frac{\mathrm{d}c}{\mathrm{d}z}\right)^2 = 4\beta. \tag{13}$$

To obtain the general solution to this equation we introduce $p = \mathrm{d}c/\mathrm{d}z$ and consider p as a function of c. Substituting this expression in Equation (13), we obtain

$$\frac{\mathrm{d}p^2}{\mathrm{d}c} - \frac{p^2}{c} = \frac{4\beta}{c}. \tag{14}$$

The solution to this linear equation is straightforward:

$$p^2 = 4(Mc - \beta), \tag{15}$$

where M is an arbitrary constant. Substituting this result in the relation $p = \mathrm{d}c/\mathrm{d}z$ and integrating the obtained equation, we obtain

$$\int \frac{\mathrm{d}c}{\sqrt{Mc - \beta}} = \pm 2z. \tag{16}$$

Now we consider two cases.

i) $M \neq 0$. In this case Equation (16) reduces to

$$c = M(z+N)^2 + \frac{\beta}{M}, \tag{17}$$

where N is an arbitrary constant. For a symmetric profile, *i.e.* when $c(z)$ is an even function, we have $N = 0$. Since $c > 0$, $\beta < 0$ when $M < 0$, while β can have arbitrary sign when $M > 0$.

ii) $M = 0$. In this case $\beta < 0$, and Equation (16) reduces to

$$c = \pm 2\sqrt{|\beta|}(z+N). \tag{18}$$

We see that we can have the Klein–Gordon equation with constant coefficients when $c(z)$ is either a linear or a quadratic function.

Note that, when $\beta = 0$, Equation (12) is the wave equation with constant coefficients, which admits the well-known D'Alambert solution. When $\beta \neq 0$, Equation (12) has a solution in the form of monochromatic wave, $\Phi \propto \exp[\mathrm{i}(k\tau - \omega t)]$, where $\omega^2 = k^2 - \beta$, so the wave is dispersive. When $\beta < 0$, there is the cut-off frequency $\omega_\mathrm{c} = \sqrt{-\beta}$, while waves with arbitrary frequencies can propagate when $\beta > 0$.

4. Application to Wave Propagation in Coronal Loops

In this section we consider the application of the general theory to the kink-wave propagation in coronal loops. We adopt a very popular model of a coronal loop, which is a coronal loop with the half-circle shape embedded in an isothermal atmosphere. We assume that the temperature is the same inside and outside the loop, which implies that the ratio of densities inside the loop [ρ_i] and outside the loop [ρ_e] is constant, $\rho_i/\rho_e = \zeta > 1$. The density is related to the height [h] in the solar atmosphere by

$$\rho_i = \rho_f \exp(-h/H), \tag{19}$$

where H is the atmospheric scale height and ρ_f is the density at the footpoints. The height in the atmosphere is related to the length along the loop counted from one of the footpoints by

$$h = \frac{L}{\pi} \sin \frac{\pi z}{L}, \tag{20}$$

where L is the loop length. Substituting this result in Equation (19), we obtain

$$\rho_i = \rho_f \exp\left(-\frac{L}{\pi H} \sin \frac{\pi z}{L}\right). \tag{21}$$

Using this equation and Equation (4), we transform Equation (3) to

$$c_k = c_f \left(\frac{R_f}{R(z)}\right)^2 \exp\left(\frac{L}{2\pi H} \sin \frac{\pi z}{L}\right), \quad c_f = B_f \sqrt{\frac{2\zeta}{\mu_0 \rho_f (\zeta + 1)}}, \tag{22}$$

where B_f, R_f, and c_f are the magnetic-field magnitude, the radius of the loop cross-section, and the kink speed at the footpoints. We assume that the velocity profile is symmetric with respect to the loop apex point that corresponds to $z = L/2$. Then, to have non-reflective kink-wave propagation, we need to take the velocity profile given by Equation (17) with $N = -L/2$. Comparing Equations (17) and (22), we arrive at the expression for $R(z)$,

$$\lambda(z) \equiv \frac{R(z)}{R_f} = \sqrt{\frac{M^2 L^2 + 4\beta}{M^2 (2z - L)^2 + 4\beta}} \exp\left(\frac{L}{4\pi H} \sin \frac{\pi z}{L}\right), \tag{23}$$

where we have used the condition $R(0) = R_f$ to express c_f in terms of M and β. The condition $c_k(z) > 0$ for $z \in [0, L]$ is satisfied when either

$$M > 0, \qquad \beta > 0, \tag{24}$$

or

$$M < 0, \quad 4\beta < -M^2 L^2. \tag{25}$$

We make the viable assumption that $\lambda(z)$ monotonically increases when $z \in [0, L/2]$ and monotonically decreases when $z \in [L/2, L]$. This condition reduces to

$$\operatorname{sgn}(\beta) \left\{ 8H(2z - L) - \cos \frac{\pi z}{L} \left[(2z - L)^2 + \frac{4\beta}{M^2} \right] \right\} < 0, \quad z \in [0, L/2]. \tag{26}$$

