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On the dynamics of the world demographic transition and financial-economic crises forecasts

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Abstract. The article considers dynamic processes involving non-linear power-law behavior in such apparently diverse spheres, as demographic dynamics and dynamics of prices of highly liquid commodities such as oil and gold. All the respective variables exhibit features of explosive growth containing precursors indicating approaching phase transitions/catastrophes/crises. The first part of the article analyzes mathematical models of demographic dynamics that describe various scenarios of demographic development in the post-phase-transition period, including a model that takes the limitedness of the Earth carrying capacity into account. This model points to a critical point in the early 2050s, when the world population, after reaching its maximum value may decrease afterward stabilizing then at a certain stationary level. The article presents an analysis of the influence of the demographic transition (directly connected with the hyperexponential growth of the world population) on the global socioeconomic and geopolitical development. The second part deals with the phenomenon of explosive growth of prices of such highly liquid commodities as oil and gold. It is demonstrated that at present the respective processes could be regarded as precursors of waves of the global financial-economic crisis that will demand the change of the current global economic and political system. It is also shown that the moments of the start of the first and second waves of the current global crisis could have been forecasted with a model of accelerating log-periodic fluctuations superimposed over a power-law trend with a finite singularity developed by Didier Sornette and collaborators. With respect to the oil prices, it is shown that it was possible to forecast the 2008 crisis with a precision up to a month already in 2007. The gold price dynamics was used to calculate the possible time of the start of the second wave of the global crisis (July–August 2011); note that this forecast has turned out to be quite correct.

1 Finite-time singularity in the dynamics of the world population and "post-singularity" scenarios

The seminal paper by Johansen and Sornette [1] has analyzed finite-time singularities in the dynamics of the world population and some economic indices, suggesting 2052 ± 10 as the critical time (t_C). It is conjectured that the explosive population growth can occur through the interaction of a number of variables, such as capital and technology. Johansen and Sornette also discuss possible scenarios of the development of humankind before and close to t_C.

The studies performed by us confirm the main conclusions of this article, and also shed additional light on the respective phenomena.

As noted in the above mentioned article [1], several authors [2,3] have convincingly shown that the data for the world population during the whole human history up to the 1970s are with surprising accuracy described by a hyperbolic function (1):

$$N_t = \frac{C}{t_C - t}, \tag{1}$$

where N_t is world population at time t, t_C is the critical time, C is a constant.

It was found in [3] that $t_C = 2025$ with $C = 200 \cdot 10^9$. Equation (1) accounts for the explosive population growth in the 20th century. Through the approximation of the empirical demographic data with log-periodic oscillations superimposed on a power-law curve Johansen and Sornette [1] find a new value of the critical time around 2052 ± 10.

Sergey Kapitsa [4] brings attention to the point that the hyperbolic growth (1) of the world population corresponds to its growth rate dynamics described with differential Eq. (2) (see all the equations in the appendix to this article).

This equation reflects the cooperative mechanism of development, when its measure is the world population squared. It has been shown that this is accounted for by the following nonlinear second order positive feedback mechanism: the more people – the more potential inventors – the faster technological growth – the faster growth of the Earth's carrying capacity – the faster population growth – with more people you also have more potential inventors – hence, faster technological growth, and so on [5–9].

The solution of Eq. (2) is the hyperbolic function (1). Equations of type (2) are well studied and their solutions (1) are known as blowup regimes. Their characteristic feature is that at some finite time t_C, which is called the point of singularity or critical point, the value of the respective variable becomes infinite. In real life there occurs a change of regime. In case of demography, the explosive growth is altered by the decrease of fertility, *i.e.* through self-organization the World System moves to a new regime of low fertility. This phenomenon is denoted as "the global demographic transition".

To describe the global demographic transition, Kapitsa regularizes Eq. (2) by introducing into it the characteristic time τ (which represents "the effective duration of human life"), this results in Eq. (3).

Equation (3) no longer gives a blowup, and the solution does not become infinite. What is more, this equation has a simple analytic solution – (4). Basing on the data from the world demographic statistics, Kapitsa calculated the numerical values of the constants (5).

This implies that the world demographic transition came to its zenith in 1995 and will continue until 2040. As regards the population growth rates $q_N = \frac{dN}{Ndt}$, they passed their peak even earlier, in 1963 ($q_{N\,\max} = 2.2\%$), and further on they are supposed to decline, approaching zero levels. Thus, the global demographic transition

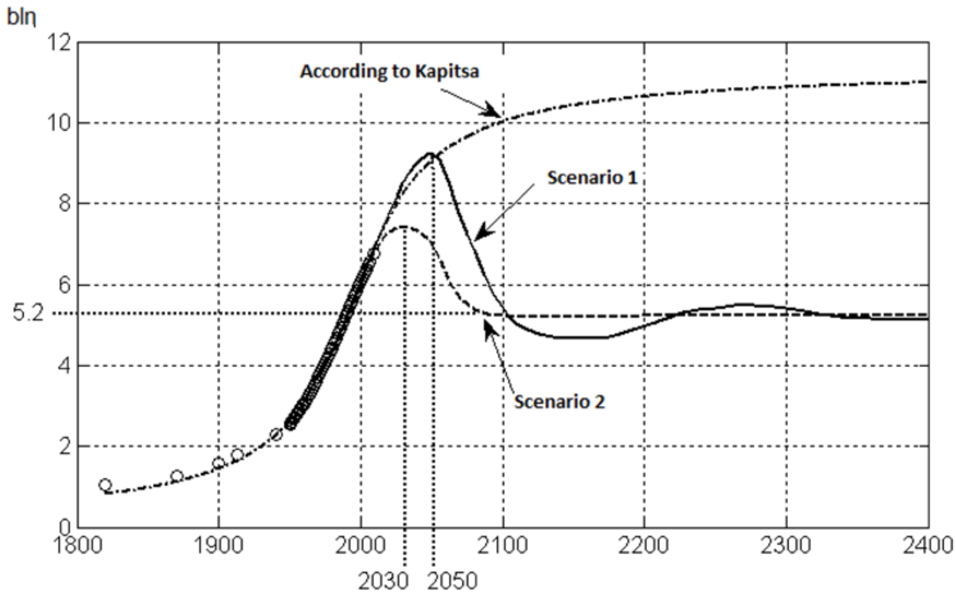

Fig. 1. World population dynamics.

Table 1. Summary of suggested carrying capacities.

