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XXXI International Seminar on Stability Problems for Stochastic Models

and

VII International Workshop "Applied Problems in Theory of Probabilities and Mathematical Statistics Related to Modeling of Information Systems"

and

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Book of Abstracts

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В сборник включены тезисы докладов, представленных на XXXI Международный семинар по проблемам устойчивости стохастических моделей (ISSPSM'2013), VII Международный рабочий семинар "Прикладные задачи теории вероятностей и математической статистики, связанные с моделированием информационных систем" (APTP + MS'2013) (весенняя сессия) и Международный рабочий семинар "Прикладная теория вероятностей и теоретическая информатика".

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On approximation of power type estimator of survival functions by censored data

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Incomplete or censored observations occur in survival analysis, especially in clinical trials and engineering when we partially observe death in biological organisms or failure in mechanical systems. There are several models of random censoring. In this work we deal only with right censoring model.

Let $X_1, X_2, ...$ be a sequence of independent and identically distributed random variables(i.i.d.r.v.-s) (the lifetimes) with common distribution function(d.f.) F. Let X_j be censored on the right by Y_j, so that observations available for us at the n-th stage consist of the sample $S^{(n)} = \{(Z_j, \delta_j), 1 \le j \le n\}$, where $Z_j = min(X_j, Y_j)$ and $\delta_j = I(X_j \le Y_j)$ with $I(A)$ meaning the indicator of the event A. Suppose that Y_j are again i.i.d.r.v.-s, the so-called censoring times with common d.f. G, independent of lifetimes X_j.

The main problem consist of nonparametrically estimating F with nuisance G based on censored sample $S^{(n)}$, where r.v.-s of interest X_j-s observed only when δ_j=1. Kaplan and Meier (1958) were the first to suggest the product-limit (PL) estimator F_n^{PL} defined as

$$F_n^{PL}(t) = \begin{cases} 1 - \prod_{\{j: Z_{(j)} \le t\}} \left[1 - \frac{\delta_{(j)}}{n-j+1}\right], & t \le Z_{(n)}, \\ 1, & t > Z_{(n)}, \delta_{(n)} = 1, \\ undefined, & t > Z_{(n)}, \delta_{(n)} = 0, \end{cases}$$

where $Z_{(1)} \le ... \le Z_{(n)}$ are the order statistics of Z_j and $\delta_{(1)}, ..., \delta_{(n)}$ are the corresponding δ_j. In statistical literature there are different versions of this estimator. However, those do not coincide if the largest Z_j is a censoring time. Gill (1980) redefined the F_n^{PL} setting $F_n^{PL}(t) = F_n^{PL}(Z_{(n)})$ when $t > Z_{(n)}$. At present there is an enormous literature on properties of the PL-estimator (see, for example [3]-[9]) and most of work on estimating incomplete observation are concentrated on PL-estimator. However F_n^{PL} is not unique estimator of F.

The second, closely related with the F_n^{PL}, nonparametrical estimator of F is the exponential hazard estimator

$$F_n^E(t) = 1 - exp\Big\{ -\sum_{j=1}^{n} \frac{\delta_{(j)} I(Z_{(j)} \le t)}{n-j+1} \Big\}, -\infty < t < \infty.$$

F_n^E plays an important role in investigating the limiting properties of the estimator F_n^{PL}.

Abdushukurov (1998,1999) proposed another estimator for F of power type:

$$F_n(t) = 1-(1-H_n(t))^{R_n(t)} = \begin{cases} 0, & t < Z_{(1)}, \\ 1-(\frac{n-j}{n})^{R_n(t)}, & Z_{(j)} \leq t < Z_{(j+1)}, 1 \leq j \leq n-1, \\ 1, & t \geq Z_{(n)}, \end{cases}$$

where

$$H_n(t) = \frac{1}{n}\sum_{j=1}^{n} I(Z_j \leq t)$$

is empirical estimator of d.f. $H(t) = P(Z_j \leq t) = 1-(1-F(t))(1-G(t))$ and

$$R_n(t) = \frac{-log(1-F_n^E(t))}{\sum_{j=1}^{n} \frac{I(Z_{(j)} \leq t)}{n-j+1}}.$$

As we see, estimator F_n is defined on whole line. Let

$$d(t) = \int_{-\infty}^{t} [(1-F)^2(1-G)]^{-1} dF.$$

For the estimator F_n of power type we have prove a consistency and Gaussian approximations results on weak and strong forms up to some large order statistics in the sample with the rates depending on the order of these statistics. In order to choose the order statistics we choose a sequence $\{k_n\}$ of integers such that $1 \leq k_n < n$ and require the condition:

($\mathbb{C}$) $k_n \geq \log n$ for all n large enough $\{\frac{k_n}{n}\}$ is asymptotically nonincreasing. We have

Theorem. Let d.f.-s F and G are continuous. Then

$$\text{I.} \quad \sup_{t \leq Z_{(n-k_n)}} \frac{|F_n(t)-F(t)|}{1-F(t)} = \begin{cases} \mathbb{O}_p(k_n^{-1/2} + k_n^{-2} n) \\ \mathbb{O}((k_{2n}^{-1} \log n)^{1/2} + k_n^{-2} n), \; a.s. \end{cases}$$

II. There exist a sequence $\{\mathbb{W}_n(\cdot), n \geq 1\}$ of Wiener processes such that

$$\sup_{t \leq Z_{(n-k_n)}} \left| \frac{n^{1/2}(F_n(t)-F(t))}{1-F(t)} - \mathbb{W}_n(d(t)) \right| = \begin{cases} \mathbb{O}_p\left(k_n^{-1} n^{1/2} \log n + k_n^{-2} n^{3/2}\right) \\ \mathbb{O}\left(k_{2n}^{-1} n^{1/2} \log n + k_n^{-2} n^{3/2}\right), a.s. \end{cases}$$

In monographies [3,5] of author one can find several extensions of estimators F_n, F_n^{PL} and F_n^E with full asymptotical results theory (weak convergence, law of itherated logarithm type strong consistency, weak and strong approximation, empirical Bayes approach ...) in competing risks models with random censorship from the right and both sides.

References

1. Abdushukurov, A.A. (1998). Nonparametric estimation of the distribution function based on relative risk function. *Commun. Statist.:Theory and Methods*, **27**, N.8, pp. 1991-2012.

2. Abdushukurov, A.A. (1999). On nonparametric estimation of reliability indices by censored samples. *Theory Probab. Appl.*, **43**, N.1, pp. 3-11.

3. Abdushukurov, A.A. (2009). *Statistics of Incomplete observations: Asymptotical Theory of Estimation for Nonclassical Models.* University Press. Tashkent.

4. Abdushukurov, A.A.(2011). Nonparametric estimation based on incomplete observation. International Enciclopedia of Statistical Sciences. Springer. Part 14. pp. 962-964.

5. Abdushukurov, A.A.(2011). Estimation of unknown distributions from incomplete data and its properties. LAMBERT Academic Publishing.

6. Akritas, M.G. (2000). The central limit theorem under censoring. *Bernoulli, 6,* N.6, pp. 1109-1120.

7. Csörgő, S. (1996). Universal Gaussian Approximations under Random Censorship, *Ann. Statist.*, **24**, N.6, pp. 2744-2778.

8. Gill, R.D. (1980). *Censoring and Stochastic Integrals*, Mathematical Centre Tracts, **124**. Amsterdam.

9. Gill, R.D. (1994). Glivenko-Cantelli for Kaplan-Meier. *Math. Meth. of Statist.*, **3**, N.1, pp. 76-87.

10. Kaplan, E.L., Meier, P. (1958). Nonparametric estimation from Incomplete observations. *J.Amer.Statist.Assoc.*, **58**, pp. 457-481.

Large deviations for queueing system with a regenerative input flow

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Our result is a generalization of the classical $GI|GI|1$ analysis in point of view large deviations theory. The strength of the theory is that one can draw broad conclusions for systems, which are otherwise hard to analyze, without relying on calculations. Besides, large deviations theory is the basic tool for studying rare and non-desirable events.

We consider a single-server queueing system with a regenerative input flow $A(t)$. Definition of regenerative flow one can see i.e. in [1]. Let $\{\theta_i\}_{i=0}^{\infty}$ ($\theta_0 = 0$), $\{\tau_i = \theta_i - \theta_{i-1}\}_{i=0}^{\infty}$ be sequences of points and intervals of regeneration respectively, $\{\xi_i\}_{i=0}^{\infty}$ be numbers of customers entering the system during the

ith period of regeneration ($\xi_i = A(\theta_i) - A(\theta_{i-1})$). We assume that $\mu = \mathsf{E}\tau_i < \infty$, $a = \mathsf{E}\xi_i < \infty$. The sequence of service times $\{\eta_i\}_{i=0}^{\infty}$ consisting of i.i.d.r.v.'s does not depend on input $A(t)$. Denote $\lambda = a/\mu$, $b(s) = \mathsf{E}e^{-s\eta_i}$, $b = \mathsf{E}\eta_i$ and introduce the function $G(z,s) = \mathsf{E}z^{\xi_i}e^{-s\tau_i}$.

Assumption 1. The greatest common divisor of numbers $j = 1, 2, \dots$ such that $\mathsf{P}(\xi_i = j) > 0$ is equal to one.

Assumption 2. There exist $\delta > 0$ and $M < \infty$ such that

$$\mathsf{P}(\xi_i \le M) = 1, \quad \mathsf{E}e^{\delta\tau_j} < \infty, \mathsf{E}e^{\delta\eta_j} < \infty.$$

We consider a workload process $W(t)$. Let t_n be the time of the nth customer's arrival ($n = 1, 2, \dots$). Introduce embedded processes $W_n = W(\theta_n - 0)$, $w_n = W(t_n - 0)$. If $\rho = \lambda b < 1$ then these processes are ergodic and there exist the limits

$$\Phi(x) = \lim_{n\to\infty} \mathsf{P}(W_n \le x), \quad F(x) = \lim_{n\to\infty} \mathsf{P}(w_n \le x).$$

Theorem. Let $\rho < 1$ and assumptions 1 and 2 fulfilled. Then there exist the limits

$$\lim_{x\to\infty} \frac{1}{x} \log(1 - \Phi(x)) = \lim_{x\to\infty} \frac{1}{x} \log(1 - \Phi(x)) = -q,$$

where q is the unique positive solution of the equation

$$G(b(-q), q) = 1.$$

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References

1. L.G.Afanasyeva, E.E. Bashtova. Limit Theorems for Queues in heavy traffic situation. Modern problems of Mathematics and Mechanics IV, No. 1, 40–54 (in Russian) (2009)

Optimal design for parameters of a shifted Ornstein-Uhlebeck sheet

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The problem of optimal design for parameter estimation and for prediction of a shifted Ornstein-Uhlenbeck sheet is studied, where for parameter estimation we consider D-optimality, while for prediction integrated mean square prediction error (IMSPE) and entropy criterion are used. For classical Ornstein-Uhlenbeck processes D-optimality was studied e.g by Kiseľák and

Stehlík [4], while optimal predictions with respect to IMSPE and entropy were investigated by Baldi Antognini and Zagariou [1]. In the present work we derive exact D-optimal designs for estimation of the shift parameter (Baran and Stehlík [2]) and optimal designs for prediction with respect to entropy criterion (Baran et al. [3]) in the cases when the design points form a regular grid or a monotonic set. This later design is motivated by problems when one has measurement on the sets with a specific geometric shape, e.g. measurement along the isotherms. We also investigate optimality with respect to IMSPE criterion.

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References

1. A. Baldi Antognini, M. Zagoraiou. Exact optimal designs for computer experiments via Kriging metamodelling. *J. Statist. Plann. Inference.*, 2010, vol. 140, p. 2607–2617.

2. S. Baran, M. Stehlík. Optimal designs for parameters of shifted Ornstein-Uhlenbeck sheets measured on monotonic sets. *Appl. Stoch. Model. Bus.*, submitted.

3. S. Baran, K. Sikolya, M. Stehlík. On the optimal designs for prediction of Ornstein-Uhlenbeck sheets. *Statist. Probab. Lett.* (2013), doi:10.1016/j.spl.2013.03.003.

4. J. Kiseľák, M. Stehlík. Equidistant D-optimal designs for parameters of Ornstein-Uhlenbeck process. *Statist. Probab. Lett.*, 2008, vol. 78, p. 1388–1396.

Trimming of dependent sequences and applications

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Trimming is a standard method to decrease the effect of large sample elements in statistical procedures used, e.g., to construct robust estimators and tests. Trimming of i.i.d. sequences has been extensively studied from the 1960's and most basic problems of the theory have been solved, except a few isolated problems, e.g. the CLT under modulus trimming. In contrast, very little is known about trimming of dependent sequences, even though results here would be very useful e.g. in the statistics of heavy tailed processes. We formulate a few new results in this direction.

(a) We prove a functional CLT for trimmed AR(1) processes with stable errors, leading to a change point test for the unknown parameter of the process.

(b) We prove the CLT for trimmed -mixing sequences, with applications in the theory of continued fractions.

Our method also gives insight into the central limit theory of modulus trimmed i.i.d. sums, showing that the difficulties in the classical theory can be removed by allowing random (but sample dependent) centering sequences in the CLT.

References

1. Berkes, I. Horváth, L. and Schauer, J.: Asymptotics of trimmed CUSUM statistics. *Bernoulli* **17** (2011), 1344–1367.

2. Berkes, I. Horváth, L.: The central limit theorem for trimmed sums with heavy tails. Submitted.

3. Bazarova, A., Berkes, I. Horváth, L.: Change point detections with stable AR(1) errors. Submitted.

4. Bazarova, A., Berkes, I. Horváth, L.: Trimmed stable AR(1) processes. Submitted.

Stein's method and a quantitative Lindeberg CLT for the Fourier transforms of random vectors

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We use a multivariate version of Stein's method to establish a quantitative Lindeberg CLT for the Fourier transforms of random N-vectors. We achieve this by deducing a specific integral representation for the Hessian matrix of the solution to the Stein equation with test function $e_t(x) = \exp\left(-i\sum_{k=1}^{n} t_k x_k\right)$ (where $t, x \in \mathbb{R}^N$).

Kernel quantile estimators in dose-effect relationships over indirect data

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In many applications, the problem of constructing efficient estimators of quantile function often arises. For example, when runway areas are optimized with a specified limitation on the landing safety, in the process of forming optimum securities portfolios, and in risk management, one should solve quantile optimization problems (see [1, 2]). It is well known (see [3, 4]) that such a popular index as the value at risk (VaR) is properly a distribution quantile of price changes. The optimal solution to these problems is based on constructing efficient estimators of the quantile function using both complete and incomplete samples. In the present paper, we consider the problem of constructing estimators of the quantile function and studying their asymptotic behaviour using incomplete samples, namely, in the dose-effect relationships over indirect data.

Let $\mathcal{U}^{(n)} = ((U_1, W_1), (U_2, W_2), ..., (U_n, W_n))$ be n independent and identically distributed with (U, W) pairs of random variables, where $W = I(X < U)$ is an indicator of event $(X < U)$, X and U are random variables with distribution functions $F(x) = \mathbf{P}(X < x)$, $G(x) = \mathbf{P}(U < x)$ and with density $f(x) > 0$, $g(x) > 0$ respectively. The examined model is interpreted as a dose-effect relationships, where U is a random dose of the substance brought into an organism, and X is the lower bound, where the effect begins ($W = 1$). This situation is called a random plan of the experiment. If the dose U is nonrandom that plan is called fixed. In this case we assume that $U_i = u_i$, $i = 0, 1, ..., n+1$, where $0 = u_0 < u_1 < ... < u_n < u_{n+1} = 1$. Let us consider the statistics $\hat{F}_n(x) = \dfrac{1}{n}\sum_{i=1}^{n} W_i K_h(i/n - x)$ – to be the kernel estimator of the distribution function $F(x)$ based on fixed plans, where $K_h(x) = \dfrac{1}{h} K\left(\dfrac{x}{h}\right)$.

Define the quantile estimator ξ_p of order $0 < p < 1$ by $\hat{\xi}_{np} = \inf\{x \in \mathbf{R}, \hat{F}_n(x) \geq p\}$. These observations are called direct data and in this case behaviour of the estimator $\hat{\xi}_{np}$ was studied in work [5].

However, quite often data measurements are made with an error ε, which is a random variable with a known or an unknown continuous distribution function $Q(x)$ and density $q(x)$. In other words, instead of the data $(U_i, W_i), 1 \leq i \leq n$ we observe $(Y_i, W_i), 1 \leq i \leq n$ where $Y_i = U_i + \varepsilon_i$, $W_i = I(X_i < U_i)$. How can limit distributions of estimator $\hat{\xi}_{np}$ change? How robust is this estimator containing measurement errors?

In this report, we show that the limit distributions of the estimators $\hat{\xi}_{np}$ in the presence of errors are asymptotically normal, but their limit variances in case of indirect observations differ from the limit variances in case of direct ones.

Indeed, let $y_i = u_i + \varepsilon_i$. We consider two situations:

(i) u_i are random or nonrandom variables ; measured errors ε_i are independent and identically distributed variables;

(ii) true doses are observed with an error, but this dose u_i is taken, so that the equality $u_i + \varepsilon_i = i/n$ was held, where i/n are specified.

It is shown that in case (i) we have $\hat{\xi}_{np} \xrightarrow[n\to\infty]{p} \int_{-\infty}^{\infty} F(\xi_p - y)q(y)\,dy$ under suitable regularity conditions.

If the density distribution $q(y)$ is an even function, i.e. $q(y) = q(-y), y \in \mathbf{R}$, that in case (ii) we also have $\hat{\xi}_{np} \xrightarrow[n\to\infty]{p} \int_{-\infty}^{\infty} F(\xi_p - y)q(y)\,dy$.

Limit variances of the considered estimators are presented.

References

1. A. I. Kibzun, E. Matveev. Optimisation of the quantile function on the basis of kernel estimates. *Automatica and Telemekhanika*, 2007. No.1. pp. 68–80.

2. U. S. Kan, A. I. Kibzun. *Problems of stochastic programming with probability criterions.* M.: Fizmatlit, 2009.

3. Holton G.A. *Value-at-Risk. Theory and Pratice.* Academic Press, 2003.

4. Araujo Santos P. *Excess, Durations and Forecasting Value-at-Risk.* Universidade de Lisboa, 2011.

5. M. S. Tikhov, T.S. Borodina. Kernel estimators of quantiles in dose-effect relationships. *Automatic Control and Computer Sciences*, 2013, No. 2, pp. 29–43.

Systems with several replenishment sources

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Multi-supplier inventory systems have always attracted attention of many researchers. An excellent review of results obtained before 2003 can be found in Minner [1]. During the last decade various methods were used in the framework of cost approach for investigation of such systems. Thus, Afanaseva and Bulinskaya [2] have studied the systems with several suppliers and seasonal demand by queueing methods. On the other hand, linear and dynamic programming methods were employed in Wang et al. [3], Fox et al. [4], Bulinskaya [5], see also references therein.

We mention in passing that multi-source replenishment is typical not only for inventory theory but also for insurance, finance and other applied probability domains, see, e.g., Bulinskaya [6]. Hence, in addition to cost approach, we use a reliability one as well, treating the inventory and insurance models.

Below we formulate some results in terms of the periodic-review one-product two-supplier inventory model introduced in Bulinskaya [7]. It was assumed there that the second supplier is unreliable delivering the order immediately with probability p and having a one-period lag with probability $q = 1 - p$. We suppose additionally that there are capacity or budget constraints. That means, the order sizes z_i, $i = 1, 2$, must satisfy either relations $0 \le z_i \le a_i$ for some $a_i < \infty$ or $c_1 z_1 + c_2 z_2 \le b$ where c_i is the unit ordering cost at the ith supplier. We take into account the unit holding cost h per period and backlogging unit penalty r.

As objective function to minimize we choose the mean discounted costs incurred in n period process. Discount factor is denoted by α and x is the initial inventory level whereas $d_i = b/c_i$, $i = 1, 2$. Let $\Delta_0 = \{0 \le c_1 \le r\}$, $\Delta^0 = \{0 \le c_2 \le pr\}$, $\Gamma = \{c_1 \ge 0, c_2 \ge 0\}$ and for $k \ge 1$

$$\Delta_k = \{(c_1, c_2) \in \Gamma : r\sum_{i=0}^{k-1} \alpha^i < c_1 \le r\sum_{i=0}^{k} \alpha^i\},$$

$$\Delta^k = \{(c_1, c_2) \in \Gamma : r(p + \sum_{i=1}^{k-1} \alpha^i) < c_2 \le r(p + \sum_{i=1}^{k} \alpha^i)\}.$$

For the case of budget constraint we prove that for some order costs it is optimal to send orders only to one supplier (even unreliable) as state the following theorems, where by $z_i^{(n)}(x)$ we denote the order sent to the ith supplier at the first step of n step process.

Theorem 1. *If* $(c_1, c_2) \in \{c_2 \ge (p+\alpha q)c_1\} \cap \Delta_k$, $k \ge 0$, *then for any* n *it is optimal to take* $z_2^{(n)}(x) = 0$. *For* $n \le k$ *one puts also* $z_1^{(n)}(x) = 0$. *There exists an increasing sequence* $\{v_n\}_{n \ge k+1}$ *such that* $z_1^{(n)}(x) = \min(d_1, (v_n - x)^+)$ *for* $n \ge k + 1$.

Theorem 2. *If* $(c_1, c_2) \in \{c_2 < c_1 - qr\} \cap \Delta^k$, $k \ge 0$, *then for any* n *it is optimal to take* $z_1^{(n)}(x) = 0$. *For* $n \le k$ *one puts also* $z_2^{(n)}(x) = 0$. *There exists an increasing sequence* $\{u_n\}_{n \ge k+1}$ *such that* $z_2^{(n)}(x) = \min(d_2, (u_n - x)^+)$ *for* $n \ge k + 1$.

For other parameters values the optimal policy involving two suppliers is much more complicated.

It is established that the models under consideration are stable with respect to small parameters fluctuations and underlying processes perturbations.

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References

1. S. Minner. Multiple-supplier inventory models in supply chain management: A review. *International Journal of Production Economics*, 2003, vol. 81-82, p. 265-279.

2. L. Afanaseva, E. Bulinskaya. Multi-supplier systems with seasonal demand. In: B.Vallespir and T.Alix (eds.) *Advances in Production Management Systems: New Approaches. Proceedings of the IFIP WG 5.7*, Springer, 2010, p. 267–274.

3. G. Wang, Zh. Jiang, Zh. Li, W. Liu. Supplier selection and order splitting in multiple-sourcing inventory systems. *Frontiers of Mechanical Engineering in China*, 2008, vol. 3, p. 23–27.

4. E. J. Fox, R. Metters and J. Semple. Optimal inventory policy with two suppliers. *Operations Research*, 2006, vol. 54, p. 389–393.

5. E. Bulinskaya. Optimal and asymptotically optimal control for some inventory models. In: A.N.Shiryaev et al. (eds.) *Prokhorov and Contemporary Probability Theory*, Springer Proceedings in Mathematics and Statistics, 2012, vol. 33, chapter 8, Springer-Verlag, Berlin.

6. E. Bulinskaya. Stochastic insurance models. Their optimality and stability. In: C.H.Skiadas (ed.) *Advances in Data Analysis*, 2010, Birkhäuser, Boston, p. 129–140.

7. E. Bulinskaya. Optimal control and stability of some inventory and insurance models. *XXX International Seminar on Stability Problems for Stochastic Models*, Svetlogorsk, September 24-30, 2012, p. 19–20.

Asymptotic behavior of statistics employed for high-dimensional data analysis

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The development of new methods for analysis of data having huge dimensions is of great importance. For example a challenging problem is to find the genetic and non-genetic (or environmental) factors which could increase the risk of complex diseases such as diabetes, myocardial infarction and others, see, e.g., [1]. In this regard recall that human genom contains more than 6 milliard nucleotide bases. The vast research domain called the *genome-wide association studies* (GWAS) requires new techniques for handling large arrays of biostatistical data.

The plan of the talk is as follows. After the brief introduction we concentrate on the modern methods such as multifactor dimensionality reduction (MDR) and its modifications, logic regression and machine learning. We deal with optimization problems for random functions defined on various graphs. The model selection is discussed as well. We apply also K-fold cross validation and permutation tests. Along with survey we present our quite recent results. In [2] the basis for application of the MDR-method was proposed when one uses an arbitrary penalty function to describe the prediction error of the binary response variable by means of a function in factors. Now we establish the asymptotic normality of appropriately normalized statistics used to justify the optimal choice of a subcollection of the explanatory variables. Moreover, we consider self-normalization in this variant of CLT.

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References

1. A. Bulinski, O. Butkovsky, V. Sadovnichy, A. Shashkin, P. Yaskov, A. Balatskiy, L. Samokhodskaya, V. Tkachuk. Statistical methods of SNP data analysis and applications. *Open Journal of Statistics*, 2012, vol. 2, No. 1, p. 73-87.

2. A. V. Bulinski. To the foundations of the dimensionality reduction method for explanatory variables. *Zapiski Nauchnyh Seminarov POMI*, 2012, vol. 408, p. 84-101 (in Russian; English translation: Journal of Mathematical Sciences).

Asymptotic of the mean absolute error of UNVUE and MLE in the case of one-parameter exponential family lattice distributions

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Uniformly minimum variance unbiased estimators (UMVUE) and maximum likelihood estimators (MLE) play all-important role in current statistical research. A choice of the best among them can be made by computing of their precision. The precision of estimators is defined by means of absolute risk function. The author proposes solution for this problem for high sample size in the case of one-parameter exponential family lattice distributions. The absolutely continuous distributions family case were considered by Chichagov and Fedoseeva [1].

The observation model description. Let $X_1, \ldots, X_n$ a repeated sample, which elements have the same lattice distribution as observation random variable ξ. The distribution of the random variable ξ belong to exponential family, which determined with the following expression:

$$\mathbf{P}(\xi = x) = \exp\left\{\Phi_1[a]\, T[x] - \kappa[\Phi_1[a]] + d[x]\right\}, \; x \in \mathbb{X}_G \subset \mathbf{R}, \; a \in \mathbb{A} \subset \mathbf{R}. \tag{1}$$

Here $\mathbb{X}_G$ is the distribution support; $a = \mathbf{E}(T[\xi])$ is a mean value parameter of given distribution; $d[x]$, $T[x]$, $\Phi_1[a]$ are known functions; $\kappa[\theta]$ is a cumulant transform of distribution (1).

Let $\widetilde{\Theta}$ be the set of values θ satisfies

$$\sum_{x \in \mathbb{X}_G} \exp\left\{\theta \cdot T[x] + d[x]\right\} < \infty.$$

Basic results. The following regularity conditions are applied.

($\mathbf{A_1}$). The support $\mathbb{X}_G$ is contained in $\mathbf{Z}$ but not in any sublattice of $\mathbf{Z}$, and not depended on the parameter a.

($\mathbf{A_2}$). The distributions family (1) is steep and $\mathbb{A} = \left\{\kappa'[\theta] : \theta \in \mathbf{Int}[\widetilde{\Theta}]\right\}$.

($\mathbf{A_3}$). $\Phi_1'[a] > 0$ on $\mathbb{A}$.

Let us denote $\check{G}[a|S_n]$ and $\widehat{G}[a|S_n]$, where $S_n = \sum_{i=1}^n T(X_i)$, corresponding the MLE and UMVUE of the given parametrical function $G[a]$, $a \in \mathbb{A}$.

Denote

$$\epsilon = \frac{G''[a]}{2G'[a]}, \; \varphi[x] = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{x^2}{2}\right], \; U^{(k)}[a]^j = (U^{(k)}[a])^j, \; U^{(k)}[a] = \frac{d^k U[a]}{da^k},$$

$\lfloor x \rfloor$ is the fractional part of x.

Theorem. *Let conditions* $(\mathbf{A_1}) - (\mathbf{A_3})$ *are satisfied and there exists a positive integer* L *such that* $\boldsymbol{V}\widehat{G}[a|S_L] < \infty$ *and* $\boldsymbol{V}\check{G}[a|S_L] < \infty$. *Then as* $n \to \infty$

and $G'[a] \neq 0$ the following expansions

$$\boldsymbol{E}\left|\widehat{G}[a|S_n] - G[a]\right| = \frac{2\varphi[0]\,|G'[a]|}{\sqrt{n\Phi_1'[a]}}\left\{1 - \frac{\Phi_1'[a]}{n}\left(\frac{\lfloor na+\epsilon\rfloor^2}{2} - \frac{\lfloor na+\epsilon\rfloor}{2} + \frac{1}{12}\right) + \right.$$
$$+\frac{1}{n}\left(\frac{G''[a]^2}{8G'[a]^2\Phi_1'[a]} + \frac{G''[a]\Phi_1''[a]}{4G'[a]\Phi_1'[a]^2} - \frac{\Phi_1''[a]^2}{12\Phi_1'[a]^3} - \frac{G^{(3)}[a]}{6G'[a]\Phi_1'[a]} + \frac{\Phi_1^{(3)}[a]}{24\Phi_1'[a]^2}\right)\Bigg\} +$$
$$+ \boldsymbol{o}\left(\frac{1}{n^{3/2}}\right), \tag{2}$$

$$\boldsymbol{E}\left|\check{G}[a|S_n] - G[a]\right| = \frac{2\varphi[0]\,|G'[a]|}{\sqrt{n\Phi_1'[a]}}\left\{1 - \frac{\Phi_1'[a]}{n}\left(\frac{\lfloor na\rfloor^2}{2} - \frac{\lfloor na\rfloor}{2} + \frac{1}{12}\right) + \right. \tag{3}$$
$$+\frac{1}{n}\left(-\frac{G''[a]\Phi_1''[a]}{6G'[a]\Phi_1'[a]^2} - \frac{\Phi_1''[a]^2}{12\Phi_1'[a]^3} + \frac{G^{(3)}[a]}{3G'[a]\Phi_1'[a]} + \frac{\Phi_1^{(3)}[a]}{24\Phi_1'[a]^2}\right)\Bigg\} + \boldsymbol{o}\left(\frac{1}{n^{3/2}}\right)$$

are hold. If $G'[a] = 0$ but $G''[a] \neq 0$, then as $n \to \infty$

$$\boldsymbol{E}\left|\widehat{G}[a|S_n] - G[a]\right| = \sqrt{\frac{2}{\pi e}}\,\frac{|G''[a]|}{n\Phi_1'[a]} + \boldsymbol{o}\left(\frac{1}{n^{3/2}}\right), \tag{4}$$

$$\boldsymbol{E}\left|\check{G}[a|S_n] - G[a]\right| = \frac{|G''[a]|}{2n\Phi_1'[a]} + \boldsymbol{o}\left(\frac{1}{n^{3/2}}\right). \tag{5}$$

The theorem proof is mainly based on the results of Chichagov, Fedoseeva, Bhattacharya and Rao [1]–[3]. By using (2)–(5), one can obtain the asymptotic expansions for arbitrary order moments and central moments of random variable $T[\xi]$ and for probability $\mathbf{P}(\xi \leq x)$.

For some parametric functions $G[a]$, the comparison of expansions (2)–(5) will be represented in the case of the Poisson and geometric distributions.

References

1. V. V. Chichagov, N. P. Fedoseeva. Comparison of UMVUE and MLE with absolute loss function in the case of a one-parameter exponential family continuous distributions, *Statistical methods of estimation and testing of hypotheses*, 2012, Perm State University, Russia, No. 24, p. 96-109.

2. V. V. Chichagov. Stochastic expansions of the unbiased estimators in the case one-parameter exponential family. *Informatics and applications*, 2008, vol. 2, No. 2, p. 62–70.

3. R. N. Bhattacharya, R. R. Rao. *Normal approximation and Asymptotic Expanions*, 1986, Robert E. Krieger Publishing Company, Malabar, Florida.

Confidence intervals for means of lattice-valued random variables constructed using split-sample methods

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We explore the properties of distributions of sums of independent means of independent lattice-valued random variables. Let $\{X_{j1}, \ldots, X_{jn_j}\}$, for $j = 1, 2$, be two independent samples of independent random variables, where n_j denotes the sample size of the random sample j. We assume that the X_{ij} have a finite fourth moment, and that X_{1j} has a lattice distribution with maximal edge e_j, for $j = 1, 2$. Let $\bar{X}_j = n_j^{-1} \sum_i X_{ji}$ be the sample mean of sample j. Put $S = \bar{X}_1 + \bar{X}_2$, and denote by β_n the standardised skewness of the distribution of S. Define $\rho = \rho(n) = (e_2\, n_1)/(e_1\, n_2)$, take $n = n_1 + n_2$ to be the asymptotic parameter, and view n_1 and n_2 as functions of n. We are interested in an Edgeworth expansion of the distribution of S under the following two assumptions. First, assume that

$$\liminf_{n\to\infty} n^{-1} \min_{j=1,2} n_j > 0\,, \tag{1}$$

which ensures that the two sample sizes n_1 and n_2 are diverging at a similar pace. Moreover, for a positive rational number ρ_0 and some $\gamma \in (0, 1/2)$, assume that

$$|\rho - \rho_0| \asymp n^{-\gamma}\,, \tag{2}$$

where the notation $a(n) \asymp b(n)$, for positive sequences $a(n)$ and $b(n)$, means that the ratio $a(n)/b(n)$ is bounded away from 0 and infinity as $n \to \infty$. Assumption (2) ensures that the ratio $(e_2\, n_1)/(e_1\, n_2)$ converges slowly to a rational number. Let ϕ and Φ denote the standard normal density and distribution functions, respectively. We have proved the following result.