It is obvious that this condition is satisfied when $\beta > 0$. When M and β satisfy condition (25), it is easy to show that the condition (26) is equivalent to

$$\beta < -\frac{1}{4} L M^2 (L + 8H). \tag{27}$$

Obviously, this inequality is stronger than the second inequality in Equation (25).

The loop expansion factor [$\lambda_0 = \lambda(0)$] is given by

$$\lambda_0 = \sqrt{1 + \frac{M^2 L^2}{4\beta}} \exp\left(\frac{L}{4\pi H}\right). \tag{28}$$

It is straightforward to see that the dependence of λ on the dimensionless length along the loop [$Z = z/L$] is determined by two dimensionless parameters: M^2L^2/β and the ratio of the loop height to the atmospheric scale height [$\kappa = L/\pi H$]. Using Equation (28) we obtain

$$\frac{M^2 L^2}{\beta} = 4\lambda_0^2 \mathrm{e}^{-\kappa/2} - 4. \tag{29}$$

Substituting this result in Equation (23), we rewrite it in terms of dimensionless variables,

$$\lambda(Z) = \frac{\lambda_0}{\sqrt{(\lambda_0^2 - \mathrm{e}^{\kappa/2})(2Z-1)^2 + \mathrm{e}^{\kappa/2}}} \exp\left(\frac{\kappa}{4}\sin(\pi Z)\right). \tag{30}$$

It follows from Equation (29) that $\beta > 0$ when $\lambda_0 > \mathrm{e}^{\kappa/4}$, and $\beta < 0$ otherwise. In the latter case the inequality (27) reduces to

$$\lambda_0 > \frac{\mathrm{e}^{\kappa/4}}{\sqrt{1 + \pi\kappa/8}} \equiv \lambda_{\mathrm{m}}. \tag{31}$$

It is easy to show that $\lambda_{\mathrm{m}} > 1$. Hence, in our model, the loop expansion factor can take any value from the interval $(\lambda_{\mathrm{m}}, \infty)$. The quantity λ_{m} is an increasing function of κ. It varies approximately from 1.036 to 1.234 when κ increases from 1/2 to 2. Hence, we can have quite realistic expansion factors that do not exceed 1.5 in our model even for large coronal loops with a height of the apex point above the atmospheric scale height.

If we take a monochromatic solution of Equation (12), the displacement of the loop axis in accordance with Equations (5) and (9) is given by

$$\eta(t, z) = C R(z) c_{\mathrm{k}}^{1/2}(z) \exp\bigl[\mathrm{i}\bigl(k\tau(z) - \omega t\bigr)\bigr], \tag{32}$$

where C is a constant. Hence, the oscillation amplitude [$a(z)$] is proportional to $R(z)c_{\mathrm{k}}^{1/2}(z)$. Then, using Equation (22), we obtain

$$a(z) = a_{\mathrm{f}} \exp\left(\frac{L}{4\pi H} \sin\frac{\pi z}{L}\right), \tag{33}$$

where $a_{\mathrm{f}} = a(0)$.

When $\lambda_0 > \mathrm{e}^{\kappa/4}$, we have $\beta > 0$, and there is no cut-off frequency. For $\lambda_0 < \mathrm{e}^{\kappa/4}$ the cut-off frequency is $\omega_{\mathrm{c}} = \sqrt{-\beta}$. We express ω_{c} in terms of the loop parameters. Using Equation (17) with $z = 0$ and $N = -L/2$, and Equation (29), we obtain

$$M^2 = c_{\mathrm{f}}^{-2} \lambda_0^4 \beta^2 \mathrm{e}^{-\kappa}.$$

Eliminating M from this equation and Equation (29), we find β and obtain

$$\omega_{\mathrm{c}} = \frac{2c_{\mathrm{f}}\mathrm{e}^{\kappa/4}}{L\lambda_0^2} \sqrt{\mathrm{e}^{\kappa/2} - \lambda_0^2}. \tag{34}$$