Authors	*Suggested estimate of the Earth's carrying capacity*	*Source*
J. K. Smail	2–3 billion	[13]
Dennis H. Medows, J. Randers, D. L. Meadows	7.7 billion	[17]
A. Matrosov	6.5 billion	[16]
B. M. Dolgonosov	5.2 billion	[18]
Charles Fourier (1772–1837)	5 billion	[19]

is connected with the limit of the world population growth rate within the blowup regime. According to Kapitsa, with the decline of the population growth rates, the world population should stabilize at a plateau level of about $N_{\max} = 11.36$ billion, which stems directly from eq. (4) with $t \to \infty$. The diagram of the world population growth calculated with Kapitsa's equation (4) is displayed in Fig. 1 that also indicates (with circles) estimates of the world population. As may be seen in this figure, Kapitsa's equation (4) approximates the World System demographic dynamics retrospectively in a very accurate way (especially, during the period of the global demographic transition). Generally speaking, it may be said that Kapitsa's model (4) describes the scenario of sustainable development of the humankind [4].

Kapitsa formulated the demographic imperative, according to which global social, historical, economic, and cultural processes adapt to changes in the world population. According to Kapitsa, this variable is the leading one; in complexity studies it is called the "order parameter" governing all the other variables [10]. Indeed, as Kremer [11] shows, the technology growth rate (q_A) is proportional to population size (6). Accordingly, before the demographic transition (1), equation (7) was true.

Comparing the Eqs. (2) and (6) we can obtain Eq. (8); hence it follows that[1]

$$A = A_0 N^{1+\delta} = A_0 C^{1+\delta}(t_C - t)^{-(1+\delta)}, \quad (9)$$

where $\delta = bC - 1$, *i.e.* δ can be calculated numerically. We showed that $\delta = 0.11$ [12].

Thus, it was namely the explosive population growth of the 20th century that predefined the explosive growth of technology and the highest rates of economic growth which occurred in the 1950s and 1960s. The same applies to the dynamics of GDP (Y), *i.e.* to the economic growth. Indeed, as is shown in [5,12]:

$$Y = \gamma N^{2+\delta}, \quad \gamma = const. \quad (10)$$

Consequently,

$$Y = \gamma C^{2+\delta}(t_C - t)^{-(2+\delta)}. \quad (11)$$

However, all this (9–10) was true before the demographic transition. After the transition all these variables (N, A, Y) started stabilizing, and we need new mathematical models that are capable to describe various scenarios of sustainable development. We denote as "sustainable" such a development, within which the world population does not exceed the economic carrying capacity of the Earth – the threshold of the anthropogenic influence on the environment, over which ecological catastrophes become inevitable. Kapitsa's model does not take into account the ceiling of the acceptable Earth's carrying capacity. If we take that limitation into account, a possible scenario of the world population dynamics may look as follows. The world population first grows in a blowup regime. Then, by inertia, it jumps over the level of the ultimate Earth's carrying capacity determined by the available technologies. The growth continues further. At this point, due to the overpopulation, human beings destroy the environment. An ecological crisis starts, due to which the population declines to the level that is lower than the one of the original carrying capacity. Gradually, through the immense efforts of the humankind the ecological crisis is overcome, the environment is recovered, which leads to a new period of population growth, and, after a few fluctuations with a declining amplitude, the population enters the phase of stabilization at a stationary level. This scenario may be denoted as a scenario of growth with return to the stationary level within a regime of damped oscillations. The model can also describe a scenario of the growth with return and stabilization at a stationary level without fluctuations.

One of the most likely scenarios of the world population dynamics appears to be the one of the growth with return and the further stabilization at a stationary level. This seems to be congruent with Smail's reasoning that the ceiling of the Earth's carrying capacity will be reached rather soon, and the further stabilization of the world population (accompanied by its decline to the stationary level) appears to be almost inevitable [13]. This possible significant decrease of the world population would be to some extent a result of a century of its explosive growth, as a result of which (as a number of indicators suggest) the carrying capacity of the Earth might have already been exceeded. According to estimates produced by some experts [14], the humankind now consumes more than 20% of the planetary biosphere output in energetic equivalent, whereas the acceptable consumption (that does not destroy the biosphere) does not exceed 1% [15]. Thus, according to such estimates, the humankind has exceeded the acceptable limit of its economic power more than 20 times and has gone beyond the limits of the biospheric stability. The study by Wackernagel *et al.* [14] suggests that the humankind was last time at the level of sustainability in the 1980s.

[1] Note that the result (9) can be also more simply obtained by $\frac{d\ln A}{dt} = \frac{aC}{(t_C - t)} \Rightarrow \ln A = -aC\ln(t_C - t)$, which gives (9) directly.

According to Smail's estimates, the stationary carrying capacity of the Earth does not exceed 2–3 billion, and this figure will not be reached earlier than in two centuries. This estimate by Smail of the Earth's stationary carrying capacity appears to be too low, taking into account estimates of other experts that will be analyzed below.

The excess of the human population over the carrying capacity is, of course, compensated to some extent by social and biological mechanisms of fertility reduction that, however, are not sufficiently strong yet in many developing countries. In any case, the process of the reduction of the world population growth rates down to zero levels is not likely to take less than half a century (about two generations), whereas during this period of time the world population will grow up to 8–9 billion [13], whereas the decline of the world population will only start afterwards.

The issue of the value of sustainable population of the Earth is one of the most fundamental issues of the present time. There are a number of ways to estimate the stationary population of the world that are reviewed in the monograph [16]. The stable population of 7.7 billion is treated as acceptable within the resource model of Dennis Medows and his colleagues [17].

Matrosov has developed further the resource model of the Medows' group, and estimated the sustainable value of the world population as being equal to 6.5 billion [16]. Dolgonosov [18] has obtained an elegant proportion for an approximate estimation of the value of N_C, demonstrating that the constant C in the hyperbolic growth Eq. (1) may be represented as the product $N_C t_C$, i.e. $C = N_C t_C$, where $t_C = \tau$ – the characteristic time of demographic transition in the regularized equation of Kapitsa (3). The adjusted value of τ is 38.5 years. This proportion suggests the estimate of N_C as being equal to 5.2 billion. We have also mentioned above Smail's estimate of the stationary world population (2–3 billion).

Thus, it seems possible to suppose that the most likely value of the world stationary population may be found in the diapason between 2 and 7 billion. While computing our forecast scenario we have chosen N_C as being equal to 5.2 billion, as this estimate occurs in many forecast computations of recent years [18]. It appears necessary to underline the point that we confront an acute necessity to develop more secure and precise methods of the estimation of the acceptable stationary population of the world, as well as of particular countries. On the other hand it appears difficult at this point not to mention a curious historical fact connected with a remarkable prediction made by one of the founders of the Utopian Socialism, Charles Fourier (1772–1837). He believed that it was needed to "establish the balance of population, a certain proportion between the number of consumers and the production forces", and he contended that in order to do this it might be necessary in future to stabilize the world population at a level of "proportionality of means and needs" that he estimated to be about 5 billion [19].