Assume that (1) and (2) hold. Then, if γ is such that $0 < \gamma < 1/4$,

$$P\left\{\frac{S - E(S)}{(\operatorname{var} S)^{1/2}} \le x\right\} = \Phi(x) + n^{-1/2}\,\frac{1}{6}\,\beta_n\,\big(1 - x^2\big)\,\phi(x) + O\big(n^{-(1/2)-\gamma}\big)\,, \tag{3}$$

whereas if $1/4 \le \gamma < 1/2$ then the remainder in (3) changes to $O(n^{\gamma-1+\epsilon})$, for any $\epsilon > 0$. In both settings, (3) holds uniformly in x as $n \to \infty$.

Expansion (3) can be specialised in the context of a single sample $\{X_1, \ldots, X_n\}$ of size n of i.i.d. random variables, rather than two distinct samples. Split the random sample $\{X_1, \ldots, X_n\}$ randomly into two parts $\{X_{j1}, \ldots, X_{jn_j}\}$ for $j = 1, 2$, where $n_1 = \langle n/2 + s_n \rangle$ and $n_2 = n - n_1$ with

$\langle x \rangle$ denoting the integer nearest to a real number x and with the sequence s_n satisfying $|s_n| \asymp n^{1-\gamma}$, for some $\gamma \in (0, 1/2)$. Let $\bar{X}_{\text{split}} = (\bar{X}_1 + \bar{X}_2)/2$ be a split-sample version of the conventional sample mean $\bar{X} = n^{-1} \sum_i X_i$. Like $\bar{X}$, $\bar{X}_{\text{split}}$ is unbiased for $E(X)$. Under assumptions (1) and (2), expansion (3) becomes

$$P\left\{ \frac{\bar{X}_{\text{split}} - E(X)}{(\operatorname{var} \bar{X}_{\text{split}})^{1/2}} \leq x \right\} = \Phi(x) + n^{-1/2} \frac{1}{6} \beta_0 \left(1 - x^2\right) \phi(x) + O\left(n^{-\xi}\right), \quad (4)$$

where β_0 denotes the standard skewness of the distribution of X, and

$$\xi = \begin{cases} 1/2 + \gamma & \text{if } \gamma \in (0, 1/4) \\ 1 - \gamma - \epsilon & \text{for any } \epsilon > 0, \text{ if } \gamma \in [1/4, 1/2) . \end{cases}$$

On the other hand, the Edgeworth approximation to the standardised distribution of $\bar{X}$ contains a uniformly bounded but highly oscillatory term of the same order as the correction for skewness, see Esseen [3]. In particular, this term does not cancel from expansions of coverage error of two-sided confidence intervals based on normal approximations.

Expansion (4) shows that the main effects of distribution smoothness can be understood in terms of ρ. In particular, when ρ converges sufficiently slowly to a rational number, the effects of the discontinuity of lattice distributions are of smaller order than the effects of skewness. This is true in many other cases too, for example when ρ converges to an irrational number, see Decrouez and Hall [2]. In other words, the normal approximation under assumptions (1) and (2) is more accurate that one would usually get for the distribution of the sum of sample means of lattice-valued random variables. Therefore, constructing those confidence intervals using the split-sample approach, with n_1 and n_2 carefully chosen, produces confidence intervals with more accurate coverage than conventional approaches. A numerical study in the context of estimating a binomial proportion or a Poisson mean shows that split-sample methods perform well when it is used to modify confidence intervals based on existing techniques that already perform very well. In particular, we compare our approach with intervals considered in [1], and show that the split-sample intervals typically have better coverage accuracy.

References

1. L. D. Brown, T. T. Cai, A. DasGupta. Confidence intervals for a Binomial proportion and asymptotic expansions. *Ann. Statist.*, 2002, vol. 30, p. 160–201.

2. G. Decrouez, P. Hall. Normal approximation and smoothness for sums of means of lattice-valued random variables. *Bernoulli*, 2013, to appear.

3. C. -G. Esseen. Fourier analysis of distribution functions. A mathematical study of the Laplace-Gaussian law. *Acta Math.*, 1945, vol. 77, p. 1–125.

A moment inequality for a certain class of weakly dependent random fields

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Consider a family $\mathcal{U} = \{(u_1, v_1] \times \cdots \times (u_d, v_d],\ u_i, v_i \in \mathbb{Z},\ i = 1, \ldots, d\}$ of blocks in $\mathbb{R}^d$, $d \in \mathbb{N}$. For $U = (u_1, v_1] \times \cdots \times (u_d, v_d] \in \mathcal{U}$ set $|U| = \prod_{i=1}^d (v_i - u_i)$. We study partial sum moments of (BL, θ)-dependent random fields (see, e.g., [1], p. 94). Using a method proposed by Shao and Yu [2] we prove

Theorem 1. *Let $X = \{X_k,\ k \in \mathbb{Z}^d\}$ be an integrable centered (BL, θ)-dependent random field with $\theta(x) \leqslant Cx^{-\lambda}$, $x \geqslant 1$, where $C, \lambda > 0$. Suppose there exists $s > 2$, such that $M_s = \sup_{k \in \mathbb{Z}^d} \mathsf{E}|X_k|^s < \infty$. Then for each $U \in \mathcal{U}$, any $p \in (2, s)$ and $\nu > 0$ the following inequality holds*

$$\mathsf{E}\left|\sum_{k \in U} X_k\right|^p \leqslant$$

$$\leqslant K\left(|U|^{1+\nu} \max_{k \in U} \mathsf{E}|X_k|^p + |U|^\gamma C^{(s-p)/(s-2)} M_s^{(p-2)/(s-2)} + |U|^{p/2} Q^{p/2}(X)\right).$$

Here $K = K(d, s, p, \nu, \lambda)$, $\gamma = \max\{(s(p-1) - p - \lambda(s-p)/d)/(s-2), 1 + \nu\}$,

$$Q(X) = \sup_{U \in \mathcal{U}} \frac{1}{|U|} \mathsf{E}\left(\sum_{k \in U} X_k\right)^2.$$

In [2] this estimate is established for $d = 1$ and $X_k = f(Y_k)$, $k \in \mathbb{Z}$, where f is a Lipschitz function and $Y = \{Y_k,\ k \in \mathbb{Z}\}$ is an associated sequence.

References

1. A. V. Bulinski, A. P. Shashkin. *Limit Theorems for Associated Random Fields and Related Systems*, 2007, World Scientific, Singapore.

2. Q-M. Shao, H. Yu. Weak convergence for weighted empirical processes of dependent sequences. *The Annals of Probability*, 1996, vol. 24, No. 4, p. 2098–2127.

Method for localization of brain activity sources

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The brain functional mapping is a challenging task posed by current techniques for non-invasive investigation of the brain. Magnetoencephalography (MEG) is a very powerful tool with a scientific and medical application potential. It allows to retrieve large datasets describing processes in the human brain. Analysing these date we face with a highly ill-posed inverse problem – precise spatial reconstruction of the MEG signal sources in the human brain. At the moment there are no tools powerful and accurate enough to analyse MEG datasets in the inverse problem context.

In general MEG inverse problem can be written in a form:

$$B_t = GJ_t + N_t, \tag{1}$$

where: $B_t \in R^{N_{sensors}}$ is the random vector representing the measured data at time t; G is the lead-field matrix; $J_t \in R^{3N_{points}}$ is the random vector representing the sources distribution at time t; $N_t \in R^{N_{sensors}}$ is the noise in the model.

Given the stochastic nature of the signal noise we can consider the inverse problem from the statistic point of view. It allows to analyse linear as well as non-linear models with a noise distributed distinctly (e.g., nongaussianity of MEG signal noise was shown earlier).

Apparently MEG data contain a superposition of multidipole signals (signal source generalising). We can firstly apply Independent Component Analysis (ICA) to these MEG data. ICA enables to separate multiple independent dipoles by finding several directions of maximum nongaussianity and decompose relevant independent signal sources.

An analytical solution of the neuroimaging inverse problem was obtained from Biot Savart equation, if simplify the model assuming head spherical shape and uniform conductivity of the brain tissue:

$$B_r = -\frac{\mu_0}{4\pi} \frac{\left[\overrightarrow{Q}, \overrightarrow{r_Q}\right] \overrightarrow{e_r}}{|\overrightarrow{r} - \overrightarrow{r_Q}|^3}, \tag{2}$$

where: $\overrightarrow{Q}$ is dipole moment, $\overrightarrow{r_Q}$ is dipole coordinates, μ_0 is magnetic constant and $\overrightarrow{e_r}$ is a unit vector, $\overrightarrow{e_r} = \frac{\overrightarrow{r}}{r}$.

Assuming monodipole model the analytical solution can be written in spherical coordinates in the form:

$$r_Q = R\frac{(3-\cos^2\theta) \pm \sqrt{(3-\cos^2\theta)^2 - 4\cos^2\theta}}{2\cos\theta}. \tag{3}$$

Obtained independent components may be treated as a single-sources models and this enables to use analytical solutions (Eq. 3).

The software based on ICA and also engaging other approaches for neuroimaging inverse problem solving is under development at the moment.

We do hope that our work and findings in this field will advance inverse problem solving and become of practical use in everyday clinical practice.

References

1. T. V. Zakharova, S. Y. Nikiforov, M. B. Goncharenko, M. A. Dranitsyna, G. A. Klimov, M. S. Khaziakhmetov, N. V. Chayanov. Signal processing methods for the localization of nonrenewable brain regions. *Informatics and Applications*, 2012. Vol. 22(2). P. 157–175.

2. J. Sarvas. Basic mathematical and electromagnetic concepts of the biomagnetic inverse problem. *Physics in Medicine and Biology*, 1987, vol. 32, p. 11–22.

3. A. Hyvärinen, J. Karhunen, E. Oja. *Independent Component Analysis*, 2001, John Wiley & Sons, New-York.

4. M. Hämäläinen, R. Hari, R. J. Ilmoniemi, J. Knuutila, O. V. Lounasmaa. Magnetoencephalography – theory, instrumentation, and applications to noninvasive studies of the working human brain. *Reviews of Modern Physics*, 1993, vol. 65, p. 413–497.

Branching processes in random environment

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Let $\zeta = \{\zeta_n,\ n = 1, 2, \ldots\}$ be an irreducible positive recurrent Markov chain with a countable state space $\Theta = \{\theta_1, \theta_2, \ldots, \theta_k, \ldots\}$. With each $\theta \in \Theta$ we associate a p-dimensional vector $\mathbf{f}^{(\theta)}(\mathbf{s}) = (f_1^{(\theta)}(\mathbf{s}), \ldots, f_p^{(\theta)}(\mathbf{s}))$, $\mathbf{s} \in \mathbf{J}^p := \{\mathbf{s} = (s_1, \ldots, s_p) : 0 \le s_i \le 1,\ i = 1, \ldots, p\}$, of probability generating functions.

Consider a Galton–Watson branching process $\mathbf{Z}(n) = (Z_1(n), \ldots, Z_p(n))$, $n = 1, 2, \ldots$, evolving in the random environment ζ, which describes the evolution of a population with p types of particles, where $Z_i(n)$, $i = 1, \ldots, p$, is the number of type i particles in the n-th generation of the process. More precisely, it is assumed that, given $\zeta_n = \theta$, all the $Z_i(n)$ type i particles of the n-th generation reproduce according to the reproduction law generated by the p-dimensional generating function $f_i^{(\theta)}(\mathbf{s})$, independently of the other particles of this generation and of the prehistory of the process.

Limit theorems for the number of particles are established for this process. The results obtained generalize and strength a number of known results established earlier for the critical branching processes (with one or several types of particles) evolving in a random environment generated by a sequence of independent and identically distributed random variables.

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References

1. E. E. Dyakonova. Multi-type Galton-Watson branching processes in Markovian random environment. *Theory of Probability and its Applications*, 2011, vol. 55, No. 3, p. 592-601.

2. E. E. Dyakonova.Multitype branching process evolving in Markovian environment. *Discrete Mathematics and Applications*, 2012, vol. 22, No. 5-6, p. 639–664.

Integro-local limit theorem for sums of independent random variables in scheme of series

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1. Introduction.

Let $X_{n1}, X_{n2}, ..., X_{nn}$ be independent random variables in scheme of series with,

$$EX_{nj} = 0, \quad 0 < \sigma_{nj}^2 = DX_{nj} < \infty, \quad B_n^2 = \sum_{j=1}^{n} \sigma_{nj}^2,$$

$$\overline{\sigma}_n^2 = \max_{j \le n} \sigma_{nj}^2, \quad S_n = \sum_{j=1}^{n} X_{nj}.$$

In what follows we will use notation from A.A.Borovkov [1].

Let Δ be any positive number and $\Delta[x)$ denotes an interval $[x, \, x+\Delta)$. In present work an asymptotic behavior of the probability

$$P\left(S_n \in \Delta[x)\right)$$

is studied, provided the following conditions hold:

1) There exists constants $0 < c_1 < c_2 < \infty$ which do not depend on n such that

$$c_1 \leqslant \overline{\sigma_n} \leqslant c_2. \tag{1}$$

2) As $n \to \infty$

$$B_n \to \infty. \tag{2}$$

Following Borovkov [1] we say that for the sequence $\{X_{nk}, k \le n\}$ of random variables (r.v.'s) integro-local limit theorem takes place, if for any $\Delta > 0$

$$P\left(S_n \in \Delta[x)\right) = \frac{\Delta}{B_n}\varphi\left(\frac{x}{B_n}\right) + o\left(\frac{1}{B_n}\right), \tag{$*$}$$

where $\varphi\left(x\right) = \left(2\pi\right)^{-1/2} e^{-x^2/2}$ is a density of standard normal distribution, the reminder term $o\left(\frac{1}{B_n}\right)$ is uniform in x .

If r.v.'s $X_{nj} = X_j \stackrel{d}{=} X$ are identically distributed, do not depend on n and nonlattice then known theorems of Shepp - Stone [2], [3] establish asymptotics of $P\left(S_n \in \Delta[x)\right)$ for any fixed $\Delta > 0$.

2. Integro-local limit theorem.

Let $F_{nj}\left(x\right)$ and $\Phi_{nj}\left(x\right)$ be distribution functions of X_{nj} and Gaussian random variable's with parameters $\left(0, \sigma_{nj}^2\right)$. Assume that the following takes place

$$\sum_{j=1}^{n} \int\limits_{|x|>\varepsilon B_n} |x| \left|F_{nj}\left(x\right) - \Phi_{nj}\left(x\right)\right| dx \to 0 \tag{L}$$

for any $\varepsilon > 0$.

We will use the condition of asymptotic nonlatticity (R) from [1]. For given $\varepsilon > 0$. and $N > 0$ set

$$q_{nj} = q_{nj}\left(\varepsilon, N\right) = \sup_{\varepsilon \leqslant |t| \leqslant N} \left|f_{nj}\left(t\right)\right|, \quad f_{nj}\left(t\right) = Ee^{itX_{nj}}$$

Condition (R): for any fixed $\varepsilon > 0$ and $N > 0$

$B_n \prod\limits_{j=1}^{n} q_{nj} \to 0$ as $n \to \infty$.

Theorem. Assume that the conditions (1), (2), (L) and (R) hold. Then integro-local limit theorem (*) takes place.

Note that in above theorem, the condition of uniform integrability of the sequence of random variable's $\left\{\frac{X_{nj}^2}{\sigma_{nj}^2},\ j \geqslant 1\right\}$ used in [1] to prove integro-local theorem, replaced by weaker condition (L).

References

1. Borovkov A.A. Integro-local and local theorems of normal and large deviations of the sums of differently distributed random variables in the scheme of series. Theory prob. and its app. 2009, v 54, 4, 625-644.

2. Shepp. L.A. A local limit theorem. Ann. Math. Statist. 1964, V.35, 419-428.

3. Stone S. A local limit theorem for nonlattice multi-dimonsional distribution function. Ann. Math. Statist. 1965, v.36, 546-551.

Reduced-bias mean-of-order-p extreme value index estimation

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Given a sample of size n of independent and identically distributed random variables (r.v.'s), $X_1, \ldots, X_n$, with distribution function (d.f.) F, let us denote by $X_{1:n} \le \cdots \le X_{n:n}$ the associated ascending order statistics. Let us further assume that there exist sequences of real constants $\{a_n > 0\}$ and $\{b_n \in \mathbb{R}\}$ such that the maximum, linearly normalized, i.e. $(X_{n:n} - b_n)/a_n$, converges in distribution to a non-degenerate random variable. Then (Gnedenko [3]), the limit distribution is necessarily of the type of the general *extreme value* d.f., given by the functional expression, $\mathrm{EV}_\gamma(x) = \exp(-(1+\gamma x)_+^{-1/\gamma})$, $\gamma \in \mathbb{R}$. The d.f. F is then said to belong to the *max-domain of attraction* of EV_γ, and we write $F \in \mathcal{D}_{\mathcal{M}}(\mathrm{EV}_\gamma)$. The parameter γ is the *extreme value index* (EVI), the primary parameter of extreme events. For *heavy-tailed* models, i.e. a positive EVI, the classical EVI-estimators are the Hill estimators (Hill [5]), which are the average of the log-excesses, $V_{ik} := \ln X_{n-i+1:n} - \ln X_{n-k:n}$, $1 \le i \le k < n$, and can thus be written as the logarithm of the *geometric mean* (or *mean-of-order*-0) of the statistics $U_{ik} := X_{n-i+1:n}/X_{n-k:n}$, $1 \le i \le k < n$. More generally, we can consider as basic statistics the *mean-of-order-p* (MOP) of U_{ik}, $p \ge 0$, denoted by $M_p(k) = \left(\sum_{i=1}^k U_{ik}^p/k\right)^{1/p}$, and the associated class of MOP EVI-estimators, introduced and studied in Brilhante *et al.* [1], dependent now on a *tuning* parameter $p \ge 0$, and with the functional expression,

$$\mathrm{H}_p(k) := \begin{cases} \left(1 - M_p^{-p}(k)\right)/p & \text{if} \quad p > 0 \\ \ln M_0(k) = \mathrm{H}(k) & \text{if} \quad p = 0, \end{cases}$$

with $\mathrm{H}_0(k) \equiv \mathrm{H}(k)$, the Hill estimator. The class of MOP EVI-estimators is highly flexible, but it is not asymptotically unbiased for large and even for moderate k-values, the ones that lead to minimum *mean square error*. After reviewing the asymptotic behaviour of the class of MOP EVI-estimators, we introduce and study asymptotically and for finite samples, a class of reduced-bias MOP (RBMOP) EVI-estimators, indeed an optimal RBMOP (ORBMOP) class of EVI-estimators, introduced in the following.

Whenever dealing with bias reduction, it is usual to consider a slightly more restrict class than $\mathcal{D}^+_{\mathcal{M}} := \mathcal{D}_{\mathcal{M}}\left(EV_\gamma\right)_{\gamma>0}$, the class of models with a reciprocal quantile function $U(t) := F^{\leftarrow}(1-1/t) = C\,t^\gamma\left[1+A(t)/\rho+o(t^\rho)\right],\ A(t) = \gamma\beta t^\rho$, as $t \to \infty$, where $C > 0,\ \gamma > 0,\ \rho < 0$ and $\beta \neq 0$ (Hall and Welsh [4]). The simplest class of corrected-Hill (CH) EVI-estimators, introduced in Caeiro *et al.* [2], is defined as

$$\mathrm{CH}(k) \equiv \widehat{\gamma}_n^{\mathrm{CH}}(k) \equiv \widehat{\gamma}^{\mathrm{CH}}_{n,\hat{\beta},\hat{\rho}}(k) := \mathrm{H}(k)\Big(1 - \hat{\beta}(n/k)^{\hat{\rho}}/(1-\hat{\rho})\Big).$$

These estimators can be second-order minimum-variance reduced-bias (MVRB) estimators, for adequate levels k and an adequate external estimation of the vector of second-order parameters, (β,ρ), i.e. the use of $\mathrm{CH}(k)$ can enable us to eliminate the dominant component of bias of the Hill estimator, $\mathrm{H}(k)$, keeping its asymptotic variance. Indeed, from the results in Caeiro *et al.* [2], we know that, asymptotically, $\mathrm{CH}(k)$ overpasses $\mathrm{H}(k)$ for all k. For the aforementioned class of models and for $p < 1/(2\gamma)$, so that the asymptotic normality of the MOP EVI-estimators holds, there is an optimal value $p \equiv p_{\mathrm{M}}$ given by $p_{\mathrm{M}} = \big(1 - \rho/2 - \sqrt{\rho^2 - 4\rho + 2}/2\big)/\gamma =: \varphi(\rho)/\gamma$, which maximizes the efficiency of the MOP EVI-estimators. Then, with the notation H^* for the MOP estimator associated with $p \equiv p_{\mathrm{M}}$, $\mathrm{H}^*(k)$ overpasses $\mathrm{H}(k)$ for all k and, with Z_k^* a standard normal r.v., we get the validity of the asymptotic distributional representation

$$\mathrm{H}^*(k) \equiv H_{p_{\mathrm{M}}}(k) \stackrel{d}{=} \gamma + \frac{\gamma(1-\varphi(\rho))Z_k^*}{\sqrt{k}\sqrt{1-2\varphi(\rho)}} + \frac{(1-\varphi(\rho))A(n/k)}{1-\varphi(\rho)-\rho} + o_p\big(A(n/k)\big),$$

which immediately suggests the consideration of the ORBMOP EVI-estimator,

$$\mathrm{CH}^*(k) \equiv \mathrm{ORBMOP}(k) := \mathrm{CH}_{p_{\mathrm{M}}}(k) = \mathrm{H}^*(k)\Big(1 - \frac{\hat{\beta}(1-\varphi(\hat{\rho}))}{1-\hat{\rho}-\varphi(\hat{\rho})}\left(\frac{n}{k}\right)^{\hat{\rho}}\Big).$$

Then, $\mathrm{CH}^*(k)$ outperforms $\mathrm{H}^*(k)$ for all k, just as $\mathrm{CH}(k)$ outperforms $\mathrm{H}(k)$. We further compare such a class of estimators with the MVRB EVI-estimators, for finite samples. An application to simulated random samples and to sets of real data in the fields of insurance, finance and environment is undertaken and some overall comments on the new ORBMOP EVI-estimation are drawn.

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References

1. M.F. Brilhante, M.I. Gomes, D. Pestana. A simple generalisation of the Hill estimator. *Computat. Statist. and Data Analysis*, 2013, vol. 57, No. 1, p. 518–535.

2. F. Caeiro, M.I. Gomes, D. Pestana. Direct reduction of bias of the classical Hill estimator. *Revstat*, 2005, vol. 3, No. 2, p. 111–136.

3. B.V. Gnedenko. Sur la distribution limite du terme maximum d'une série aléatoire. *Ann. Math.*, 1943, vol. 44, p. 423–453.

4. P. Hall, A.H. Welsh. Adaptive estimates of parameters of regular variation. *Ann. Statist.*, 1985, vol. 13, p. 331–341.

5. B.M. Hill. A simple general approach to inference about the tail of a distribution. *Ann. Statist.*, 1975, vol. 3, p. 1163–1174.

Localisation of brain activity regions using MEG-signals

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Localization of areas of brain activity is a cornerstone in the study of the brain, as well as in helping people with various pathologies. Many scientific teams engaged in similar research, from physiologists and neurosurgeons, to mathematicians and physicists. Over many years of research in this area many revolutionary and evolutionary changes have been done. In particular the method was invented to register the magnetic activity of the brain, using ultra-sensitive sensors - quantum interference devices allowing to catch the field inductances 10^15 T (10^10 times weaker than the geostationary Earth's field). Apparatus for magnetoencephalography (MEG) research is very complicated and sensitive. Nowadays in Russia there is only one such device in the MEG-center at Moscow State University of Psychology and Education and about 30 items are in use all over the world (for more details about the MEG-research see [1, 2]).

Classically forward MEG problem stated as follows:

$$Y_t = L \cdot J_t + \epsilon,$$

where Y_t contains MEG-signals, L - leadfield matrix (i.e., the Biot-Savart operator, generated using mesh, obtained from patient MRI), J_t - dipoles activation vector and ϵ is noise vector.

The main difficulties in the inverse problem solving are that the matrix L is far from the square one and the solution is extremely unstable. We have studied the classical methods (see [2]), as well as some new ones (see [3]). Using this knowledge we theoretically proved the inability of active brain regions' precise localisation using these methods in the current form, because there are situations in which they do not converge. Also we started to develop the method that will use an adaptive segmentation of the brain (grouping of similar areas into one) and iteratively specify the location of active regions.

References

1. T.V.Zakharova, S.Y.Nikiforov, M.B.Goncharenko, M.A. Dranitsyna, G.A.Klimov, M.S.Khaziakhmetov, N.V.Chayanov. Signal processing methods for the localization of nonrenewable brain regions. *Systems and Means of Informatics*, 2012, vol. 22, No. 2, p. 157–176.

2. M.Hamalainen, R.Hari, R.J.Ilmoniemi, J.Knuutila, O.V.Lounasmaa. Magnetoencephalography theory, instrumentation, and applications to noninvasive studies of the working human brain. *Rev. Mod. Phys.*, 1993, vol. 65, p. 413–497.

3. Karl Friston, Lee Harrison, Jean Daunizeau, Stefan Kiebel, Christophe Phillips, Nelson Trujillo-Barreto, Richard Henson, Guillaume Flandin, and Jeremie Mattoutf. Multiple sparse priors for the M/EEG inverse problem. *NeuroImage*, 2008, vol. 39, p. 1104–1120.

On information technology for the plasma turbulence research

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Nowadays the design of methods for the analysis of stochastic processes is very important for the evaluation of turbulence characteristics when conditions of the plasma confinement are changed. Finite local-scale normal mixtures are the basis of model to describe the fine structure of the chaotic processes [1]. The paper aims at review of creation the information technology to identify the specific plasma turbulence structures by spectral analysis. Development and application of new methods of analyzing spectra are based on a special probabilistic bootstrap-like approach. There are two reasons for the methodology. First of all, bootstrap is well-known procedure in the research of complex processes in various areas. Secondly, the compliance of the implemented method results with experimental ones is quite good.

Some ideas with application could be found in paper [2]. For example, lets consider the short-wave fluctuations of the plasma near the centre of the plasma filament (see one-sided spectrum in Fig. 1). There are four dominating components forming the spectrum. It corresponds with transmission of energy between various turbulence types. Another examples of spectra decompositions

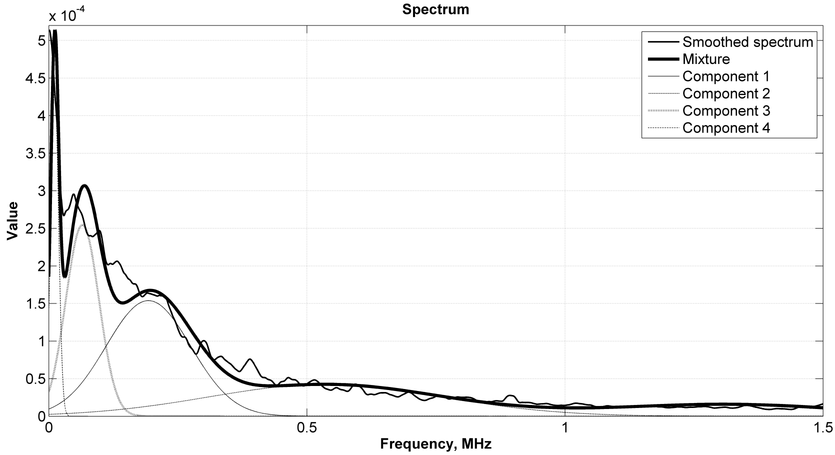

Figure 1: Normal components in spectra

for the low-frequency plasma fluctuations under different external conditions are in paper [2] too.

Note, to find the maximum likelihood estimates, classic EM-algorithm is used in the paper [2]. But creation of information technology should not be restricted by the only one method. For example, we can use different modifications of EM-algorithm for improving accuracy and computational efficiency. The grid methods in situations, when parameter range can be specified, are suitable too. Not only finite local-scale normal mixtures can be used as valid model but mixtures of another distributions (for example, gamma distributions) are convenient too. Implementation of new methods entails theoretical exploring for stability, computational efficiency, etc.

Integration of different methodologies, algorithms and techniques in the united information technology make it possible to analyse stochastic processes in plasma turbulence more precise and to obtain new physical results.

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References

1. V. Yu. Korolev. Probabilistic and Statistical Methods of Decomposition of Volatility of Chaotic Processes. – M.:Moscow University Publishing House, 2011 (in Russian).

2. A. K. Gorshenin, V. Yu. Korolev, D. V. Malakhov, N. N. Skvortsova. On the investigation of plasma turbulence by the analysis of the spectra // Computer Research and Modeling, 2012, vol. 4, no. 4, pp. 793–802. (in Russian).

Stochastic approach for big data analysis

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There is the global trend of large data set processing in different fields. So it leads to actuality of big data problematic. It should be pointed out that special official commissions try to standardize conception "big data" in the USA and European countries. Global leading IT-companies (IBM, Microsoft, HP, Google, etc) suggest some solutions for big data working and at the same time they look for data scientists. So, urgency of methodology design for big data analysis is obvious.

Generally term "big data" in information technologies implies collection of data sets so large and complex that traditional data storage and processing tools (for example, using data bases) are inefficiency. And the most important and critical problem for big data is development of methods which could be worked with the specific data. Leaving out the question of big data storage the paper suggests some ideas for big data analysis based on stochastic approach.

The main advantage of the approach is combination of statistical analysis and data mining methods. We use basic assumption about stochastic character of data and suppose that data can be modelled by mixture of probability distributions. For the investigation of the fine structure of data flow we assume the total sample to be locally homogeneous and suggest that within the window (number of elements) the sample is homogeneous. Then the window moves in the direction of the astronomic time making it possible to trace the evolution of the mixture parameters in time. This idea is the essence of a data mining method which is called "moving separation of mixtures" (MSM method) [1]. Accordingly, the original sample is split into smaller subsamples (windows), and the system is analysed within each window.

Undoubtedly, computational efficiency problems can be arose for big data. Methodology should be tested on diagnostic data with smaller volume, but the data must be real. Some applications for special information system and its information flows could be found in paper [2]. The limit order book for a some time is the example of real financial big data. But the subsample from the limit order book (for example, its behaviour during one day) can be used for testing and adjustment methodology of analysis. Fig. 1 shows some results for one-day subsample which were obtained by MSM method based on classic EM-algorithm [1] and finite mixtures of gamma distributions.

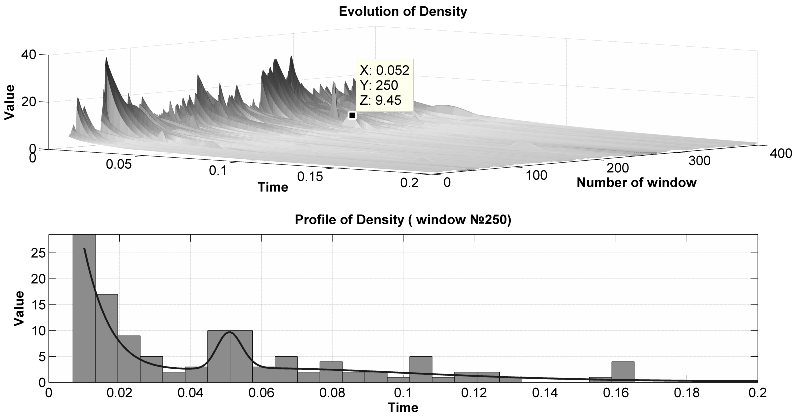

Figure 1: Evolution of density and profile for one of window possition

Consider reasons for mixtures of gamma distribution. Classical stochastic models of information systems are based on the hypothesis that the data flows are Poisson. The assumption of the Poisson character of flow entails the fact that the development of a random process in future does not depend on its past and is determined only by its value at the current time. But this model is ideal, because real processes do not satisfy the ideal conditions that imply Poissonity. Because of heterogeneity of chaos in real information systems, compound Cox process [1] should be used instead of Poisson process. So,we have special reasons to examine finite gamma mixtures for modelling information flows.

Fig. 1 shows good compliance between histogram and profile of density in different window locations. So, we expect identical results for various data.

Note, the methodology can be applied to dissimilar information systems. For example, such models can be used for various sample for web-traffic analysis. Instead EM-algorithm the grid methods is suitable for separation of mixtures in some situations. Moreover, paper [3] shows prospectivity of intensity flows research for exploring fine process structure (in terms of financial systems). Thus the methodology admits different opportunities for improving.

