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Interpretation of Shewhart Control Charts: New Challenges and Opportunities

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Summary
This work shows that an interpretation of signals on Shewhart control charts (SCC) can be different from commonly used as soon as one takes into account that an assignable cause of variation can change not only the parameters of the underlying distribution but its type as well. For example, the lack of points beyond 3σ level can mean not the problems of rational subgroup formation as it is usually being explained in literature but it may be caused by a change of the distribution function (DF). Or a point beyond the control limits may signal not about the change of average or standard deviation (SD) but about the DF's change while mean and SD may remain unchanged. As a result the significance of formal approach to SCC interpretation decreases and new opportunities for process analysis emerge. At the end of this paper we stated some problems that should be discussed by all interested in the development of Statistical Process Control (SPC) and statistical thinking.

Key words: *Interpretation of Shewhart control chart, statistical process control.*

1. INTRODUCTION

Practically all processes and phenomena people encounter during their daily experience are influenced by the variability which affects the results of our activity, the decisions we make, the data we collect, the methods we control whatever and whoever, the ways of teaching and learning, healing and bringing up children and so on. The main tool of primary variability analysis is Shewhart Control Chart (SCC) which "great contribution …is to separate variation by rational methods into two sources: (1) the system itself ("chance causes," Dr. Shewhart called them), the responsibility of management; and (2) assignable causes, called by Deming "special causes," specific to some ephemeral event that can usually be discovered to the satisfaction of the expert on the job, and removed. A process is in *statistical control* when it is no longer afflicted with special

causes. The performance of a process that is in statistical control is predictable"[1]. If special causes are present the process is uncontrolled and unpredictable. That is why process improvement always starts with the analysis of process stability: if one wants to improve any process first of all he/she must ensure that it is in the state of statistical control[2-6]. Many authors interpret this state statistically as the state in which the process has a distribution function (DF) that does not change in time and that describes the probabilities to get some values of the process outcome[5-10].
But if the process becomes unpredictable after the interference of a Special Cause of Variation (SCV) and if before this the process was statistically predictable (a permanent DF did exist) then there are only three opportunities for this:

(i) the interference of SCV has changed the parameters of DF,
(ii) the interference of SCV has changed the DF itself,
(iii) the interference of SCV has made it impossible to describe the process by using the notion of DF.

The variant (i) is the most widespread in literature especially for analysis of the comparative performance of SCC. In respect to such simple charts as charts for averages and ranges ($\overline{X} - R$), charts for individuals and moving range ($x - mR$), charts for averages and standard deviations ($\overline{X} - s$) just this opportunity has been considered for the last 50 years[11] and by now it had been already investigated rather deeply.

The variant (iii) seems to be the most plausible from the viewpoint of common sense because if the interference of SCV does not limited with only one point of the process then it automatically means that in fact our system had been already changed (the old system plus the interference), and if the interference makes the process unpredictable then most probably process outcomes can not be described by using DF at all. We won't discuss this case because it is not clear how to do this even in principle.

So only the second variant is discussed below: the change of DF due to the interference of SCV. Just this case was discussed in our previous paper[11] where we suggested to divide SCV into two groups: type I (Intrinsic) and type X (eXtrinsic). SCV of type I do not change the type of the process DF but change the values of DF parameters, while SCV of type X change DF itself. This work is devoted to the consequences which follow from the opportunity of changing DF under the influence of some SCV.

2. TWO EXAMPLES OF UNUSUAL INFLUENCE OF SCV

Below we consider two very simple examples of SCV when such sample parameters as mean and SD do not change but the DF changes its functional form. This leads to the conclusions which are principally different from traditional explanations and raise some questions about the existing approach.

Example 1. The change of the DF under the influence of SCV does not cause a point beyond the control limits.

1.1 The first 100 points in Figure 1 present the chart for individuals taken from the standard normal distribution $gau\{x \mid m, s\}$ with m = 0, s = 1. The same 100 points were used for calculation of control limits shown by dotted lines. The next 100[1] points were simulated from the uniform distribution $uni\{x \mid m, s\}$ with m = 0, s = 1.

[1] The number of points – 100 - was taken because we anticipated the construction of DF on the probability paper and wanted to have a step for DF of about 1%. Actually the number of points before interference as well as after it may be whatever a process is and what is more important we never know at what moment the interference occurred.

