Electron waves in the passbands and stopbands of periodic slow-wave systems

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Abstract: With the help of the finite-difference equation of excitation of periodic waveguides obtained the characteristic equation of the electron waves formed by the interaction of an electron beam with the forward and backward electromagnetic waves periodic slow-wave systems (SWS). The resulting characteristic equation describes the electron-wave interaction in the passband and stopband of periodic SWS. Found a number of analytical solutions of the characteristic equation, which allowed to compare the properties of the amplification and propagation of electron waves inside and on the border of passband and stopband of periodic slow-wave systems.

Keywords: slow-wave system, electron beam, interaction, TWT, passband, stopband.

Introduction

The equations of the discrete electron-wave interaction in TWT with periodic slow-wave structures (SWS) of the type of a coupled cavities system or of a disk-loaded waveguide have been analyzed in [1,2] and the interaction equations of small-signal theory have been obtained. Unlike in the previously known works, the equation of excitation of periodic waveguides with second-order finite differences and the local interaction impedance instead of Pierce's coupling impedance are used in the suggested theory. In this paper obtained the characteristic equation of the electron waves describes the electron-wave interaction in the passband and stopband of periodic SWS. Thsis universal characteristic equation includes as special cases of well-known equations for the "smooth" SWS (for example helical SWS) [3,4] and equation by using the equivalent circuit for the resonator SWS [5-8] . Found a number of analytical solutions of the characteristic equation, which allowed to compare the properties of the amplification and propagation of electron waves inside and on the border of passband and stopband of periodic SWS.

Universal characteristic equation of electron waves and its particular cases

Let us consider a straight electron beam in a SWS with interaction gaps placed periodically with step *L*. In [1,2] using the finite-difference equation excitation found the coefficient matrix that relates the dimensionless variables RF beam current I, the electron velocity (kinetic potential) V and the field F in q +1- m gap interaction with their values in the two previous gaps.

$$\begin{aligned} I_{q+1} &= a_{11} I_q + a_{12} V_q + a_{13} F_q, \\ V_{q+1} &= a_{21} I_q + a_{22} V_q + a_{23} F_q, \\ F_{q+1} &= a_{31} I_q + a_{32} V_q + a_{33} F_q + a_{34} F_{q-1}, \end{aligned} \qquad (1)$$

The solution can be found in the form of electron waves, which $I_{q+1} = \lambda I_q$, $V_{q+1} = \lambda V_q$, $F_{q+1} = \lambda F_q$, $\lambda = e^{i\psi}$. Equating to zero the determinant of the system, we get the equation of the fourth degree on their eigenvalues, due to a finite- difference excitation equations , which relates the value of the field on the three steps of the SWS. As a result, opening the determinant of the system, we obtain a universal characteristic equation of electron waves in periodic structures, obtained in [9]:

$$\begin{aligned} &(cos\psi - cos\theta_q)[cos\varphi_s - \mu \cdot cos(\varphi_e + \psi) - \\ &-i(\varepsilon\varphi_e)^3 (Y_1 - iY_2)] + G = 0. \end{aligned} \qquad (2)$$

This equation defines a complex perturbation of the phase shift ψ of the electron wave at the step of SWS. Value ε have the meaning of the gain parameter C of TWT with the difference that it is expressed through the local interaction impedance, and therefore has no singularities on the cutoff frequency. G-interaction function, $\mu = \pm 1$ for SWS with normal and anomalous dispersion, respectively θ_q is the step and transit angles, measured in plasma wavelengths. For a small space charge and fine gaps interaction: $\theta_q = 1, Y_{1,2} = 0$. Because usually gain parameter is small $\varepsilon \ll 1$, $\psi \ll 1$, then, and from (2) algebraic characteristic equation of the fourth degree for perturbations. In the theory electron devices a small perturbation is usually normalized: according L.A.Vainshtein $\eta = \dfrac{\psi}{\varepsilon\varphi_e}$ according J.Pierce $\delta = x + iy = -i\eta^* = \dfrac{-i\psi^{/} - \psi^{//}}{\varepsilon\varphi_e}$.