The results mentioned above imply that described approach represents one of the perspective technique for big data analysis.

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References

1. V. Yu. Korolev. Probabilistic and Statistical Methods of Decomposition of Volatility of Chaotic Processes. – M.:Moscow University Publishing House, 2011 (in Russian).

2. A. Gorshenin, A. Doynikov, V. Korolev and V. Kuzmin. Statistical Properties of the Dynamics of Order Books: Empirical Results // VI International Workshop "Applied Problems in Theory of Probabilities and Mathematical Statistics Related to Modeling of Information Systems." M.: Institute of Informatics Problems, RAN, 2012. P. 31–51.

3. V. Yu. Korolev, A. V. Chertok, A. Yu. Korchagin and A. K. Gorshenin. Probability and Statistical Modeling of Information Flows in Complex Financial Systems Based on High-Frequency Data // Informatics and Its Applications, 2013. Vol. 7. Iss. 1. P. 12–21.

Asymptotic properties of sign estimation of the autoregressive field's coefficients

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Let us consider an autoregressive field X_{ij} described by an equation

$$X_{ij} = a_{10}X_{i-1,j} + a_{01}X_{i,j-1} + a_{11}X_{i-1,j-1} + \varepsilon_{ij}, \qquad i,j = 0, \pm1, \pm2, \dots, \quad (1)$$

where ε_{ij} are independent and identically distributed random variables with an unknown distribution function $F(x)$ and $a = (a_{10}, a_{01}, a_{11})$ is an unknown vector of the parameters.

The processes of spatial autoregression are used in the theory of pattern recognition, economy, geology, geography, biology, agriculture, and so on [1,2]. The traditional methods of studying (1) rely on the principles of maximum likelihood (see [3] and their bibliographies). They suggest that the distribution of the ε_{ij} is known. However, this is not always true in practice. Therefore there is a need for methods oriented to a wide class of distributions ε_{ij}. One of such methods is a sign method which appeared as early as in the XVIII century and showed itself to good advantage in the recent decades [4]. Sign method uses not the observation X_{ij}, but only signs $S_{ij}(a) = \mathrm{sign}(\varepsilon_{ij}(a))$, $\mathrm{sign}(x) = x/|x|$, of residuals

$$\varepsilon_{kl}(a) = X_{ij} - a_{10}X_{i-1,j} + a_{01}X_{i,j-1} + a_{11}X_{i-1,j-1},$$

and relies on the assumption that the distribution function $F(x)$ of ε_{ij} should satisfy the condition $F(0) = 1/2$.

In this paper sign estimate for the parameter a of field (1) is constructed and investigated. The sign estimate is shown to be consistent and asymptotically normal. It allows to calculate the asymptotic relative efficiency of sign estimate with respect to least squares estimate for the main types of probability distributions of the innovation field ε_{ij}.

We define the set $\{\delta_{ij}(a)\}$ by the recurrent relation

$$\delta_{ij}(a) = a_{10}\delta_{i-1,j}(a) + a_{01}\delta_{i,j-1}(a) + a_{11}\delta_{i-1,j-1}(a), \quad i, j = 1, 2, \dots$$

with the boundary conditions $\delta_{00}(a) = 1$, $\delta_{k0}(a) = (a_{10})^k$, $\delta_{0l}(a) = (a_{01})^l$, $k > 0$, $l > 0$, $\delta_{ij}(a) = 0$, $i < 0$ or $j < 0$.

We denote $\phi(x) = -\dfrac{f'(x)}{f(x)}$,

$$Z_{ij}(a) = \sum_{k=i+1}^{m} \sum_{l=j+1}^{n} S_{kl}(a) S_{k-i,l-j}(a),$$

$$W_{pq}(a) = \sum_{i=0}^{m-1-p} \sum_{j=0}^{n-1-q} \delta_{ij}(a) Z_{i+p,j+q}(a).$$

In [5] are shown that $W(a) = (W_{10}(a), W_{01}(a)W_{11}(a))$ — the statistics of the locally most powerful tests for checking $H^0 : a = a^0$. Small values of $|W_{pq}(a^0)|$ attest to H^0. Therefore, according to the idea of J.L.Jr. Hodges and E.L. Lehmann [6], we propose to estimate the parameter a by the solution $\hat{a}$ of the equation system $W(a) = 0$.

Theorem. *Let the distribution $F(x)$ and density $f(x)$ functions of independent identically distributed random variables ε_{ij} in (1) satisfy the conditions $F(0) = \frac{1}{2}$, $f(0) > 0$, $\mathsf{E}\varepsilon_{ij} = 0$, $\mathsf{E}\varepsilon_{ij}^3 < \infty$, $\mathsf{E}[\phi^2(x)] < \infty$, $\int_{-\infty}^{\infty} |f'(x)|\, dx < \infty$,*

$$\mathsf{E}[|f(\theta u X_{11}) - f(0)||X_{11}|] \to 0 \text{ under } u \to 0 \text{ for all } \theta \in (0, 1),$$

$$|\phi(x + y) - \phi(x)| \le C|y|, \qquad y \in \mathbb{R}, \quad C > 0, \text{ for a.s. } x \in \mathbb{R}.$$

Then, as $m, n \to \infty$ the random vector $\sqrt{mn}(\hat{a} - a^0)$ is asymptotically normal with zero mean and covariance matrix $(4f(0)\mu)^{-2}\mathcal{K}^{-1}$, where a^0 — true parameter of (1), $\mu = \int_{-\infty}^{0} x f(x)\, dx$,

$$\mathcal{K} = \begin{pmatrix} \mathcal{K}_{00} & \mathcal{K}_{11} & \mathcal{K}_{01} \\ \mathcal{K}_{11} & \mathcal{K}_{00} & \mathcal{K}_{10} \\ \mathcal{K}_{01} & \mathcal{K}_{10} & \mathcal{K}_{00} \end{pmatrix}, \quad \mathcal{K}_{kl} = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \delta_{ij}(a^0) \delta_{i+k,j+l}(a^0).$$

Corollary. *The asymptotic relative efficiency of sign estimates with respect to least squares estimates is $e(f) = 16 f^2(0)\mu^2$.*

References

1. C. Gaetan, X. Guyon. *Spatial statistics and modeling*, 2010, Springer, New York.
2. A. E. Gelfand et al. (Eds.) *Handbook of spatial statistics*, 2010, CRC, Boca Raton.
3. Q. Yao, P. J. Brockwell. Gaussian Maximum Likelihood Estimation for ARMA Models II Spatial Processes. *Bernoulli*, 2006, vol. 12, No. 3, p. 403–429.
4. M. V. Boldin, G. A. Simonova, Yu. N. Tyurin. *Sign-Based Methods in Linear Statistical Models*, 1997, AMS, Providence.
5. V. B. Goryainov, E. R. Goryainova. Nonparametric identification of a spatial autoregression model under conditions of a priori stochastic uncertainty. *Autom. Remote Control*, 2010, vol. 71, No 2, p. 198–208.
6. J. L. Jr. Hodges, E. L. Lehmann. Estimates of location based on rank tests. *Ann. Math. Stat.*, 1963, vol. 34, No 2, p. 598–611.

On a Bahadur-Kiefer representation of von Mises statistic type for intermediate sample quantiles

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Let $X_1, X_2, \ldots$ be a sequence of independent identically distributed real-valued nondegenerate random variables with common distribution function (*df*) F, and for each integer $n \geq 1$ let $X_{1:n} \leq \cdots \leq X_{n:n}$ denote the order statistics based on the sample $X_1, \ldots, X_n$.

Introduce the left-continuous inverse function F^{-1} defined as $F^{-1}(u) = \inf\{x : F(x) \geq u\}$, $0 < u \leq 1$, $F^{-1}(0) = F^{-1}(0^+)$, and let F_n and F_n^{-1} denote the empirical *df* and its inverse respectively, put $f = F'$ to be a density of F, when it exists.

Let k_n be a sequences of integers, such that $k_n \to \infty$, whereas $p_n := k_n/n \to 0$, as $n \to \infty$. Let $\xi_{p_n} = F^{-1}(p_n)$, $\xi_{p_n\, n:n} = F_n^{-1}(p_n)$ denote p_n-th population and empirical quantile respectively.

Let $SRV_\rho^{-\infty}$ be a class of regularly varying in $-\infty$ functions such that $g \in SRV_\rho^{-\infty}$ if and only if:

(i) $g(x) = \pm|x|^\rho\, L(x)$, for $|x| > x_0$, with some $x_0 < 0$, $\rho \in \mathbb{R}$, and $L(x)$ is a positive slowly varying function at $-\infty$;

(ii) $\Big| g(x+\triangle x) - g(x) \Big| = O\Big(|\, g(x)| \, \Big|\frac{\triangle x}{x}\Big|^{1/2} \Big)$, when $\triangle x = o(|x|)$, as $x \to -\infty$.

Here is one of our main results.

THEOREM 1. *Let $k_n \to \infty$, $p_n \to 0$, as $n \to \infty$, and suppose that F^{-1} is differentiable in $(0, \varepsilon)$ for some $\varepsilon > 0$ and that $f \in SRV_\rho^{-\infty}$ with $\rho = -(1+\gamma)$, $\gamma > 0$. Let G be some function differentiable in $F^{-1}((0,\varepsilon))$, and $g = G' \in SRV_\rho^{-\infty}$ with some $\rho \in \mathbb{R}$. Then*

$$\int_{\xi_{p_n n:n}}^{\xi_{p_n}} (G(x) - G(\xi_{p_n}))\, d\, F_n(x) = -\frac{1}{2}[F_n(\xi_{p_n}) - p_n]^2 \frac{g}{f}(\xi_{p_n}) + R_n,$$

where

$$\mathbf{P}\big(|R_n| > A\, p_n^{3/4}(\log k_n/n)^{5/4} \frac{|g|}{f}(\xi_{p_n})\big) = O(k_n^{-c})$$

for each $c > 0$ and some positive constant A, which depends only on c.

Moreover, if, in addition, $k_n^{-1} \log n \to 0$, as $n \to \infty$, then

$$\mathbf{P}\big(|R_n| > A\, p_n^{3/4}(\log n/n)^{5/4} \frac{|g|}{f}(\xi_{p_n})\big) = O(n^{-c})$$

for each $c > 0$ and some positive constant A, which depends only on c.

Although we assume that $k_n \to \infty$ but $k_n/n \to 0$, it is evident that a similar result holds for the case $n - k_n \to \infty$ but $(n - k_n)/n \to 0$, as $n \to \infty$.

In particular (when $G(x) = x$), Theorem 1 provides a Bahadur – Kiefer type representation for the sum of order statistics lying between the intermediate population p_n-quantile and the corresponding sample quantile by a von Mises type statistic approximation, especially useful in establishing second order approximations for (slightly) trimmed means (cf. Gribkova & Helmers [1, 2, 4]).

The talk is based on a joint work [3] with R. Helmers (CWI, Amsterdam, The Netherlands).

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References

1. N. V. Gribkova, R. Helmers. The empirical Edgeworth expansion for a Studentized trimmed mean, *Mathematical Methods of Statistics*, 2006, vol. 15, p. 61-87.

2. N. V. Gribkova, R. Helmers. On the Edgeworth expansion and the M out of N bootstrap accuracy for a Studentized trimmed mean, *Mathematical Methods of Statistics*, 2007, vol. 16, p. 142-176.

3. N. V. Gribkova, R. Helmers. On a Bahadur-Kiefer Representation of von Mises Statistic Type for Intermediate Sample Quantiles, *Probab. Math. Statist.*, 2012, Vol. 32, N. 2, p.255-279.

4. N. V. Gribkova, R. Helmers. Second order approximations for slightly trimmed means, *Theory of Probability and its Applications*, 2013, vol.58 (to appear); arXiv:1104.3347v1 [math.PR].

Bayesian analysis of multivariate time dependent models

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Multivariate models under Bayesian formulation have attracted the attentions of many researchers over the years. These models have extensive applications in many real world situations. One of the reasons is that with the computational advantages over classical models, Bayeasian methodolody can accommodate complicated practical scenarios into their models without being overly-simplified by too many unnecessary or even unrealistic assumptions.

In our research, our interest is to study models which demonstrate nonlinear behaviors and are under the influence of autocorrelation, especially with time as a factor. These curves are often also called growth curves. Also intuitively, we know that the growth of organisms are normally determined by more than one variable (in addition to time) and these variables are usually correlated or even autocorrelated, such as weight and height, etc. Our research provides a unique as well as comprehensive model which incorporates all of the above mentioned conditions. Users will then have the flexibility to make impartial choices of priors to sample from the posteriors for estimating the model parameters.

References

1. D. Barry. A Bayesian model for growth curves analysis. *Biometrics*, 1995, vol. 51, p. 639–655.

2. T. Fearn. A Bayesian approach to growth curves. *Biometrika*, 1975, vol. 62, p. 89–100.

3. S. Geisser. Bayesian analysis of growth curves. *Sankhya A*, 1970, vol. 32, p. 53–64.

4. S. Geisser. *Growth curve analysis*, 1980, In Handbook of Statistics (P.R. Krishnaiah ed.) North-Holland, Amsterdam, p. 89–115.

5. J. E. Grizzle, D. M. Allen. Analysis of growth curves and responses. *Biometrics*, 1969, vol. 25, p. 357–381.

6. C. G. Khatri. A note on a MANOVA model applied to problems in growth curves. *Annals of Institute of Statistical Mathematics*, 1966, vol. 18, p. 75–86.

7. R. F. Potthoff, S. N. Roy. A generalized multivariate analysis of variance model useful especially for growth curve problems. *Biometrika*, 1964, vol. 51, p. 313–326.

8. S. J. Press. *The Subjective and Objective Bayesian Statistics Principles, Models and Applications*, 2002, 2nd edition, John Wiley & Sons.

9. C. R. Rao. The theory of least squares when the parameters are stochastic and its application to the analysis of growth curves. *Biometrika*, 1965, vol. 52, p. 447–458.

10. G. A. F. Seber, C. J. Wild. *Nonlinear Regression*, 1989, John Wiley & Sons.

11. D. von Rosen. Maximum likelihood estimates in multivariate linear normal model. *Journal of Multivariate Analysis*, 1989, vol. 31, p. 187–200.

12. D. von Rosen. Moments for a multivariate linear normal models with application to growth curve model. *Journal of Multivariate Analysis*, 1990, vol. 35, p. 243–259.

13. D. von Rosen. The growth curve model: a review. *Communication in Statistics: Theory and Methods*, 1991, vol. 20, p. 2791–2882.

Generalized convolutions in the non-commutative probability

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We consider the following binary operations from the non-commutative probability theory:

- q- convolution defined by G. Carnovale and T. Koornwinder introduced in [1],
- (p, q)-convolution introduced by A. Kula and E. Ricard in [6].

We show that the above convolutions on the set of the sequences of moments $\mathcal{M}_+$ of probability measures on $(0, \infty)$ are generalized convolutions defined on $\mathcal{M}_+ \times \mathcal{M}_+$. The relationship between q- convolution and (p, q)-convolution is similar to the weak stability property under generalized convolution introduced by J. Kucharczak and K. Urbanik in [5]. We follow the method of defining of weakly stable probability measures under generalized convolution given in [5] and apply this construction in the Kendall convolution case. This way we obtained a new classes of heavy tailed distributions.

References

1. G. Carnovale, T. H. Koornwinder. A q-analogue of convolution on the line. *Methods Appl. Anal.*, 2000, vol. 7, p. 705–726.

2. B. Jasiulis. Limit property for regular and weak generalized convolutions. *Journ. of Theor. Probab.*, 2010, vol. 23, No. 1, p. 315–327.

3. B. Jasiulis-Gołdyn, A. Kula. The Urbanik generalized convolutions in the non-commutative probability and a forgotten method of constructing generalized convolution. *Proceedings of the Indian Academy of Science - Math. Sc.*, 2012, vol. 122, No. 3, p. 437–458.

4. J. Kucharczak, K. Urbanik. Transformations Preserving Weak Stability. *Bull. Pol. Acad. Sci. Math.*, 1986, vol. 34, p. 475–486.

5. A. Kula. The q-deformed Convolutions: Examples and Applications to Moment Problem. *Operators and Matrices*, 2010, vol. 4, No. 4, p. 593–603.

6. A. Kula, E. Ricard. On a convolution for q-normal operators. *Inf. Dim. Anal. Quantum Prob. Rel. Topics*, 2008, vol. 11, p. 565–588.

7. J. K. Misiewicz, K. Oleszkiewicz, K. Urbanik. Classes of measures closed under mixing and convolution. Weak stability. *Studia Math.*, 2005, vol. 167, No. 3, p. 195–213.

8. K. Urbanik. Generalized convolutions I-V. *Studia Math.*, 1964, vol. 23, p. 217–245, 1973, vol. 45, p. 57–70, 1984, vol. 80, p. 167–189, 1986, vol. 83, p. 57–95, 1988, vol. 91, p. 153–178.

9. V. Vol'kovich, D. Toledano-Ketai, R. Avros. On analytical properties of generalized convolutions. *Banach Center Publications, Stability in Probability*, 2010, vol. 5, No. 3, p. 243–274.

Dividend payments in discrete time model

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The study of dividend payments problems goes back to De Finetti [1]. He was first to consider an insurance company as a stock company and suggest that the main goal of the company should be maximizing the expected discounted dividends paid to the shareholders until ruin. De Finetti worked with a discrete time model and proved that the optimal dividend strategy must have been a barrier one.

We consider discrete time model to describe work of an insurance company with the initial surplus $x \in N$ and assume that a dividend strategy with a constant barrier $n \geq x$ ($n \in N$) is applied. It means that when the surplus attains n, dividends are paid out to the shareholders until the next claim occurs. The surplus at the i-th moment can be defined as

$$S_x(i) = min(S_x(i-1) + 1 - z_i, n), \quad S_x(0) = x,$$

where z_i is the claims amount paid to policy holders at i-th moment. Those amounts are i.i.d. random variables with

$$P(z_i = 0) = p, \quad P(z_i = 1) = r, \quad P(z_i = 2) = q, \quad p + q + r = 1.$$

Let the time until ruin be $\eta_x = min(i : S_x(i) < 0)$. Then the expected discounted dividends paid to shareholders until ruin can be determined as

$$m_x(n) = \mathsf{E} \sum_{i=1}^{\eta_x} v^i D_x(i),$$

where $D_x(i) = max(S_x(i-1) + 1 - z_i - n, 0)$ is the amount of dividends paid to shareholders at the i-th moment and $0 < v < 1$ is a force of interest.

Lemma 1. *Let an insurance company use dividend strategy with a constant barrier $n \geq x$, $x, n \in N$. Then the amount of expected discounted dividends paid to shareholders until ruin is*

$$m_x(n) = \frac{a_2^{x+1} - a_1^{x+1}}{\Delta_n},$$

where $\Delta_n = a_2^{n+1}(a_2 - 1) + a_1^{n+1}(1 - a_1)$ and $0 < a_1 < 1 < a_2$ are the roots of the quadratic equation $vpa^2 + (vr - 1)a + vq = 0$.

Knowing the expression of dividend payments we can find the optimal barrier which maximizes $m_x(n)$.

Theorem 1. *If $p < q\left(1 + \frac{1-v}{vq}\right)^2$ then the optimal barrier should be equal to the initial surplus x. Otherwise it is either one of the natural numbers closest to $n^* = log_{\frac{a_2}{a_1}} \frac{(1-a_1)^2}{(a_2-1)^2} - 1$ or x in case $n^* < x$.*

De Finetti proved that if a barrier strategy was applied then the ruin would take place with certainty. Thus it makes sense to investigate the time η_x until ruin more closely.

Lemma 2. *Let $u_{x,k}$ be the probability that the ruin will occur at k-th moment. Then generating function $U_x(s) = \sum_{k=0}^{\infty} u_{x,k} s^k$ for the variable η_x can be described by following expression*

$$U_x(s) = \left(\frac{q}{p}\right)^{x+1} \frac{(\lambda_1(s) - 1)\lambda_1^{n-x}(s) + (1 - \lambda_2(s))\lambda_2^{n-x}(s)}{(\lambda_1(s) - 1)\lambda_1^{n+1}(s) + (1 - \lambda_2(s))\lambda_2^{n+1}(s)},$$

where $\lambda_{1,2}(s)$ are the roots of the equation $ps\lambda^2(s) + (rs - 1)\lambda(s) + qs = 0$.

We consider the normalized random variable $\tau_x = \frac{\eta_x}{\mathsf{E}\eta_x}$ and find its limit behavior when the probability of the company surplus to stay intact converges to 1.

Theorem 2. *If it is equiprobable for the surplus to increase or decrease per unit time, i.e. $p = q$, then limiting distribution of τ_x (while $r \to 1$) is a mixture of $(n - x + 1)$ distributions where the k-th ($k = 0, \ldots, n - x$) of it is a convolution of $(n - k + 1)$ exponential distributions with parameters $(-x^2 + (2n+1)x + 2(n+1))\left(1 - \cos\frac{2j+1}{2n+3}\pi\right)$, $j = k, \ldots, n$.*
In the case of $p \neq q$ the form of limiting distribution depends on the correlation of p and q:
1) if $qp^{-1} \to 0$ then limiting distribution of τ_x is exponential with parameter 1;
2) if $qp^{-1} \to \infty$ then it is a sum of $(x + 1)$ exponential distributions with parameter $(x + 1)$;
3) if $qp^{-1} \to d \neq 0{,}1$ then the density of limiting distribution while $r \to 1$ can be found as

$$p(u) = \sum_{k=1}^{n+1} \frac{S_{n-x}(\gamma_{k,n+1})}{H_k(\gamma_{k,n+1})} e^{\gamma_{k,n+1} u},$$

where $H_k(u) = \frac{S_{n+1}(u)}{u-\gamma_{k,n+1}}$ *and* $\gamma_{k,n+1}$, $k = 1, \ldots, n+1$, *are solutions of the equation* $S_{n+1}(u) = 0$. *The expression* $S_{n+1}(u)$ *is calculated according to the formula*

$$S_m(u) = \frac{(\mu_1 - 1)\mu_1^m + (1-\mu_2)\mu_2^m}{\mu_1 - \mu_2}$$

with $c = (d-1)^2((d-1)(x+1) + d^{-n-1} - d^{x-n})^{-1}$, $\mu_{1,2} = w \pm \sqrt{w^2 - d}$ *and* $w = \frac{d+1+cu}{2}$.

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References

1. B. De Finetti. Su un'impostazione alternativa della teoria colletiva del rischio. *Transactions of the XVth International Congress of Actuaries,* 1957, vol. 2, p. 433–443.

Robustness of sequential decision making on parameters of stochastic data under distortions

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Sequential approach [1] to the problem of decision making on parameters of stochastic data is an effective method to minimize the expected number of observations provided the requested small values of error probabilities are satisfied [2]. Optimality properties stated by the theory for sequential tests are often broken in practice as data do not follow a hypothetical model exactly, in other words, the hypothetical model is distorted [3] – [6], and the performance characteristics of sequential procedures (error probabilities values, expected sample sizes) demonstrate their instability, and increase significantly [7].

The problem of robustness [3] of sequential statistical procedures for decision making is analyzed theoretically for simple and composite hypotheses cases for several models of data: independent observations, Markov chains, high order Markov chains, time series with trends, autoregressive time series. The following types of distortions are considered: "outliers" in observations, "contamination" of prior probability distributions of parameters, neighborhoods in L1- and C- metrics. Asymptotic expansions for the sequential test performance characteristics are constructed with respect to the distortion level value. With the use of main terms of these expansions, the deviations of the performance characteristics under distortions from the hypothetical values are evaluated.

A parametric family of robustified sequential procedures is proposed, and within this family the robust sequential procedures are constructed by the total error probability maximal value minimization provided the expected sample size is constrained. The theory is applied to the problems of incidence data monitoring and to the problems of medical diagnostics.

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References

1. A. Wald. *Sequential analysis*, 1947, Wiley, New York.
2. N. Mukhopadhyay, B. de Silva. *Sequential methods and their applications*, 2009, Chapman & Hall / CRC, London.
3. P. J. Huber. *Robust statistics. Theory and methods*, 2004, John Wiley and Sons, New York.
4. A. Yu. Kharin, P. A. Shlyk. Robust multivariate Bayesian forecasting under functional distortions in the chi-square metric. *Journal of Statistical Planning and Inference*, 2009, vol. 139, p. 3842–3846.
5. A. Yu. Kharin. Robust Bayesian prediction under distortions of prior and conditional distributions. *Journal of Mathematical Sciences*, 2005, vol. 126, p. 992–997.
6. A. Yu. Kharin. Robustness of sequential testing of hypotheses on parameters of M-valued random sequences. *Journal of Mathematical Sciences*, 2013, vol. 189, p. 924–931.
7. A. Kharin. Robustness analysis for Bayesian sequential testing of composite hypotheses under simultaneous distortion of priors and likelihoods. *Austrian Journal of Statistics*, 2011, vol. 40, p. 65-74.

Multivariate fractional Levy motion

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We propose a multivariate analog of fractional Levy motion. One-dimentional variant of this process has been represented in De Nicola [1], where some application to network traffic modelling has been considered.

Let $(B_H(t), t \geq 0)$ be multivariate fractional Brownian motion, i.e.

1) B_H has Gaussian distributions in R^p,
2) B_H is H-self-similar, $H \in (0,1)^p$,
3) B_H has stationary increments.

(see, for example, Amblard [2] and Stoev [3]).

Next let $(L_\alpha(t), t \geq 0)$ be Levy motion with one-sided α-stable distributions, $\alpha < 1$.

Stochastic process

$$X(t) := B_H(L_\alpha(t)) \ , \ t \geq 0.$$

is said to be **Multivariate Fractional Levy Motion**.

In our report we investigate the properties of this process and consider its applications to actuarial and financial mathematics and teletraffic modelling.

References

1. C. De Nikola, Y. S. Khokhlov, M. Pagano and O. I. Sidorova. Fractional Levy motion with dependent increments and its application to network traffic modeling, *Informatics and Applications*, 2012, vol 6, No. 3, p. 58-63.

2. P. O. Amblard, J. F. Coeurjolly, F. Lavancier, A. Philippe. Basic properties of the multivariate fractional Brownian motion. *ArXiv 1007.0828, to appear in Bulletin Society Mathematique de France*, 2011.

3. S. Stoev and M. Taqqu. How rich is the class of multifractional brownian motions? *Stochastic Processes and their Applications*, 2006, vol 116, p. 200221.

Parallel minimax control in the two-armed bandit problem, one arm known

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We consider a problem of rational adaptive control in a random environment which is also well-known as the two-armed bandit problem (see e.g. Sragovich [1], Berry and Fristedt [2]) in application to processing a large number T items of data. Two universal methods of data processing are available, numbered by $\ell = 1, 2$. All data are partitioned into N groups each containing K items of data, so $T = NK$. Data in the same group are processed by the same method and this processing can be implemented in parallel. According to the central limit theorem the result of the processing of the group of data has often close to normal distribution even though original distributions were not those.

So, in the sequel we consider the two-armed bandit problem for the process ξ_n, $n = 1, \ldots, N$, which values (usually interpreted as incomes) are normally distributed with unit variances and mathematical expectations equal to m_1, m_2 if methods $\ell = 1, 2$ are used. We assume that mathematical expectation

m_1 is known and without loss of generality $m_1 = 0$ (otherwise the process $\xi_n - m_1$ can be considered). The goal is to maximize (in some sense) the total expected income. The core of the problem is that the best method is not known in advance because it may be different for different data. So, it should be estimated meanwhile the control process.

A direct determination of the minimax strategy and minimax risk is practically impossible. However, it is shown in Kolnogorov [3, 4] that they can be found as Bayes' ones corresponding to the worst prior distribution on the set of parameters. For considered setting we use the idea of Bradt, Johnson, and Karlin [5]. Since application of the first method does not give any additional information, the optimal strategy is as follows. At the first stage it tries the second method until optimal stopping condition is fulfilled and then applies the first method till the end of the control. We present a sequential design of optimal minimax control. The results of numerical experiments and Monte Carlo simulations are given.

Note, that usual approach to the control is to process data sequentially, one by one. However, if the problem is considered in minimax setting it turned out that the control may be implemented in parallel almost without the lack of its quality, i.e. under mild conditions minimax risks in both cases of parallel and sequential controls have close values.

References

1. V. G. Sragovich. *Adaptive Control*, 1981, Nauka, Moscow. (In Russian)

2. D. A. Berry, B. Fristedt. *Bandit problems*, 1985, Chapman and Hall, London, New York.

3. A. V. Kolnogorov. Finding Minimax Strategy and Minimax Risk in a Random Environment (the Two-Armed Bandit Problem). *Automation and Remote Control*, 2011, vol. 72, No. 5, p. 1017-1027.

4. A. V. Kolnogorov. Parallel Design of Robust Control in the Stochastic Environment (the Two-Armed Bandit Problem). *Automation and Remote Control*, 2012, vol. 73, No. 4, p. 689-701.

5. R. N. Bradt, S. M. Johnson, and S. Karlin. On Sequential Designs for Maximizing the Sum of n Observations. *Ann. Math. Statist.*, 1956, vol. 27, p. 1060–1074.

Analysis of a repairable redundant system with PH distribution of restoration times of its elements

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A reliability model of homogeneous multi-redundant repairable hot standby systems with PH distribution restoration is developed. Stationary and non-stationary reliability properties of such systems are worked out.

Many stochastic models involve, in one way or another, phase-type probability distributions. Many known probability distributions (exponential, Erlang, hyper-exponential etc.) are all considered special cases of a continuous PH distribution. Moreover, a continuous phase-type distribution can be used to approximate any positive-valued distribution.

Consider a homogeneous hot standby n-unit system. The operational times of its elements follow exponential distribution with parameter λ. Their repair times follow the PH distribution, i.e. their distribution function (d.f.) $B(x)$ is given by

$$B(x) = 1 - \mathbf{q}' e^{\mathbf{M}x}\mathbf{1},\ x > 0,\ \mathbf{q}'\mathbf{1} = 1,$$

and it admits the irreducible PH-representation $(\mathbf{q}', \mathbf{M})$ of order m [1], where

- $\mathbf{q}' = (q_1, ..., q_m)$ is a vector of dimension m, for which $\sum\limits_{j=1}^{m} q_j \le 1$, $q_j \ge 0$, $j = \overline{1,m}$,
- $\mathbf{M} = (M_{ij})_{i,j=\overline{1,m}}$ is a square matrix of order m with the following properties: $\sum\limits_{j=1}^{m} M_{ij} \le 0$; $M_{ij} \ge 0, i \ne j$; $M_{ii} < 0$, $i, j = \overline{1,m}$, and for at least one i it holds, that $\sum\limits_{j=1}^{m} M_{ij} < 0$.

According to the probabilistic interpretation of the PH-representation, stated in [1], this system admits description by means of a homogeneous Markov process $\{X(t),\ t \ge 0\}$ defined on the state set $E = \{(0); (k,j),\ k = \overline{1,n},\ j = \overline{1,m}\}$, where

- k is the number of failed system elements at time t,
- j is the number of a recovery phase in which the element under repair is at time t,
- m is the total number of recovery phases; the recovery of the current element under repair can be finished at any phase at that.

The number of states $N = nm + 1$ of the process $X(t)$ is finite, and all the states are communicating. So the process is ergodic and there exist limit probabilities $p_0 = \lim\limits_{t\to\infty} \mathbf{P}\{X(t) = (0)\} = \lim\limits_{t\to\infty} p_0(t) > 0$ and

$p_{kj} = \lim_{t\to\infty} \mathbf{P}\{X(t) = (k, j)\} = \lim_{t\to\infty} p_{kj}(t) > 0$, that are independent of the initial state of the process $X(0)$ and coincide with its stationary probabilities.

The stationary distribution of state probabilities $\{p_0, \mathbf{p}_k, k = \overline{1,n}\}$ in the considered reliability system $\langle M_n|PH|1\rangle$ admits the representation as the truncated matrix-geometric progression:

$$\mathbf{p}'_k = \begin{cases} p_0\mathbf{w}'_0, & k = 1, \\ p_0\mathbf{w}'_0 \prod\limits_{i=2}^{k} \mathbf{W}_i, & k = \overline{2, n-1}, \\ p_0\mathbf{w}'_0 \prod\limits_{i=2}^{n-1} \mathbf{W}_i\mathbf{W}_n, & k = n, \end{cases} \tag{1}$$

where the vector $\mathbf{w}'_0$ and matrices $\mathbf{W}_k$, $k = \overline{1,n}$ are expressed in terms of parameters of operational and repair times distributions. In order to find p_0 we make use of the normalisation condition: $\sum\limits_{k=0}^{n} p_k = 1$, where $p_k = \mathbf{p}'\vec{1}$ is the stationary probability that k elements of the system are functioning properly.