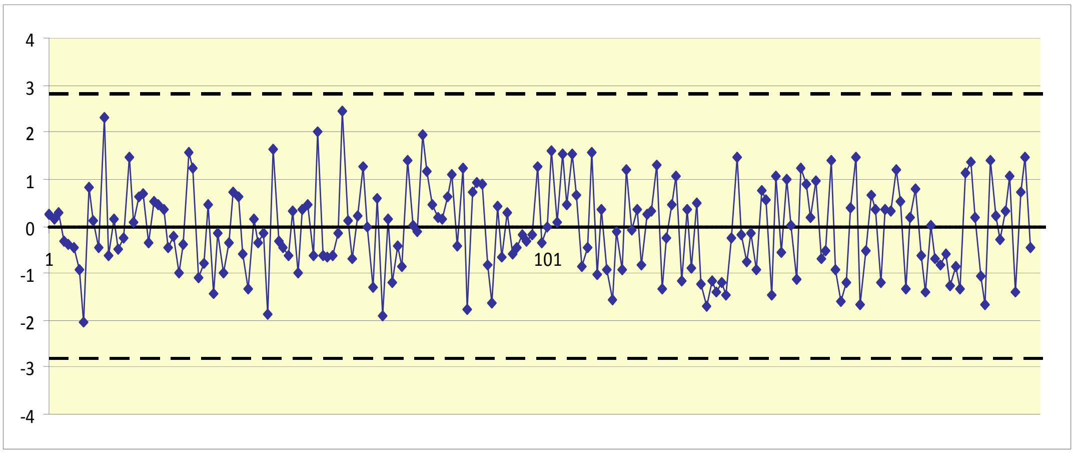

Figure 1. SCC: the first 100 points ~ gau{x|0,1}, the second ~ uni{x|0,1}

In spite of the fact that *all points are within the calculated limits* our process has changed. Having looked attentively one can see that the points stopped falling beyond two-sigma limits what is quite natural because in our model the limits of spread are equal to $\pm\sqrt{3}$ for the last 100 points. As a result we see the so-called hugging of the central line. Within the frames of traditional analysis such process behavior does not disturb the process owner (if he does not use additional rules of SCV searching) because all points lie within the control limits. And if he uses additional rules they usually interpret hugging the central line as a result of wrong stratification[7,12,13]. But in fact there are no any stratification problems here and our system has evidently changed though from fig.1 it is unclear when.

Moreover our DF corresponds now to the uniform distribution so all probabilities for points to fall beyond both of the control limits and tolerances (if they are established and wider than control limits) are equal to zero. And this is principally another situation than we had before. Besides if we found out the root cause of the process change and made it permanent then both control limits and tolerances could be made more narrow. In practice we can encounter such situation when, for example, our supplier has changed something in his technology but not informed us about this change.

1.2 The first 100 points are again a sample from $gau\{x\,|\,0,1\}$ and x-chart is constructed by these values. Then we simulated 100 additional points taken from the exponential distribution with the same values of mean and standard deviation: *exp{x|0, 1}*.

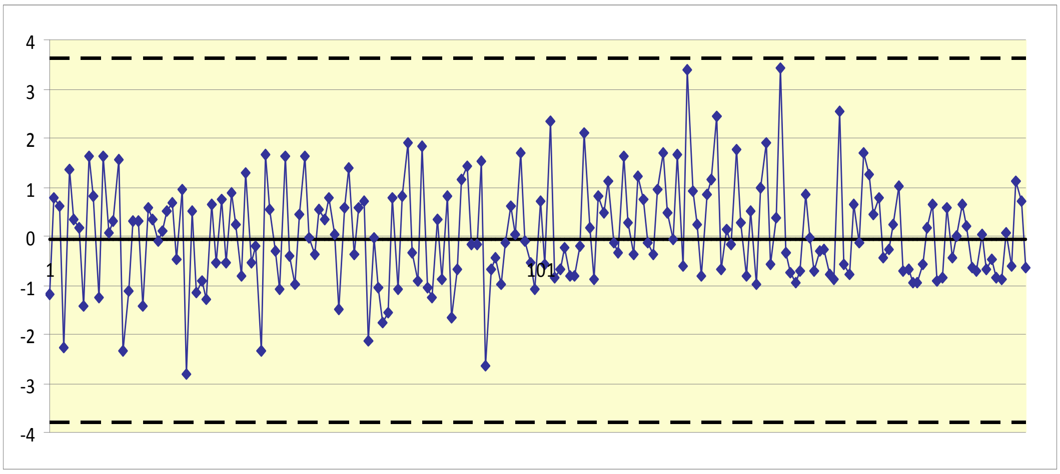

Figure 2. SCC: the first 100 points ~ gau{x|0,1}, the second ~ exp{x|0, 1}.

