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Finitely Smooth Local Equivalence of Autonomous Systems with One Zero Root

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Abstract—In this paper, in a neighborhood of a singular point, we consider autonomous systems of ordinary differential equations such that the matrix of their linear part has one zero eigenvalue, while the other eigenvalues lie outside the imaginary axis. We prove that the problem of finitely smooth equivalence can be solved for such systems by using finite segments of the Taylor series of their right-hand sides.

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Key words: *autonomous system, ordinary differential equations, finitely smooth equivalence, singular point, zero eigenvalue, Taylor series, normal form, N-jet of a function.*

1. INTRODUCTION

This paper continues the studies started in [1], [2], where we analyzed systems of ordinary differential equations whose linear part matrix has one eigenvalue, while the other eigenvalues lie outside the imaginary axis. For brevity, such systems are called *systems with one zero root*. We study the problem of local finitely smooth equivalence between systems of equations of this type such that the Taylor series of their right-hand sides differ in higher-order terms (the so-called problem of finite-definite germs of vector fields). Most of the papers concerned with such problems deal with systems with a nondegenerate singular point (these problems were reviewed to a large extent in the book [3]). For partially degenerate systems, see [4]. The problem of infinitely smooth equivalence between formally equivalent systems with one zero root or two pure imaginary roots of the matrix of the system linear part was studied in [5], [6]. Our approach to solving the problem of finitely smooth equivalence is based on the reduction of the systems to a certain special normal form (see [2]). Although we use transformations with singularities, the proposed method still permits establishing a criterion for finitely smooth equivalence between systems under study.

We consider the real autonomous system

$$\dot{\xi} = \frac{d\xi}{dt} = Q(\xi), \tag{1}$$

where $\xi, Q(\xi) \in \mathbb{R}^{n+1}$, $n > 0$, and $Q(\xi)$ is a function of class C^∞ in a neighborhood of the origin, $Q(0) = 0$, and the matrix $\widetilde{A} = Q'(0)$ has n eigenvalues lying outside the imaginary axis and one zero eigenvalue.

Let $\lambda, \dots, \lambda_n$ be eigenvalues of the matrix $\widetilde{A}$ that lie outside the imaginary axis, and let $\lambda_0 = 0$. An argument in [2] shows that, without loss of generality, we can assume that system (1) has the form

$$\begin{aligned} &\dot{x} = f(x, y), \\ &\dot{y}_i = \varepsilon_i y_{i+1} + \lambda_i y_i + g_i(x, y), \qquad i = 1, \dots, n. \end{aligned} \tag{2}$$

Here $\varepsilon_i = 0$ or 1, $f(x,y), g_i(x,y) = o(\|(x,y)\|)$, $x \in \mathbb{R}^1$, and $y = (y_1, \dots, y_n)$ is a complex vector, $g_i(x,0) = 0, 1 \le i \le n$, $f(x,0) = bx^{m+1} + cx^{2m+1}$, $b = \pm 1$, and $m \ge 1$ is an integer. The coordinates y_i

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