For this system, the failure-free time distribution function, the reliability function and mean failure-free time are worked out by examining the corresponding process $\hat{X}(t)$ with absorbing set of failure states. For this purpose, the solution of the Kolmogorov differention equations system in terms of Laplace-Stieltjes transform is found:

$$\tilde{\mathbf{p}}^{\mathbf{T}}(s)(\mathbf{I}s - \mathbf{\Lambda}) = \mathbf{b}^{\mathbf{T}}, \tag{2}$$

where $\mathbf{I}s - \hat{\mathbf{\Lambda}}$ is a square non-degenerate block matrix of order $(nm + 1)$, $\hat{\mathbf{\Lambda}}$ is an infinitesimal matrix of the modified process with absorbing set of failure states, $\mathbf{I}$ is an identity matrix, and $\mathbf{b}$ is a zero vector of dimension $(nm + 1)$ with its first component equal to 1.

Studying the performance of redundant systems during their life-cycle, i.e. till the total failure, is a problem of a peculiar interest. Therefore, in addition to stationary distribution and performance reliability measures in transient and stationary regime, the quasi-stationary distribution [2] is calculated.

In the main speech a numerical example is given to illustrate the model, the results of numerical analysis are presented along with the plot of the system reliability function and the table of values of high-reliability time quantiles.

References

1. P. P. Bocharov, A. V. Pechinkin. *Queueing theory*, 1995, PFUR Publishing house, Moscow.

2. V. V. Rykov. *Multidimensional Alternative Processes as Reliability Models* // Proc. of BWWQT-2013. Pp.147–156.

Asymptotic confidence interval for quantile of Gamma distribution, constructed from samples with random size

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Quantiles are frequently used as measures of risk of portfolios of asserts and in finacial mathematics are known as values-at-risk. Let us recall that for financial position X under a given probability measure P value-at-risk at level α is defined as

$$\mathrm{VaR}_\alpha(X) = \inf\{m | P(X+m<0) \le \alpha\}.$$

So estimation of quantiles is surely of interest. We propose an asymptotic interval estimator of a quantile of Gamma distribution under assumption of random size of sample.

Let X_1, X_2,... be i.i.d. observations from Gamma distribution $\Gamma_{\alpha,\lambda}$ with density function

$$g(x;\alpha,\lambda) = \frac{\alpha^\lambda}{\Gamma(\lambda)} x^{\lambda-1} e^{-\alpha x}, \qquad x>0, \quad \alpha>0, \quad \lambda>0.$$

Parameter α is unknown, λ is assumed to be known. It is easy to show that the p-quantile $x_p(\Gamma_{\alpha,\lambda}) = \frac{1}{\alpha} x_p(\Gamma_{1,\lambda})$.

Let $S_n = \sum\limits_{i=1}^{n} X_i$. Define quantile estimator as follows:

$$\hat{x}_{p,n}(\Gamma_{\alpha,\lambda}) = \frac{S_n}{\lambda n} x_p(\Gamma_{1,\lambda}).$$

Statistics $\hat{x}_{p,n}(\Gamma_{\alpha,\lambda})$ is unbiased and asymptotically normal estimator of $x_p(\Gamma_{\alpha,\lambda})$ with asymptotic dispersion $\sigma^2(\alpha) = \dfrac{x_p^2(\Gamma_{1,\lambda})}{\lambda\alpha^2}$.

Assume that for each $n \ge 1$ the random variable (r.v.) N_n has the negative binomial distribution $NB(r, \frac{1}{n})$, i.e.

$$P(N_n = k) = \begin{pmatrix} k+r-2 \\ k-1 \end{pmatrix} \left(1-\frac{1}{n}\right)^{k-1} \frac{1}{n^r}, \qquad k=1,2,\ldots$$

and is independent on the sequence X_1, X_2,.... We will regard N_n as the random sample size.

Proposition. *Let r.v.'s N_n, X_1, X_2,... satisfy conditions mentioned above. Then for any $\alpha>0$ and for $\varepsilon>0$*

$$\lim_{n\to\infty} P_\alpha\left(\frac{\hat{x}_{p,N_n}(\Gamma_{\alpha,\lambda})\sqrt{\lambda r n}}{\sqrt{\lambda r n} - t_{\varepsilon/2}} < x_p(\Gamma_{\alpha,\lambda}) < \frac{\hat{x}_{p,N_n}(\Gamma_{\alpha,\lambda})\sqrt{\lambda r n}}{\sqrt{\lambda r n} + t_{\varepsilon/2}}\right) = 1-\varepsilon,$$

where $t_{\varepsilon/2}$ is a quantile of Student distribution of order $\varepsilon/2$.

The result is essentially based on results and methods of Bening and Korolev [1] and Gnedenko [2].

References

1. V. E. Bening, V. Yu. Korolev. On application of Student distribution for problems in probability theory and mathematical statistics, *Theory of Probability and its Applications*, 2004, vol. 49, No. 3, p. 417–435.

2. B. V. Gnedenko. On estimation of unknown distribution parameters under random size of independent observations. *Proceedings of Proceedings of A. Razmadze Mathematical Institute*, 1989, vol. 92, p. 146–150.

Generalized Cross Validation for Vaguelette-Wavelet Signal Decomposition

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Many physical problems involve indirect noisy measurements where one faces a linear inverse problem in the presence of noise. We consider the following model:

$$X_i = (Kf)_i + \varepsilon_i, \tag{1}$$

where X_i are the observed data, K is some linear operator, f is the unknown signal, and ε_i are independent normal variables with zero mean and variance equal to σ^2. We suppose that K is homogeneous with index α.

Nonlinear wavelet methods of signal processing are very popular because of their ability to deal with non-stationarity and capture local features of the signal. One possibility is to use the following approximate signal decomposition:

$$f = \langle Kf, \varphi_{0,0}\rangle K^{-1}\varphi_{0,0} + \sum_{j=0}^{J-1}\sum_{k=0}^{2^j-1} \beta_{j,k}\langle Kf, \psi_{j,k}\rangle u_{j,k}, \tag{2}$$

where $\varphi_{0,0}$ is a scaling function, $\{\psi_{j,k}\}$ is a wavelet basis generated by a certain mother wavelet ψ, and $\{u_{j,k}\}$ is a corresponding "vaguelette" basis, which is stable if K is homogeneous. This kind of decomposition is called vaguelette-wavelet decomposition (see [1]).

To filter out the noise we use thresholding method with soft-thresholding function $\rho_{T_j}(x) = \mathrm{sgn}(x)\,(|x| - T_j)_+$, and obtain an estimate of the signal:

$$\hat{f} = Y^A_{0,0}K^{-1}\varphi_{0,0} + \sum_{j=0}^{J-1}\sum_{k=0}^{2^j-1} \beta_{j,k}\rho_{T_j}(Y^W_{j,k})u_{j,k}, \tag{3}$$

where $Y^A_{0,0}$ is a noisy approximation coefficient and $Y^W_{j,k}$ are noisy wavelet coefficients of the signal. Here we use individual threshold T_j for each decomposition level j.

Risk of soft thresholding method is defined as

$$r_J = \sum_{j=0}^{J-1} \sum_{k=0}^{2^j-1} \beta^2_{j,k} \mathrm{E}(2^{J/2}\langle Kf, \psi_{j,k}\rangle - \rho_{T_j}(Y^W_{j,k}))^2. \tag{4}$$

This expression contains unknown values $\langle Kf, \psi_{j,k}\rangle$, so it cannot be calculated and has to be estimated. In [2] D. Donoho and I. Johnstone proposed to use SURE estimate

$$\hat{r}_J = \sum_{j=0}^{J-1} \sum_{k=0}^{2^j-1} \beta^2_{j,k} R_{T_j}(Y^W_{j,k}), \tag{5}$$

where $R_{T_j}(x) = (x^2-\sigma^2)\mathrm{I}(|x| \le T_j)+(\sigma^2+T_j^2)\mathrm{I}(|x| > T_j)$. This estimate is unbiased, i.e. $\mathrm{E}\hat{r}_J = r_J$. To choose the thresholds T_j we use so-called generalized cross validation (see [3]). Let

$$\hat{G}_j(T_j) = \frac{\sum\limits_{k=0}^{2^j-1} \left(Y_{j,k} - \rho_{T_j}(Y_{j,k})\right)^2}{\mu^2_{T_j}}, \text{ where } \mu_{T_j} = \frac{1}{2^j}\sum_{k=0}^{2^j-1} \mathrm{I}(|Y_{j,k}| \le T_j). \tag{6}$$

The threshold T^G_j minimizes function $\hat{G}_j(T_j)$. In this way we avoid the necessity to estimate variance σ^2.

We prove that under certain conditions risk estimate with the thresholds T^G_j is asymptotically normal. Let $r_{J,Min}$ be the 'ideal' (minimal) risk. The following theorem holds.

Theorem. *Let K be a homogeneous linear operator with index $\alpha > 0$. Let mother wavelet ψ have sufficient number of vanishing moments and satisfy certain conditions, which ensure that basis $\{u_{j,k}\}$ is stable. Let Kf have support in $[0,1]$ and be Lipschitz continuous of order $\gamma > (8\alpha+2)^{-1}$. Then*

$$\frac{\hat{r}_J - r_{J,Min}}{\sqrt{2\sigma^4\beta^4_{0,0}(2^{4\alpha+1}-1)^{-1}\ 2^{(2\alpha+1/2)J}}} \Longrightarrow N(0,1) \quad as \quad J\to\infty. \tag{7}$$

In (7) we do not use traditional normalization which involves variance of $\hat{r}_J$, because this variance depends on the unknown values $\langle Kf, \psi_{j,k}\rangle$. Proposed normalization allows to construct asymptotic confidence intervals for $r_{J,Min}$.

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References

1. Abramovich F., Silverman B. W. Wavelet Decomposition Approaches to Statistical Inverse Problems. *Biometrika*, 1998. Vol. 85. No. 1. P. 115–129.

2. Donoho D., Johnstone I. Adapting to Unknown Smoothness via Wavelet Shrinkage. *J. Amer. Stat. Assoc.*, 1995. Vol. 90. P. 1200–1224.

3. Jansen M., Malfait M., Bultheel A. Generalized Cross Validation for wavelet thresholding // Signal Processing, 1997. Vol. 56. No. 1. P. 33–44.

Gaussian limits for multi-channel stochastic networks

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The main object of the investigation in this paper is a multi-channel stochastic $[\overline{M_t}|GI|\infty]^r$-network consisting of r service nodes. Each of "r" nodes operates as a multi-channel stochastic system. In the i-th node the service time is distributed with distribution function $G_i(t)$, $i = 1, 2, \ldots, r$. The service process in the network is of r-dimensional type form $Q'(t) = (Q_1(t), \ldots, Q_r(t))$, where $Q_i(t)$ is the number of calls in the i-th node at the moment of time t. Our goal is to study the process $Q(t)$ under conditions of heavy traffic.

The heavy traffic regime for the $[\overline{M_t}|GI|\infty]^r$-network is determined by the following behavior of network parameters.

Condition 1. The input flows depend on n (series number) so in any finite interval $[0, T]$ we have

$$n^{-1}\Lambda_i^{(n)}(nt) \overset{U}{\Rightarrow}_{n\to\infty} \Lambda_i^{(0)}(t) \in C[0,T], \quad i = 1, 2, \ldots, r,$$

where $C[0, T]$ is a set of continuous functions on the interval $[0, T]$, let us note that the symbol $\overset{U}{\Rightarrow}$ means convergence in the uniform metric.

Condition 2. $G_i^{(n)}(nt) \Rightarrow^d_{n\to\infty} G_i(t), \quad i = 1, 2, \ldots, r.$

In order to formulate the main result, we need a semi-Markov processes $x^{(i)}(t)$, $i = 1, \ldots, r$, in the set of states $\{1, ..., r, r+1\}$, which are defined by the following semi-Markov matrix $\|G_{ij}(t)\|_1^r$:

$$G_{ij}(t) = \begin{cases} p_{ij}\, G_i(t), \ i = 1, \ldots, r; \ j = 1, \ldots, r, r+1; \\ \delta_{r+1j}\, G_{r+1}(t), \ i = r+1; \ j = 1, \ldots, r, r+1; \end{cases}$$

$$G_{r+1}(t) = \begin{cases} 0, \ t < 1; \\ 1, \ t \geq 1. \end{cases}$$

where $P = \|p_{ij}\|_{i,j=1}^r$ is a switching matrix of the network.

At the initial time moment $t=0$, $x^{(i)}(0) = i$ and the distribution function of the permanence in the initial state "i" coincides with $G_i(t)$.

Let us denote for the transitional probabilities: $p_{ij}(t) = P\left(x^{(i)}(t) = j\right)$, $P(t) = \|p_{ij}(t)\|_1^r$, and also $p_{ij}^{(m)}(s,t) = P\left(x^{(m)}(s) = i, x^{(m)}(t) = j\right)$, $E^{(m)}(s,t) = \|p_{ij}^{(m)}(s,t)/p_{mi}(s)\|_{i,j=1}^r$, where $0 \le s < t$.

Under Conditions 1, 2 we consider a normalized service process, that is:

$$\xi^{(n)}(t)^{-1/2}\left(Q^{(n)}(nt) - \int_0^{nt} [d\Lambda^{(n)}(\tau)]' P^{(n)}(nt-\tau)\right),$$

where $[d\Lambda^{(n)}(\tau)]' = \left(d\Lambda_1^{(n)}(\tau), \dots, d\Lambda_r^{(n)}(\tau)\right)$, $P^{(n)}(t) = \|p_{ij}^{(n)}(t)\|_{i,j=1}^r$, $p_{ij}^{(n)}(t) = P\left(x^{(i,n)}(t) = j\right)$, $x^{(i,n)}(t)$ is a semi-Markov process which is defined as $x^{(i)}(t)$ with the replacement of the distribution functions from $G_i(t)$ to $G_i^{(n)}(t)$, $i = 1, 2, \dots, r$.

In order to construct the approximate process for $\xi^{(n)}(t)$, we need two independent Gaussian processes $\xi^{(1)}(t)$ and $\xi^{(2)}(t)$ which have zero means and correlation matrices of the following form

$$\begin{aligned}
R^{(1)}(t) &= \int_0^t P'(t-\tau)\Delta[d\Lambda^{(0)}(\tau)]P(t-\tau),\\
R^{(1)}(s,t) &= \int_0^s P'(s-\tau)\Delta[d\Lambda^{(0)}(\tau)]P(t-\tau),\ s<t,\\
R^{(2)}(t) &= \int_0^t \left[\Delta[(d\Lambda^{(0)}(\tau))'P(t-\tau)] - P'(t-\tau)\Delta[d\Lambda^{(0)}(\tau)]P(t-\tau)\right],\\
R^{(2)}(s,t) &= \sum_{m=1}^r \int_0^s \left[\Delta(p_m(s-\tau)) - p_m(s-\tau)p_m'(s-\tau)\right] E^{(m)}(s-\tau, t-\tau)\, d\Lambda_m^{(0)}(\tau),\\
& \quad s<t,
\end{aligned}$$

where $p_m'(t) = (p_{m1}(t), \dots, p_{mr}(t))$ is the m-th row of the matrix $P(t)$, $\Delta(x) = \|\delta_{ij}x_i\|_{i,j=1}^r$ is a diagonal matrix with a vector $x' = (x_1, \dots, x_r)$ on the principle diagonal.

Theorem. *Let the stochastic network of $[\overline{M_t}|GI|\infty]^r$-type satisfy conditions 1, 2 and at the initial time, $t=0$, the network is empty. Then on any finite interval $[0,T]$ the sequence of random processes $\xi^{(n)}(t)$ converges weakly, in the uniform topology, to $\xi^{(1)}(t) + \xi^{(2)}(t)$.*

Note that the part $\xi^{(1)}(t)$ of the limit process is associated with fluctuations of the input flows and $\xi^{(2)}(t)$ with the fluctuations of the service times at the network nodes.

The assertion of the theorem generalizes the main result of the paper given in [1], obtained for networks of a Markov type.

References

1. A. V. Livinska, E. A. Lebedev. A Limit Theorem for Multi-channel Networks in Heavy Traffic. *Cybernetics and Systems Analysis*, 2012, No. 6, p. 106–113.

A note on "A goodness of fit test for skew normal distribution based on the empirical moment generating function"

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There are several methods for goodness of fit test for the skew normal distribution. This work focused on method of Meintanis [7] which is based on the empirical moment generating function. This test is discussed for the known and the unknown shape parameter. Meintanis [7] claimed that power of his test is higher than the Kolmogorov-Smirnov test. But this claim is true only for the known shape parameter. In this paper, we provide a new method for finding his test statistic that has more efficiency. Also Meintanis [7] not determine the size of himself test for the known shape parameter which in this paper we will determine it.

References

1. A. Azzalini. *A class of distributions which includes the normal ones*, 1985, Scandinavian Journal of Statistics.

2. J. Clark, L. Horvath, M. Lewis. *On the estimation of the spread rate for a biological population*, 2001, Statistics and Probability Letters.

3. T. W. Epps, K. J. Singleton, L. B. Pulley. A test of separate families of distributions based on the empirical moment generating function. *Biometrika*, 1982, p. 391–399.

4. A. Feuerverger. On the empirical saddlepoint approximation. *Biometrika*, 1989, p. 457–464.

5. R. C. Gupta, N. Brown. *Reliability studies of the skew-normal distribution and its application to a strength-stress model*, 2001, Communication in Statistics, Theory and Methods.

6. S. Kots, T. Kozubowski, K. Podgrski. The Laplace Distribution and Generalizations: A Revisit with Applications to Communications, Economics, Engineering, and Finance. *Birkhauser, Boston*, 2001.

7. S. G. Meintanis. . *A Kolmogorov-Smirnov type test for skew normal distributions based on the empirical moment generating function*, 2007, Journal of Statistical Planning and Inference.

Poisson processes under generalized convolution

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We discuss two classical definitions of the Poisson process applied to the case of generalized convolutions. First definition states that the Poisson process has independent stationary increments with the Poisson distribution, where the Poisson distribution is the exponent of a Dirac measure.

The second based on the direct construction by a sequence of i.i.d. random variables with exponential distributions (which is the only one distribution with the lack of memory property) and their partial sums. Adapting this construction to the case of generalized convolutions we show first that each generalized convolution admits its own distribution with lack of memory property, but only some of them are non-trivial. Next show that only some of generalized convolutions have monotonicity property, which is also important in classical case. Finally, we give the construction of generalized Poisson process based on generalized random walk with steps having the monotonicity and the lack of memory property on the example of α-convolution and Kendall convolution.

It turns out that in the case of generalized convolutions these two definitions of Poisson process do not coincide. Moreover, the second construction lied to a process which does not have even Markov property.

References

1. B.H. Jasiulis-Gołdyn, On the weak random walk under the Kendall convolution, submitted, 2012.

2. B.H. Jasiulis-Gołdyn, J. K. Misiewicz, Classical definitions of the Poisson process do not coincide in the case of generalized convolutions, submitted, 2013.

3. B.H. Jasiulis-Gołdyn, J. K. Misiewicz, Weak Lévy-Khintchine representation for weak infinite divisibility, *Theory of Probabability and Its Applications*, in press, 2012.

4. J.F.C. Kingman, *Random Walks with Spherical Symmetry. Acta Math.* **109**(1), 11–53, 1963.

5. J.K. Misiewicz, K. Oleszkiewicz and K. Urbanik, Classes of measures closed under mixing and convolution. Weak stability., *Studia Math.* **167**(3), 195–213, 2005.

6. Urbanik, K., Generalized convolutions I-V, *Studia Math.* **23**(1964), 217–245, **45**(1973), 57–70, **80**(1984), 167–189, **83**(1986), 57–95, **91**(1988), 153–178.

7. N. Van Thu, A Kingman convolution approach to Bessel Process, *Probab. and Math. Statist.* **29**(1), 119–134, 2009.

8. C. Vignat, A. Plastimo, Geometry of the Central Limit Theorem in the Nonextensive Case, *Physics Letters A* **373**(20), 1713–1718, 2009.

9. V. Vol'kovich, D. Toledano-Ketai, R. Avros, On analytical properties of generalized convolutions, *Banach Center Publications, Stability in Probability* **5**(3), 243–274, 2010.

M - order Markov logarithmic series distribution

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In this article, we derived Markov logarithmic series distribution to model the distribution of success runs in a time homogeneous sequence of Markov dependent Bernoulli trials. We generalize this results to a time homogeneous m - order Markov dependent sequence of Bernoulli trials. We will estimate the parameters of model, and proceed to fit the corresponding model by computing the expected frequencies of various wet spell lengths. Finally, we carry out a chi-square test for goodness of fit for model.

Inspection paradox: an application to loss and optical queues

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The inspection (or renewal time) paradox means that in the limit, as time goes to infinity, the mean remaining (and attained) renewal time approaches the ratio containing the 1st and the 2nd moments of the original interrenewal time, Asmussen [1], Feller [2]. The inspection paradox is also formulated for the mean interval covering instant t in limit as $t \to \infty$. In this work, we use numerical simulation to study the rate of convergence in the paradox for various interrenewal time distributions.

Then we apply the inspection paradox to analyze the loss probability for a class of the non-conventional loss systems (Tikhonenko [4]). In such a system, each customer has both service time and size, and the system has infinite capacity for the queue-size, but a finite capacity M for the total size of the awaiting customers. Thus, the arriving customer is lost if he meets total size N in the system and his size v is such that $v + N \geq M$.

Then a system with the optical buffers is considered. In the system, signals travel from host to host in the form of light and buffering by means of a set of fiber delay lines (FDL) *with deterministic values.* Thus the set of possible waiting times is not a continuum (like in a classic queueing system), but a denumerable set, with each value corresponding to the length of a delay line. As a result, in general arriving signals have to wait for service longer than in the classic case. A sufficient stability condition for the systems with optical buffers has been recently obtained (Rogiest *et al.* [3]). In this work we present and verify by simulation a tighter sufficient condition which stems from the inspection paradox.

Another contribution of this work is that we also consider the optical system *with the independent identically distributed differences between fiber line lengths.* This extension is based on the following motivation. For heavily-loaded modern large networks, a large number of the lines is required. These lines constitute a huge number of possible paths between hosts and users. As a first-order approximation, it seems appropriate to describe the differences between their lengths as random variables to reflect a variability of the paths. On the other hand, to guarantee a high QoS requirement and avoid a dramatic difference in transmission time, it seems reasonable to assume that the difference $\Delta > 0$ between the *adjacent increasing* link lengths has the same distribution reflecting a "homogeneity" of the network. Indeed, an "inhomogeneity" (different distributions of Δ) may cause a huge loss of the capacity because of dramatic increasing of the idle time after completion of a transmission. In particular, it may be important in the problem of reducing the reordering in the multi-path transmission of a big file by means of separate fragments. One more argument to support a common distribution of Δ is that the modern networks are very well-connected containing a huge number of links, and a path is collected from a number of optical cables with *comparable* lengths.

Moreover, in this work, we confirm by simulation that the inspection paradox can be effectively applied to estimate the mean loss size and obtain a tighter stability region for the above described systems. In particular, the results seem to confirm the conjecture that for M large the mean loss size approaches to the mean covering interval obtained from the inspection paradox for the renewal process generated by the customer sizes.

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References

1. S. Asmussen *Applied probability and queues*, 2003, Springer-Verlag New York.

2. W. Feller *An introduction to probability theory and its applications*, 1971, Wiley.

3. W. Rogiest, E. Morozov, D. Fiems, K. Laevens, H. Bruneel *Stability of single-wavelength optical buffers*, 2010, 21, p. 202–212, European Transactions Telecommunications.

4. O. M. Tikhonenko *Queuing systems with processor sharing and limited resources*, 2010, vol. 71, No. 5, p. 803–815, Automation and Remote Control.

Generalized Symmetric Covariation Coefficients for Random Variables with Finite First Moments: simulation and application for Indonesia stock market

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We discuss two new linear dependence measures between two random variables that we call the *generalized covariation* coefficient and the *generalized symmetric covariation* coefficient, introduced recently in Rosadi [1], [2]. These measures can be applied for two random variables with finite first moments, satisfying a linearity property. The *generalized covariation* contain the covariance function and the covariation coefficient (dependence measure applicable for stable distributed random variable, see e.g., Nikias and Shao [3]) as the special cases. The *generalized symmetric covariation* function is the symmetricized and normalized version of generalized covariation, and it will satisfy the properties of the classical Pearson correlation coefficient and contain symmetric covariation (dependence measure applicable for stable distributed random variable, see Garel, d'Estampes and Tjøstheim [4]) as the special cases. We extend the theoretical studies in Rosadi [1], [2] by providing simulation studies and applications of the measures for analyzing financial data from Indonesian stock market.

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References

1. D. Rosadi, Generalized Covariation Function for Stochastic Process with Finite First Moments: Some numerical properties, 2008, *Presented in 7th World Congress in Probability and Statistics, Singapore.*

2. D. Rosadi, Generalized Covariation and Generalized Symmetric Covariation Coefficients for Random Variables with Finite First Moments, 2012, *Presented in 8th World Congress in Probability and Statistics, Istanbul, Turkey.*

3. C. L. Nikias, C.and Shao, M. *Signal Processing with α-Stable Distributions and Applications*, 1995, Wiley, New York.

4. B. Garel, L. d'Estampes, L. and D. Tjøstheim, Revealing some unexpected dependence properties of linear combinations of stable random variables using symmetric covariation, *Communications in Statistics: Theory and Methods* 33 (4): 769786.

Sums of independent poissonian subordinators and a family of Ornstein-Uhlenbeck type processes

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Poissonian Stochastic Index process (PSI - process ψ) is defined by a subordination of the random sequence (ξ_n), $n = 0, 1, \ldots$, to an independent Poisson process $\Pi(s)$, $s \geq 0$, with intensity $\lambda > 0$ and spacings $\tau_1, \tau_2, \ldots$ that are i.i.d. r.v's $\in Exp(\lambda)$; i.e. $\psi(s) = \xi_{\Pi(s)}$. We focus on the case of a strictly stationary sequence (ξ_n), mainly, when (ξ_n) consists of i.i.d. random variables.

We consider the appropriate normalized sums of independent copies $(\psi^j(s))_{j\in\mathbf{N}}$ of the process ψ which are the subordinators for the sequences $(\xi_n^j)_{j\in\mathbf{N}}$, respectively. The basic result is that under the standard assumptions for the CLT, these sums tend to the Ornstein-Uhlenbeck process with the same viscosity parameter λ.

Let us define the multi-index process, $t \in [0, 1]$, $s \geq 0$, prelimit random field:

$$\Psi_N(t, s) = \frac{1}{\sqrt{N}} \sum_{i=1}^{[Nt]} \psi_i(s)\,.$$

Let us introduce the Wiener-Ornstein-Uhlenbeck (WOU) random field $Z(t, s)$:
1) centered gaussian function defined on $\mathbf{R}_+ \times \mathbf{R}$;
2) $cov(Z(t_1, s_1), Z(t_2, s_2)) = \exp\{-\lambda|s_2 - s_1|\} \min(t_1, t_2), \quad \lambda > 0.$

The WOU field is the tensor product of the Brownian motion (Bm) and the O-U process. The time t we name *the extrinsic time* and the time s – *the intrinsic time.*

Basic Theorem. *The following convergence of the finite dimensional distributions takes place as $N \to \infty$; $t \in [0,1]$, $s \geq 0$;*

$$\Psi_N(t,s) \Rightarrow Z(t,s)\,.$$

When the distribution of ξ_n is α-stable, we obtain in the limit an α-stable generalization of the O-U process (O.V.Rusakov (2009)). The existence of the limit random fields follows from the representation of the telecom processes (R.L.Wolpert, M.S.Taqqu (2005)).

For all cases $\alpha \in (0,2]$ limiting processes of O-U type are Markovian ones, and for $\alpha \in (1,2]$ there exists the transition expectation

$$E\{U(s)\,|\,U(0)=x\} = xe^{-\lambda s}$$

of the standard form, which is obtained by the properties of Brownian bridge ($\alpha = 2$ case, (Fig.1.)) and by their fat tailed analogues: Harnesses processes ($\alpha \in (1,2)$ case, see M.Roger, M.Yor (2005))

In the non-homogeneous case, when the leading Poissonian processes have the intensities λ ranging in the interval $(0,\infty)$, we distribute the total variance of the normalized random variables (ξ_0^j) over all existing intensities λ. Let for simplicity this total variance be equal to 1. Then the given distribution ν of this total variance is a probabilistic one and the corresponding covariance function $cov(s)$, $s \geq 0$, of the examined limit stationary process is explicitly the Laplace transform of ν. We apply our approach to processing the American Treasures financial data and to the LIBOR rates.

Examples

0. The classical O-U process is a particular case of the measure ν which is degenerated at the point λ.

1. A Simple curious example is as follows. Let the measure ν be the exponential distribution $Exp(\mu)$, $\mu > 0$. Then the covariance function of the limit stationary process has the following long-memory property: $cov(s) = \mu/(s+\mu)$.

References

1. R. L. Wolpert, M. S. Taqqu. *Fractional Ornstein-Uhlenbeck Lévy Processes and the Telecom Process: Upstairs and Downstairs*, Signal Processing 85(8), Aug 2005, 1523-1545.

2. O. V. Rusakov. *Sums of Independent Poissonian Subordinators and Their Relation with Strictly α-stable Processes of the Ornstein-Uhlenbeck Type*, Journal of Math Sci. Volume 159, No 3, 2009, 350-357.

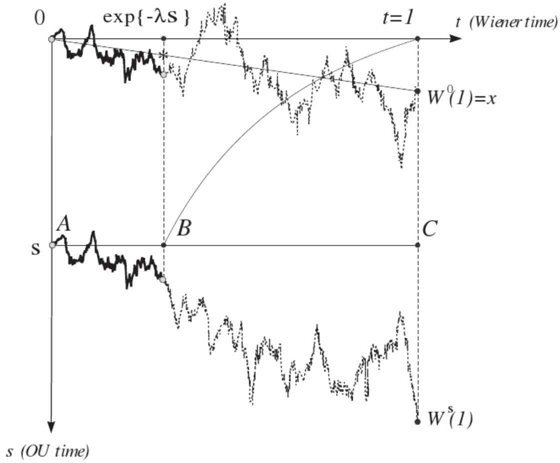

Figure 1: Transition Characteristics and Bridges

3. M. Roger, M. Yor. *Harnesses, Levy Bridges and Monsieur Jourdain*, Stochastic Processes and their Applications. Volume 115, Issue 2, February 2005, 329-338.

On some invariant under group of affine transformations optimal criteria

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Let ξ be a random vector in $(R^m, \mathfrak{B})$, $f(x)$ be it probability density function with respect to Lebesgue measure μ and G be a group of nondegenerate transformations on R^m. The problem is in discrimination of two hypotheses $H_i : f(x) \in \mathcal{F}_i$, $i = 1, 2$ on the base of a sample from a population $X = g\xi$, $g \in G$, when g is unknown. We shall suppose that G is a group of affine transformations on R^m or some of its subgroups.

Let $\mathbf{X}_n = (X_1, X_2, \ldots, X_n)$ be a repeated sample from the population X. Every sample element $\mathbf{x}_n$ is in the fact $(m \times n)$-matrix, with columns $x_1, x_2, \ldots, x_n$. One possibility for elimination of the nuisance parameter $g \in G$ can be realized via replacement of original observations on some invariants of transformations $\mathbf{X}_n \to g\mathbf{X}_n$, $g \in G$, where $g\mathbf{X}_n = (gX_1, gX_2, \ldots gX_n)$. For brevity one term "invariant" will be use as for $\mathbf{U}_n = \mathfrak{u}(\mathbf{X_n})$, so and for $\mathfrak{u}(\mathbf{x_n})$.

The most informative are the maximal invariants (MI-s), these functions have constant values on the orbits of the group and distinguish the different orbits Leman~[1]. MI doesn't define by the unique way. It is possible that MI is a function on some manifold, embedded in the space R^{mn}, but for the evaluations are more preferable MI-s, which have absolutely continuous distributions with respect to Lebesgue measure in Euclidian subspace of R^{nm}.

Let $m(r) = \dfrac{r}{m}$, where r is a dimension of G and $nm > r$. In my report it is proposed one approach to the construction of functionally independent MI such that supports of their distributions belong to Euclidian space $R^{m(n-m(r))}$ and a method for obtaining of the probability density function with respect to Lebesgue measure in $R^{m(n-m(r))}$ for such invariants.

If $\sigma(\mathbf{x_n})$ is an equivariant estimate of the group shift's parameter g, and it is continuously differentiable in the natural region of definition, then a matrix $\mathbf{u}_n(\mathbf{x}_n) = \sigma^{-1}(\mathbf{x}_n)\mathbf{x}_n$ is the maximal invariant. Moreover matrix of the partial derivatives of vector-column composed from columns of matrix $\mathbf{u}_n(\mathbf{x}_n)$ under transposed vector-column of matrix $(\mathbf{x}_n)$ has rank $n - m(r)$. Suppose that $\mathbf{u}_{n-m(r)}(\mathbf{x}_n)$ are functional independent and an equation $\sigma(\mathbf{u}_{n-m(r)}, \mathbf{u}_{n-m(r)+1,\,n}) = e$ has some solution $\mathbf{u}_{n-m(r)+1,n} = \varphi(\mathbf{u}_{n-m(r)})$, where e is the unite of the group G, and $\mathbf{u}_{n-m(r)+1,n}$ is submatrix of matrix $\mathbf{u}_n$ composed from $m(r)$ last columns.