According to Gorshkov [15], the Earth's biota can only regulate and stabilize the environment if the value of the human consumption of the primary biological output does not exceed about 1% of all the output of the biosphere. He has also calculated that this value of the acceptable biological consumption corresponds to about 1 billion of the world population, which corresponds to the Earth's population in about the 1820s. According to Gorshkov's estimates, at present the humans consume about 22–23% of the biosphere output, which is very close to the abovementioned estimates by Wackernagel *et al.* [14]. Hence, such estimates imply that the human population has already exceeded the acceptable limit of the natural biosphere stability more than 20 times, whereas it first exceeded it already in the early 19th century.

For describing the demographic dynamics in post-transitional period the model offered in [20] seems the most suitable. This model is based on a non-linear differential equation with three lags (12) where the expression for the environment (the Earth biosphere) instant carrying capacity takes into account the level of technological development (13).

Two scenarios of world population dynamics generated by the model (12)–(13), along with the growth trajectory according to Kapitsa model (4), are presented in Fig. 1. The first scenario forecasts the maximum population number of 9 bln in 2050 with a subsequent fall and stabilization at the stationary level of 5.2 bln in oscillation regime. Let us note that the maximum population value in this scenario is achieved at the critical time calculated by Johansen and Sornette [1] in the mode of log-periodic oscillations ($t_C = 2052 \pm 10$ years). The second scenario describes the aperiodic return to the stationary level with the maximum population number of 7.7 bln being achieved in 2030. With the assumptions specified above, the sustainable development scenario by Kapitsa model with asymptotic achievement of maximum population number of 11.4 bln does not appear to be very plausible, as it exceeds the stationary biosphere capacity (estimated at 5.2 bln [18]) more than twice [20] (for possible mechanisms that may lead to respective catastrophic scenarios see, *e.g.*, [21]). In order to avoid catastrophic scenarios, serious amendments should be made to the world economic, ecological, and demographic policies that could increase the number of people that the Earth could support, on the one hand, and slow down the population growth in the World System periphery, on the other [22].

Against this background it appears necessary to note that the non-linear differential equation with lags (12) has been employed above for the modeling of smooth continuous trajectories of demographic macrodynamics (see, for example, above the diagrams of world population dynamics at Fig. 1). The article by Yukalov, Yukalova, and Sornette [23] suggests an interesting approach to the transformation of logistic equations of type (12) with certain time lags and variable carrying capacity of the biosphere to the shape that allows to model punctuated staircase-like growth (or decline), which corresponds better to real processes in natural and social macrosystems. In the near future, we plan to use this approach in order to investigate the punctuated equilibria regimes on the basis of the further development of model (12)–(13) along the lines suggested by Yukalov, Yukalova, and Sornette [23].

2 Influence of the world demographic phase transition on the global socioeconomic and geopolitical processes

Consider now socioeconomic and geopolitical problems generated by the above described demographic phenomena. We have already mentioned that at present the humankind is in the state of the global demographic transition that started in the 19th century [4,5]. The developed countries have already finished it; however, the majority of the developing countries have not finished it yet. As is well known, the demographic transition goes from the regime of high fertility and high mortality to the regime of low fertility and low mortality. During the first phase of the demographic transition the mortality decline takes place whereas the fertility decline tends to lag behind and is mostly observed during the second phase of the demographic transition (see, e.g., [5]). Those processes will result in the world population reaching its maximum in the 2050s and the start of the world demographic decline. This implies the acceleration of the world population aging, which is already observed in the demographically developed countries of the world, and this could affect negatively the economic development. In the USA and West Europe the numerous baby-boom generation starts to retire. As the birth (and total fertility) rates were at their peak in the 1950s, the new generation of pensioners will be the most numerous in history. That is why in the forthcoming 20–30 years we are likely to deal with an avalanche-like process of retirements and the growth of expanses on pensions and health care for the elderly. The current low fertility rates and high continuously growing life expectancies in the developed countries just aggravate the situation.

According to the OECD, at present in the world we have nine working age adults per one pensioner in developing countries, whereas in the developed countries (with their aging populations) there are only four. By 2050 the number of the elderly in this proportion will double, and for the world on average we will have just four working age adults per a pensioner, whereas in the developed countries this proportion will drop to 2 : 1 [24].

For example, in Japan the working-age population peaked in 1995 at 87.3 million, whereas afterwards it contracted to 81.5 million with the number of pensioners growing from 18.3 million to 29 million. In the near future the situation will aggravate and by 2050 the number of pensioners in Japan will reach 35 million, whereas the working-age population will contract to 54 million, that is just 1.5 working age adult per a pensioner!

Countries of the European Union and the USA will get into the "Japanese" demographic trap a bit later. Developed countries try to solve the forthcoming problems by the raising of the retirement age. However, at this point the governments tend to confront a strong resistance on the part of employees, as was observed recently in France. But the rise of the retirement age does not solve the main problem. The fact is that the productivity of labor tend to decrease with age (after a certain point), whereas the salaries tend to increase. On the other hand, the elderly in this case tend to occupy some of the jobs that could be taken by the young, which could lead to an increase in the unemployment among the youth.

Within such a context, in order to provide for the pensioners at the acceptable level a country should secure rather high rates of economic growth. However, it turns out to be rather difficult to secure high rates of economic growth in conditions of the general population aging.

The life expectancy at birth (and the number of the elderly) has been increasing more or less steadily for about 300 years. However, the contraction of the young population is a much younger phenomenon. However, even now many young people suspect that by the moment of their retirement the workers : pensioners ratio might be too low to secure for them acceptable level of pensions [24].

A continuous (for the last 50 years) growth of the working age cohort (generated by the baby boom) in the USA and Europe is coming to its end; its contraction is beginning as a result of the fertility decline of recent decades. In order to mitigate the problem of population aging these countries need an additional influx of the young population, and in the context of low fertility this can only be produced by the immigration. Thus the need to accept this influx is determined for the North Americans and Europeans by their own internal conditions.

In most Asian countries, a rather fast growth of the working age population continues with its peak forecasted for the 2020s after which more and more Asian countries will start entering the contraction phase. This contraction will be slower than in Europe as the initial postwar level of fertility in Asia was generally much higher than in Europe. As a result, the share of the working age population in most Asian countries will exceed more and more the one in the countries of Europe and Asia. Such changes in the age structure are of major importance, as at the phase of the growth of the working age population proportion one can observe the formation of especially favorable conditions for the economic growth. And this implies that in the forthcoming decades Asian countries will have an evident advantage with respect to the West as regards the demographic dividend [25,26]. Afterwards the turn of Sub-Saharan Africa will come. The contribution of the demographic dividend to the economic growth turns out to be rather substantial. According to calculations made by David Bloom and Jeffry William [27] on the basis of data on 78 countries in and outside Asia, the increase in the working age population by 1% tends to lead to the growth of per capita income by 1.46%.

