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# UNIFIED ENDOGENOUS GROWTH MODEL FOR MINERAL ABUNDANT COUNTRIES

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## Introduction

This paper analyses an extended version of creative destruction model with strategic complementarity between R&D investment of firms and education investment of households [Aghion, Howitt, 1992; 2005]. The new assumption is that the probability of innovation in the productive sector is determined endogenously because it depends on the level of human capital in the economy. This model provides a framework to explain the coexistence of two long-run equilibria: zero growth equilibrium and sustainable positive growth equilibrium transition as well as the transition from zero growth to high-growth equilibrium. The model provides explanation for a convergence club phenomenon: some developing countries experience the absence of growth during the years, while others are characterized by catching-up process to the income level of developed countries.

We apply the model for analyzing the effect of natural resource rents on economic development process. The extended version of a model makes a contribution to a debate between supporters and opponents of resource curse thesis. In the model the final (positive or negative) effect of resource rent on economic growth is determined by the stage of country development (the initial level of human capital) and policy parameters.

This model can be considered as unified growth theory model, This type of models describes the process of transition from stagnation to modern growth in developed countries [Galor, Weil, 2000; Galor, 2005] while our model analyses the transition from stagnation to growth in modern time's developing countries.

The paper is organized as follows. In the next section we propose a benchmark for analysis of poverty traps in a simple endogenous growth model, then we propose an extended version of a model with rent-seeking and make a conclusion.

## Basic framework

The time is discrete:  $t = 1, 2, \dots$  The interest rate is zero. We assume that an economy is populated by a sequence of two-period-lived overlapping generations.

The number of agents born in the period  $t$  equals  $L_t$ . At each point in time agents are endowed by 1 unit of labor. Young agents chose to get education or not. Each old agent is endowed by 1 unit of labor, skilled or unskilled in dependence of the decision to get an education in the previous period. The units of labor supply inelastically on the market.

Each period a final good is produced according to the Cobb–Douglas technology:

$$Y = Ax^\alpha, \quad (1)$$

where  $x$  denotes the quantity of intermediate input used in final good production, and  $A$  is a productivity parameter that reflects the current quality of the intermediate good.

The market for final good is perfectly competitive. A firm problem in a consumption sector look like

$$\begin{aligned} p_t Y - p_x x &\rightarrow \max_x, \\ p_t A \alpha x^{\alpha-1} - p_x &= 0. \end{aligned}$$

In this case the price of consumption good ( $p_t$ ) in the economy equals to the marginal product of intermediate good:

$$p_t = \frac{p_x}{A \alpha x^{\alpha-1}}. \quad (2)$$

The intermediate good is itself produced using labor according to a simple one-for-one technology, with one unit of labor producing one unit of the current intermediate good. Thus  $x$  also denotes the amount of labor currently employed in manufacturing.

Labor can also be employed in research to generate innovations. Each innovation improves the quality of the intermediate input, from  $A$  to  $\gamma A$ , where  $\gamma > 1$  measures the size of the innovation. Innovations result from research investment. More specifically, there is an innovator who innovates with probability  $\lambda n$  after recruiting  $n$  units of skilled workers for innovation purposes  $\lambda(H) \cdot n$ .

The probability of innovation  $\lambda$  depends on the level of human capital in the economy ( $H$ ).  $\lambda(H)$  is monotonically increasing function from  $0+$  to  $\lambda_{\max}$ . Without a loss of generality we assume that a lower bound of the level of human capital in the economy is 1:

$$\lambda(1) \rightarrow 0, \quad \lim_{H \rightarrow \infty} \lambda(H) = \lambda_{\max} \leq 1.$$

The causality between the level of human capital in the economy and the probability of innovation for individual firm is the crucial element in our model. This

assumption is based on the idea that the success of innovator depends not only from the skills of their researchers but also from the environment for an innovation activity, the level of trust and cooperation in society. So,  $H$  can also be considered as a social capital.

The innovator enjoys a monopoly power in the production of the intermediate good, but faces a competitive fringe. The profit of innovator equals to

$$\pi = (\mu - 1)w\chi, \quad (3)$$

where  $\mu$  is innovator's mark-up,  $w$  is a wage in productive sector. We suppose that the innovator is a monopolist only for one period of time. A market for research is competitive. Research arbitrage equation (4) shows that in the equilibrium at any date  $t$  the amount of research undertaken by the innovator must equate the marginal cost of a unit of research labor with the expected marginal benefit. The marginal cost is the wage in the research sector  $w_t^S$

$$w_t^S = \lambda(H_t)(\mu - 1)w_t\chi_t. \quad (4)$$

A firm employs a skilled labor force in the beginning of the period  $t$  and with some probability  $\lambda(H)$  make a profit from successful innovation at the end of the period  $t$ . This assumption is needed for a tractability of the model.