We shall speak that a function $H(\mathbf{u}_{n-m(r)}, \mathbf{x}_{n-m(r)+1,n})$ is obtained from the function $\tilde{H}(\mathbf{x}_n)$ via incomplete change of variables, if it can be presented in the form $\tilde{H}(\mathbf{x}_{n-m(r)}(u_{n-m(r)}, \mathbf{x}_{n-m(r)+1,n}), \mathbf{x}_{n-m(r)+1,n})J(\mathbf{u}_{n-m(r)}, \mathbf{x}_{n-m(r)+1,n})$, where $J(\mathbf{u}_{n-m(r)}, \mathbf{x}_{n-m(r)+1,n})) = \left|\frac{D(\mathbf{x}_{n-m(r)}(\mathbf{u}_{n-m(r)}, \mathbf{x}_{n-m(r)+1,n}))}{D(\mathbf{u}_{n-m(r)})}\right|$.

Let ν be a right invariant measure on G and $\Delta(g)$ be Jacobian of transformation $x \to g^{-1}x$, then the density function of random matrix $\mathbf{u}_{n-m(r)}(\mathbf{X}_n) = \sigma^{-1}(\mathbf{X}_n)\mathbf{X}_{n-m(r)}$ with respect to Lebesgue measure in $R^{m(n-m(r))}$ exists and can be obtained from the function

$$\frac{\int\limits_G \prod\limits_{j=1}^{n} [f(g^{-1}x_j)\Delta(g)]d\nu_r(g)}{\int\limits_G \prod\limits_{j=n-m(r)+1}^{n} [f(g^{-1}x_j)\Delta(g)]d\nu_r(g)} \tag{1}$$

via incomplete change of variable $\mathbf{x}_n \to (\sigma(\mathbf{x}_n)\mathbf{u}_{n-m(r)}, \mathbf{x}_{n-m(r)+1,n})$, where $\sigma(\mathbf{x}_n)$ defined by the equation $\sigma(\mathbf{x}_n)\varphi(\mathbf{u}_n) = \mathbf{x}_{n-m(r)+1,n}$.

In spite of the presence of "superfluous" arguments $\mathbf{x}_{n-m(r)+1,n})$ at the function (1) and in the formula change of variables, the final result doesn't contained them. Denominators in the formula (1) and Jacobins at any permissible incomplete change of variables don't depend on choice of the density function so ratios of MI densities for two distributions are defined exclusively by numerators of correspondent formulas, and Jacobins it is sufficiently evaluate for the normal case.

In monograph R.Wijsman [2] was done an justification with survey and analysis of former results of the integral formula $\int_{\mathfrak{G}} \mathfrak{f}(gx)\chi(g)d\nu_l(g)$ under left invariant measure ν_l on the group for the density function of MI distribution under rather general superpositions on invariant sample space $(\mathfrak{X}, \mathfrak{B}, \mathfrak{G})$ and invariant family $\{\mathfrak{f}(gx)\chi(g),\ g \in \mathfrak{G}\}$ with respect to quasiinvariant measure μ with a modulator $\chi(g)$ on it. But this function is the density function with respect to suitable measure on the some smooth manifold only. The partial case of this formula is numerator in formula (1).

In my report are considered an applying of obtained formulas to the construction for tests of some parameter's hypotheses in comparison with tests of complete likelihood ratio.

References

1. E. Lehmann. *Test of statistical hypotheses*, 1963, Science, Moscow.

2. R. Wijsman. *Invariant Measures on Groups and Their Use in Statistics.* 1990, Inst. Math. Statist. Hayward, California.

Adaptive segmentation of piecewise stationary stochastic processes based on the statistical testing for homogeneity

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In this paper, we discover the problem of adaptive segmentation of piecewise stationary stochastic processes which spectral characteristics abruptly change [1]. Though it is better to use here the word "regularity" instead of "stationarity", we will retain he latter for clarity [2]. Such time series are usually modeled by autoregressive (AR) processes with Gaussian distribution, where the autocorrelation matrix (ACM) of the process remain constant for certain time intervals and then jump to new values (with possible transition period) [1]. These jump processes [2] are usually handled by splitting the signal into small segments $\{\mathbf{x}(t)\}, t = \overline{1,T}, \mathbf{x}(t) = [x_1(t), ..., x_\Lambda(t)]$ of fixed size $\Lambda = const$, where T is the total number of segments ("fixed-window" approach). Thus, the segmentation problem is usually reduced [3] to the statistical hypothesis testing for distribution $\mathbf{P}(t)$ of segment $\mathbf{x}(t)$:

$$W_0 : \quad \mathbf{P}(t) = \mathbf{P}_h, \tag{1}$$

where $\mathbf{P}_h$ is the distribution of previous regular set of segments $\mathbf{x}_h$.

Theorem 1. If signals $\mathbf{x}(t)$ and $\mathbf{x}_h$ are AR processes (order $p = const$), has a Gaussian distribution with zero mean and unknown ACM $K(t)$ and K_h, respectively, and all vectors $\mathbf{x}_i(t) = [x_i(t), ..., x_{i+p}(t)], i = \overline{1, \Lambda - p}$ are independent ("naive" assumption), then the optimal in Bayesian terms decision is made in favor of hypothesis W_0 (1) by the following rule

$$\rho(\mathbf{x}_h, \mathbf{x}(t)) < \rho_0 = const, \tag{2}$$

where ρ_0 is a fixed threshold, and the similarity measure $\rho(\cdot)$ is the Kullback-Leibler (KL) divergence between ACMs of signals $\mathbf{x}(t)$ and $\mathbf{x}_h$.

In this paper we suppose to use alternative approach [4] and reduce the segmentation problem to the task of homogeneity testing

$$W_0' : \quad signals\, \mathbf{x}(t)\, and\, \mathbf{x}_h\, are\, homogeneous. \tag{3}$$

Theorem 2. If conditions of Theorem 1 are satisfied, then the optimal in Bayesian terms decision is made in favor of hypothesis W_0' by criterion

$$\rho(\mathbf{x}_h, \mathbf{x}) + \frac{\Lambda - p}{\Lambda \cdot \Delta t_h - p} \rho(\mathbf{x}(t), \mathbf{x}) < \rho_1 = const, \tag{4}$$

where $\mathbf{x} = [\mathbf{x}_h, \mathbf{x}(t)]$, Δt_h is the length (in segments) of signal $\mathbf{x}_h$, $\rho(\cdot)$ is again the KL discrimination.

Experimental results. The experiment deals with automatic speech recognition [3], namely, with recognition of a vowel in a syllable for Russian language. Five speakers formed the phoneme database by pronouncing 10 vowel sounds of Russian language with a close-speaking microphone A4Tech HS. The training set was filled with 1000 realizations of various syllables (100 realizations per each vowel). The phoneme recognition was performed by the nearest-neighbor rule with the KL divergence which is calculated as the Itakura-Saito distance [3] between power spectral densities of signals. The proposed criterion (4) was compared with a traditional segmentation (2) and the "no segmentation" case. The parameters were fixed as follows: PCM (8 kHz, mono, 16 bits), $\Lambda = 120$, $p = 12$, $\rho_0 = 0.8$, $\rho_1 = 0.25$. Accuracy and FRR (False-Reject Rate) for speaker-dependent and -independent modes are presented in Table 1.

Mode	No segmentation		Conventional segmentation (2)		Proposed segmentation (4)	
	FRR	Accuracy	FRR	Accuracy	FRR	Accuracy
Speaker-dependent	0.009	0.727	0.082	0.727	0.055	0.764
Speaker-independent	0.273	0.618	0.082	0.591	0.064	0.664

Table 1: Results of isolated words recognition

Based on these results we could draw the conclusion that the proposed criterion (2) to adaptive segmentation outperforms either conventional algorithm

(2) or the recognition algorithm without segmentation. The accuracy of (4) is 4-5% higher and the FRR is 2-3% lower than the same indicators of (2).

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References

1. U. Appel, A. V. Brandt. Adaptive sequential segmentation of piecewise stationary time series. *Information Sciences*, 1983, vol. 29, No. 1, p. 2756

2. P. Prandoni, M. Vetterli. Approximation and compression of piecewise smooth functions. *Philosophical Transactions of the Royal Society*, 1999, vol. 357, No. 1760, p. 2573-2591

3. J. Benesty, M. Sondh, Y. Huang (eds.). *Springer Handbook of Speech Recognition*, 2008, Springer, New York.

4. A. A. Borovkov. *Mathematical Statistics*, 1998, Gordon and Breach Science Publishers, Amsterdam.

Threshold theorems for generalized epidemic size in a new Markovian epidemic model with immunization

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As the epidemic model we consider a continucus-time Markov procecc $\xi(t) = (R(t), S(t))$ with transition probalilities given by

$$\begin{cases} P(\xi(t+\Delta t) = (r-1, s+1)/\xi(t) = (r,s)) = \lambda r^{\alpha} s\Delta t + o(\Delta t), \\ P(\xi(t+\Delta t) = (r, s-1)/\xi(t) = (r,s)) = \mu s\Delta t + o(\Delta t), \\ P(\xi(t+\Delta t) = (r-1, s)/\xi(t) = (r,s)) = \theta s\Delta t + o(\Delta t), \end{cases} \tag{1}$$

and initial condition $\xi(0) = (n, m)$, where $0 \leqq s \leqq n + m - r$, $0 \leqq r \leqq n$, ρ_1 *and* ρ_2 are the relative infection and remuval rate respectively. The components of $\xi(t)$ represent respectively the number of susceptibles and infectives at time t. If $a = 1$ and $\rho_2 = 0$ we obtain the classical general stochastic epidemic in closed, homogeneously mixing population. If $0 < a < 1$, then model takes into accound a non-homogeneous mixing in a population.

The third transition of the model (1) reflects a possibility of susceptibles immunization in some sense.

If this probability has the form $\rho_2 rs\Delta t + o(\Delta t)$, then that model is well-known as Downton model [1] or Nagaev-Rakhmanina model with natural immunization [2].

Note that states of the form $(k, 0)$ are absorbing.

Absorption at $(n-k, 0)$ means that epidemic size is equal to k for genaral stochastic epidemic or generalized epidemic size $\nu = \nu_1 + \nu_2$ is equal to k, where ν_2 is the immunization size and ν_1 is equal to initial susceptibles that are ultimately infected. Much of the work on the standard epidemic process has been directed toward finding the distribution of epidemic size, ν. However, explicit expressions are, in general, very cumbersome. In connection with this in the wok of Nagaev and Startsev [3]. Was proposed a metod of asumptotical analysis, as $n \to \infty$ in that the problem of epidemic size reduce to boundary crossing problem for sums of independent random variables.

This report is devoted to obtaining of the limit distributions for the generalized epidemic size in the model (1). We suppose that $m \to \infty$ as $n \to \infty$ and parameters of the model are changed together with n ("series scheme"). The parameter $\theta_1(n) = \rho_1/n^a$ plays a regulating role and similar to the parameter ρ_1/n in the general stochastic epidemic. The threshold theorems are concerned of the case when $\theta_1(n) \to 1$, $m(1-\theta_1(n)) = O(1)$ and $m^3 = O(n)$. In this case limit boundary $g(t)$ (in corresponding boundary problem) is a continuous function of parabolic form. In other cases this boundary is degenerate, namely, it is infinite at $t \in (0, 1)$, and consequently the normal distribution appears as a limit law.

Theorem. *If* $\theta_1 \to 1$, $\beta \equiv m(1-\theta_1) \to \beta_0$, $\frac{m}{n} \to \gamma_0 < \infty$, $|\beta_0| < \infty$, *then for* $\forall$ *fixed* $x > 0$

$$P\left(\nu > \frac{(1+\theta_2)m^2}{2}\right) \Longrightarrow$$
$$P\left(w(t) < \frac{1}{\sqrt{x}} + \frac{\sqrt{x/2}}{\sqrt{1+\theta_{20}}}\beta_0 t - \frac{(\theta_{10}+\theta_{20})\alpha\gamma_0(\frac{x}{2})^{3/2}}{4(1+\theta_{20})^{5/2}} t^2, 0 \le t \le 1\right).$$

References

1. Dawton F., *The ultimate size of carrier-born epidemic*, Biometrika, 1968, v. 55, 2, p.277-289.

2. Nagaev A.V., Rakhmanina G.I., *Threshold theorems for a stochastic epidemic model with natural immunization.* Math. notes, 1970,v.III,N2

3. Nagaev A.V., Startsev A.N. *The asymptotic analysis of a stochastic model of an epidemic.* Theory Prob Appl , 1970, V. 15, p.98-107.

On an approach to approximation in the CLT

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The communication deals with an approach to the construction of the approximations to the distributions of normalized sums of independent identically distributed random variables related to the use of the asymptotic expansions of high accuracy. In the local forms of CLT for random variables with unite variances and finite moment of order $6, 8, 10, \ldots$ under rather relaxed conditions these expansions allow to obtain approximations whose accuracy is equivalent to the following one:

$$\frac{\beta_6}{120n^2}, \frac{\beta_8}{960n^3}, \frac{\beta_{10}}{9600n^4}, \ldots,$$

where n is the number of summands in the sums. The main part of the report is the discussion of the conditions mentioned above. A substantial part of the communication is devoted to numerical illustrations.

Incomplete data problems in tomography

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Mathematical methods of computer tomography are based on the inversion of Radon-type transforms, which describe different settings of tomographic experiments. Reconstruction techniques allow to reconstruct density function if all (an infinite number) of projections are known. However in practice one can only obtain a finite number of projections and unique reconstruction is not possible in this case. It was proved in [1] that for any density function $f(x)$ ($x \in \mathbb{R}^2$) and any finite number of directions $\theta_1, \ldots, \theta_N$ in the plane there exists another density function $g(x)$ with the same Radon projections in the directions $\theta_1, \ldots, \theta_N$ as $f(x)$ and such that $g(x)$ has only two values: 0 and 1. This result gives the following paradox: for any human object and corresponding projection data there exist many different reconstructions, including reconstruction consisting only of bone and air (0 or 1), but still having the same projections as the original object. Similar examples of nonuniqueness are familiar in tomography, but are usually ignored because tomography machines produce useful images.

In [2] this paradox is solved using estimates of the distances between windowed reconstructions of density functions having finite number (N) of identical Radon projections. We obtain similar estimates for other Radion-type transforms, including exponential Radon transform, attenuated Radon transform (these are used in emission tomography models), circular Radon transforms (used in thermocoustic tomography and reconstruction of SAR images) and fan-beam Radon transform (used in x-ray tomography to speed up data collection process). These estimates have order of $O(N^{-1})$. We also obtain some estimates in case when projections are not identical but may differ by some level ε in uniform metric (these situations may occur due to inaccuracies in projection data). These estimates have order of $O(\varepsilon) + O(N^{-1})$.

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References

1. Gutman S., Kemperman J.H.B., Reeds J.A, Shepp L.A. Existence of probability measures with given marginals // Ann. Prob., 1991. Vol. 19. No. 4. P. 1781–1797.

2. Khalfin L.A., Klebanov L.B. A solution of the computer tomography paradox and estimating the distances between the densities of measures with the same marginals // Ann. Prob., 1994. Vol. 22. P. 2235–2241.

Some moment inequalities of probability theory

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Let X be a random variable (r.v.) such that $\mathsf{E}|X|^r < \infty$ for some real and positive r. Denote

$$\alpha_k = \mathsf{E}X^k,\ k = 1, 2, \dots, [r], \quad \beta_s = \mathsf{E}|X|^s,\ 0 < s \leqslant r, \quad \alpha_0 \equiv \beta_0 \equiv 1.$$

Theorem 1. *If the r.v. X is lattice with span $h > 0$ and $\alpha_1 = 0$, $\beta_{2+\delta} < \infty$ for some $0 < \delta \leqslant 1$, then*

$$h \leqslant (\beta_{2+\delta}/\beta_2 + \beta_\delta)^{1/\delta};$$

if, in addition, the r.v. X has a symmetric distribution, then

$$h \leqslant \max\left\{(\beta_{2+\delta}/\beta_2)^{1/\delta},\ 2\sqrt{\beta_2}\right\}.$$

Theorem 1 improves and generalizes the well-known Mises inequality:

$$h\beta_2 \leqslant 2\beta_3.$$

Theorem 2. *If $\alpha_1 = 0$, then for all $\lambda \geqslant 1$*

$$|\alpha_3| + 3\beta_1\beta_2 \leqslant \lambda\beta_3 + M(p(\lambda), \lambda)\beta_2^{3/2},$$

where

$$p(\lambda) = \frac{1}{2} - \sqrt{\frac{\lambda+1}{\lambda+3}} \sin\left(\frac{\pi}{6} - \frac{1}{3}\arctan\sqrt{\lambda^2 + 2\frac{\lambda-1}{\lambda+3}}\right),$$

$$M(p,\lambda) = \frac{1 - \lambda + 2(\lambda+2)p - 2(\lambda+3)p^2}{\sqrt{p(1-p)}}, \quad 0 < p \leqslant \frac{1}{2},\ \lambda \geqslant 1,$$

with the equality attained for each $\lambda \geqslant 1$ at the special two-point distribution.

Theorem 2 improves Esseen's moment inequality [1]

$$|\alpha_3| + 3h \leqslant (\sqrt{10} + 3)\beta_3,$$

which was used by Esseen when he was solving the problem on the asymptotically best constant in the central limit theorem

Theorem 3. *If $\beta_3 < \infty$, then*

$$\mathsf{E}|X - \alpha_1|^3 \leqslant \frac{17 + 7\sqrt{7}}{27}\beta_3 < 1.3156 \cdot \beta_3,$$

with the equality attained at the special two point distribution.

Theorem 4. *Take any $b \geqslant 1$. If $\mathsf{E}X = 0$, $\mathsf{E}X^2 = 1$, $\mathsf{E}|X|^3 = b$, then*

$$\left|\mathsf{E}X^3\right| \leqslant c(b)\mathsf{E}|X|^3, \qquad c(b) = \sqrt{\frac{1}{2}\sqrt{1 + 8b^{-2}} + \frac{1}{2} - 2b^{-2}} < 1,$$

with the equality attained for each $b \geqslant 1$ at the special two-point distribution.

Theorem 5. *For any r.v. X with the characteristic function $f(t) = \mathsf{E}e^{itX}$, $t \in \mathbf{R}$, and $\alpha_1 = 0$, $\beta_2 = 1$, $\beta_3 < \infty$, the following estimates hold for all $t \in \mathbf{R}$:*

$$\begin{aligned}
|f(t) - 1 + \alpha_2 t^2/2| &\leqslant \gamma_3(\beta_3) \cdot \beta_3 |t|^3, \\
|f(t) - 1 + \alpha_2 t^2/2| &\leqslant \frac{8}{\beta_3^2}\left(\frac{\beta_3|t|}{2} - \sin\left(\frac{\beta_3|t|}{2} \wedge \frac{\pi}{2}\right)\right) + \left(\left(|t| - \frac{\pi}{\beta_3}\right)^+\right)^2, \\
|f(t) - 1 + \alpha_2 t^2/2| &\leqslant \varkappa_3\beta_3|t|^3 \leqslant 0.0992 \cdot \beta_3|t|^3, \ \textit{if}\ \ \alpha_3 = 0;
\end{aligned}$$

$$\begin{aligned}
|f'(t) + \alpha_2 t| &\leqslant \gamma_2(\beta_3) \cdot \beta_3 t^2 \wedge \left[\frac{8}{\beta_3}\sin^2\left(\frac{\beta_3|t|}{4} \wedge \frac{\pi}{4}\right) + 2\left(|t| - \frac{\pi}{\beta_3}\right)^+\right]; \\
|f'(t) + \alpha_2 t| &\leqslant \pi^{-1}\beta_3 t^2 \leqslant 0.3184 \cdot \beta_3 t^2, \quad \textit{if}\ \ \alpha_3 = 0; \\
|f''(t) + \alpha_2| &\leqslant \gamma_1(\beta_3) \cdot \beta_3|t| \wedge 2\sin\left(\frac{\beta_3|t|}{2} \wedge \frac{\pi}{2}\right), \\
|f''(t) + \alpha_2| &\leqslant \varkappa_1\beta_3|t| \leqslant 0.7247 \cdot \beta_3|t|, \quad \textit{if}\ \ \alpha_3 = 0; \\
|f'(t) + \alpha_2 t| &\leqslant \gamma_2(\beta_3) \cdot \beta_3 t^2 \wedge \left[\frac{8}{\beta_3}\sin^2\left(\frac{\beta_3|t|}{4} \wedge \frac{\pi}{4}\right) + 2\left(|t| - \frac{\pi}{\beta_3}\right)^+\right]; \\
|f'(t) + \alpha_2 t| &\leqslant \pi^{-1}\beta_3 t^2 \leqslant 0.3184 \cdot \beta_3 t^2, \quad \textit{if}\ \ \alpha_3 = 0;
\end{aligned}$$

where

$$\varkappa_3 \equiv \sup_{x>0}(\cos x - 1 + x^2/2)/x^3 = 0.0991\ldots, \quad \varkappa_1 \equiv \sup_{x>0} \frac{1-\cos x}{x} = 0.7246\ldots,$$

$$\gamma_n(b) = \inf_{\lambda \geqslant 0} \frac{\lambda c(b) + q_n(\lambda)}{n!}, \quad q_n(\lambda) = \sup_{x>0} \frac{n!}{x^n} \left| e^{ix} - \sum_{k=0}^{n-1} \frac{(ix)^k}{k!} - \lambda \frac{(ix)^n}{n!} \right|, \ \lambda \geqslant 0.$$

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References

1. C.-G. Esseen. A moment inequality with an application to the central limit theorem. *Skand. Aktuarietidskr.*, 1956, vol. 39, p. 160–170.

2. I. Shevtsova. On the accuracy of the approximation of the complex exponent by the first terms of its Taylor expansion with applications. *arXiv:1301.2783*, 2013.

Discrete analogues of stable distributions

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Stable distributions play an important role both in the theory and applications. A lot of phenomenas are modeled by continuous stable distributions, however the character of the data would suggests a discrete approach. An analogue of the stability property may be obtained also in the discrete case when we chose a different normalization procedure. The aim of this talk is to introduce two possible definitions of stability for integer valued random variables, one for symmetric random variables and one for a general case.

The characteristic function of the symmetric discrete stable distribution is given by

$$f(t) = \exp\{-\lambda(1-\cos t)^\gamma\}, \quad \lambda > 0, \ \gamma \in (0, 1].$$

We give the probabilities of the distribution, and we show how this distribution converges to absolutely continuous symmetric stable distribution. The index of stability of the limiting stable distribution is $\alpha = 2\gamma$. For $\gamma = 1$ we thus obtained a discrete analogue of Gaussian distribution.

The discrete stable distribution of integer valued random variables is defined, and we show its characteristic function takes form

$$f(t) = \exp\left\{-\lambda_1 \left(1 - e^{it}\right)^{\gamma} - \lambda_2 \left(1 - e^{-it})\right)^{\gamma}\right\}, \quad \lambda_1, \lambda_2 > 0,\ \gamma \in (0, 1].$$

The limiting distribution is absolutely continuous stable distribution with index of stability $\alpha = \gamma$ for $\gamma < 1$ and $\alpha = 2$ for $\gamma = 1$.

References

1. L. Klebanov, L. Slámova. Integer valued stable random variables. *Statistics & Probability Letters*, 2013, accepted for publication.

Control fractional dynamics

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Firstly, you will be introduced to fractional calculus which has recently gained popularity in a wide range of fields, in particular establishing itself useful in modeling anomalous diffusion by suitable continuous time random walks (CTRWs). Secondly, you will see how to write a dynamic programming equation for the optimal payoff for a process in our consideration which is derived from a scaled controlled CTRW. You will see the new equations derived in my research for the different versions on the process. We will then discuss existence and uniqueness of a classical solution to a simple version of the resulting fractional Hamilton Jacobi Bellman type equation for the optimal payoff function.

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References

1. V. N. Kolokoltsov. *Markov processes, semigroups and generators*, 2011, Walter de Gruyter, Berlin.

2. M. M. Meerschaert, A. Sikorskii. *Stochastic Models for fractional calculus*, 2012, Walter de Gruyer, Berlin.

3. I. Podlubny. *Fractional differential equations*, Mathematics in science and engineering, 1999, vol. 198, Academic press.

4. M. A. Veretennikova, V. N. Kolokoltsov. *Controlled Continuous Time Random Walks and Fractional Hamilton Jacobi Bellman Equations*, Arxiv 2012

The Zolotarev polynomials revisited

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Linnik [2], Skorokhod [3] and Zolotarev [7] established important asymptotic properties of the extreme stable laws, which were originally anticipated by Kolmogorov (see also Ibragimov and Linnik [1], and Zolotarev [8]). In particular, the "*exponentially small*" Poincaré series for the probability density functions of the extreme stable laws were constructed therein.

The members of these series are expressed in terms of the polynomials which we call the "*Zolotarev polynomials*" and denote by $\{\mathcal{Z}_k(\rho),\ k \geq 0\}$. Here, $\rho \in \mathbf{R}^1$. In contrast, Zolotarev [8] employed $\alpha = -\rho$.

The first five Zolotarev polynomials are as follows:

$$\mathcal{Z}_1(\rho) = -\frac{1}{24} \cdot (\rho+2) \cdot (2\rho+1),$$

$$\mathcal{Z}_2(\rho) = \frac{1}{1152} \cdot (\rho+2) \cdot (2\rho+1) \cdot (2\rho^2 - 19\rho + 2),$$

$$\mathcal{Z}_3(\rho) = \frac{1}{414720} \cdot (\rho+2)(2\rho+1)(556\rho^4 + 1628\rho^3 - 9093\rho^2 + 1628\rho + 556),$$

$$\mathcal{Z}_4(\rho) = -\frac{1}{39813120} \cdot (\rho+2) \cdot (2\rho+1) \cdot (4568\rho^6 - 226668\rho^5 - 465702\rho^4 + 2013479\rho^3 - 465702\rho^2 - 226668\rho + 4568),$$

$$\mathcal{Z}_5(\rho) = -\frac{(\rho+2)(2\rho+1)}{6688604160}(2622064\rho^8 + 12598624\rho^7 - 167685080\rho^6 - 302008904\rho^5 + 1115235367\rho^4 - 302008904\rho^3 - 167685080\rho^2 + 12598624\rho + 2622064).$$

Also, $\mathcal{Z}_0(\rho) \equiv 1$.

We construct analogous closed–form saddlepoint-type approximations having an arbitrary fixed number of refining terms for members of the *power-variance family* of distributions, which are indexed by the *power parameter* p, whose domain is $\mathbf{R}^1 \setminus (0,1)$. (We refer to Vinogradov [4] as well as to Vinogradov, Paris and Yanushkevichiene [5], [6] for more detail on this family, which recently has become popular in stochastic modelling.) Specifically, we demonstrate that the successive terms of the corresponding "exponentially small" Poincaré series are also expressed in terms of the Zolotarev polynomials.

We discuss an interesting relationship between the Zolotarev polynomials and the so-called *Stirling coefficients*, which are denoted by $\{\gamma_k,\ k \geq 0\}$ and constitute the coefficients of the following (divergent) Poincaré series for the reciprocal of the gamma function as $z \to +\infty$:

$$\frac{1}{\Gamma(z)} \sim \frac{z^{1/2-z} \cdot e^z}{\sqrt{2\pi}} \cdot \sum_{k=0}^{\infty} \gamma_k \cdot z^{-k}.$$

In particular,

$$\gamma_0 = 1,\ \gamma_1 = -\frac{1}{12},\ \gamma_2 = \frac{1}{288},$$

$$\gamma_3 = \frac{139}{51840},\ \gamma_4 = -\frac{571}{2488320},\ \gamma_5 = -\frac{163879}{209018880}.$$

We conjecture that for each fixed $k \geq 0$,

$$\mathcal{Z}_k(0) = \gamma_k.$$

We verified the validity of this hypothesis numerically for all non-negative integer values of $k \leq 30$.

We conclude by considering a new important special case for which $p = -1$.

References

1. I. A. Ibragimov, Yu. V. Linnik. *Independent and stationary sequences of random variables.* 1971, Wolters-Noordhoff, Groningen.

2. Yu. V. Linnik. On stable probability laws with exponent less than one. *Dokl. Akad. Nauk SSSR*, 1954, vol. 94, p. 619–621 (in Russian).

3. A. V. Skorokhod. Asymptotic formulas for stable distribution laws. *Dokl. Akad. Nauk SSSR*, 1954, vol. 98, p. 731-734 (in Russian).

4. V. Vinogradov. On the power-variance family of probability distributions. *Commun. Statist. Theor. Meth.*, 2004, vol. 33, p. 1007–1029 (errata p. 2573).

5. V. Vinogradov, R. B. Paris, O. Yanushkevichiene. New properties and representations for members of the power-variance family. I. *Lith. Math. J.*, 2012, vol. 52, p. 444-461.

6. V. Vinogradov, R. B. Paris, O. Yanushkevichiene. New properties and representations for members of the power-variance family. II. *Lith. Math. J.*, 2013, vol. 53, p. 103-120.

7. V. M. Zolotarev. Expression of the density of a stable distribution with exponent α greater than one by means of a density with exponent $1/\alpha$. *Dokl. Akad. Nauk SSSR*, 1954, vol. 98, p. 735-738 (in Russian).

8. V. M. Zolotarev. *One–dimensional stable distributions.* 1986, AMS, Providence, RI.

The rate of convergence for a class of Markovian queues

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A class of Markovian non-stationary queueing models with batch arrivals and group services was introduced and studied in our recent papers, see [2,3].

Erlang-type queueing model with group services was introduced and studied in [7]. Namely, in this paper criteria for weak ergodicity and bounds on the rate of convergence have been obtained.

Another popular and one of simplest queuing systems is $M/M/S$ queue. There is a large number of investigations for this model in stationary and non-stationary situations, see for instance, [1,4-6].

Here we consider a natural generalization of this model for the queue with possible simultaneous services and obtain general bound on the rate of convergence in weak ergodic situation.

Namely, we suppose that there are S servers and infinitely many waiting rooms in the queueing system, an intensity of arrival of a customer to the queue is $\lambda(t)$, and an intensity of departure (servicing) of a group of k customers is $\mu_k(t) = \frac{\mu(t)}{k}$ for all $1 \le k \le S$.

Let $X = X(t)$, $t \ge 0$ be a queue-length process for the queue.

Then the probabilistic dynamics of the process is represented by the forward Kolmogorov system:

$$\frac{d\mathbf{p}}{dt} = A(t)\mathbf{p}(t),$$

where $A(t)$ is transposed intensity matrix,

$$A(t) = \begin{pmatrix} a_{00}(t) & \mu_1(t) & \mu_2(t) & \mu_3(t) & \cdots & \mu_r(t) & \cdots \\ \lambda(t) & a_{11}(t) & \mu_1(t) & \mu_2(t) & \cdots & \mu_{r-1}(t) & \cdots \\ 0 & \lambda(t) & a_{22}(t) & \mu_1(t) & \cdots & \mu_{r-2}(t) & \cdots \\ \cdots & & & & & & \\ 0 & 0 & \cdots & 0 & \lambda(t) & a_{rr}(t) & \cdots \\ \cdots & & & & & & \end{pmatrix},$$

where $\mu_k(t) = \mu(t)/k$, for $k \le S$, $\mu_k(t) = 0$, $k > S$ and $a_{ii}(t)$ are such that all column sums in $A(t)$ equal zero for any $t \ge 0$.

Theorem 1. Let there exist $\delta < 1$ such that

$$\int_0^\infty \alpha^*(t)\, dt = +\infty,$$

where

$$\alpha^*(t) = \sum_{k=1}^{S} \frac{\left(1-\delta^k\right)}{k} \mu(t) - \left(\frac{1}{\delta} - 1\right) \lambda(t).$$

Then queue-length process $X(t)$ is weakly ergodic, and the following bound on the rate of convergence holds:

$$\|\mathbf{p}^*(t) - \mathbf{p}^{**}(t)\|_1 \le 4\, e^{-\int_s^t \alpha^*(u)\, du} \sum_{i \ge 1} g_i |p_i^*(s) - p_i^{**}(s)|$$

for any initial conditions $\mathbf{p}^*(s)$, $\mathbf{p}^{**}(s)$ and any s, t, $0 \le s \le t$, where $g_i = \sum_{n=0}^{i-1} \delta^{-n}$.

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References

1. B. L. Granovsky, A. I. Zeifman. Nonstationary Queues: Estimation of the Rate of Convergence. *Queueing Systems*, 2004, vol. 46, p. 363–388.

2. Ya. A. Satin, A. I. Zeifman, A. V. Korotysheva, S. Ya. Shorgin. On a class of Markovian queues. *Informatics and its Applications*, 2011, vol. 5. No. 4, p. 6–12 (in Russian).