Such a demographic perspective leads one to expect the further slow-down of the economic growth rates in the developed countries and further acceleration of the economic growth in the developing countries. This implies that the gap in per capita income between the center and semiperiphery of the World System is likely to decrease more and more fast at least up to the mid-21st century [26]. There is a strong likelihood that in conditions of globalization technological and other innovations will flow from the center to the semiperiphery and periphery of the World System much faster than before. Hence, the technological gap between them will continue to decrease. And in combination with the demographic dividend, this could support the abovementioned accelerated economic growth in the World System periphery and semiperiphery [26].

Within this context we still observe the growing surplus of labor force in the World System periphery and its growing deficit in the World System center, which makes the flow of labor from the periphery quite a regular and inevitable phenomenon. For the forthcoming decades the forecasted annual migration gain of the European Union exceeds 0.7 million per year, whereas the long-term demographic forecast for the USA envisages that the migration gain of the USA in the first half of this century will be about 45 million [28], that is about one million a year.

The migration to rich countries allows the migrants from poor countries to reach a higher standard of living, to secure a better education for their children, and so on. In addition, the migration serves as a certain mechanism of redistribution of financial resources between the rich North and the poor South. The transfers of funds earned by migrants bring to developing countries significant gains, including the growth of investment activities. These transfers increase substantially the well-being of their families in their homeland and bring down levels of poverty there. According to some estimates, the overall amount of these transfers in 2004 (about $160 billion) was just a bit smaller than the amount of foreign direct investment ($166 billion), and it exceeded twice the official developmental aid ($79 billion) on the part of the developed countries [28]. All these make the periphery – center migration more and more attractive for the "southerners" and the migration pressure on the rich North increases.

The potential supply of the labor force on the part of the developing countries is still enormous, whereas the demand on the part of the developed countries is rather limited. There are also additional problems of sociocultural adaptation. There is a growing antipathy toward immigrants due to the fear of the immigration as a serious factor of destabilization.

Yet, the demographic pressure of overpopulated South on the World System center is growing and every year it becomes more and more difficult to control it. The demographic mass of the periphery and semiperiphery (about 6 billion) is many times larger than the one of the center (about 1 billion). Vishnevsky compares this demographic pressure with the tectonic movements of geological plates producing earthquakes and tsunamis, maintaining the following: "The current international migrations should be regarded not only as a mechanism of demographic… 'feeding' of the depopulating countries of the 'Northern Ring' dosed in accordance with their demands. It may be more important that this is one of those factors that somehow counteract the demographic and economic misbalance between the North and the South letting the excessive steam out from the overheated world 'boiler'. Thus, does it make sense to counteract the growing migration pressure on the developed countries, to erect a dam against this migration flow, a dam that may not be able to withstand the pressure? Will not it be more farsighted to think over the upgrading of the migration 'valve' and to consider the expansion of the immigration capacity of the 'Northern Ring' countries, including Russia, as an independent task put forward by history and stemming not only from the internal needs of those countries, but from the whole global situation?" [28]. This task is directly connected with the processes of the global demographic and economic transition and it demands strategic decisions on the part of the world community.

3 Explosive growth of oil and gold prices as a precursor of deep cyclical crises in the global economy

The value of the approach outlined in [1], is that it allows for the forecast of the economic crisis. This is important for the dynamics of capital (physical, financial, aggregate). Indeed, according to the Kondratieff theory of cycles and crises [29], it is the explosive growth of capital, the so-called overaccumulation of capital that causes the economy to collapse in a crisis recession. Note also an article by Sornette and Woodard [30] where the leverage leading to the "illusion of the perpetual money machine" is advanced as the main cause of instability and non-sustainability in the economic sphere.

In a number of seminal works by Didier Sornette, Anders Johansen and their colleagues [11,31–34] it has been demonstrated that accelerating log-periodic oscillations superimposed over an explosive growth trend that is described with a power-law function with a singularity (or quasi-singularity) in a finite moment of time t_C,

$$x(t) = a - b(t_c - t)^{\beta}\{1 + c\cos[\omega \ln(t_c - t) + \phi]\}, \tag{14}$$

are observed in situations leading to crashes and catastrophes. They can be analyzed as their precursors which allow the forecasting of such events. One can mention such examples as the log-periodic oscillations of the Dow Jones Industrial Average (DJIA) that preceded the crash of 1929 [32], or the changes in the ion concentrations in the underground waters that preceded the catastrophic Kobe earthquake in Japan on the 17th of January, 1995 [33], which are also described mathematically rather well with log-periodic fluctuations superimposed over a power-law growth trend.

In this article, we suggest that an explosive growth of prices of such highly liquid commodities as gold and oil can be regarded as a precursor of deep cyclical crises in the world economy and propose a possible algorithm for the calculation of a probable time of the start of a next crisis based on the approximation of prices of some key commodities with a power-law function with accelerating log-periodical oscillations superimposed over it.

2008 evidenced the start of the global recession of the world economy precipitated by the subprime mortgage crisis that blew up in the USA in 2007. The World System in general confronted the deepest crisis in the world economy since the Great Depression. For the majority of the expert community such a deep economic crisis turned out to be totally unexpected, almost nobody was able to predict it in a convincing way. Only those, who analyzed the situation on the basis of the theory of Kondratieff long waves predicted a few years in advance the world economic crisis of 2008–2010 at the downswing phase of the 5th Kondratieff cycle [35]. As global cyclical crises in the world economy are connected with the Kondratieff-wave (K-wave) dynamics, their advance can be predicted if we know the duration of the forthcoming K-wave. Due to the long-term trend toward the shortening of the period of Kondratieff waves, the duration of the current (5th) Kondratieff wave (that started around 1982) was estimated to be about 36 years, whereas a major economic crisis was expected about 8–10 years before the end of the 5th long wave at its downswing phase [35] (for example, see the K-waves in the global GDP growth rates).

Note that at the transition from A- to B-phases of the recent K-waves we observe the pattern whereby typically we observe a first (energy crisis) [36] and a second (capital) wave [40].