Though all points are lying within the control limits it is obvious that everything is not OK with our process. One can see a sharp asymmetry in the process behavior in the right part of the panel. Besides the last 20 points have notably diminished zone of variability. But neither the main rule of Shewhart nor most of additional rules reveal the presence of SCV except the rule of hugging the central line again. And again there is no any problems with data stratification here. But in fact Figure 2 shows us an example of one of the most general rule for detecting the lack of control: the rule of WOW. This rule is working every time when the process change is obvious at the first glance though we have no special rule for it and do not know what is the cause of the intervention.

Example 2. The change of the DF under the influence of SCV does cause a point beyond the control limits.

2.1 The first 100 points in the Figure 3 again present a chart of individual values taken from the standard normal distribution $gau\{x|0,1\}$. The next 100 are simulated from the lognormal distribution biased one unit left and having mean m = 0 and standard deviation s = 1. Thus the process setting has not changed. But now our chart gives us a clear signal of SCV presence – there is a point outside the control limits. Besides the data showings an obvious process change – the spread of data has evidently changed.

Traditionally such change is being interpreted as the change of either process average or standard deviation or both. But as we pointed out earlier these parameters have not changed at all – everything is OK with them! As a result one could try to search for the root cause of a SCV which does not exist. This could facilitate to divert him/her from making a right decision.

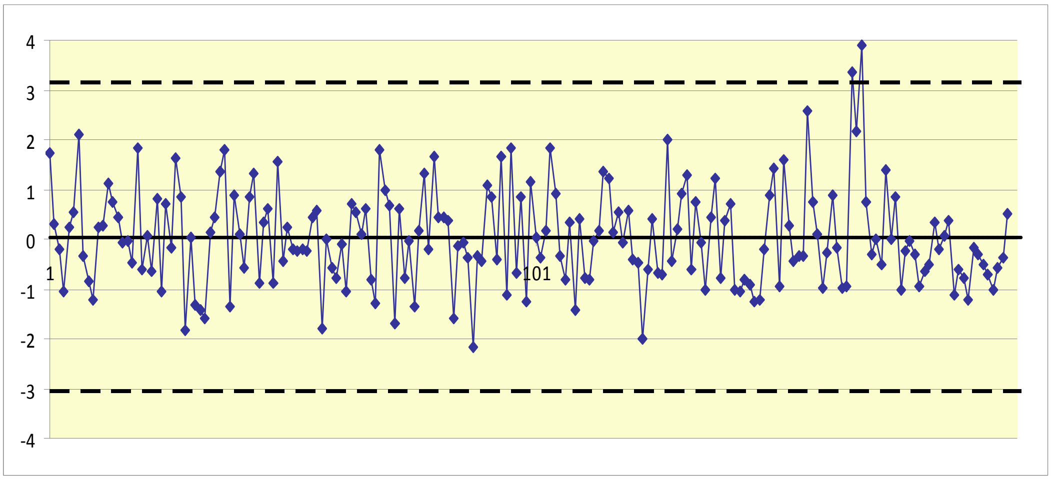

Figure 3. SCC: the first 100 points ~ gau{x|0,1}, the second ~ lgau{x}

2.2 Now (Figure 4) the first 100 points are taken from $gau\{x|0,1\}$, the second 100 – from gamma distribution with the same mean and SD, so the parameters are constant for all data. It is obvious that this chart "is crying" that our process is in uncontrollable state. Of course this situation demands to stop this process straightway and start searching for the root causes of such behavior.

3. DISCUSSION AND CONCLUSIONS

All abovementioned simulated situations relate to the SCV caused by the change of DF. As soon as we acknowledge the possibility of such interference we have to think carefully about the rules of SCC interpretation.