Then the characteristic equation of electron waves in the periodic structure of the form:

$$\frac{\varepsilon}{2}\cos\varphi_e \eta^4 + \frac{\sin\varphi_e}{\varphi_e}\eta^3 + \frac{\cos\varphi_s - \cos\varphi_e}{\varepsilon\varphi_e^2}\eta^2 + \mu = 0. \qquad (3)$$

Electron wave in passband of SWS

SWS type for disk-loaded waveguide with normal dispersion fundamental spatial harmonic wave in the middle of the bandwidth at synchronism

($\varphi_e = \varphi_s = \frac{\pi}{2}$) obtain a cubic equation. On the border with the bandwidth of one of the spatial harmonics both direct and counter wave of SWS is synchronized with the electron beam, and we come to the fourth power of the equation that determines the 4 electronic wave

$$\eta^4 = \frac{2}{\varepsilon}, \ \psi^4 = (\varepsilon\varphi_s)^4 \cdot \eta^4 = 2\varepsilon^2\pi^4,$$

$$\psi_{1,3} = \pm 2^{\frac{1}{4}}\pi \cdot \varepsilon^{\frac{3}{4}}, \ \psi_{2,4} = \pm i \cdot 2^{\frac{1}{4}}\pi \cdot \varepsilon^{\frac{3}{4}} \quad (4)$$

We see that the constant increase of the electron waves on near cut-off frequency $\sim \varepsilon^{\frac{3}{4}}$ (and not ε, as in the middle of the passband, or "smooth" SWS). Similar results were obtained for the SWS type coupled cavity systems (CCS) with anomalous dispersion fundamental spatial harmonics, when working in a TWT is its 1st spatial harmonic($\mu = -1$, $\pi \le \varphi_s = \varphi_{s,1} \le 2\pi.$).

Electron waves in the stopbands of SWS

Periodic SWS in stopbands phase of the field throughout the volume of the SWS or the same, or jumps to π in some sections, as in the cavity, or standing waves. It is possible synchronous interaction of the electron beam and the spatial harmonics of the field. Consider different options SWS with normal dispersion fundamental spatial harmonic *(* $\mu = 1$*)*. In the low-frequency stopband have locked on the m-th spatial harmonic synchronized with the electron beam: $\varphi_s = 2\pi m + i\varphi_s^{//}$, $\varphi_e = 2\pi m$, $m = 0,1,2...$ were $\varphi_s^{//}$ determines the reactive attenuation. In the case m = 0 synchronism with nonrelativistic electron beams impossible because field in phase in the whole volume of the SWS. In other cases, we obtain from (2) the characteristic quadratic equation for $Cos\psi$, solutions are of the form:

$$cos\psi = 1 + \frac{1}{2}(ch\varphi_s^{//} - 1) \mp$$
$$\mp \frac{1}{2}\sqrt{(ch\varphi_s^{//} - 1)^2 - 2\varepsilon^3(2\pi m)^4}. \quad (5)$$

With the growth of the gain parameter ε $Cos\psi$ are real, up to a point defined by the relation

$$\sqrt{2\varepsilon_{\grave{e}\eth}^2}(2\pi m)^2 = ch\varphi_s^{//} - 1. \quad (6)$$

When ε> ε_{kr} values $Cos\psi$ are complex and thus may gain the electron waves in the low-frequency stopband. In the high-frequency stopband have locked at exact synchronism

$$\varphi_s = \pi(2m+1) + i\varphi_s^{//},$$
$$\varphi_e = \pi(2m+1), \ m = 0,1,2... \quad (7)$$

$$cos\psi = 1 + \frac{1}{2}(ch\varphi_s^{//} - 1) \mp$$
$$\mp \frac{1}{2}\sqrt{(ch\varphi_s^{//} - 1)^2 + 2\varepsilon^3\pi^4(2m+1)^4}. \quad (8)$$

In contrast to (5), we obtain the real values $Cos\psi$ and apparently no gain for any ε. SWS with anomalous dispersion fundamental spatial harmonics (μ = -1) in the low-frequency stopband of locking, the relations (5,6), ($2\pi m \to \pi(2m+1)$)

$$\sqrt{2\varepsilon_{\grave{e}\eth}^2}\pi^2(2m+1)^2 = ch\varphi_s^{//} - 1. \quad (9)$$

conclusions about the possible increase in ε> ε_{kr}.
In the high-frequency band with locking obtain solutions similar to (8) is the real value and the gain appears to be no gain for any ε.

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