The cut-off frequency [ω_{c}] takes the highest value when $\lambda_0 = \lambda_{\mathrm{m}}$. This value is given by

$$\omega_{\mathrm{cM}} = \frac{2c_{\mathrm{f}}\mathrm{e}^{\kappa/4}}{L\lambda_{\mathrm{m}}^2} \sqrt{\mathrm{e}^{\kappa/2} - \lambda_{\mathrm{m}}^2} = \frac{c_{\mathrm{f}}}{4H} \sqrt{1 + \frac{8}{\pi\kappa}}. \tag{35}$$

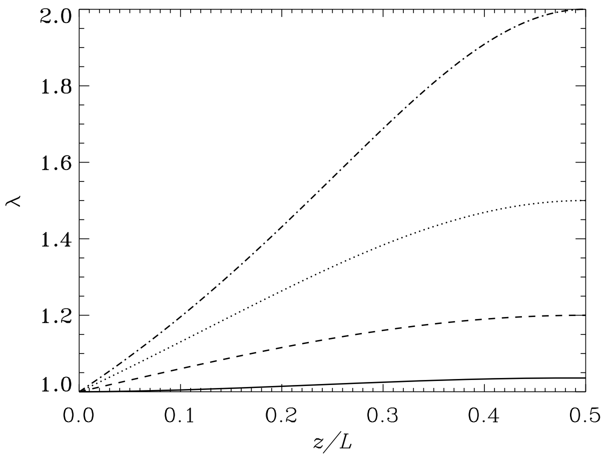


Figure 1 The dependence of the dimensionless radius of the loop cross-section [λ] on the length along the loop [z] for $\kappa = 1/2$. Since the loop is symmetric, only half of the loop is shown. The solid, dashed, dotted, and dash–dotted curves correspond to $\lambda_0 = 1.036$, 1.2, 1.5, and 2. The value $\lambda_0 = 1.036$ is close to λ_m.

When κ varies from 1/2 to 2, the dimensionless quantity $\omega_{cM}H/c_f$ decreases from 0.617 to 0.377. If we take as typical coronal values $H = 60$ Mm and $c_f = 1000\ \mathrm{km\,s^{-1}}$, we obtain a value for the shortest cut-off period $2\pi/\omega_{cM}$ that is between 10 and 17 minutes. We see that the shortest cut-off period is longer than the typical periods of observed propagating waves in coronal loops, although, quite rarely, oscillations with periods longer than ten minutes are observed (*e.g.* Aschwanden *et al.*, 2002).

Schrijver, Aschwanden, and Title (2002) described 17 events with transverse oscillations of coronal loops observed with the *Transition Region and Coronal Explorer* (TRACE). Aschwanden *et al.* (2002) presented the study of the geometrical parameters of the loops involved in these oscillations. The lengths of all but one of these loops vary from 72 Mm to 200 Mm. If we assume that the loops have a half-circle shape, the heights of their apex points vary from 23 Mm to 127 Mm. If, in addition, we take the atmospheric scale height in the corona to be equal to 60 Mm, we obtain κ between 0.4 and 2.1. Hence, we have chosen $\kappa = 1/2$, 1, and 2 as representative values. One coronal loop studied by Aschwanden *et al.* (2002) was extremely large: 582 Mm long. However, the shape of this loop was very different from a half-circle, so we disregarded it. In Figures 1, 2, and 3 the dependence of λ on z/L is shown for various values of κ and λ_0.

To complete this section, we add that standing kink waves in coronal loops with the quadratic velocity profile symmetric with respect to the loop apex point have been studied by Dymova and Ruderman (2006).

5. Summary and Conclusions

We have studied the non-reflective propagation of fast kink waves along magnetic-flux tubes in the solar atmosphere. We restricted our analysis to kink oscillations of straight thin flux tubes in cold plasma. The density was assumed not to vary across the tube cross-section, but both the density and cross-section radius were allowed to vary along the tube. To describe the kink oscillations we used the wave equation derived by Ruderman, Verth, and Erdélyi (2008).

We briefly described the general theory of one-dimensional wave propagation in inhomogeneous media and showed that the wave equation with a variable phase speed can always be reduced to the Klein–Gordon equation with one variable coefficient. Moreover, we showed

Figure 2 The dependence of the dimensionless radius of the loop cross-section [λ] on the length along the loop [z] for $\kappa = 1$. Since the loop is symmetric, only half of the loop is shown. The solid, dashed, dotted, and dash–dotted curves correspond to $\lambda_0 = 1.09$, 1.2, 1.5, and 2. The value $\lambda_0 = 1.09$ is close to λ_m.