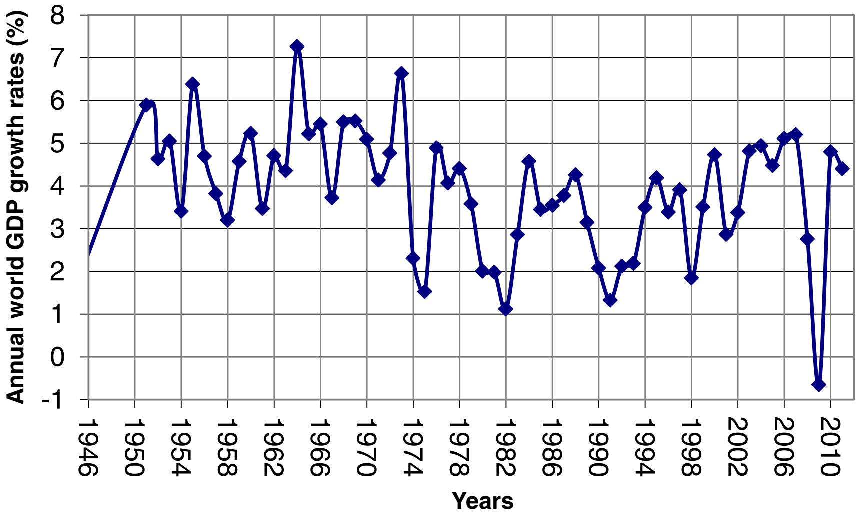

Fig. 2. Post-World War II global GDP annual growth rate dynamics. *Sources:* World Bank. *World Development Indicators Online.* Washington, DC: World Bank, 2011. URL: http://data.worldbank.org/indicator/NY.GDP.MKTP.PP.KD[2]; Maddison A. *World Population, GDP and Per Capita GDP, A.D. 1–2008.* 2010. URL: www.ggdc.net/maddison[3]; Conference Board. *The Conference Board Total Economy Database*, January 2011. URL: http://www.conference-board.org/data/economydatabase/.[4]

We suggest that the moments of the start of cyclical crises can be forecasted in a more accurate way through the analysis of the dynamics of oil and gold prices. Energy is the main production resource; that is why the structure and level of the energy consumption characterize the state of the world and national economies, as well as the level and quality of human lives. There is a tight connection between energy consumption and economic growth. In the present time the oil is still the dominant energy carrier, though the era of gas seems to be forthcoming.

The basic argument is that industrial production depends first and foremost on energy (*e.g.*, oil). The industry is stimulated by low energy prices and will increase its production, leading to more demand and investment and hence, increased production again. In this self-reinforcing cycle, the price of oil will grow at a non-sustainable level (bubble). If investor believes that industrial production peaks, demand for energy will fall very quickly, leading to a crash in the price of oil (corresponding to the first wave of global economic crisis).

Marchetti and Nakicenovic were the first to pay attention to periodic sharp increases in dominant energy carriers' prices that tend to correlate with passages from one K-wave to another [36]. Those price surges tend to last about 10 years and manifest the start of a major structural shift in energy consumption. We suggest that those price surges can be regarded as precursors of global cyclical crises in the world economic and financial system. Indeed, when the world economy goes through a K-wave upswing phase, the principal energy carrier price (in accordance with the Kondratieff cycle theory) tends to be at a low stationary level determined by the costs of production and transportation. However, with the transition to the K-wave downswing,

[2]Estimates of the world GDP annual growth rates for 2004–2009.

[3]Estimates of the world GDP annual growth rates for 1940–2003.

[4]Estimate of the world GDP annual growth rate for 2010 and a forecast for 2011.

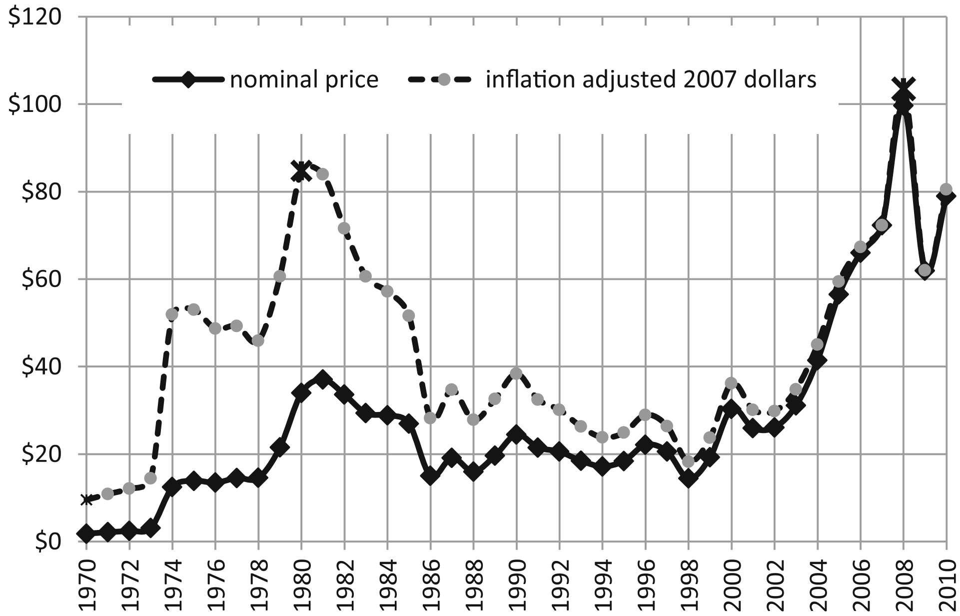

Fig. 3. Yearly oil price dynamics, 1970–2010.
Note: 1970–1973 prices are the official price of a barrel of Saudi Light, 1974–1985 prices are refiner acquisition costs of imported crude oil, 1986–2010 prices are spot prices for West Texas Intermediate at Cushing, OK.
Sources: Earth Policy Institute (Washington, DC, 2010) database (URL: www.earth-policy.org/datacenter/xls/update67_5.xls, oil prices for 1970–2006); U. S. Energy Information Administration database. URL: http://www.eia.doe.gov/dnav/pet/pet_pri_spt_s1_a.htm (oil prices for the period 2007–2010); World Development Indicators Online (Washington, DC: World Bank, 2010), URL: http://data.worldbank.org/data-catalog/world-development-indicators (data on the inflation in the U.S.).

capitals tend to move to such commodities with an almost absolute liquidity as oil and gold, producing an explosive growth of respective prices, as can be seen in Figs. 3 and 4: As can be seen in Fig. 3, immediately after the oil shock of the early 1970s, the oil prices grew up to $50/barrel; afterwards they jumped in a rather sharp way up to the maximum level of $95/barrel in 1979, which manifested the start of the 1980–1982 economic crisis. Afterwards they declined up to the mid-1980s, then they more or less stabilized in $25–35/barrel corridor up to 2003. The growth restarted afterwards. In about 2004–2006 the world economy entered the downswing phase of the 5th K-wave, which is likely to continue till 2017–2018. This led to the acceleration of the growth of oil and gold prices (see Figs. 3 and 4). Since 2006 within a year and a half the oil prices grew in a rather sharp way from $60 to $145/barrel. Almost immediately after that the World System entered the acute phase of the global economic crisis whose start is usually dated to September 15, 2008. After July 2008 the oil prices dropped in an equally precipitous way to $30; however, rather soon afterwards they returned to a rather stable diapason between $70 and $90/barrel (they only went out of it in 2011 in a direct connection with the 2011 Arab Revolution). It may be maintained that the oil prices that exceed significantly the level of 2006–2007 can be regarded as critical, as the ones that generate global economic crises (note that in spring 2011 we were dealing just with such a price level).