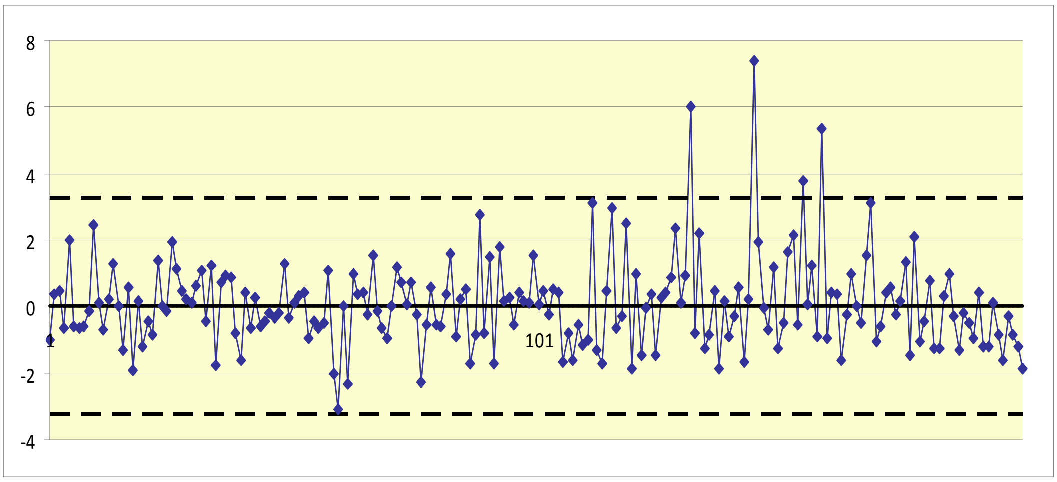

Figure 4. SCC: the first 100 points ~ gau{x|0,1}, the second ~ gam{x}

The first example shows that there may be many situations when an appropriate SCC does not signal about the lack of stability and the sample parameters of the process are unchanged while in fact a SCV present and affects our process. It is clear that in some cases we will be able to see that something is wrong though in case of Figure 1 it is not very noticeable. And in other cases we can hardly hope to discover some abnormalities – as an example one can look at Figure1a where we simulated the first 100 points from standard normal distribution then 50 points from uniform distribution with the same average and SD and then 50 points from standard normal again. If one uses only the main Shewhart rule the process on this chart will be stable. If one uses additional rule for seven or more points above or under the central line then the process will be unstable though for points 76-82 and 160-168 this conclusion is wrong and for points 132-138 it is right. (Ryan-Joiner test for normality rejects the null hypothesis for data of Figure1 and accepts it for Figure1a).

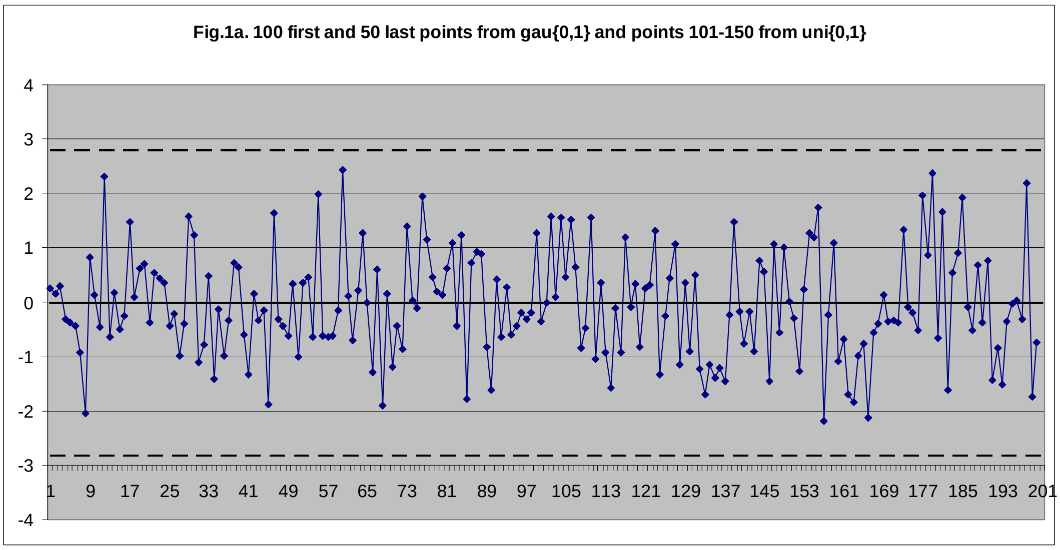

Figure 1a. 100 first and 50 last points from gau{0,1} and points 101-150from uni{0,1}

The second example shows that there may be many situations when an appropriate SCC does signal about the lack of stability but the parameters remain constant and process behavior change is caused by the change of the DF.