3. Ya. A. Satin, A. I. Zeifman, A. V. Korotysheva. On the rate of convergence and truncations for a class of Markovian queueing systems. *Theory of Probability and its Applications*, 2012, vol. 57, No. 3, p. 611–621 (in Russian).

4. A. I. Zeifman. Upper and lower bounds on the rate of convergence for nonhomogeneous birth and death processes. *Stochastic Processes and Their Applications*, 1995, vol. 59, p. 157–173.

5. A. Zeifman, S. Leorato, E. Orsingher, Ya. Satin, G. Shilova. Some universal limits for nonhomogeneous birth and death processes. *Queueing Systems*, 2006, vol. 52, p. 139–151.

6. A. Zeifman, A. Korotysheva. Perturbation bounds for $M_t/M_t/N$ queue with catastrophes. *Stochastic Models*, 2012, vol. 28, p. 49–62.

7. A. I. Zeifman, A. V. Korotysheva, Ya. A. Satin, G. N. Shilova, T. L. Panfilova. On a queueing model with group services. *Lecture Notes in Communications in Computer and Information Science*, 2013, vol. 356, p. 198–205.

Class of random vectors with strictly geometric stable marginal distributions

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We consider a class of random vectors of the form

$$V = (Z_1^{\frac{1}{\alpha_1}} Y_1, Z_2^{\frac{1}{\alpha_2}} Y_2, \dots, Z_k^{\frac{1}{\alpha_k}} Y_k),$$

where $Y = (Y_1, \dots, Y_k)$ is a vector with independent components. Each component of the $Y_i (i = 1, \dots, k)$ has a strictly stable distribution with the characteristic function $g_i(\theta_i)$ and parameters $\alpha_i, \eta_i, \beta_i$. $Z = (Z_1, Z_2, \dots, Z_k)$ is the independent from Y random vector having Marshall-Olkin multivariate exponential distribution with the parameters $\lambda_\varepsilon \geqslant 0$; $\{\varepsilon\}$ is a set of $k-$dimensional indices $\varepsilon = (\varepsilon_1, \dots, \varepsilon_k)$, and each component of ε_i is 0 or 1. Vector ε will be used for the coordinate hyperplane selection in k-dimensional space.

Let us $\Psi_V(\theta), \theta = (\theta_1, \dots, \theta_k)$ is the characteristic function of V, and $\Psi_V(\varepsilon\theta)$ are its projections on the hyperplane ε. Let (ε, z) means the scalar product of vectors ε and z; εz is their coordinate-wise product. The sign $\bullet$ denotes summation on some coordinate.

Theorem. *Characteristic functions of V and its projections on the coordinate hyperplanes can be found by the formulas*

$$\Psi_V(\theta) = Ee^{i(\theta,V)} = \frac{1}{\sum_{\varepsilon\in\{\varepsilon\}} \lambda_\varepsilon - \sum_{i=1}^k \ln g_i(\theta_i)} \sum_{\varepsilon\in\{\varepsilon\}} \lambda_{\bar\varepsilon} \Psi_V(\varepsilon \ln g(\theta)),$$

$$\Psi_V(\varepsilon\theta) = \frac{1}{\sum_{\gamma:\gamma\varepsilon>0} \lambda_\gamma - (\varepsilon, \ln g(\theta))} \sum_{\delta:\delta\varepsilon>0} \lambda_\delta \Psi_V(\bar\delta\varepsilon \ln g(\theta)).$$

Corollary. For the most frequently used case $k = 2$, $\beta_i = 0$ $(i = 1, 2)$, $0 < \alpha_1 < 2$, $\alpha_2 = 2$, the characteristic function can be written as

$$\Psi_V(\theta_1, \theta_2) = \frac{\lambda_{\bullet\bullet}}{\lambda_{\bullet\bullet} + \eta_1^{\alpha_1}|\theta_1|^{\alpha_1} + \eta_2^2\theta_2^2} \left(\frac{\lambda_{11}}{\lambda_{\bullet\bullet}} + \frac{\lambda_{10}}{\lambda_{\bullet\bullet}} \frac{\lambda_{\bullet 1}}{(\lambda_{\bullet 1} + \eta_2^2\theta_2^2)} + \frac{\lambda_{01}}{\lambda_{\bullet\bullet}} \frac{\lambda_{1\bullet}}{(\lambda_{1\bullet} + \eta_1^{\alpha_1}|\theta_1|^{\alpha_1})} \right).$$

In this case

$$\Psi_V(\theta_1, 0) = \frac{\lambda_{1\bullet}}{\lambda_{1\bullet} + \eta_1^{\alpha_1}|\theta_1|^{\alpha_1}}, \ \Psi_V(0, \theta_2) = \frac{\lambda_{\bullet 1}}{\lambda_{\bullet 1} + \eta_2^{\alpha_1}|\theta_2|^{\alpha_1}},$$

where $\lambda_{1\bullet} = \lambda_{11}, \lambda_{\bullet 1} = \lambda_{01} + \lambda_{11}, \lambda_{\bullet\bullet} = \lambda_{10} + \lambda_{01} + \lambda_{11}$.

Stochastic model for dynamics of financial flows in savings-and-loans institutions

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Work of the bank can be interpreted as the work of a queuing system, with the incoming flow of contributions corresponding to the incoming requirements stream. Number of deposits is the amount of requirements serviced at some moment.

Let the contributor number i invest contribution η_i for the time τ_i, then he return his money back. It is assumed that

1. Contributions form Poisson flow with the constant intensity λ.
2. Times of contributions τ_i are i.i.d. variables with the known reliability function $\bar{F}(t) = P(\tau > t)$ and finite expectation $E\tau = s$.
3. The amounts of contributions η_i are also independent random variables with the characteristic function of $\phi(\theta) = Ee^{i\theta\eta}$, the expectation of $E\eta = a$, with the variance of $var\ \eta = \sigma^2$ and with the finite third moment of $E\eta^3 = C < \infty$.

For stationary mode, number of deposits N, lying in the bank, has the Poisson distribution. Then the amount of money in the bank at the moment t is equal to

$$U(t) = \sum_{i=1}^{N(t)} \eta_i.$$

Theorem. *Under the assumptions 1,2,3 the normalized process*

$$\overset{\circ}{U}(t) = \frac{U(t) - EU(t)}{\sqrt{var\ U(t)}}$$

for stationary mode weakly converges to process $Y(t)$ as $\lambda \to \infty$.

Here $Y(t)$ denotes stationary Gaussian normalized process with the normalized correlation function

$$\varepsilon(\tau) = \frac{1}{s} \int_{\tau}^{\infty} F(u)du.$$

In this case, $EU(t) = \lambda sa, DU(t) = \lambda s(\sigma^2 + a^2)$.

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Task access procedures in multiprocessor system

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The multiprocessor computer system (CS) of homogeneous processors (PR), which receives the tasks of random length, that may require parallel execution on multiple processors was considered in this paper. The number of allocated processors to each incoming task is predetermined so that the all tasks occupied the PR in average are performing the same time (ie, proportional to the length of tasks). If the tasks are assigned to be performed on the same number of PR then we say that they have a same type of stream. Types of tasks are numbered $1 \dots M$ in ascending order of the resources allocated for task performing. Group of processors allocated to perform a task of type i are called a computing resource (CR) of type $i = 1, ..., I$. Tasks that require multiple CR are named as "long" tasks, otherwise as "short" tasks. Each group of CR has its own hard drive of limited capacity to store the tasks which are assigned to this CR.

Our problem is to investigate the dependence of output intensity of executed tasks in CR on different access procedures [1]. Four types of tasks access procedures for CS were considered in this paper:

1. **PREV** procedure. Long tasks are not allowed in CR and the short tasks are allowed if there is a free CR of the same type or there is a free space in the appropriate drive;

2. **FULL** procedure. Long tasks are allowed in CR if there is a required number of free CR of appropriate type or there is a free space in the appropriate drive;

3. **APR** procedure. Long tasks are allowed in CR if there is a free space in the appropriate drives and there is a CR of appropriate types which are free or in use by short tasks;

4. **DPR** procedure. Long tasks are allowed in CR depending on the number of tasks that are on CR and in storages.

In the cases of 1) and 2) the tasks are performed according to the FIFO discipline. In the cases of 3) and 4) the long tasks are allowed to perform if there are free necessary CR or by dropping the short tasks to the storage if there are not. Interrupted short tasks can be re-entered to the CR in order they received.

To get the DPR procedure we used the model with following simplifying assumptions: the execution times for all tasks is exponentially distributed random variables, the tasks streams of each type are Poisson.

Lets introduce the notations: k_i– the number of tasks of i–stream in the system,$\overline{k} = (k_1, ..., k_M)$ – vector describing the state of the system, m_j– the total number of tasks at the $j-$st CR or its storage, R_j- the number of processors on j-st type of CR, w_j– the storage capacity of j–st type of CR, c_{ji}– the number of CR of type j allocated for tasks from stream i, d_j– the cost of providing the j–st CR, ρ_j – the load of $j-$tasks, $j = 1, ..., I$,$i = 1, ..., M$.

For this case we suggest the following rule defining thresholds in the DPR procedure: if $d_i(\bar{k}) \geq d_i$ then task of i stream is not allowed to the system; else if $d_i(\bar{k}) < d_i$ then task is allowed to the system, where $\bar{k}$ is the current system state; $d_i(\bar{k}) = \sum_{j=1}^{I} \sum_{l=1}^{c_{ji}} u_{m_l+1}$–is the cost function of providing the CR for tasks of i stream; $u_{m_l} = d_j E_{R_j+w_j}(\rho_j)/E_{m_j-1}(\rho_j)$, where $E_{m_j}(\rho_j)$ – is the Erlang's first formula for QS with m_j devices and load ρ_j in case when $m_j < R_j$ and the Erlang's second formula for QS with R_j devices, $m_j - R_j$ free space in storage device and load ρ_j in case when $m_j \geq R_j$, $m_j = 1, \ldots, R_j + w_j$, $j = 1, \ldots, I$, $i = 1, \ldots, M$.

We proved that DPR procedure always more effective than **PREV** in this model. Numerical experiments for a comparative analysis of the effectiveness of the above access tasks procedures were carried out using a computer model of a the above computer system. The results show that none of the procedures has advantages over the other in the whole range of the parameters of input streams, but the **DPR** procedure is preferable to the other in terms of providing both high-intensity of total output stream of tasks and a relatively high probability of long tasks execution.

References

1. M. Tim Johns. *Planning in Hadoop. Introduction to the pluggable schedulers*, http://www.ibm.com/developerworks/ru/library/os-hadoop-scheduling

A discrete-time $Geo/G/1/\infty$ with service interruptions

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In this paper we analyze a discrete-time queueing system in which an arriving customer can decide, with a certain probability, to go directly to the server expelling out of the system the customer that is currently in service or to join the queue in the last place. We carry out an extensive analysis of the system.

1. **The Mathematical model**

   Customers arrive according to a geometric arrival process with rate a. If, upon arrival, the service is idle, the service of the arriving customer begins immediately, otherwise, the arriving customer either with probability θ expels the customer that is currently being served out of the system and starts immediately its service, or with complementary probability $\bar{\theta} = 1 - \theta$ joins the last place of the queue.

   Service times are governed by an arbitrary distribution $\{s_i\}_{i=1}^{\infty}$, with generating functions $S(x) = \sum_{i=1}^{\infty} s_i\, x^i$.

2. **The steady state probabilities**

   Let π_0 be the stationary probability that in the moment immediately after a potential arrival, that we denoted by m^+, the system is empty and $\pi_{i,k};\ \ i \geq 1,\ \ k \geq 1$ the stationary probability that there are k customers in the system and that the customer just being served needs i more slots to finish its service. We define $\pi_k = \sum_{i=1}^{\infty} \pi_{i,k},\ \ k \geq 1$ as the probability that there are $k;\ \ k \geq 1$, customers in the system. The corresponding generating function (GF) is given by $\pi_k(x) = \sum_{i=1}^{\infty} x^i \pi_{i,k};\ \ \ k \geq 1$. The main results of this paper are summarized in the following: theorem

   The probability that the system is empty is given by

   $$\pi_0 = \frac{S(\bar{a} + a\bar{\theta}) - \bar{\theta}}{1 - \bar{\theta}}.$$

   The stability condition of the system is $S(\bar{a} + a\bar{\theta}) > \bar{\theta}$.

   The joint generating function of the number of customers in the system and the sojourn time spent in the server is given by

   $$\begin{aligned} \pi(x, z) &= \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} x^i z^k \pi_{i,k} = \\ &= \frac{S(x) - S(\bar{a} + a\bar{\theta}z)}{x - (\bar{a} + a\bar{\theta}z)} \cdot \frac{axz(1 - \bar{\theta}z)}{S(\bar{a} + a\bar{\theta}z) - \bar{\theta}z}\, \pi_0. \end{aligned}$$

3. **Busy period**

The generating function of the busy period is

$$h(x) = \frac{[1-(1-a\theta)x]\,S(x-a\theta x)}{1-x\,[1-a\theta\,S(x-a\theta x)]}.$$

4. **Sojourn time of a customer in the server and in the queue**

The generating functions of sojourn time of a customer in the server and in the queue are given by

$$\begin{aligned} b(x) &= \frac{1}{1-a\theta}S(x-a\theta x) + \frac{a\theta}{1-a\theta}\cdot\frac{(1-a\theta)x - S(x-a\theta x)}{1-(1-a\theta)x} \\ w(x) &= \pi_0^+ + \theta(1-\pi_0^+) \\ &+ (1-\theta)\,\frac{[1-(1-a\,\theta)\,x][1-a\,\theta+a\,\theta\,h(x)] - a\,\theta\,h^2(x)}{(1-a\,\theta)[1-(1-a\,\theta)\,x]\,h(x)} \\ &\times \pi^+((1-a\,\theta)\,x, h(x)) + (1-\theta)\,\frac{a\,\theta\,x\,h(x)}{1-(1-a\,\theta)\,x}\,\pi^+(1,h(x)), \end{aligned}$$

where π_0^+ and $\pi_{i,k}^+$; $i \geq 1$, $k \geq 1$ are the stationary probabilities that a customer arrives to an empty system and that an arriving customer finds k other customers in the system and the customer that its currently being served needs i more slots to finish its service.

The GF of the stationary distribution of the sojourn time of a customer in the system is given by

$$v(x) = w(x)b(x).$$

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References

1. I. Atencia, A. V. Pechinkin. A discrete-time queueing system with optional LCFS discipline. *Annals Operation Research*, 2012. DOI 10.1007/s10479-012-1097-2.

2. P. P. Bocharov, C. D'Apice, A.V. Pechinkin and S. Salerno. Queueing Theory. *Utrecht, Boston: VSP*, 2004, 22-29.

3. A. Pechinkin, S. Shorgin. A $Geo/G/1/\infty$ system with a one non-standard discipline for the service. *Informatics and it applications*, 2008, vol. 2. p. 55–62. (In russian)

On the regenerative splitting method for effective bandwidth estimation

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We consider the buffered stationary queue with service constant rate C (the amount of work which leaves into every moment of discrete time). When the buffer size b is fixed, and the service rate C is allowed to choose, then the effective bandwidth (EB) estimation problem arises. The natural criterion for the EB choice follows from issues of QoS. So, the EB problem is to find out the minimal service rate which will ensure an overfull probability less than given value Γ

$$P(W > b) \leq \Gamma,$$

where W is stationary workload.

The EB estimation problem is reduced to estimation of Limiting Scaled Cumulant Generating Function (LSCGF) of arrival process

$$\Lambda_V(\theta^*) = \lim_{n\to\infty} \frac{1}{n} \ln Ee^{\theta V(n)},$$

where $V(n)$ is the total work which arrives in the interval $[0, n-1]$ and parameter θ^* is unknown.

The large deviation theory [1, 2] gives us the exponential relation between the buffer size and overfull probability and the following approximation

$$\theta^* = -\ln\Gamma/b.$$

The regenerative method for EB estimation was offered in recent work [3, 4] and demonstrated the advantages in comparison with the traditional batch means methods [5].

This work presents the splitting technique for regenerative EB estimation in the cases when the variance of regenerative cycle length is large.

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References

1. A. Ganesh, N. O’Connell and D. Wischik. *Big Queues*, 2004, Springer-Verlag, Berlin.

2. P. W. Glynn, W. Whitt. *Logarithmic asymptotics for steady-state tail probabilities in a single-server queue*, JAP, 31, 1994, pp. 131-156.

3. P. Rabinovitch. *Statistical estimation of effective bandwidth*, 2000, M.Sc.thesis, University of Cambridge.

4. . ., . . , 2012, , No 5, . 8-17.

5. I. Dyudenko. E. Morozov, M. Pagano. *Regenerative estimator for effective bandwidth*, Proceedings of the International Conference "Mathematical methods for analysis and optimization of information telecommunication networks, Minsk, Belarusian State University, . 58 - 60, 2009.

A simple queueing model of loss-based overload control in a SIP-servers network

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For analysis of overload control in SIP server signaling networks [1] we propose a single-server queueing system $M|\,\mathrm{M}|\,1|\,L|\,B$ with a finite buffer of capacity B and with hysteretic overload control with the overload abatement threshold L, $1 < L < B$ (Figure 1).

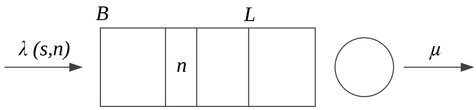

Figure 1: Queueing model of $M|\,\mathrm{M}|\,1|\,L|\,B$ with hysteretic load control

The input stream is Poisson with an intensity of λ, the service time has exponential distribution with parameter μ. The system can function in one of two modes: normal load mode (s=0) and overload mode (s=1). When in the normal load mode the buffer occupancy increases and reaches the value B the system switches to the overload mode (s=1) and new customers arent accepted in the system. In order to prevent oscillations the intensity of the input stream is restored to the normal value λ only when the buffer occupancy decreases to the overload abatement threshold L.

The state of the system is (s, n), where $s \in \{0,\ 1\}$ is overload status, n is buffer occupancy, $n = \overline{0, B}$. The service process is described by the Markov process (the continuous time Markov chain) $X(t)$, $t > 0$, with a finite state space $X = X_0 \bigcup X_1$, where $X_0 = \{(s, n) : s = 0,\ 0 \le n \le B - 1\}$ the set of normal load states, and $X_1 = \{(s,n) : s = 1,\ L \le n \le B\}$ the set of discard states.

The input stream intensity $\lambda(s,\ n)$ depends on a state of the system (Figure 2): when in the set of normal load states X_0 the intensity is equal to $\lambda > 0$, when in the set of discard states X_1 the intensity is equal to 0.

The steady state probabilities of the process $X(t)$ are

$$p_{0,k} = \rho^k p_{0,0},\ k = \overline{0, L-1};$$

$$p_{0,k} = \frac{\rho^k - \rho^B}{1 - \rho^{B-L+1}} p_{0,0},\ k = \overline{L, B-1};$$

$$p_{1,k} = \frac{\rho^B (1-\rho)}{1 - \rho^{B-L+1}} p_{0,0},\ k = \overline{L, B},$$

where $p_{0,0} = \dfrac{1 - \rho - \rho^{B-L+1} + \rho^{B-L+2}}{1 - \rho^{B-L+1} - (B-L+1)\rho^{B+1} + (B-L+1)\rho^{B+2}}$ and $\rho \ne 1$.

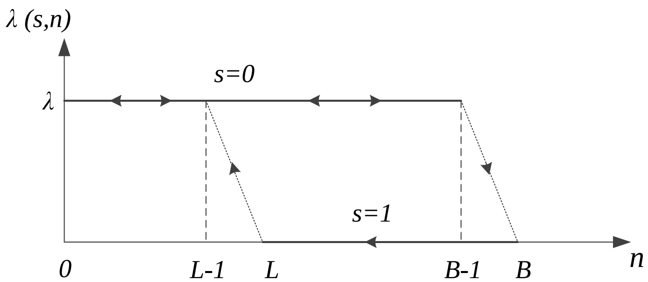

Figure 2: The intensity $\lambda(s,n)$ of the input stream

We got formulas for calculation of the following characteristics of the system.

The blocking probability $P(X_1)$ corresponding to probability of that the SIP server is overloaded and doesn't accept SIP messages:

$$P(X_1) = \sum_{k=L}^{B} p_{1,k}.$$

The mean value MQ of the buffer occupancy corresponding to average number of messages, waiting for service by the SIP server:

$$MQ = \sum_{k=0}^{B-1} k p_{0,k} + \sum_{k=L}^{B} k p_{1,k}.$$

The mean value $M\tau_1$ of the return time τ_1 to the set of normal load states corresponding to the average time interval of functioning of the SIP server in an overload mode when control is switch on:

$$M\tau_1 = \mu^{-1} (B - L + 1).$$

The mean value $M\tau$ of the control cycle time τ of SIP-server:

$$M\tau = \frac{M\tau_1}{P(X_1)}.$$

In addition we formulated and solved two optimization problems: for the mean value $M\tau_1(L)$ and for the 95% quantile value $\tau_1^{0.95}(L)$ of the return time τ_1 with respect to the choice of the threshold L:

$$\begin{aligned} &M\tau_1(L) \to min; \\ &R1 : P(X_1) \le \gamma_1; \\ &R2 : M\tau \ge \gamma_2, \end{aligned} \qquad \begin{aligned} &\tau_1^{0.95}(L) \to min; \\ &R1 : P(X_1) \le \gamma_1; \\ &R2 : M\tau \ge \gamma_2. \end{aligned}$$

The requirements for both the problems were the same: the blocking probability shouldn't exceed the level γ_1 defined by the international standards (R1) and control cycle time should exceed the level γ_2 (R2). To solve the optimization problem for $\tau_1^{0.95}(L)$ we got the distribution function $F_{\tau_1}(t)$ of the random variable τ_1 as it was done in [2,3].

For $B = 100$, $L = 50$, $\rho = 1.2$, $\mu = 24\ s^{-1}$, $\gamma_1 = 0.169$, $\gamma_2 = 16\ s$ we got the following values of systems characteristics: $P(X_1) = 0.16668$, $MQ = 70$, $M\tau_1 = 2.125\ s$, $M\tau = 12.749\ s$. For both optimization problems we got the same value of the overload abatement threshold $L = 37$ with corresponding values $M\tau_1(37) = 2.655\ s$, $\tau_1^{0.95}(37) = 3.2\ s$.

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References

1. V. Hilt, E. Noel, C. Shen, A. Abdelal. *Design Considerations for Session Initiation Protocol (SIP) Overload Control*, RFC 6357. August, 2011.

2. P. Abaev, Yu. Gaidamaka, K. Samouylov. *Queueing Model for Loss-Based Overload Control in a SIP Server Using a Hysteretic Technique*, Lecture Notes in Computer Science, Germany, Heidelberg, Springer-Verlag. 2012, vol. 7469, p. 371-378.

3. P. Abaev, Yu. Gaidamaka, K. Samouylov, A. Pechinkin, R. Razumchik, S. Shorgin. *Hysteretic control technique for overload problem solution in network of SIP servers*, Computing and Informatics. In print.

Method of statistical decision based on bans

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Let X_i, $i = 1, 2, ..., n,...$, be a sequence of finite sets, $\prod_{i=1}^{n} X_i$ be a Cartesian product of X_i, $i = 1, 2, ..., n$, X^∞ be a set of all sequences where i-th element belongs to X_i. Define $\mathcal{A}$ be a σ-algebra on X^∞, generated by cylindrical sets. $\mathcal{A}$ is also Borel σ-algebra in Tichonof product X^∞, where X_i, $i = 1, 2, ..., n,...$, has a discrete topology [1]. On $(X^\infty, \mathcal{A})$ a probability measure P_0 is defined. Assume $P_{0,n}$ is a project of P_0 on the first n coordinates of sequences from X^∞. It is clear that for every $B_n \subseteq \prod_{i=1}^{n} X_i$

$$P_{0,n}(B_n) = P_0(B_n \times X_n^\infty),$$

where $X_n^\infty = \prod_{i=n+1}^{\infty} X_i$. Let $D_{0,n}$ be a support of measure $P_{0,n}$.

Denote $\Delta_{0,n} = D_{0,n} \times X_n^\infty$. The sequence $\Delta_{0,n}$, $n = 1, 2, ...$, is nonincreasing and

$$\Delta_0 = \lim_{n\to\infty} \Delta_{0,n} = \bigcap_{n=1}^{\infty} \Delta_{0,n}.$$

The set Δ_0 is closed and it is a support of P_0. We also have a set of probability measures $\{P_\theta, \theta \in \Theta\}$ on $(X^\infty, \mathcal{A})$. Then as before we define $P_{\theta,n}$, $D_{\theta,n}$, $\Delta_{\theta,n}$, Δ_θ. If $\overline{\omega}^{(k)} \in \prod_{i=1}^{k} X_i$, then $\widetilde{\omega}^{(k-1)}$ is obtained from $\overline{\omega}^{(k)}$ by dropping the last coordinate.

Definition 1. Ban [1] in measure $P_{0,n}$ is a vector $\overline{\omega}^{(k)} \in \prod_{i=1}^{k} X_i$, $k \le n$, such that

$$P_{0,n}\left(\overline{\omega}^{(k)} \times \prod_{i=k+1}^{n} X_i\right) = 0.$$

If $P_{0,k-1}(\widetilde{\omega}^{(k-1)}) > 0$ then $\overline{\omega}^{(k)}$ is the smallest ban.

If there exists $\overline{\omega}^{(n)} \in \prod_{i=1}^{n} X_i$ such that $P_{0,n}(\overline{\omega}^{(n)}) = 0$ then there exists the smallest ban. It follows that for every n the set $\overline{D}_{0,n}$, $\overline{D}_{0,n} \neq \emptyset$, is uniquely determined by least bans in such sense that all elements of $\overline{D}_{0,n}$ are obtained by all possible extensions of smallest bans to the length n.

Let on $(X^\infty, \mathcal{A})$ $\{P_0, , P_{m-1}\}$ be a set of probability measures. Let's denote $S_{k,n}(j)$ be a number of smallest bans of length k in measure $P_{j,n}$, $S_n(j)$ be a set of smallest bans which lengths don't exceed n in measure $P_{j,n}$. Let $\vec{x}_n \in \prod_{i=1}^{n} X_i$ be an initial section of sequence $x, x \in X^\infty$.

Definition 2. Statistical decision function on space $\prod_{i=1}^{n} X_i$ is a random function $d(\vec{x}_n)$ determined on $\prod_{i=1}^{n} X_i$ with values in set $\{0, ..., m-1\}$. Consistent decision function is defined by the following two conditions: for every i

$$\lim_{n\to\infty} P_{i,\,n}(d(\vec{x}_n) = i) = 1,$$

$$\lim_{n\to\infty} P_{i,\,n}(d(\vec{x}_n) = j, i \neq j) = 0.$$

Definition 3. Decision function is defined by the minimal bans if the algorithm of its computation is defined by sets $S_{k,\,n}(j)$, $j = 0, ..., m-1$, $k = 1, ..., n$, $n = 1, 2, ...$

Theorem. *Let measures* $P_0, , P_{m-1}$ *be such that* $P_i(\Delta_j) = 0$, $i \neq j$. *Then there is the consistent decision function* $d(\vec{x}_n)$ *defined by bans.*

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References

1. Grusho, A., Timonina, E.: Prohibitions in discrete probabilistic statistical problems. Discrete Mathematics and Applications. **21**(3), 275–281 (2011).

Optimization technique for flow control parameters in computing system

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The performance of existing and designed computing systems depends essentially on workflow control strategy, that include task distribution and resource allocation. Parameter optimization for such strategies is an important and complex mathematical and engineering problem. This report is aimed to demonstrate one approach to problem solving, which may be proposed for a wide range of application.

The technique is demonstrated on a relatively simple example of a server on which to run job from the random flow. The server has a finite number of serving places and infinite queue with FIFO service discipline. There is a possibility to regulate the incoming workflow: the server can accept the task (and get some fee), but it can also reject the task (and lose the payment). It is dangerous to accumulate a great queue because of heavy fine in case of task deadline excess. Thus, there is a problem of finding an optimal congestion control strategy that maximize the total mean reward. That is, there must be defined the probability of rejection of the new tasks de-pending on the size of the queue. It should be noted that the approximate mathematical solution of the problem is possible only in the case of a Poisson input flow. In a more adequate description, for example in terms of long-tailed distributions, the analytical approach is practically unrealizable.

The approach proposed in the report, involves the implementation of two of the main action: 1) creation of an adequate simulation model of the computing facilities and 2) implementation of simulation experiments with the use of adaptive algorithms, engaged in setting the parameters of congestion control strategy. Details of the technology are contained in [1, 2]. In the case of a simple server simulation does not cause any special difficulties. For more complex system a special framework has been developed, based on the ideas of parallel communicating processes. Adaptive algorithm based on the theory of partially observed Markov decision process found the ability to optimize an objective function, respectively carrying out the optimum setting of congestion control parameters. The report provides the results of applying the optimization technique to the congestion control in the simple server under non-Poisson workload.

References

1. M. G. Konovalov. Methods of adaptive information processing and their application, Moscow, Institute of Informatics Problems RAN, 2007, ISBN 978-5-902030-59-9 (in Russian).

2. 2. M. G. Konovalov. Computer system optimization using simulation model and adaptive algorithms, 2012, vol. 6, is. 1, p. 37-48 (in Russian).

On the overflow probability asymptotics in a Gaussian queue

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We consider the so-called fluid queue with a constant service rate C driven by the input process $A(t)$ which is defined as follows:

$$A(t) = mt + X(t),$$

where $m > 0$ is the mean input rate and the process X is a sum of the independent fractional Brownian motions (fBm's), in general, with different Hurst parameters. Physically, $A(t)$ describe the amount of data (input traffic) arrived into a communication node within time interval $[0, t]$, $t \geq 0$. To motivate our interest to such systems, we consider N_i independent identical heavy-tailed on-off sources of type $i = 1, ..., n$. It then follows that the appropriately scaled cumulative workload arrived during period $[0, Tt]$ converges weakly to a sum of independent fBm's provided first, number of sources $N_i \to \infty$ for each i, and then scaling factor $T \to \infty$, see Taqqu et *al.* [1].

Denote by $r := C - m$ the utilization coefficient. If $r > 0$ then the system is stable and stationary workload Q exists. The present work is focused on the asymptotic analysis of the overflow probability $P(Q > b)$, that is the probability that the stationary workload exceeds a (large) threshold b. Such a probability is an important ingredient of the QoS analysis of the telecommunication systems. We present the logarithmic asymptotics of the overflow probability in the described system. The proof is mainly based on the technique developed by Duffield and O'Connell [2].

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References

1. *Taqqu M., Willinger W., Sherman R.* Proof of a fundamental result in self-similar traffic modellling. Computer Communication Review, Vol. 27, 1997, pp. 5-23.

2. *Duffield N., O'Connell N.* Large deviations and overflow probabilities for the general single server queue, with applications. Proceedings of the Cambridge Philosophical Society, 1995, vol. 116, p. 363–374.

Stationary distribution of $M_2|M_2|1|R$ queue with bi-level hysteric policy

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Queueing systems with hysteresis have recently begun to draw attention of researchers due to their possible application in next generation networks, namely for overload control in servers that deal with signalling processing [1]–[3]. Among recent papers that deal with analysis of queues with hysteric policies one can mention [4]–[10]. The utilized methods (including potential method) allow one to obtain different stationary performance characteristics under different assumptions about service time distribution and incoming flow. In this study we analyze queueing system $M_2|M_2|1|R$ with bi-level hysteric policy which is the generalization of model, presented in [4], and develop new effective approach for calculation of its joint stationary distribution.

Consider the queueing system with two poisson incoming flows of customers (say type 1 and type 2) with rate λ_1 and λ_2 respectively, finite queue of size $R < \infty$, and one server. If arriving customer sees R customers in the system, it is considered to be lost. Type 1 customers have relative priority over customers of type 2. Customers of type 1 and type 2 are served exponentially with different service rates. The hysteric mechanism operates as follows. Suppose we have numbers L, H such that $0 < L < H < R$. When the system starts to work it is empty and as long as the total number of customers in the system remains below $(H-1)$, system is considered to be in "normal" state. When total number of customers exceeds $(H-1)$ for the first time, the system changes its state to "overload" and stays in it as long as the number of customers remains between L and $(R-1)$. In "overload" state system accepts only type 1 customers till the number of customers drops down below L after which it changes its state back to "normal", or exceeds $(R-1)$ after which it changes its state to "blocking". In the "blocking" state systems does not accept new arriving customers until the total number of customers drops down below $(H+1)$, after which system's state changes back to "overload".

The proposed method allows computation of main performance characteristics of the system and calculation of joint stationary distribution of number of type 1 and type 2 messages in the queue and system's state for relatively high values of thresholds.

In order to check theoretical results there was built a simulation model using GPSS software. The comparisons of numerical and simulation results showed good accuracy.

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References

1. Abaev, P., Gaidamaka, Yu., Pechinkin, A., Razumchik, R., Shorgin, S. Simulation of overload control in SIP server networks. *Proc. of the 26th European Conference on Modelling and Simulation, ECMS 2012.* Pp. 533–539.