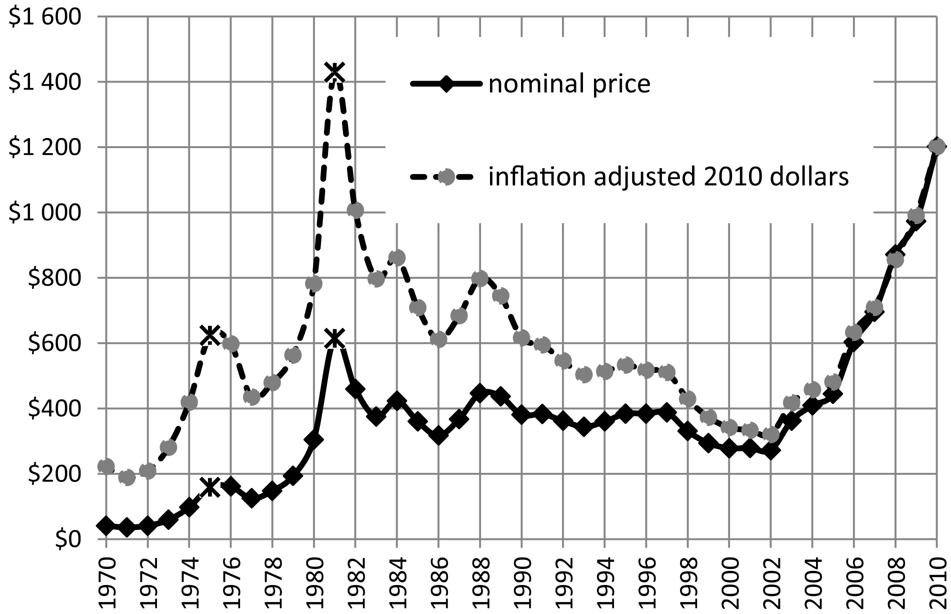

Fig. 4. Yearly gold price dynamics, 1970–2010. *Note:* yearly London fixing averages per a troy ounce of gold. *Sources*: World Gold Council database. URL: http://www.research.gold.org/prices/ (gold prices for 1970–2009); USA Gold Reference Library database. URL: http://www.usagold.com/reference/prices/history.html (average price for January 4–November 12, 2010); World Development Indicators Online (Washington, DC: World Bank, 2010), URL: http://data.worldbank.org/data-catalog/world-development-indicators (data on USA inflation).

4 Mathematical models of power-law trends with finite-time singularities and accelerating log-periodical fluctuations superimposed over them as a tool to forecast financial-economic crises

Approximate empirical data on the oil and gold prices for relevant periods with a power-law function characterized by a quasi-singularity with log-periodical oscillations superimposed over it:

$$p(t) = p_0 + p_1(t_C - t)^\beta \{1 + p_2 \cos[\omega \ln(t_C - t) + \phi]\}. \tag{15}$$

Figs. 5 and 6 display empirical data on the oil and gold price dynamics in the years that preceded respectively the first and second waves of the current global financial-economic crisis. They also display the approximation curves described by function (10). It can be seen how the accelerating log-periodic oscillations are superimposed over the explosive power-law trend, whereas the steady decrease of the period of those oscillations makes it possible to forecast the moment of the start of the crisis recession in the economy in a more accurate way in comparison with the use of the power-law trend only (this methodology is described by us in more detail in [37]).

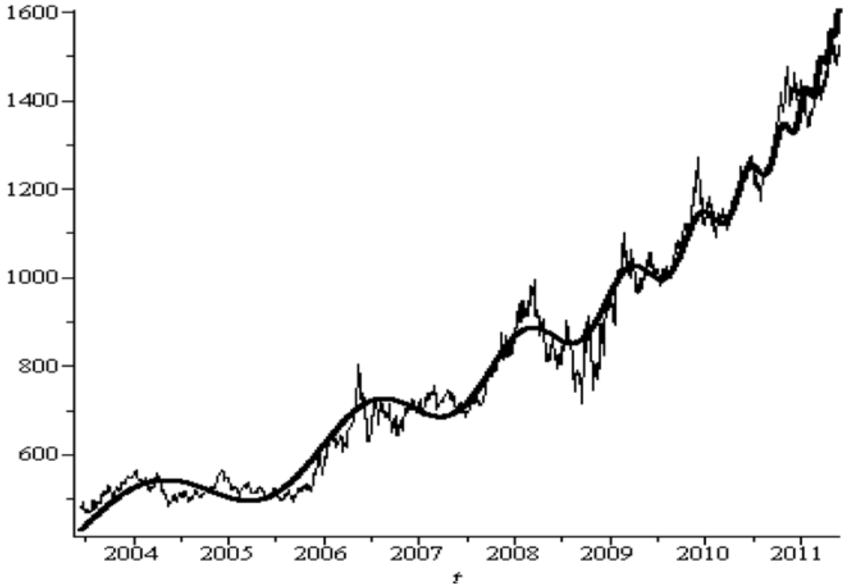

Fig. 5. Log-periodic oscillations in the gold price dynamics, November 8, 2003–May 26, 2011, inflation adjusted March 2011 US dollars/troy ounce.