And of course there may be many situations when the situation is mixed: DF has changed and mean and SD have changed as well.

Certainly if the change of DF occurred and stayed for a long time we must say that our system has changed and this cause of the interference is not a special one but this is just a change of terminology which does not alter the essence of the problem: how can we know about this?

It is worth noting that Shewhart understood that there might exist situations "where common sense suggests the use of a criteria other than one of the five"[14*] Saying this he referred to the picture where the smooth downward trend within the control limits was shown. This and many other similar situations led to the appearance of the well-known additional rules for chart interpretation though different authors give various recommendations for their use[6-8,12,13,15,16] and there is no commonly accepted agreement on what set of rules should be used. But the first Shewhart rule – a point outside control limits – turned out to be the main rule of SCC interpretation. It has always kept its leading role as the basic tool for detecting the lack of process stability. But now we see that the number of cases when even the main Shewhart rule does not work may be much more diverse that it seemed to us earlier. So we suggest the following.

3.1 Conclusion 1

We should add a very general rule of WOW to the existing list. The rule of WOW means that before any calculations of control limits one must depict a run chart of the process and look at it very attentively searching for any patterns, structures, abnormalities and so on (see, for example, a paper by W. Scherkenbach on www.spcpress.com/pdf/only_reason_to_collect.pdf). In fact this rule existed in practice[6,16] but unfortunately was not discussed enough in statistical textbooks and theoretical papers. But we think that it should be the leading rule and SCC interpretation should start from this rule.

Secondly, it is necessary to refuse to use some single-valued recommendations that are widely recommended in literature. For example: a point beyond the control limit on a chart for averages indicates the change in the process mean, and a point beyond the control limit on a chart for ranges indicates the change in the process standard deviation. Though these recommendations may be true they do not describe all possible situations in reality and as such they may lead to the wrong decisions.

3.2 Conclusion 2

We should leave all possible explanations of the causes of the process behavior change to the process owners – people who control their processes and are accountable for them. There may be many more different causes of some type of process behavior in reality and our statistical recommendations may divert people from searching in the right direction. SCC is a very delicate tool for talking with a process and this talking should be made by a person who knows this process in all its details. In other words SCC is a badly formalizable analysis tool and just this bad formalizability is the source of its prognostic power.

Thirdly we need to check up our looks at the comparative performance of different types of charts within the frames of wider look at the origin of special causes of variation? For example we know that Cusum-chart is an excellent alternative to the SCC in the phase II (monitoring)[8] but will this assertion be true when the DF changes? How will small process shifts will affect the properties of Cusum or EWMA charts if simultaneously the DF is changing?

* Shewhart introduced five criteria of SCV presence in his first book but only his first criterion became worldwide used.

3.3 <u>Conclusion 3</u>

We should start a wide investigation of influence of SCV with different DF on the properties of SCC. We think that such work can not be done by the efforts of one or two groups of professionals, so wide participation of many experts is necessary.

At last we would like to formulate some problems for further discussion.
(i) What is the relationship between the process stability and the behavior of DF parameters? In other words, if SCC shows that our process according to a corresponding operational definition is stable what can we say - if can – about the process parameters?
(ii) How can we reveal the change of DF if SCV was "ephemeral"? And what is the ephemeral SCV from the viewpoint of operational definitions? In other words as we have already mentioned if SCV came and stayed then we have a change of a system. But if it appeared and then disappeared (as in Figure 1a) how can we reveal this when, for example, the average and SD have not changed? What is the best way to discover such a change?

We hope that participation of many experts in the discussion on these issues could encourage the development of Shewhart-Deming ideas in the area of SPC.

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