2. Abaev, P., Gaidamaka, Yu., Samouylov, K. Queuing Model for Loss-Based Overload Control in a SIP Server Using a Hysteretic Technique // *Germany, Heidelberg, Springer-Verlag: Lecture Notes in Computer Science.* 2012. Vol. 7469. Pp. 371-378.

3. Abaev, P., Gaidamaka, Yu., Pechinkin, A., Razumchik, R., Samouylov, K., Shorgin, S. Hysteretic control technique for overload problem solution in network of SIP servers. // *Computing and Informatics.* 2013.(accepted).

4. Abaev, P., Pechinkin, A., Razumchik, R. On analytical model for optimal sip server hop-by-hop overload control. *Proc. of the 4th International Congress on Ultra Modern Telecommunications and Control Systems ICUMT-2012*. Pp. 303–308.

5. Bekker, R. Queues with Levy input and hysteretic control. *Queueing Systems*. 2009. Vol. 63. Issue 1. Pp. 281–299.

6. Bekker, R., Boxma, O.J. An $M|G|1$ queue with adaptable service speed. *Sochastic Models*. 2007. Vol. 23. Issue 3. Pp. 373–396.

7. Choi, D.I., Kim, T.S., Lee, S. Analysis of an $MMPP|G|1|K$ queue with queue length dependent arrival rates, and its application to preventive congestion control in telecommunication networks. *European Journal of Operational Research*. 2008. Vol. 187. Issue 2. Pp. 652–659.

8. Chydzinski, A. The oscillating queue with finite buffer. *Performance Evaluation*. 2004. Vol. 57. No. 3. Pp. 341–355.

9. Taremi, M., Reza Salehi Rad, M. Limit analysis of oscillating batch arrival $M^{[x]}|G|1$ sysmtes with finite capacity: EMC approach. *Journal of Economic Theory*. 2012. Vol. 6. No. 1. Pp. 29–36.

10. Zhernoviy, K.Yu., Zhernoviy, Yu.V. Queueing system $M^{\theta}|G|1|m$ with bilevel hysteric strategy of service rates switch-over. *Journal of Information processes*. 2012. Vol. 12. No. 2. Pp. 127–140. (in Russian)

Symbolic software tools for distributions parametrization in nonlinear stochastic systems

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In [1–2] one- and multidimensional distributions for analytical modeling methods and symbolic software tools based on parametrization by moments, quasimoments, coefficients of orthogonal expansions etc for nonlinear Euclidian stochastic systems (StS) are described. Corresponding means for multichannel circular and spherical StS are considered in [3–5]. The paper is devoted to symbolic software tools for StS on manifolds (MStS) based on mathematical foundation given in [6]. Special attention is paid to normal and ellipsoidal approximation methods.

The original software tools “MStS-Analysis” are instrumented in MATLAB for nonlinear continuous and discrete MStS. Its current experimental version uses functions of MATLAB Symbolic Math toolbox and presents the set of open program functions with numerical and graphic output.

Applications: Stochastic mechanical nonholonomial and control devices.

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References

1. V. S. Pugachev, I. N. Sinitsyn. *Stochastc Systems. Theory and Applications*, 2001, World Scientific, Singapore.

2. I. N. Sinitsyn, V. I. Sinitsyn. *Stochastic Systems. Normal and Ellipsoidal Distributions Approximations*, 2013, TORUS PRESS, Moscow (in Russian).

3. I. N. Sinitsyn. Mathematical software for nonlinear multichannel circular stochastic systems analysis based on distribution parametrization. *Informatics and Applications*, 2012, vol. 6, No. 1, p. 11–17 (in Russian).

4. I. N. Sinitsyn, V. V. Belousov, T. D. Konashenkova. Software Tools for Circular Stochastic Systems Analysis. // *In Book of Abstracts “XXIX International Seminar on Stability Problems for Stochastic Models” and V International Workshop “Applied Problems in Theory of Probabilities and Mathematical Statistics related to modeling of information systems”*, 10–16 October 2011, Svetlogorsk, Russia, p. 86–87.

5. I. N. Sinitsyn, V. V. Belousov, T. D. Konashenkova. Software Tools for Spherical Stochastic Systems Analysis and Filtering. // *In Book of Abstracts “XXX International Seminar on Stability Problems for Stochastic Models” and V International Workshop “Applied Problems in Theory of Probabilities and Mathematical Statistics related to modeling of information systems”*, 24–30 September 2012, Svetlogorsk, Russia, p. 91–93.

6. I. N. Sinitsyn. Parametric statistical and analytical modeling of distributions in nonlinear stochastic systems. *Informatics and Applications*, 2013, vol. 7, No. 2 (in print).

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Choice of optimal portfolio with transaction costs for one-period deterministic model

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Unlike one-period transaction with a single asset, the scheme for calculating the real net return on a one-period portfolio transaction has a number of features. Consider n assets $A_1, \dots, A_n$ and the portfolio π, defined by the position vector $z = (z_1, \dots, z_n)$. Let $S_0 = \sum_{k=1}^{n} z_k P_k^0$, (resp. $S_1 = \sum_{k=1}^{n} z_k P_k^1$) be the initial (resp. final) portfolio prices, where P_k^0 (resp. P_k^1) is the initial (resp. final) price of A_k. Assume that $S_0 > 0$. It is well-known [1, 3] that the price return of π without commission is of the form

$$r^{(p)} = \frac{S_1 - S_0}{S_0} = r_1^{(p)} w_1 + \ldots + r_n^{(p)} w_n.$$

Here $r_k^{(p)}$ (resp. w_k) is the price return (resp. the weight) of A_k.

Now consider the portfolio π with commission α. Let $C_0^{(\alpha)} = \alpha \sum_{k=1}^{n} |z_k| P_k^0$ (resp. $C_1^{(\alpha)} = \alpha \sum_{k=1}^{n} |z_k| P_k^1$) be the value of initial (resp. final) commission. In this case the price return of π is defined by the formula

$$r_\alpha^{(p)} = \frac{S_1 - S_0 - C_0^{(\alpha)} - C_1^{(\alpha)}}{S_0 + C_0^{(\alpha)}}.$$

This formula can be written as

$$r_\alpha^{(p)} = \frac{\sum\limits_{k=1}^{n} r_k^{(p)} w_k - \alpha \sum\limits_{k=1}^{n} (2 + r_k^{(p)}) |w_k|}{1 + \alpha \sum\limits_{k=1}^{n} |w_k|} = \frac{r^{(p)} - \alpha \sum\limits_{k=1}^{n} (2 + r_k^{(p)}) |w_k|}{1 + \alpha \sum\limits_{k=1}^{n} |w_k|}. \quad (1)$$

According to (1), there is no simple linear relationship between $r_\alpha^{(p)}$ and w_k, $k = 1, \dots, n$. Moreover, when choosing the optimal portfolio, taxes further complicate the picture, especially if the current income tax $\tau^{(c)}$ and price income tax $\tau^{(p)}$ are separated. In this case for the total income return we have

$$r_{\alpha,\tau} = (1-\tau^{(p)})r_\alpha^{(p)} + \frac{(1-\tau^{(c)})I^{(c)}}{S_0 + C_0^{(\alpha)}} = (1-\tau^{(p)})r_\alpha^{(p)} + (1-\tau^{(c)})r_\alpha^{(c)}.$$

Here $I^{(c)} = \sum_{k=1}^n z_k I_k^{(c)}$ is the portfolio total current income, $I_k^{(c)}$ is the current income of A_k and $r_\alpha^{(c)}$ is the portfolio current return.

Note that the complexities encountered here are only analytical but not computational.

Now let us consider the special case in which $n = 2$ (two dimensional portfolios). From (1) it follows that $r_\alpha^{(p)}$ is a fractional-linear function on w_1, w_2 for a given type of transaction, i.e. for a given position type (long or short) for each asset. Namely

$$\overset{+}{r}{}_\alpha^{(p)} = \frac{1-\alpha}{1+\alpha} r^{(p)} - \frac{2\alpha}{1+\alpha}, \quad \text{for } w_1 \ge 0,\ w_2 \ge 0,$$

$$\overset{\pm}{r}{}_\alpha^{(p)} = \frac{w_1(r_1^{(p)}(1-\alpha) - 2\alpha) + w_2(r_2^{(p)}(1+\alpha) + 2\alpha)}{1 + \alpha(w_1 - w_2)}, \quad \text{for } w_1 \ge 0,\ w_2 < 0,$$

$$\overset{\mp}{r}{}_\alpha^{(p)} = \frac{w_2(r_2^{(p)}(1-\alpha) - 2\alpha) + w_1(r_1^{(p)}(1+\alpha) + 2\alpha)}{1 + \alpha(w_2 - w_1)}, \quad \text{for } w_1 < 0, w_2 \ge 0,$$

It should be noted that the optimal portfolio without transaction costs may be not optimal if the transaction costs are taken into account.

Consider the problem of finding the optimal two-dimensional portfolio (denoted by $\pi^* = (w_1^*, w_2^*))$ with commission for the case $\max(r_1^{(p)}, r_2^{(p)}) > 0$ and $r_1^{(p)} > r_2^{(p)}$. We shall search for the optimal portfolio in the admissible region $a \le w_1,\ b \le w_2,\ a, b \le 0$. The optimal portfolio return is equal to $r_\alpha^{(p)}(\pi^*) = \max(\overset{+}{r}{}_\alpha^{(p)}, \overset{\pm}{r}{}_\alpha^{(p)}, 0)$ and

$$\begin{cases} \pi^* = (1-b, b), & \text{if } \overset{\pm}{r}{}_\alpha^{(p)} \ge \max(\overset{+}{r}{}_\alpha^{(p)}, 0); \\ \pi^* = (1, 0), & \text{if } \overset{\pm}{r}{}_\alpha^{(p)} \ge \max(\overset{+}{r}{}_\alpha^{(p)}, 0); \\ \text{the optimal portfolio does not exist}, & \text{if } \max(\overset{+}{r}{}_\alpha^{(p)}, \overset{\pm}{r}{}_\alpha^{(p)}, 0) \le 0. \end{cases}$$

The last case means that the investor refuses to invest.

References

1. Noel Amenc , Veronique Le Sourd. *Portfolio Theory and Performance Analysis*, 2003, John Wiley & Sons.

2. Carl R. Bacon. *Practical Portfolio Performance Measurement and Attribution*, 2008, John Wiley & Sons.

3. Yu. F., Kasimov. *Finance and Investment*, 2008, Ankil, Moscow.

4. John L. Maginn, Donald L. Tuttle. *Managing Investment Portfolios*, 2007, John Wiley & Sons.

5. Jean-Luc Prigent. *Portfolio Optimization and Performance Analysis*, 2007, Chapman & Hall/CRC.

Investigation of the stability region of a general retrial queueing system with constant retrial rate

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We consider a finite capacity retrial queueing system Σ with renewal input rate λ and a constant retrial rate. Rejected customers join an infinite-capacity orbit and then try to rejoin the primary queue after an exponentially distributed time with (retrial) rate μ_0. Thus, unlike classical retrial models, the orbit rate in Σ does not depend on the orbit size (the number of orbit customers). Such a system can be successfully applied to model the multi access protocol ALOHA with restrictions for individual retrial rates and TCP protocols provided most of the transfers are short.

We call this system unstable if the orbit size increases unlimitedly. Otherwise, the system is stable. Indeed, under mild conditions the stable system, being regenerative, obeys the stationary regime (the stationary distributions of the basic processes describing the dynamics of the system, such like workload, orbit size, etc.). The following sufficient stability condition of system Σ has been found in Avrachenkov and Morozov [1]:

$$(\lambda + \mu_0)P_{loss} < \mu_0,$$

where P_{loss} is the stationary loss probability in an auxiliary loss system with input rate $\lambda + \mu_0$. At the same time, stability criteria of such a system with renewal input and with *exponential service time* have been found in Lillo [2].

In this regard, a contribution of our work is the studying by simulation the stability region given by presented condition and comparing the results with that given by the stability criteria, for the bufferless system Σ with exponential service time. For a non-Poisson input of primary customers, P_{loss} is typically unknown, and we use a regenerative simulation to obtain a sample mean estimator of P_{loss} instead, to check the fulfilment of presented condition. Moreover, we use antithetic (negatively correlated) variables to reduce variance of the estimator, see Ross [3].

Furthermore, we consider a rather new retrial queueing system with N classes of customers and N orbits. Class-i primary customers are characterized by input rate λ_i, service rate μ_i and exponential retrial times with rate $\mu_0^{(i)}$, $i = 1, \dots, N$. Such a system is motivated by multiple telecommunication applications, for instance wireless multi-access systems, and transmission control protocols.

The stability analysis of this multi-class model is more complicated than that of the single-class system. The multi-class Σ is stable if no orbit has infinite growth. Again, using a regenerative approach we investigate the stability region of the multi-class retrial system. For the single-server bufferless system with Poisson inputs of primary customers, the following *necessary stability conditions* are obtained

$$\lambda_i P_b < \mu_0^{(i)}(1 - P_b),\ i = 1, \dots, N,$$

where P_b is the stationary busy probability in an auxiliary loss system with input rate $\sum_i(\lambda_i + \mu_0^{(i)})$. Presented conditions have rather clear probabilistic interpretation. Indeed, by the PASTA property, P_b is also the stationary blocking probability for arriving primary customers. Then, in stable regime, the left hand side of conditions is the rate of the blocked (primary) customers going to orbit, while the right hand side is the rate of the orbit customers which successfully enter the server.

As simulation results show, obtained conditions allow to delimit the stability region of a 2-orbit system with a remarkable accuracy. It means that presented necessary conditions are in fact also sufficient conditions (that is stability criteria), at least for the considered 2-orbit system with Poisson inputs and generally distributed service times.

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References

1. K. Avrachenkov, E. Morozov. *Stability analysis of $GI/G/c/K$ retrial queue with constant retrial rate*, 2010, INRIA Research Report No. 7335. Available online at http://hal.inria.fr/inria-00499261/en/.

2. L. E. Lillo. A $G/M/1$-queue with exponentional retrial. *Top*, 1996, vol. 4, No. 1, p. 99–120.

3. S. M. Ross. *Simulation*, 2006, Elsevier, USA.

Closed queueing network with non-active customers and bypasses

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Queueing network model. Closed queueing network with M customers and set of systems $J = \{1, 2, \dots, N\}$ is considered. There are input Poisson flows of signals at rates ν_i and φ_i, $i \in J$. When arriving at the system $i \in J$ the signal at rate ν_i induces an ordinary customer at system, if any, to become non-active. When arriving at the system $i \in J$ the signal at rate φ_i induces an non-active customer, if any, to become an ordinary. Non-active customers are in a system queue and can not get service. Signals do not need service. Service times are independent exponentially distributed random values with parameters μ_i, $i \in J$. Let $n_i(t), n'_i(t)$ are numbers of ordinary and non-active customers at system $i \in J$ at time t accordingly. Consider $X(t) = \Big((n_1(t), n'_1(t)), \dots, (n_N(t), n'_N(t))\Big)$. $X(t)$ is a continuous-time Markov chain. States space for process $X(t)$ is $Z = \{(n, n') = ((n_1, n'_1), \dots, (n_N, n'_N)) | \, n_i, \, n'_i \geq 0, \, i \in J, \sum_{k=1}^{N}(n_k + n'_k) = M\}$. When arriving at the system i customer queues up to the system with the probability $f_i(n_i + n'_i)$ and with the probability $1 - f_i(n_i + n'_i)$ the customer bypasses the system $i \in J$ (such customer is considered to be served). After finishing of service process at system $i \in J$ customer is routed to system $j \in J$ with the probability $p_{i,j}$ $(\sum_{j=1}^{N} p_{i,j} = 1)$, $i \in J$. Let $p_{i,i} = 0$, $i \in J$.

A traffic equations system is:

$$\varepsilon_i = \sum_{j=1}^{N} \varepsilon_j p_{j,i}, \quad i \in J.$$

Theorem. $X(t)$ is ergodic and has stationary distribution:

$$\pi(n, n') = \frac{1}{G(M, N)} \prod_{i=1}^{N} \Big[\Big(\frac{\varepsilon_i}{\mu_i}\Big)^{n_i} \Big(\frac{\varepsilon_i \nu_i}{\mu_i \varphi_i}\Big)^{n'_i} \prod_{l=1}^{n_i + n'_i} f_i(l-1) \Big].$$

Here ε_i, $i \in J$ – is a traffic equations system solution, $G(M, N)$ is found from the normalization condition $G(M, N) = \sum\limits_{(n,n') \in Z} \pi(n, n') = 1$.

References

1. G. Sh. Tsitsiashvili, M. A. Osipova. *Distributions in stochastic network models*, 2008, Nova Publishers.

2. Yu. V. Malinkovsky. Queueing networks with bypasses of order nodes. *Automat. Remote Control*, 1991, vol. 52, No. 2, p. 225–231.

Econometric analysis of Russian stock market

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When we consider stock market one of the main problem is to find the relations between different stocks and try to explain how to predict the values of the stocks using so called market index.

In our report we investigate Russian stock market. We consider the stocks of the following russian companies:
1) Gasprom (GAZN);
2) Uralkalii (URKA);
3) Aeroflot (AFLT);
4) Lukoil (LKOM);
5) Noriskii nikel (GMKN);
6) Severstal (CHMF).

As market index we consider RTS STD.

First we investigate the applicability of CAPM to russian stock market (see Sharp [1]). Next we find the cointegrated pairs of stocks.

References

1. Sharpe W. F.Ċapital Asset Prices: a Theory of Market Equilibrium Under Conditions of Risk, *Journal of Finance*, 1964, vol. 19, No. 3, p. 425-442.

Percentage of prolongation of an insurance portfolio as an indicator of its structure deterioration

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Rate making and reserving in motor insurance (casco) are quite complex and time-consuming tasks, one of the main steps of which is to identify and evaluate all the risk factors influencing an insurance portfolio at any given period of time. When analyzing the insurance portfolio of one of the companies operating in the Russian market it has been suggested that one of such factors can be the percentage of prolongation of the portfolio. This hypothesis suggests that there is a direct correlation between the percentage of prolongation and the "quality" of the portfolio due to the fact that drivers who believe them to be more susceptible to the insured event will be more likely to prolong their policies than those who believe them to be more accurate and self-confident on the road. Thus, the percentage of prolongation below expectations in one year (or equivalently higher than expected level of termination) will result in higher claim frequency of the prolongation portfolio next year.

On the other hand it can be assumed that at the moment of policy renewal a group of insured, which feels less confident about driving a car, will already have some claims, what results in a higher premium for the next year. In this situation, many insured simply decide to buy an insurance policy in another company for much lower price. In this case our hypothesis will be rejected.

For the purpose of hypothesis testing data for the years 2008-2010 of the above mentioned company was used. These data include information on the percentage of policy termination and claim frequency broken down into major risk factors: driver age and experience, vehicle sum insured and lifetime and etc.

One of the possible ways to test our hypothesis can be based on the calculation of correlation coefficients between the percentage of policy termination and claim frequency of the prolongation portfolio and further test whether this coefficient is significantly greater than zero. But the data was available just for three consecutive years, which is not enough to use the method.

Therefore, this analysis was built on the theory of generalized linear models. Two GLMs were built for claim frequency: the first one includes percentage of termination as a risk factor and the second one doesn't. These models were compared using likelihood-ratio-test. All calculations were made on the basis of the statistical package R.

The calculations were as follows: for the policies that have been prolonged in 2008-2009 analysis revealed the importance of the percentage of termination, while for policies, renewed in 2010, it showed no significant influence on claim frequency.

One possible explanation of the result is the following: peaks of the world economic crisis came just exactly on 2008-2009, so the only least confident insured decided to renew their policies in those years in the face of financial difficulties. And from the beginning of 2010 there was an economic recovery and this hypothesis was no longer so important.

References

1. E. Ohlsson, B. Johansson. *Non-life insurance pricing with generalized linear models*, 2010, Springer, Berlin.
2. S. A. Aivazian, V. S. Mkhitarian. *Probability theory and applied statistics. Essentials of econometrics*, 2001, Unity, Moscow.

Ruin probability estimation in the model with investments

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In actuarial mathematics is very important to investigate the influence of investments to indices of insurance company.

In our model we assume that the capital of insurance company at the moment t is ruled by the classical Cramer-Lundberg risk process:

$$R(t) = u + c \cdot t - \sum_{j=1}^{N(t)} X_j \ ,$$

where

1) u – initial capital;
2) $(N(t), t \geq 0)$ – homogeneous Poisson process;
3) $\{X_j, j \geq 1\}$ - i.i.d.r.v.

Next we assume that part $\alpha(t)$ of the capital is invested in risky asset $S(t)$ and part $1 - \alpha(t)$ – in riskless asset $B(t)$. The dynamics of these assets is represented by equations:

$$dS(t) = S(t)(\mu dt + \sigma dW(t)) \ ,$$

$$dB(t) = rB(t)dt \ ,$$

where $0 < r < \mu$, $\sigma > 0$, $(W(t), t \geq 0)$ – Brownian process.

Then the full capital $X(t)$ of insurance company is described by the equation:

$$dX(t) = [\alpha(t)\mu + (1 - \alpha(t))r)dt + \alpha(t)\sigma dW(t)] \cdot X(t) + dR(t) \ ,$$

$X(0) = u$.

In our report we represent the analog of Gerber formula for ruin probability.

Networks with multimode strategies of service and several types of customers

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Open queueing networks with several types of customers, Poisson incoming flow, exponential service in the nodes and Markov routing are studied. In each of the nodes there is the only device, which can operate in several regimes.

Queueing networks with multiregime service strategies have been investigated relatively recently. The necessity of their study was caused by practical considerations, because such networks allow us consider models with partially nonreliable devices. In practice the situation when devices in the network nodes are unreliable or partially unreliable often meets. Therefore search of the models in which devices in the nodes can work in several regimes answering different degrees of their working capacity is very important. When the node transits to the regime with the bigger number (in less "reliable" regime) productivity of the node decreases. The device can partially lose working capacity (the case of complete loss of working capacity isn't considered here) both during customer service, and in a free condition from customers.

Transitions of the node from one operating regime to another one are caused by internal possibilities of devices and also existence of the signals circulating in the network and reducing the number of an operating regime.

Abstract description of network states is used. Such description was considered in the article [1].

Conditions of ergodicity, sufficient conditions of multiplicativity and an analytical view of stationary distribution of network states probabilities are found.

References

1. Y. V. Malinkovsky. A criterion of representability for the stationary states distribution of Markov open queueing network with several classes of customers in the product form. *Automation and Remote Control*, 1991, No. 4, p. 75–83.

Modelling of fractional Levy motion

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Recently some generalization of fractional Brownian motion has been represented in De Nicola [1].

Let $(B_H(t), t \geq 0)$ be fractional Brownian motion with Hurst parameter H, $(L^1_\alpha(t), t \geq 0)$, $(L^2_\alpha(t), t \geq 0)$ be α-stable subordinators, $0 < \alpha \leq 1$, and B_H, L^1_α and L^2_α are independent. Consider the new process

$$X(t) := \begin{cases} B_H(L^1_\alpha(t)) & , \quad t \geq 0, \\ -B_H(L^2_\alpha(t)) & , \quad t < 0, \end{cases}$$

This model has the properties of long-range dependence and heavy tails of distributions. But it is very difficult to calculate in explicite form the most important characteristics of the teletfaffic when it is modelled by such model. So we need to use the simulation.

In our report we represent the collection of algorithms and programs for simulation of such process.

References

1. C. De Nikola, Y. S. Khokhlov, M. Pagano and O. I. Sidorova. Fractional Levy motion with dependent increments and its application to network traffic modeling, *Informatics and Applications*, 2012, vol 6, No. 3, p. 58-63.

Optimal investment strategy in the risk model with capital injections

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In this research we consider a discrete time risk model, i.e. the surplus of an insurance company is considered only at times 0, 1, 2, ... Let Y_k be the total amount of claims in k-th period, occuring at the end of the period. We assume that $Y_1, Y_2, \ldots$ are i.i.d. r.v. with absolutely continuous distribution function $Q(y)$ and probability density function $q(y)$. We denote the insurance premium income per each time unit as c. We also assume that whenever a claim causes the surplus of the company fall below fixed level $L > 0$, additional capitals should be injected in order that the company never goes to ruin. Note that this kind of modification of the discrete time risk model was initially proposed by Dickson and Waters in [1].

In order to minimize the amount of capital injections the insurance company invest money into some risky asset. Let Z_k be the return of a risky asset in period k, i.e. if we invest one monetary unit at time $k-1$, we get back $1+Z_k$ units at time k. We suppose that $R_1, R_2, \ldots$ — are i.i.d. and independent of $Y_1, Y_2, \ldots$ r.v. with common probability distribution function $H(z)$. Moreover, we assume that market is arbitrage-free, so $0 < P(Z_k \geq 0) < 1$ and $EZ_k > 0$. In this paper we consider adopted investment strategies $A = \{A_0, A_1, \ldots\}$, where $A_k \geq 0\, a.s.$ is the amount invested into the risky asset at the beginning of k-th period. In this scenario the surplus process R_k of the insurance company becomes

$$R_k^A = \max(L, R_{k-1}^A + c + A_{k-1}Z_k - Y_k),\; k = 1, 2, \ldots,\;\; R_0 = s,$$

where $s > 0$ is the initial reserve. Suppose that the company works for $n \leq \infty$ time periods, then the total amount of capital injected is

$$W_n^A(s) := E(\sum_{k=1}^{\infty} v^k \max(0, L - R_{k-1}^A - c - A_{k-1}Z_k + Y_k)|R_0^A = s),$$

where $v \in (0,1)$ is the unit time discount factor. And our goal is to minimize this amount over all admissible investment strategies $W_n(s) := \inf_A W_n^A(s)$. In this case the value function $W_n(s)$ satisfies the Bellmann equation:

$$W_n(s) = v \inf_{\alpha \geq 0} \{E \max(0, L - s - c - \alpha Z_1 + Y_1) + E W_{n-1}(\max(L, s + c + \alpha Z_1 - Y_1))\} \quad (1)$$

with $W_0(s) = 0$. We prove the following

Theorem. 1) For each $n \leq \infty$ the function $W_n(s) \in C^2[0,\infty)$. Moreover, $W_n'(s) \in [-1,0]$ and $W_n''(s) \geq 0$.

2) For each n the infimum in Bellmann equation is taken at point $\alpha_n^*(s)$, which is the unique solution to the following equation

$$E[Z(Q(s + c + \alpha Z_1 - L) - 1 + W_{n-1}'(s + c + \alpha Z_1 - Y_1))] = 0.$$

3) For $n < \infty$ the optimal investment strategy is $A_k^* = \alpha_{n-k}^*(R_k)$, $k = 0, \ldots, n-1$, where $\alpha_k^*(s)$ minimizes the right-hand side of the equation (1) for n=k. For $n = \infty$ the optimal investment strategy is $A_k^* = \alpha^*(R_k)$, where $\alpha^*(s)$ minimizes the right-hand side of the equation (1) for $n = \infty$.

In this paper we also prove some properties of the optimal investment level $\alpha_n^*(s)$ and provide some numerical examples to illustrate the theory.

References

1. D.C.M. Dickson,H.R. Waters. Some optimal dividend problems, *Astin Bulletin*, 2004, 34(1), 49–74.

2. H. Schmidli. *Stochastic Control in Insurance*, Springer–Verlag, London, 2008.

3. H. Wua, J. Guoa, L. Tang. Optimal dividend strategies in discrete risk model with capital injections, *Appl. Stochastic Models Bus. Ind.*, 2011, 27, 557-566.

A continuous inventory optimal control problem for discrete semi-Markov model

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A system used for storing and delivery some inventory is considered. The exact value of inventory at the moment $t > 0$ is defined by stochastic process $x(t) \in (-\infty, \tau]$, where τ - is the maximum storage capacity. A negative value of the inventory is related to inventory deficiency. We assume that the inventory consumption in the model goes with constant rate $\alpha > 0$.

We partition the set $(-\infty, \tau]$ into finite number of subsets in the following way:

$$\left[0, \tau_1^{(0)}\right), \left[\tau_1^{(0)}, \tau_2^{(0)}\right), \dots, \left[\tau_{N_0-1}^{(0)}, \tau_{N_0}^{(0)}\right], \quad \text{where} \quad \tau_{N_0}^{(0)} = \tau;$$

$$\left(-\infty, \tau_{N_1}^{(1)}\right], \left(\tau_{N_1}^{(1)}, \tau_{N_1-1}^{(1)}\right], \left(\tau_1^{(1)}, \tau_0^{(1)}\right], \quad \text{where} \quad \tau_0^{(1)} = (0-).$$

If at the moment $\tilde{t} > 0$ right after an inventory replenished the process $x(\tilde{t})$ is within $\left[\tau_i^{(0)}, \tau_{i+1}^{(0)}\right)$ then the moment when a replenish order of inventory has been done is planed after time $\xi_i^{(0)}$, where $\xi_i^{(0)}$ is a random variable having distribution function $G_i^{(0)}(t)$, $i = \overline{0, N_0 - 1}$.

When an inventory replenish gets ordered the delivery period gets started.

Denote by $\eta_k^{(0)}$ a random variable which describes a delivery duration if at the order moment $\hat{t} > 0$ the system inventory value $x(\hat{t}) = x - \alpha\xi_i^{(0)} \in \left[\tau_k^{(0)}, \tau_{k+1}^{(0)}\right)$; if the system inventory value $x(\hat{t}) = x - \alpha\xi_i^{(0)} \in \left(\tau_{k+1}^{(1)}, \tau_k^{(1)}\right]$ then duration of the delivery is described by random variable $\eta_k^{(1)}$. Functions $H_k^{(0)}(t), k = \overline{0, N_0 - 1}$ and $H_k^{(1)}(t), k = \overline{0, N_1}$ are the distribution functions for random variables $\eta_k^{(0)}$ and $\eta_k^{(1)}$ respectively.

Let $\mu_k^{(0)} = M\eta_k^{(0)} < \infty, k = \overline{0, N_0 - 1}$ and $\mu_k^{(1)} = M\eta_k^{(1)} < \infty, k = \overline{0, N_1}$ are given values which describes mathematical expectations of delivery duration. At the end of a delivery duration period the inventory is replenished.

The system inventory replenished is related to process $x(t)$ transition from one of the admissible subsets to another. To describe this procedure we denote the next probabilistic characteristics of the system:

$\left\{\beta_{kl}^{(0)}\right\}_{l=k}^{N_0-1}$ - transition probabilities from $\left[\tau_k^{(0)}, \tau_{k+1}^{(0)}\right)$ to $\left[\tau_l^{(0)}, \tau_{l+1}^{(0)}\right)$, where $k = \overline{0, N_0 - 1}$.

$\left\{\beta_{kl}^{(1)}\right\}_{l=0}^{N_0-1}$ - transition probabilities from $\left(\tau_{k+1}^{(1)}, \tau_k^{(1)}\right]$ to $\left[\tau_l^{(0)}, \tau_{l+1}^{(0)}\right)$, where $k = \overline{0, N_1}$.

For the considered model we assume that an inventory value after replenished moment is a positive value always.

An exact inventory value after replenished is defined by probabilistic distributions $B_l(x)$, where $l = \overline{0, N_0 - 1}$ for each of the subsets $\left[\tau_l^{(0)}, \tau_{l+1}^{(0)}\right)$.

Characteristics $\left\{\beta_{kl}^{(0)}\right\}_{l=k}^{N_0-1}$, where $k = \overline{0, N_0 - 1}$; $\left\{\beta_{kl}^{(1)}\right\}_{l=0}^{N_0-1}$, where $k = \overline{0, N_1}$ and $B_l(x)$, where $l = \overline{0, N_0 - 1}$ are assumed as given.

A process $x(t)$ evolution after an order for replenish as well as after a replenished moment depends on a subset in which the process has got into. In other words, for the rules of system functioning described above the process

$x(t)$ has Markov condition in the moments of orders for replenish and in the moments right after replenished.

Along with the main process $x(t)$ we denote an auxiliary semi-Markov process $\zeta(t)$, $t \geq 0$ having the finite set of states.

Let $\{t_n\}_{n=0}^{\infty}$ is a sequence of inventory replenished moments. Let ζ_n - subset in which process $x(t)$ has got into at the moment $t_n + 0$: $\zeta_n = k$ if $x(t_n + 0) \in \left[\tau_k^{(0)}, \tau_{k+1}^{(0)}\right)$, where $k = \overline{0, N_0 - 1}$. The defined sequence $\{\zeta_n\}_{n=0}^{\infty}$ is the Markov chain. Let's define the process related to sequence $\{\zeta_n\}_{n=0}^{\infty}$ by the following interrelation:

$$\zeta(t) = \zeta_n \quad \text{where} \quad t_n \leq t < t_{n+1}; \quad n = 0, 1, 2, \ldots$$

The process $\zeta(t)$ is a controlled semi-Markov process having finite set of states $E = \{0, 1, \ldots, N_0 - 1\}$. The process $\zeta(t)$ is controlled in the moments t_n; the controlled parameter u_n - a time interval between a moment right after inventory replenished and a moment when the next replenish of the inventory is to be ordered. Exactly, $u_n = \xi_k^{(0)}$ if $\zeta_n = k$. The set of admissible controlled decisions u_n is $U = [0, \infty)$.