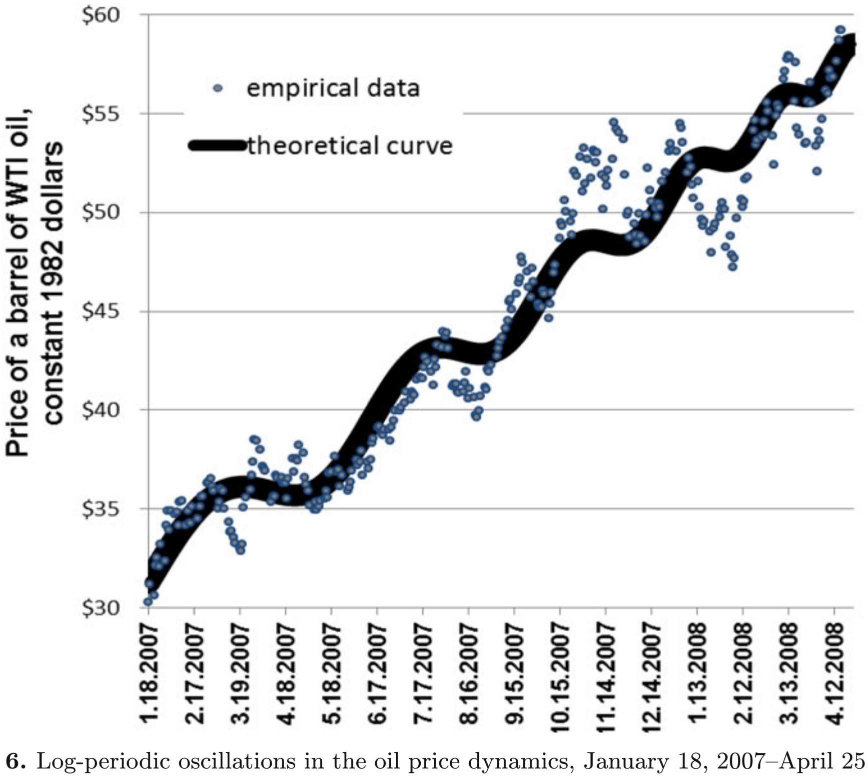

Fig. 6. Log-periodic oscillations in the oil price dynamics, January 18, 2007–April 25, 2008 (taking inflation into account; constant 1982 dollars).

On the basis of empirical data we have calculated with the least-squares method (see [37] for more detail) the following values of calibrating parameters of the approximating function (10):

a) for oil prices: $p_0^{(o)} = 66.9$; $p_1^{(o)} = 22.84$; $p_2^{(o)} = 0.0557$; $\beta^{(o)} = 0.915$; $\omega^{(o)} = 22.097$; $\phi^{(o)} = 0.471$; $t_C^{(o)} =$ August 31, 2008;

b) for gold prices: $p_0^{(g)} = 1978.2$; $p_1^{(g)} = -734.8$; $p_2^{(g)} = 0.024$; $\beta^{(g)} = 0.36$; $\omega^{(g)} = 16.5$; $\phi^{(g)} = -36.3$; $t_C^{(g)} =$ July 27, 2011.

It appears necessary to note at this point that the best-fit periodic component identified by our calculations has nothing to do with Kondratieff waves and Juglar cycles as the periodicity (omega) we find is much shorter than K-wave or J-cycles.

The calculated critical time for the oil price series in question corresponds to August 31, 2008 (note that this is quite close to the moment of the beginning of the acute phase of the world financial-economic crisis that is usually dated to September 15, 2008). According to Sornette [38], the actual start of a bubble's burst tends to begin about 1.4 months prior to a critical time, which in this case corresponds to approximately July 18, 2008. As has been already mentioned, the actual collapse of the oil prices started on July 14, 2008. Thus, a calculation with equation (10) could have allowed to forecast in 2008 the start of the oil price collapse in a rather accurate (within a week) way approximately 80 days before its actual beginning (somehow similar results have been also arrived at by Sornette, Woodard, and Zhou [39]).

In the meantime, our calculations indicate that with the use of the data for an earlier period the error starts growing [37]. This is connected with the point that log-periodic oscillations, preceding the oil crash of July 2008 can only be traced since early 2007, and for an accurate calculation of the critical time on the basis of data for earlier periods (*e.g.*, for January 1–December 31, 2007) we lack a sufficient number of log-periodic oscillations and data points that is necessary for an exact forecast of the critical time (the situation with the dynamics of gold prices was somehow different in this respect, which apparently allowed to forecast the gold price crash for a longer period in advance).

In the meantime the gold prices did not reach their maximum by the start of the first wave of the crisis, and after a relatively modest decline they continued to grow. The point is that during economic crises the gold enjoys an especial demand on the part of investors, as it serves as the most secure substitute for national currencies and shares. When investors see energy prices falling (and the economy contracting), they would try to save their wealth by fleeing into more secure assets. As fiat money is not safe in hazard crisis, people fall back buying gold which is traditionally believed to be very crisis resistant. Hence, as industrial production plunged, demand for gold sharply rose. As more and more people were attracted by gold, gold prices rose very quickly (bubble) and subsequently, in August 2011, ended in a crash that marked the second wave of global economic crisis (note that this happened after the gold prices had exceeded the $1900 level, just as was predicted by our calculations [41]).

That is why already in December 2011 we indicated that the explosive increase in the amount of investments in gold indicated the forthcoming advent of the second wave of the crisis in August 2011 [42]. We drew attention to the fact that the gold price dynamics experienced in the recent years a clear explosive power-law pattern with a finite-time singularity that indicated a fundamental illness of the world economy due to the overproduction of money that could not be invested effectively in real economy and moved in explosively accelerating fashion to the gold assets, and suggested that we should expect the start of the second wave of the global crisis around the time of the respective singularity. Within the gold price dynamics (see Fig. 5) log-periodic oscillations can be quite confidently traced since 2003, and by December 2011 we had a sufficient number of data points and already attested log-periodic oscillations in order to forecast that the second wave of recession would start in summer 2011, as appears to have been indicated by the critical point in the gold price dynamics.

The explosive growth of gold prices also indicated a precipitous decline of dollar. The weakening of the American currency was caused by the emission of trillions of dollars that did not have any adequate support. In its turn it provoked a race of devaluations of national currencies that led to the threat of a global currency war. In the world economy the money supply without any adequate support is growing in the avalanche-like manner, which is likely to lead to hyperinflation, and, consequently, a deep recession (not excluding a world currency war).

As nominal wealth and real wealth had already decoupled after the first wave (inflation), people tried to save some of their nominal wealth by moving to a seemingly save asset (gold). As more and more people replaced the fiat money (US dollar) by gold (with only limited supply), the economy started experience movement toward hyperinflation and money stopped properly playing its function.

In order to mitigate the consequences of the second wave of crisis, it appears necessary to secure the transition to the basket of internationally recognized currencies, to introduce certain elements of the gold standard or to put the dollar emission under the international control. Only such determined steps of the international community can secure a scenario of a smooth blow out of the bubble.

This, of course, raises a number of very important questions: Could joint international control not lead to more synchronization of the countries business cycles and make the problem even more severe? Would such a regulation not just treat the symptoms, but does not solve the problem at the root? Are there any serious grounds to really believe that the US as a sovereign country will accept any inference with its currency policy?

We believe that such questions are very appropriate, but to answer them adequately we would need one more article of no smaller size than the present one.