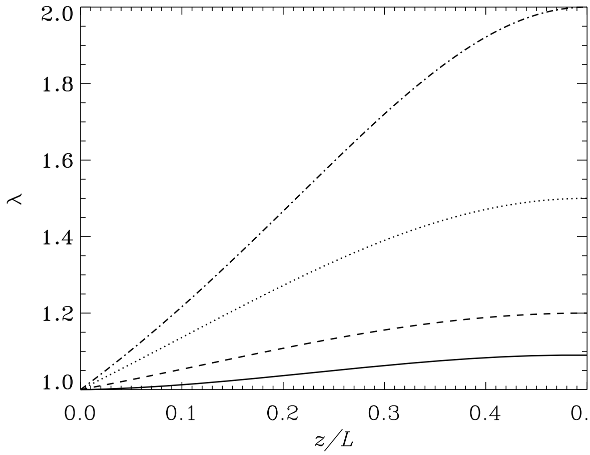
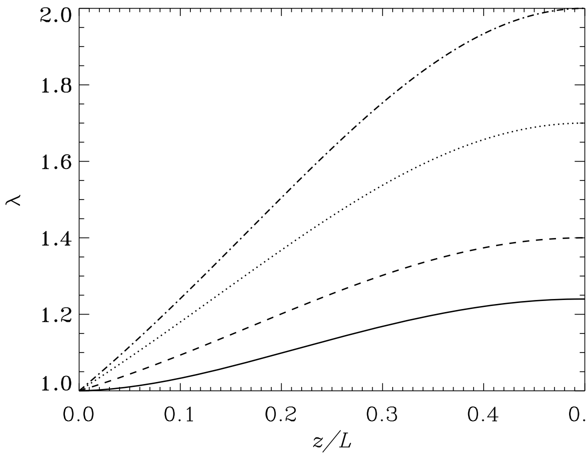


Figure 3 The dependence of the dimensionless radius of the loop cross-section [λ] on the length along the loop [z] for $\kappa = 2$. Since the loop is symmetric, only half of the loop is shown. The solid, dashed, dotted, and dash–dotted curves correspond to $\lambda_0 = 1.24$, 1.4, 1.7, and 2. The value $\lambda_0 = 1.24$ is close to λ_m.



that this Klein–Gordon equation has constant coefficients if the dependence on the phase speed on the spatial variable is either linear or quadratic.

The general theory was applied to fast kink-wave propagation along coronal magnetic loops. We considered magnetic loops with a half-circle shape in an isothermal atmosphere. We assumed that loops are symmetric with respect to the apex point, and that the dependence of the kink speed on the distance along the loop is quadratic. This assumption dictates the dependence of the loop cross-section radius on the distance along the loop. Our model contains two free parameters: the ratio of the loop height to the atmospheric scale height κ, and the loop expansion factor λ_0, which is the ratio of the loop cross-section radii at the apex and footpoints.

We made the viable assumption that the loop cross-section radius increases with the height. This assumption imposes the restriction that λ_0 has to be larger than the lowest values $\lambda_m > 1$, which is a monotonically increasing function of κ. This value is not very high. Even for $\kappa = 2$ we have $\lambda_m \approx 1.234$, therefore our model can describe quite realistic coronal loops. We plot the dependence of the loop cross-section radius on the length along the loop for various values of κ and λ_0. An interesting property of our model is that there is a cut-off frequency when $\lambda_0 < e^{\kappa/4}$, while there is no cut-off frequency when $\lambda_0 > e^{\kappa/4}$. For typical coronal-loop parameters the cut-off period is longer than the typical periods of observed propagating kink waves.

It is important to note that the condition that the Klein–Gordon equation has constant coefficients is a sufficient condition for having non-reflective wave propagation, but it is not a necessary condition. Recently, Cally (2012) investigated the propagation of Alfvén waves in the solar atmosphere. He showed that even the atmosphere with the exponentially increasing Alfvén speed is transparent for Alfvén waves, and the total absorption of Alfvén wave energy at infinity is possible. He also showed that the same is true for a power-law profile of the Alfvén speed. Although Cally (2012) considered Alfvén waves, his analysis is applicable to any type of waves that are described by the wave equation. An important distinctive property of waves described by the Klein–Gordon equation with constant coefficients is that they can propagate without changing shape. Only the wave amplitude changes with the distance from the wave driver. There are other differences related to the appearance of oscillating wakes in the case of a general wave profile. However, discussing these differences is beyond the scope of our article.

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