Equation (4) can also be considered as a demand on skilled labor force. More precisely, a demand is described by system of equations (5).

$$\begin{cases} \text{if } w_t^S < \lambda(H_t)(\mu - 1)w_t\chi_t & \text{then } n_t = 0 \\ \text{if } w_t^S = \lambda(H_t)(\mu - 1)w_t\chi_t & \text{then } n_t \in [0, \infty) \\ \text{if } w_t^S > \lambda(H_t)(\mu - 1)w_t\chi_t & \text{then } n_t = \infty \end{cases} \quad (5)$$

## Household problem

Each household lives two periods of time and proposes one unit of labor in the first and second period of time. At the first period of life households decide to invest in education or not.

Assume that agents have different capabilities for education. The agent type  $j$  spends on education  $c \cdot j$  units of time, where  $c$  is the parameter of the model,  $j$  is distributed uniformly from 0 to 1. Then the share of educated workers can be determined from the participation constraint

$$w_{t+1}^S - cw_t i_t = w_{t+1}, \quad (6)$$

where  $i_t$  is a share of young who get an education,  $c$  – alternative costs of education (in units of time). Then, we can rewrite a participation constrain as

$$i_t = \frac{w_{t+1}^S - 1}{c}. \quad (7)$$

Equation (7) shows that the difference between the expected wages in skilled and unskilled sector provides incentives to get an education. The more distance between the wage in skilled and unskilled sector in the period  $t + 1$  the higher proportion of households desired to get an education in the period  $t$ .

Old agents employ in R&D or manufacturing sector according to his own skills level

$$L_{t-1} = x_{2,t} + n_t, \quad (8)$$

where  $L_{t-1}$  is a number of agents born in a period  $(t - 1)$ ,  $x_{2,t}$  is a number of old agents, employed in manufacturing,  $n_t$  – number of agents employed in R&D sector.

The dynamics of human capital are modeled as in the papers of Lucas and Redding [Lucas, 1988; Redding, 1996]:

$$H_{t+1} = H_t(1 + i_t - \delta), \quad (9)$$

where  $\delta$  is the frontier for the human capital dynamics. If the share of people that get an education is higher then  $\delta$  then the general level of human capital in the society is increasing, otherwise the general level of education is deteriorating or remain constant at a level 1.

## Solution of a model

From research arbitrage equation (4), education participation constraint (7) and labor force constraints we derive a dynamic equation for optimal share of education (10). The dynamic equation for evolution of human capital is given by eq. (9).

$$i_t = \frac{2\lambda(H_t)(\mu - 1)L - 1}{c + \lambda(H_t)(\mu - 1)L} - \frac{\lambda(H_t)(\mu - 1)L}{2[c + \lambda(H_t)(\mu - 1)L]} i_{t+1}^2. \quad (10)$$

There is two steady-state equilibriums for which a share of educated workforce remain constant. A first is a poverty trap equilibrium for which

$$i_t = 0 \text{ for all } t.$$

The precondition for poverty trap equilibrium is the absence of incentives for innovation. From eq. (10) poverty trap equilibrium exists if and only if

$$\lambda(1) \leq \frac{1}{2(\mu - 1)L}. \quad (11)$$

In this case the lowest probability of innovation with minimum level of human capital ensures that the marginal benefit for innovation for given  $L$  and  $\mu$  is lower than the marginal costs. In the case of poverty trap, nobody works in R&D sector, so that

$$n = 0, \quad x = 2L, \quad H = 1.$$

And the pace of economic growth is zero.

Modern growth path equilibrium is characterized by maximum level of probability of innovation. So, it can be achieved with sufficiently high level of human capital

$$\lambda(H) = \lambda_{\max}, \quad H \rightarrow \infty.$$

In this equilibrium the level of people employed in high-skilled sector as well as growth rate of productivity are constant,

$$i_t = i_{t+1} = i_{\max}. \tag{12}$$

Steady-state rate of growth equals

$$\frac{g_Y}{L} = \gamma i_{\max} L.$$

## Model dynamic with static expectations

We can analyze the dynamic of model in a simple environment assuming that the expectations of agents are static. In this case there is only one plausible dynamics from stagnation to growth