Thus, oil and gold prices can be regarded as indicators whose dynamics may point to global cyclical crises connected with the Kondratieff cycles. Such global cyclical crises include, for example, the world economic crisis of the 1970s, as well as the current crisis that started in 2008. Yet, how could one interpret intermediate recessions connected with the mid-range Juglar cycles? Juglar cycles are a specific outcome of dynamics that is typical for the manufacturing industries, from which the flourishing or depression diffuse to other spheres of modern economy with its highly developed communication system. That is why the changes in the output of manufacturing industries producing durable goods may be regarded as indicators of phases of the Juglar cycle, and as a means for the forecasting of local economic crisis (like the one that was observed in 2001 in the USA, and that was caused be serious problems in the sphere of IT technologies).

In the world of the present day the hopes for the economic upswing are connected with a wide introduction of innovations on the basis of nanotechnologies, biotechnologies, gene engineering and regeneration of human organs. It cannot be excluded that it is this sphere where in the forthcoming decade one will see the emergence of problems that will be able to generate the next economic crisis if the potential commercial capabilities of the abovementioned technologies are overestimated.

The situation is further aggravated by the current phase of the global demographic transition discussed in the previous section of this article, the consequent population aging of the World System core and the growth of the age dependency ratios that contribute to the tendency toward the decline of the economic growth rates in the World System core.

Thus, in the forthcoming years it will be necessary to monitor in a rather thorough way directions of the main investment currents and to control constantly their changes. On the other hand, it appears necessary not to forget that the trigger mechanism of most economic crisis of the recent decades were produced by financial crises connected with bursts of "financial bubbles". For example, the main contribution to the generation of recession in the USA economy in 2001 was made by the stock market crash that began in the late 2000. Note, however, that this still occurred within the A-phase of the 5^{th} Kondratieff wave, and, hence, this was accompanied by a significantly different pattern of commodity price dynamics. Indeed the crash of the dot-com bubble was accompanied by a little crash in oil. But, gold price was already falling for several years and then from 2002 raising till August 2011.

5 Conclusions

1. Dynamic processes involving non-linear power-law behavior are observed in such apparently diverse spheres, as demographic dynamics and dynamics of prices of highly liquid commodities such as oil and gold. All the respective variables exhibit features of explosive growth containing precursors indicating approaching phase transitions/catastrophes/crises.
2. Our analysis of mathematical models of demographic dynamics that describe various scenarios of demographic development in the post-phase-transition period (including a model that takes the limitedness of the Earth carrying capacity into account) points to a critical point in the early 2050s, when the world population, after reaching its maximum value may decrease afterward stabilizing then at a certain stationary level.
3. The global demographic transition generates a considerable number of socioeconomic and geopolitical problems. For example, the current phase of the global demographic transition is characterized by the population aging of the World System core and the growth of the age dependency ratios that contribute to the tendency toward the decline of the economic growth rates in the World System core, which acts as one of major factors of the World System restructuring that could results in a series of particular severe crises, including the current global crisis.
4. Our study of the phenomenon of explosive growth of prices of such highly liquid commodities as oil and gold demonstrates that at present the respective processes could be regarded as precursors of waves of the global financial-economic crisis that will demand the change of the current global economic and political system.
5. We have shown that the moments of the start of the first and second waves of the current global crisis can have been forecasted with a model of accelerating log-periodic fluctuations superimposed over a power-law trend with a finite singularity developed by Didier Sornette and collaborators. We suggest that the 2008 wave of the world economic crisis could have been predicted through the approximation of the oil price dynamics with a power-law function with a finite quasi-singularity and log-periodic oscillations superimposed over it.
6. The proposed method has allowed us to forecast the start of the second wave of the crisis in July–August 2011 through the approximation of the gold price dynamics with the above described function and the calculation of the critical time with the least squares method.
7. In order to mitigate the consequences of the second recession wave, it appears necessary to put an end to the currency wars through the limitation of the dollar emission under the international control, the introduction of some elements of the gold monetary standard, or the transition to a basket of internationally recognized currencies.

We would like to express our gratitude to two anonymous referees of our article for their invaluable comments and recommendations.

Appendix. Equations used in the article

$$N_t = \frac{C}{t_C - t}, \tag{1}$$

where N_t is world population at time t, t_C is the critical time, C is a constant.

$$\frac{dN}{dt} = \frac{N^2}{C}. \tag{2}$$

$$\frac{dN}{dt} = \frac{C}{(t_1 - t)^2 + \tau^2}, \tag{3}$$

where the characteristic time τ represents, according to Kapitsa [4], "the effective duration of human life".

$$N = K^2 \mathrm{arcctg}\left(\frac{t_1 - t}{\tau}\right), \quad K^2 = \frac{C}{\tau}. \tag{4}$$

$$K = 60100; \quad t_1 = 1995; \quad \tau = 45\,\text{years}. \tag{5}$$

$$q_A = \frac{dA}{Adt} = bN, \tag{6}$$

where q_A is the technology growth rate, $b = const.$

$$q_A \sim (t_C - t)^{-1} \tag{7}$$

$$\frac{dA}{Adt} = bC\frac{dN}{Ndt}. \tag{8}$$

$$A = A_0 N^{1+\delta} = A_0 C^{1+\delta}(t_C - t)^{-(1+\delta)}, \tag{9}$$

where $\delta = bC - 1$.

$$Y = \gamma N^{2+\delta}, \quad \gamma = const \tag{10}$$

$$Y = \gamma C^{2+\delta}(t_C - t)^{-(2+\delta)}. \tag{11}$$

$$\frac{dN}{dt} = rN^2(t - \tau_1)\left\{1 - \frac{N(t)}{K(N,\tau_2,\tau_3)}\right\}, \tag{12}$$

where τ_1 is the medium time of the onset of reproductive capacity and the expression for the environment (the Earth biosphere) instant carrying capacity $K(N,\tau_2,\tau_3)$ takes into account the level of technological development.

$$K(N,\tau_2,\tau_3) = N_C + \gamma\,[N(t - \tau_2) - N_0]\exp\{-\kappa\,[N(t - \tau_3) - N_0]\}, \tag{13}$$

where τ_2 is the time of basic technologies diffusion; τ_3 is the lag of biosphere reaction to the anthropogenic load ($\tau_1 = 25$ years, $\tau_2 = 25$–30 years, $\tau_3 = 100$ years); N_C is the stationary number of population ($N_C = 5.2$ bln people); γ, κ and r are constants. N_0 ($= 1$ billion) is the world population corresponding to the acceptable bioconsumption.

$$x(t) = a - b(t_c - t)^\beta\{1 + c\cos[\omega\ln(t_c - t) + \phi]\}. \tag{14}$$

$$p(t) = p_0 + p_1(t_C - t)^\beta\,\{1 + p_2\cos[\omega\ln(t_C - t) + \phi]\}. \tag{15}$$

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