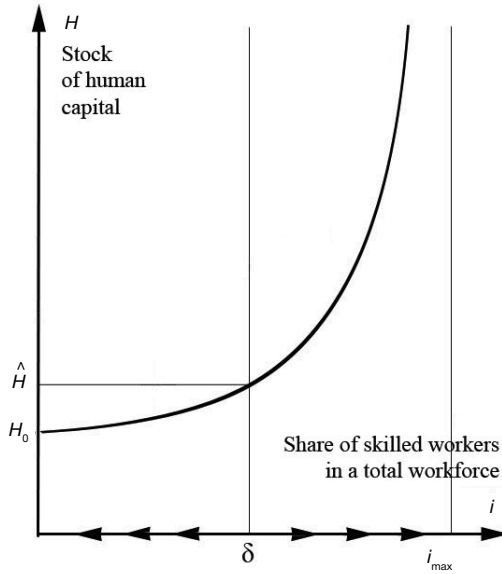
$$i_{t+1}^e = i_t. \tag{13}$$

Figure 1 describes the dynamics in the basic model with static expectations.

There is a threshold level of human capital for which economy locks in a poverty trap. However, from eq. (11) the existence of poverty trap equilibrium is determined by fundamental factors like a quantity of labor force ( $L$ ) and mark-up for successful innovator ( $\mu$ ). The rise of market size ( $L$ ) or innovator's mark-up ( $\mu$ ) leads to the transition from zero growth to positive growth equilibrium.

## Model extension with rent-seeking activities

We suppose now that a government redistributes some part of output collecting value-added taxes and providing lump-sum transfer. A government sector is ineffective and it provides possibilities for rent-seeking behavior in form of corruption,



**Fig 1.** Model dynamics with static expectations

grants for bureaucrats. A fixed part of government revenue  $\xi$  is used as a rent for officials.

Suppose that a tax rate on consumption good equals  $\tau$ . Then we can reformulate a firm problem in a competitive consumption sector as

$$(1 - \tau)P_Y Y - p_x x \rightarrow \max_x. \quad (14)$$

Therefore, from a first-order condition a price of consumption good equals

$$p_Y = \frac{P_x}{A\alpha x^{\alpha-1}(1-\tau)}. \quad (15)$$

We suppose that a government has two sources of income: value-added taxes and a fixed quantity of natural resources rent, defined by  $R$ . Final output in this economy consist of final good  $Y$  plus a resource rent  $R$ , which totally redistribute by government [Torvik, 2002]. A part  $\xi$  of total government revenue is supposed to be used as rent for corrupted officials. Another part  $(1 - \xi)$  is used as a lump-sum transfer for households.

From eq. (15) we can derive a sum of resource rents and total amount of taxes

$$R + \frac{\mu x}{\alpha} \cdot \frac{\tau}{1-\tau}.$$

As previously we assume that agents decide in a first period to get education or not. If agents get an education they benefit for skilled sector salary in a second period. In the opposite case, they receive a wage plus with probability of  $\eta$  agents receive a part of corruption rent. We assume that only uneducated persons have an access to corruption rents because there is a strict division between productive and rent-seeking sector. If an agent is employed in a R&D sector and produces a fruitful innovation it is not possible for him to be an official and engaged in rent-seeking activities. This division of labor is typical for rent-seeking literature [Mehlum et al., 2003; Torvik, 2002]).

Thus, an expected quantity of rent received by an educated person in a second period equals

$$\frac{\xi \left( R + \frac{\mu x}{\alpha} \cdot \frac{\tau}{1-\tau} \right)}{x_{2,t}}$$

We can now rewrite a participation constraint for choice of education level as

$$w_{t+1}^{e^s} - ci_t = 1 + \frac{\xi \left( R + \frac{\mu x_{t+1}}{\alpha} \cdot \frac{\tau}{1-\tau} \right)}{x_{2,t+1}}. \quad (16)$$

An innovator problem and human capital dynamics are the same as in a basic model.

In a rent-seeking model as in a basic model it exists a poverty trap equilibrium

$$i_t = 0 \text{ for all } t. \quad (17)$$

As previously poverty trap equilibrium exists only if profits for innovation do not exceed marginal costs for innovation

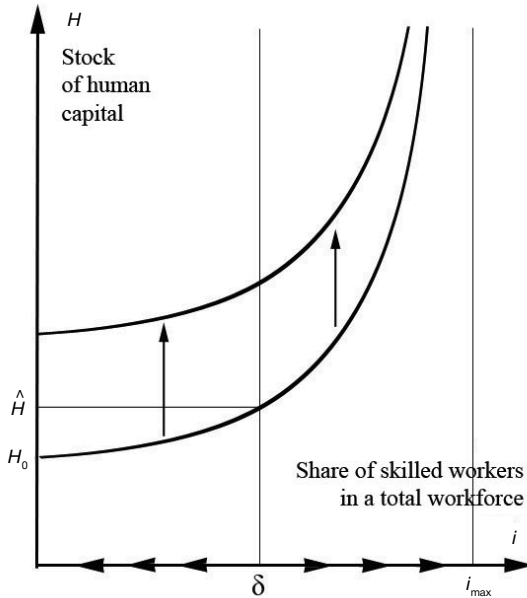
$$w_{t+1}^s < \lambda(H)(\mu - 1)x_{t+1}. \quad (18)$$

The condition (18) holds if and only if

$$\lambda(H) < \frac{1 + \xi \left( \frac{R}{L} + \frac{2\mu\tau}{\alpha(1-\tau)} \right)}{2L(\mu - 1)}. \quad (19)$$

Thus, in a rent-seeking model ( $\xi > 0$ ) a critical level for total human capital ( $H$ ) under which a poverty trap occurs is higher than in a basic model.

As shown in a Fig. 2 the appearance of natural resources influences negatively on the incentives for education.



**Fig. 2.** A model with natural resources

Moreover, in more developed countries (with larger level of human capital) the adverse effect of natural resources on incentives to innovate as well as incentives to education should be smaller according to the model. Higher level of human capital corresponds to higher value of wages, that's why relative profit for rent-seeking activities in a developed countries would be smaller than a profit from innovation.

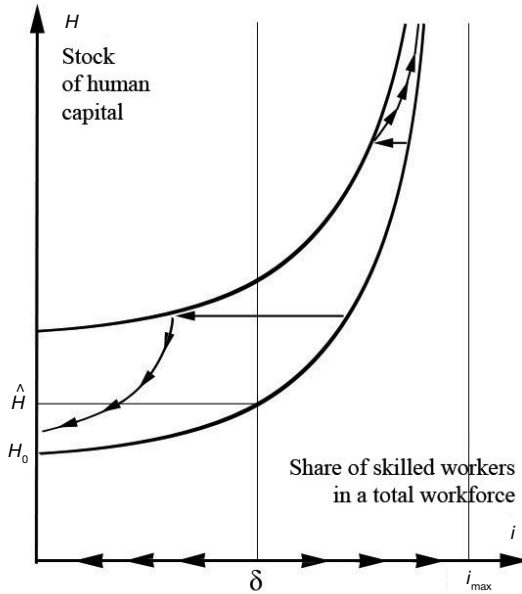
The appearance of rent-seeking activities leads to very different outcomes depending on initial characteristics of countries.

In an economy staying in a poverty trap a positive natural resources shock (for example, a rise of raw-materials world price) leads to a temporary rise in a growth rate but do not influence on long run zero rate of growth. Indeed, if the gross domestic product (GDP) consists of output in productive sector and also an output in resource exporting sector then the temporary rise of resource rents would lead to boost in GDP growth but only temporarily. This case is described the experience of growth in 1970–1980 in African raw-materials exporters (Nigeria, Angola).

If human capital is sufficiently high then a positive resources shock only partially influences on incentives to education. Thus, the economy will stay on a stable growth path (Fig. 3) (Canada, Australia and Norway).

For intermediate level of  $H$ , the influence of natural resources on economic development is very unstable. For some value of parameters the economy eventually





**Fig. 3.** The effect of natural resource shock on economic development

returns to a stable growth path, for other values of parameters the natural resources shock would drive the economy to a poverty trap. Indeed, positive natural resources shock would lead to a boost in a resource output but also decrease the incentives for education. This case can be described as a tragedy of resource-abundant country which gains temporary output growth on the price of long-run development (Venezuela).

## Conclusion

We construct an endogenous growth model with endogenous probability of innovation which describes the features of world economy development process. Some countries (developed) enjoy long run stable growth path equilibrium, other countries lie in a poverty trap equilibrium with near zero rate of growth, eventually, a third group of countries converges to a stable growth path equilibrium. We show that changes in size of market and property rights protection variables determine endogenous transition from stagnation to growth.

Then we apply the model for analyses the effect of natural resource rents on economic development process. We assume that agents, not employed in R&D sec-

tor, can be engaged in rent-seeking activities with a fixed probability. In this case an influence of natural resources stock on economic growth will depend crucially on the initial stage of development for economy. For developed countries the appearance of exhaustible natural resources do not influence on long-run rate of growth, for countries, which lie in a poverty trap, natural resources rent would lead to a temporary rise of output. Eventually, for countries with intermediate level of human capital the appearance of natural resources crucially influences long-run equilibrium (steady-state with sustainable growth or poverty trap). Therefore, for this type of countries even little changes in policy variables (private property rights, education costs, a size of a market) lead to different long-run outcomes.

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