The optimal control problem of the described stochastic model is related to choosing distribution functions $G_k^{(0)}(t) = P(\xi_k^{(0)} < t), k = \overline{0, N_0 - 1}$ so that the quality index of controlling the system $I(G_k^{(0)}(\cdot), k = \overline{0, N_0 - 1})$ approaches to an absolute extremum.

The main results obtained in the research.

The functionals related to the quality of the system control are obtained in the analytic form. The statement that the optimal strategy of controlling the system is a deterministic strategy is proved. Analytic form representation for the function the absolute extremum of which is determined as the optimal control strategy is obtained also.

Pair trading

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One of the main problem in financial mathematics is how to predict the value of one security if we can observe the other one only. In such situation we can use the effect of so called of cointegration of two time series.

Let we have two time series $(X_t, t \geq 0)$ and $(Y_t, t \geq)$ which are not stationary (for example they are 1-order integrated) but for some $\beta \in R^1$ the series $Z_t = Y_t - \beta \cdot X_t$ is stationary. In this case we say that X_t and Y_t are cointegrated. It means that these series have some common long-term tendency.

Next using the methods from the theory of stationary process we can construct formally some prediction for process $(Z_t, t \geq 0)$. If we have the trajectory of $(X_t, t \geq 0)$ we can find some prediction for $(Y_t, t \geq 0)$.

In our report we represent several procedures which realized this plan for some special type of processes.

References

1. Ganapthy Vidyamurthy. *Pair Trading*, 2004, John Wiley & Sons, New Jersey.

Portfolio of options with dependent underlying assets

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In paper Cox [1] it was represented a simple discrete-time option price formula. It is assumed that the stock price follows a multiplicative binomial process over discrete periods. In such model some Binomial Option Pricing Formula has been derived. Next it was shown that famous Black-Scholes formula can be derived as a limit one.

We propose some multivariate generalization of this result. It is considered the European call option on two stocks without dividents. The prices of these stocks follows some multivariate generalization of multiplicative binomial process. The main feature of our models is that the prices of stocks are dependent.

In our report we represent some analog of Binomial Option Pricing formula and show that some multivariate analog of Black-Scholes formula can be derived as a limit one.

References

1. J. C. Cox , S. A. Ross, M. Rubinstein. Option pricing: A simplified approach. *Journal of Financial economics*, 1979, vol. 7, p. 229–263.

The preliminary analysis and the data processing, intended for creation of a mathematical market model of grain crops in Russia

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The purpose of the present paper is a preliminary analysis of the available information related to the grain market, based on the concept of mathematical model of control of the Russian market of cereals. Here is a description of the basic parameters of a mathematical model characterizing the state and ontrol.

As a basic parameter characterizing the state of the object (grain market) would naturally consider price per unit volume (tonne) grain, which is formed as a result of trades on the stock exchange.

It is necessary to clarify whether the definition of the state take into account additional factors, namely, a kind of grain culture and region of Russia. If Yes, then you must build mathematical models for each region and each type of grain separately.

The general structure of these models will be the same, but their specific characteristics can vary significantly. Each model you must set specific values for maximum and minimum levels of rates. These values should be set, on the basis of economic considerations. Price is valid if it takes values from the set between the minimum and maximum levels.

Prices above the maximum permissible values conditionally corresponds to unacceptable levels, the values of the top, below the minimum allowed conditionally unacceptable level indicate the lower. On the basis of the broad economic patterns of behaviour of prices in commodity markets, it can be assumed that control in this economic system is connected to the external impact on the market, sold in the form of interventions. Under the intervention as a whole requires understanding the following two possible types of influences on the market:

1) Supply to the market lump of significant volumes of grain from State intervention fund. Such action should lead to lower prices in a short period of time.

2) Buying on the market lump of significant volume of grain and put it in the State Interventional Fund. Such action should lead to an increase in prices in a short period of time.

Used grain prices, trades and volumes of interventions. The following shows the association of price and volume of intervention.

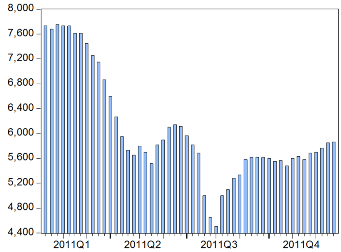

Figure 1: Price. Center - Wheat 3rd class. Annual schedule of prices (2011)

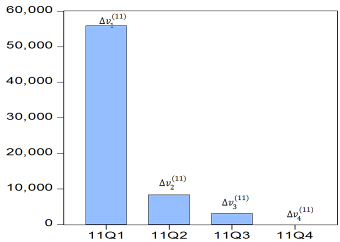

Figure 2: The annual quarterly volume chart of interventions (2011)

The designations employed

$\Delta\nu_i^{(\cdot)}$ - of intervention for the i-th quarter of the year

An analysis of the available data generally confirms the above associations between external (control) impacts, implemented in the form of interventions, and the price of the cereals. Next, you must determine which rule will change the price of grain in any possible relevance of intervention (volume of supply or procurement). The idea of such a rule must be probabilistic in nature as to define deterministic way to change prices in a market where there are random factors is not possible. This will be the next stage of the development of a mathematical model.

References

1. P.V. Shnurkov , T.N. Doynikova. *Optimal control in semi-Markov model with periodically occurring external impacts.* The international conference "Probability Theory and Its Applications" is devoted to the 100 anniversary since the birthday of B. V. Gnedenko. Moscow,June 26 - 30, 2012. Theses of reports/Under A.N.Shiryaev, A.V.Lebedev edition - M. LENAND, 2012. - . 265-266

Market network construction: choice a measure of association and probability of the errors

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In this report the problem of market network construction is considered. A vertices of the market network correspond to the stocks and edges of the network correspond to the interaction between every pair of the stocks. Interaction between stocks is measured by correlation. The complexity of the system is reflected in the associated graph. The minimum spanning tree (MST), planar maximally filtered graph (PMFG), market graph (MG), maximum cliques (MC) and maximum independent sets (MIS) of the market graph give an interesting information about financial market structure. The prices of stocks on financial market have a large element of randomness. Any measure of interaction between stocks therefore has to be extracted from the joint distributions of corresponding stochastic process. Once the measure is defined one can use a statistical procedures to estimate its value from observations. To study the statistical uncertainty of the network market analysis we introduce the concepts of true network model and sample network model. True network model is generated by the stochastic process using the true value of the measure of interaction and sample network model is obtained by statistical estimation of the measure of interaction. The aim of the present report is to discuss a different ways for the network construction and estimate the statistical uncertainty of the network market analysis. We propose a general approach to the network construction on the base of idea of measure of association introduced by Kruskal and developed by Lehman and show that existing network models can be obtained from this approach. We show that the statistical uncertainty of the MC and MIS is essentially lower than the statistical uncertainty of MG which is essentially lower than statistical uncertainty of MST and PMFG

Tests of the Neyman structure for the marker graph construction

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In this report we consider the problem of market graph construction from the mathematical statistics point of view. Construction of these graph is based on the analysis of simultaneous behavior of the stocks. As a statistical model of the financial market we use the Markowitz type model. We consider construction of the sample market graph as a multiple decision statistical procedures. In our investigation we consider the class of unbiased multiple decision statistical procedures in the sense of Lehmann. We construct the conditional multiple statistical procedure for the identification of the true market graph. This procedure is based on tests of the Neyman structures and Pearson tests for generating hypothesis. The result is obtained by application of the Lehmann's theory of multiple decision problems to the method of construction of the market graph. The equations for calculating the thresholds for tests of the Neyman structure are given and analyzed. The numerical results of comparison for Pearson test and conditional test are given.

Development of mathematical model for description of grain market of Russia

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The purpose of this paper is the presentation of the ideas and concepts that form the basis of the concept of mathematical model control some processes occurring in the Russian market of cereals. The estimated model must have a stochastic nature, i.e. constitute some random process. Indeed, in a free market there are objectively random factors that cannot be described by deterministic.

In a basis of the concept of control processes occurring in the market of grain are supposed to put developed by P.V.Shnurkov stochastic Semi-Markov model with periodic external impacts.

In this model assumes the existence of periodic external impacts, which are controlled. Once the process reaches some specified set of boundary conditions, which will be known as valid, it is subjected to external impact, which consists in forcibly transfer inside the set limit (the importance of the process of forcibly transferred from one state to another in accordance with specified discrete probabilistic distributions, which describe the system control). Then the process begins to evolve again without external influences (control) until again won't be out of the permissible limits.

In theory, the problem of optimal control in the Semi-Markov model is a task of identifying two discrete probability distributions, which describe the transfer process from the top or bottom of the inside the plenty of states. Optimal distributions of some fixed functionality are extreme-quality score - this is an indicator of control quality. It is proved that the optimal distributions are confluent, but optimum control strategy-deterministic. Such optimal strategy can be defined as the point of extremum of a specified function of two integer variables $I(k_0, k_1)$, where k_0, k_1 - states from plenty of internal (valid)

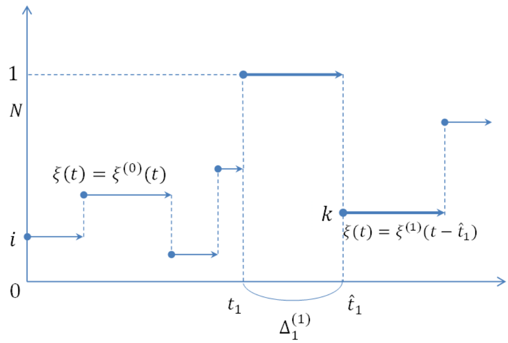

Figure 1: The graph characterizing theoretical model - trajectory of semi-Markov process

states, which should be transferred from the lower or upper boundary levels respectively.

The main components of the mathematical model of the market of cereals are parameters describing the condition and control.

Now we shall provide the short description of application of this mathematical model. As a basic parameter characterizing the state of the object (grain market) would naturally consider price per unit volume (tonne) grain, which is formed as a result of trades on the stock exchange. Price control is carried out by means of intervention. There are two types of interventions-purchasing and inventory. According to the contents they are characterized as follows: either a purchase of grain available on the market (purchased grain is placed in the Interventional Fund) or selling grain from the intervention fund.

The general concept of the proposed mathematical model of control is as follows. The control is at times when the price goes from a given set of allowed values and takes either the top or bottom is invalid an invalid value. Direct control action leads to return valid values in a variety of prices, i.e. the one allowable levels.

Optimal control of the price of grain is carried out in accordance with the above theoretical results for semi-Markov model with periodic external impacts.

References

1. P.V. Shnurkov , T.N. Doynikova. *Optimal control in semi-Markov model with periodically occurring external impacts.* The international conference "Probability Theory and Its Applications" is devoted to the 100 anniversary since the birthday of B. V. Gnedenko. Moscow,June 26 - 30,

2012. Theses of reports/Under A.N.Shiryaev, A.V.Lebedev edition - M. LENAND, 2012. - . 265-266

Developing a new approach to the problem of optimal control in the open dynamical model of a three-sector economy

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In some works of V. A. Kolemaev it was developed and analyzed a dynamic model of three-sector economy. The zero sector produces job objects, the first- means of labor, the second-consumables. In particular, in [1] there was considered an open version of the model, taking into account the impact of foreign trade.

Here is a list of the main indicators of this model:

Y_j - the volume of output in the j-th sector;
K_j - the main production funds (capital) in the j-th sector;
L_j - the number of employed in the j-th sector;
I_j - the volume of investments in the j-th sector;
X_j - imports of goods-sector j (j = 1, 2);
Z_0 - the export volume of materials.
v - the growth rate of employment;
q_0 - the world price of exported materials;
q_1^+, q_2^+ - world prices of imported consumer goods and investment;
μ_j - wear out factor of MPF j-th sector
$\lambda_j = \mu_j + v$ - the coefficient of reduction of assets through depreciation of physical capital and the increase in the number of employed j-th sector.

An analytical study of dynamic model of three-sector economy will be produced in the unit settings. This small Latin letters are indicated by the appropriate amount related to the volume of labour resources in the sector and small Latin letters with cover are related to the total. (In addition $\theta_j = \frac{L_j}{L}$ to the j-th-share sector in the allocation of labor resources).

The basic dynamic and balance sheet ratios describing the open model of three-sector economy are given in [1].

New mathematical optimal control problem

New statement of a problem of optimal control may be developed. In order to do this, we introduce some additional assumptions in the original model.

Problem management is considered in the specified target timescale $[0, T]$.

State of the system is described by a dynamic set of parameters specific to sectors of the capital. Control settings are specific investments $i_1(t)$ and specific volume of imports $x_1(t)$ in the first sector.

We introduce several additional assumptions regarding key ratios of the models:

2.1. The distribution of investments: $I_0 = \rho\,[Y_1 + X_1 - I_1]$,

$I_2 = (1-\rho)\,[Y_1 + X_1 - I_1]$, where $\rho, 0 < \rho < 1$ - the specified number.

2.2. Specific import volumes in the first and second sector are in a constant ratio: $x_2 = wx_1$, where w - given a positive number.

2.3. Limit on the share of export: Suppose that $z_0 \le z_0^*$, where z_0^* - the maximum share of exports. Then from the assumption 2.2. it should be: $x_1 \le \frac{q_0\theta_0 z_0^*}{q_1^+\theta_1 + q_2^+\theta_2 w}$.

2.4. Restrictions on specific investments in the establishing fund sector: $i_{\min} \le i_1 \le \varepsilon_1(y_1 + x_1)$.

As a criterion of optimality is considered a mixed target functionality consisting of integral and terminal components. Integral component is the discounted consumption at a given time interval. Terminal element characterizes the effect on the efficiency of the system parameter values of capital at the end of the process of the management .

We can obtain the differential equations on the functions of system states $k_0(t), k_1(t), k_2(t)$, that will play the role of differential due to the optimal control problem.

It is expected that the initial values of parameters of system states are set.

Restrictions on management are based on assumptions 2.3., 2.4.

As a result, we get a new setting of optimal control problem in canonical form:

1. $\int\limits_0^T e^{-\delta t}\theta_2\left(A_2k_2^{\alpha_2} + wx_1\right)dt + \psi\left(k_0(t), k_1(t), k_2(t)\right) \to \max$

2. Differential association:

$$\begin{cases} \dot{k}_0 &= -\lambda_0 k_0 + \rho l_{1,0}\left(x_1 + A_1k_1^{\alpha_1} - i_1\right) \\ \dot{k}_1 &= -\lambda_1 k_1 + i_1 \\ \dot{k}_2 &= -\lambda_2 k_2 + (1-\rho)\, l_{1,2}\left(x_1 + A_1k_1^{\alpha_1} - i_1\right) \end{cases}$$

3. Initial conditions: $k_0(0) = k_{0,0}, k_1(0) = k_{1,0}, k_2(0) = k_{2,0}$

4. Restrictions on management:

$$\begin{cases} i_{\min} \le i_1 \le \varepsilon_1(y_1 + x_1) \\ 0 \le x_1 \le \min\left(\gamma_1 A_1 k_1^{\alpha_1}, \frac{q_0\theta_0 z_0^*}{q_1^+\theta_1 + q_2^+\theta_2 w}\right) \end{cases}$$

The subject of our future study will be the task of optimal control. This issue is examined through the principle of Pontryagin's maximum. From the condition of maximum Pontryagin's function defines the structure of optimal control. Further we study adjoint equations and differential association.

References

1. V. A. Kolemaev. *Optimal balanced growth of open a three-sector economy*, 2008, Applied econometrics, No. 3.

Trajectory analysis of control process for optimal control of investments in the model of a three-sector economy

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In this research, we consider the optimal control problem for an economic system whose behavior is described by the dynamical model of a three-sector economy. The results are summarized in [1,2]. For the state parameters of the system we take the capital-labor ratio functions (specific capital) in each sector and for the control parameter we take the amount of specific investments in the capital generating sector. The solution of the optimal control problem under consideration is based on the Pontryagin maximum principle. We find the optimal control structure depending on some auxiliary function, which is expressed in terms of conjugate variables. Analytic solutions of the systems of differential equations for the state variables and the conjugate variables are obtained. The system of differential equations (differential association), describing the dynamics of the system states has the form:

$$\begin{cases} \dot{k}_0(t) = -\lambda_0 k_0(t) + l_0^{(1)}\rho(A_1 k_1^{\alpha_1}(t) - i_1(t)), \\ \dot{k}_1(t) = -\lambda_1 k_1(t) + i_1(t), \\ \dot{k}_2(t) = -\lambda_2 k_2(t) + l_2^{(1)}(1-\rho)(A_1 k_1^{\alpha_1}(t) - i_1(t)) \end{cases}$$

In previous papers [1, 2] there were obtained optimal control structure, and solutions of differential equations and conjugate parameters. Four basic modes of optimal control were analyzed on the time interval $[0, T]$. For each of the options issued to differential equation $k_0(t)$, $k_1(t)$, $k_2(t)$ for relative depending on the control structure.

Further it is shown how $k_0(t)$, $k_1(t)$, $k_2(t)$ behave at different optimal control regimes. Below we state stationary solutions of a differential system association for the first option of the structure of optimal control with additional condition. Suppose that $k_0(t) = k_0$, $k_1(t) = k_1$, $k_2(t) = k_2$, k_0, k_1, k_2 some values. Then the only stationary solution of the system of differential equations is:

$$\begin{cases} k_0^{(0)} = \frac{1}{\lambda_0} l_0^{(1)} \rho(1-\gamma) A_1 \left(\frac{\gamma A_1}{\lambda_1}\right)^{\frac{\alpha_1}{1-\alpha_1}}, \\ k_1^{(0)} = \left(\frac{\gamma A_1}{\lambda_1}\right)^{\frac{1}{1-\alpha_1}}, \\ k_2^{(0)} = \frac{1}{\lambda_2} l_2^{(1)} (1-\rho)(1-\gamma) A_1 \left(\frac{\gamma A_1}{\lambda_1}\right)^{\frac{\alpha_1}{1-\alpha_1}}. \end{cases}$$

Consider the behavior of the function $k_1(t)$. If, for a specific set of values for the parameter t inequality $k_1 = k_1(t) < k_1^{(0)}$ holds, then $\Delta k_1 > 0$, the function $k_1(t)$ is decreasing on t. At the same time, as can be seen from the explicit representation of function $k_1(t)$:

$$\lim_{t\to\infty} k_1(t) = k_1^{(0)} = \left(\frac{\gamma A_1}{\lambda_1}\right)^{\frac{1}{1-\alpha_1}}.$$

Based on the findings, you can depict the character of solutions of differential equation about $k_1(t)$ from the above-mentioned system, depending on the initial values $k_{1,0}$, i.e. the phase trajectory of this equation.

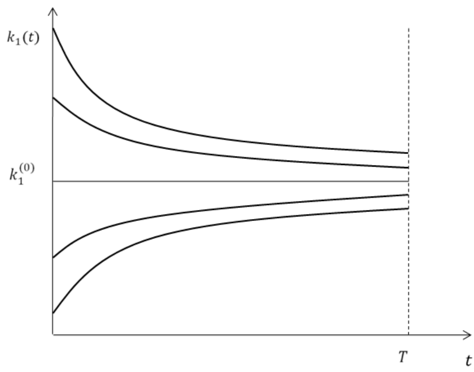

Figure 1: The behavior of functions of the capital-labor ratio of the first sector in a given interval of time

There has been stability on the trajectories of stationary solutions at corresponding control regimes. By analogy we study behavior of trajectories for functions $k_0(t)$ and $k_2(t)$ for the four best optimal control structure options.

References

1. P. V. Shnurkov, V. V. Pisarenko. *Optimal control of investments in the capital generating sector in the dynamical model of a three-sector economy*, All-Russian conference "Applied Probability Theory and Theoretical Informatics" , April 17 - 18, 2012. Theses of reports.

2. P. V. Shnurkov, V. V. Pisarenko. *Optimal control of investments in the capital generating sector in the dynamical model of a three-sector economy*, The international conference "Probability Theory and Its Applications" is devoted to the 100 anniversary since the birthday of B. V. Gnedenko. Moscow,June 26 - 30, 2012. Theses of reports.

The estimation of financial stability of insurance company

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In actuarial mathematics one of the main problem is to investigate how the capital of insurance company depends on some parameters of its activity.

In our report we consider the following model of surplus of insurance company:

$$U(t) = C_0 + Y_1(t) - R(t) - Y_2(t) + PI(t) \ ,$$

where

1) C_0 – initial capital;
2) $Y_1(t) = a \cdot t$ – the premium process;
3) $R(t) = b \cdot t$ – own expences;
4) $Y_2(t) = \sum\limits_{j=1}^{N(t)} X_j$ – claims process;
5) $PI(t)$ – return from investments.

We assume that $(N(t), t \geq 0)$ is a homogeneous Poisson process, $\{X_j, j \geq 1\}$ are i.i.d.r.v.

In our report we consider the following problems:

1) optimal investment control;
2) ruin probability as a function of the parameters of insurance company activity;
3) calculating of financial stability characteristics as a function of the parameters of insurance company activity.

References

1. L. A. Orlanyuk-Malitskaya. *Financial stability of insurance company*, 1994, ANKIL, Moscow. (In Russian).

Modelling of multivariate Cox process

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We consider the multivariate analog of generalized Cox process (one dimensional process see in Bening [1])

Let $N(t) = (N_1(t), \dots, N_m(t))$ be a multivariate Poisson process (with dependent components in general), $\{X_j = (X_{j1}, \dots, X_{jm})\}$ be a sequence of i.i.d. random vectors woth finite second moments, $\Lambda(t) = (\Lambda_1(t), \dots, \Lambda_m(t))$ be a multivariate random process such that: $\Lambda_k(0) = 0$, $\Lambda_k(t)$ has nondecreasing paths, $E(\Lambda_k(t)) = b_k \cdot t$, $Var(\Lambda_k(t)) = s_k^2 \cdot t$, $b_k > 0$, $s_k^2 > 0$ for all $k = \overline{1, m}$. The processes $(N(t), t \geq 0)$ and $(\Lambda(t), t \geq 0)$ are independent.

We consider the following variant of multivariate generalized Cox process: $C(t) = (C_1(t), \dots, C_m(t))$:

$$C_k(t) := \sum_{j=1}^{N_k(\Lambda_k(t))} X_{jk} \ .$$

If we want to use this model in practical problems we need to simulate this process.

In our report we propose the algorithms and programs for simulation of such processes.

References

1. Bening V.E., Korolev V.Yu. *Generalized Poissin Models and Their Applications in Insurance and Finance*, 2002, Utrecht, VSP.

On Markov reliability model of a system, operating in Markov random environment

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Keywords: *Reliability Models, Random Environments*

Most of complex technical systems and biological objects are usually operating in changing environment, which can have a regular (seasons etc.) as well as random (weather, rein, smog, etc.) character. At that the mean time of environment changing can be co-measured with that failures and repair or to be both smaller or grater it. Therefore the influence of this circumstances to their reliability characteristics is an important problem.

There are some papers, devoted to queueing systems in random environment investigation. One of the first it was the paper of Eisen and Tainiter, who investigated the system $M/M/1(ME)$ under assumption that the random environment takes only two states. Here and later the notation (ME–Markov Environment in commas) denoted that the system operates in Random Environment. The same system has been considered in Naor and Yehiali, and than generalized for the case of any finite number of environment's states by Yechiali. Newts used matrix-analytical method for investigation of one and multi-channel queueing systems in random environment. Than the models $M/M/1(ME)$ and $M/M/\infty(ME)$ have been considered in papers of Purdue and O'Cinneide & Purdue. For biblography see, fro example [1]. In further these investigations developed in different directions connected with generalization of input flows, service mechanisms and environment processes. For the contemporary results and the bibliography see, for example, [2]. However the problem of renewable systems reliability operating in random environment is not enough studied yet. One of aspects of the talk consists in description and studying of this problem.

The long time system behavior is usually described with steady state probabilities. However, because there is no infinitely long existing systems in most real situation it is necessary to study their life time (before entering in sone full failure subset of states) as well as their behavior during this time. The system behavior before its full failure is described with its conditional states probabilities given life time didn't end. Closed form representation of these probabilities in general case are hardly possible. But really, because a system usually many times visit any of its not absorbing states during its life time, an interesting problem is study limits of these probabilities for $t \to \infty$. The problems of these limits existence for Markov processes and especially for birth and death processes have been considered by several authors. Evaluation of the convergence rate to the quasi-limiting probabilities for queueing models by Granovsky and Zeifman has been studied. For bibliography see for example [3] and the bibliography therein.

In [4] generalized birth and death processes as a model for systems degradation has been introduced and studied, where also the problem of quasi-limiting probabilities has been discussed. Therefore the another aspect of the paper is devoted to studying of a system life time under random environment as well as their behavior during this time.

In the paper a simple Markov reliability model of a system operating in Markov environment is proposed. The system behavior is described by two-dimensional Markov process with block-wise infinitesimal matrix. Algorithms for calculation of steady state and quasi-stationary probabilities as well as reliability function based on this special form of infinitesimal matrix are given. The influence of random environment to the system reliability characteristics is numerically investigated for a special cases.

Some examples of proposed approach for study of reliability of hybrid information transmission systems, operating in random environment, has been considered in [5, 6].

References

1. *Purdue P.* The $M/M/1$ Queue in Markovian Environment // Opens. Res. 1974. V. 22. No. 3. P.562–569.

2. *Che Soong Kim, Valentina Klimenok, Vilena Mushko, Alexander Dudin.* The $BMAP/PH/N$ retrial queueing system operating in Markovian random environment // Computers & Operations Research, 2010. No 37. 1228–1237.

3. *Rykov V.* Multidimensional Alternative Processes as Reliability Models // Moderm Probabilistic Methods for Analysis of Telecommunication Networks. (BWWQT 2013) Proceedings. Edds: A.Dudin, V.Klimenok, G.Tsarenkov, S.Dudin. Series: CCIS 356. Springer, 2013.P.147-157.

4. *Rykov V.V.* Generalized birth and death processes and their applications to degradation models. // Autom. and Remote Control. 2006. No 3 P.103-120.

5. *Rykov V., Efrosinin D.* Queueing Model of the non-Reliable Hybrid Data Thransmission Channel with Heterogeneous Links // Transection of 7-th Interbational Conference in Mathematical Methods in Reliability (MMR-2011) 2011.

6. *Vishnevsky V., Kozyrev D., Rykov V.*On the reliability of hybrid system information transmission evaluation. // Queues: flows, systems, networks (Modern Probabilistic Methods for Analysis of Telecommunication Networks. Proceedings of BWWN-2013. Edds: A.Dudin, V.Klimenok, G.Tsarenkov, S.Dudin.) //Minsk 28-31 January, 2013.192-202.

Development of the semi-Markov stock management model with a discrete set of states and a random delay of delivery

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This paper examines the stochastic semi-Markov stock management model of some goods with possible deficit. The flow of the applications for this product is determined as Poisson stream with the intensity λ.The amount of product in the system can take discrete values in the range $[-N_1, N_0]$, where N_0, $N_1 > 0$ are predefined integers. Value N_0 - is the highest level of real stock. The amount of stock in the range $[-N_1, N_0]$ characterizes the deficit when bids for the good are accepted for registration. Applications received at the time when the deficit is set to $(-N_1)$ are lost. The decisions are taken at the time of replenishment. If at this moment the state of the process is equal to i then the level of reserve for next query is determined as r_i where $-N_1 \le r_i \le i-1$. Next time when the process hits the value r_i new level of reserve r_{i+1} is set. And so on. Random time τ between the moment of the order and the moment of replenishment has a specified distribution $H(x)$. The size of replenishment is random and it has discrete distribution with probabilities $\beta_{kj}^{(0)}$, $\beta_{kj}^{(1)}$,$-N_1+1 \le k \le r_i - 1$, $j > 0$.

Replenishment procedure is designed in the way that the basic process is transferred from the state (k) which it takes directly before the replenishment to the state j, $j = 1, \ldots, N_0$ the shortage of product is always filled in.

Let us introduce random processes describing the functioning of the system. Let $\xi(t)$ – the basic process that describes the level of stock at the moment t. The set of spaces is determined as $X = \{-N_1, -N_1+1, \ldots, -1, 0, 1, \ldots, N_0\}$. Denote $\{t_n^{(0)}, n = 0, 1, \ldots\}$ - sequential moments of replenishment; $t_0^{(0)} = 0$; $\xi_n^{(0)} = \xi(t_n^{(0)} + 0)$ - the state of the process directly after replenishment. We introduce a random process $\xi^{(0)}(t)$ determined by the ratios $\xi^{(0)}(t) = \xi_n^{(0)}$ when $t_n^{(0)} \le t \le t_{n+1}^{(0)}$. From now the process $\xi^{(0)}(t)$ will be referred to as the "maintainer". The set of states for this process is $X^{(0)} = \{0, 1, \ldots, N_0\}$, $\{\xi_n^{(0)}\}$ and it constitutes the embedded Markov chain for the main process.

Denote p_{ij}, $i, j, = 0, 1, \ldots, N_0$ transition probabilities of Markov chains, embedded in main semi-Markov process $\xi^{(0)}(t)$.

$$p_{ij} = P(\xi_{n+1}^{(0)} = j \,|\xi_n^0 = i), i, j = 0, 1, \ldots, N_0$$

Adduce explicit form for transition probabilities for unembedded Markov chain:

1) Let i - to be afixed state, $0 \le r_i \le i$, $i, j = 0, 1, \ldots, N_0$

$$p_{ij}^{(I)} = \sum_{k=0}^{r_i} \beta_{kj}^{(0)} \int_0^\infty \frac{(\lambda x)^{r_i - k}}{(r_i - k)!} e^{-\lambda x} dH(x) + \sum_{k=-N_1+1}^{-1} \beta_{kj}^{(1)} \int_0^\infty \frac{(\lambda x)^{r_i - k}}{(r_i - k)!} e^{-\lambda x} dH(x) +$$

$$+ \left[\sum_{k=r_i+N_1}^{\infty} \int_0^\infty \frac{(\lambda x)^k}{k!} e^{-\lambda x} dH(x) \right] \beta_{-N_1,j}^{(1)}. \tag{1}$$

2) Let i -to be a fixed state, $r_i = -N_1+1, -N_1+2, \ldots, -1$

$$p_{ij}^{(II)} = \sum_{k=-N_1+1}^{r_i} \beta_{kj}^{(1)} \int_0^\infty \frac{(\lambda x)^{r_i - k}}{(r_i - k)!} e^{-\lambda x} dH(x) +$$

$$+\sum_{k=r_i+N_1}^{\infty}\int_0^{\infty}\frac{(\lambda x)^k}{k!}e^{-\lambda x}dH(x)\beta^{(1)}_{-N_1,j}. \qquad (2)$$

3) Let i-to be afixed state,$r_i = -N_1$
Because $-N_1$ is the minimum level of stock in the system we get ratio:

$$p^{(III)}_{ij} = \beta^{(1)}_{-N_1,j}. \qquad (3)$$

Let us obtain the equation for the stationary probabilities of embedded Markov chain. Note that the transition probabilities p_{ij} for each fixed value i depends on the values r_i.

So, let us fix the control parameters corresponding to the statesof embedded Markov chain $(r_0, r_i, \ldots, r_{N_0})$. For each kind of set $(r_0, r_i, \ldots, r_{N_0})$, expression for the transition probabilities are determined by ratios (1)-(3). For a given set $(r_0, r_i, \ldots, r_{N_0})$ define the following set S_0, S_1, S_2, that represent a subset of the states $(0, 1, \ldots, N_0)$.

$$S_0 = \{i \in \{0, 1, \ldots, N_0\} : r_i \geq 0\};$$
$$S_1 = \{i \in \{0, 1, \ldots, N_0\} : -N_1 + 1 \leq r_i \leq -1\};$$
$$S_2 = \{i \in \{0, 1, \ldots, N_0\} : r_i = -N_1\};$$

Then the system of equations for stationary probabilities takes the form:

$$\pi_j = \sum_{i\in S_0}\pi_i p^{(I)}_{ij} + \sum_{i\in S_1}\pi_i p^{(II)}_{ij} + \sum_{i\in S_2}\pi_i p^{(III)}_{ij} \; j = 0, 1, \ldots, N_0$$

Next step is to obtain the indicators of quality management the semi-Markov model, which depends particularly from the stationary distribution.

The estimation of ruin probability in multivariate collective risk model

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In actuarial mathematics great importance has always been given to estimations of ruin probability of insurer. The multivariate collective risk model allows to consider the dependence between claims of different fields of insurance, operated by insurance company. Claims happened in different fields of insurance are mutually dependent very often, that affects the process of changing the value of the insurance reserve. The process of reserve changing can usually be presented in a form:

$$U(t) = u + c \cdot -S(t) \;,$$

where $U(t) = (U_1(t), \ldots U_m(t))$ - the process of insurer reserve; $u = (u_1, \ldots, u_m)$ - the initial insurer capital distributed between m fields of insurance; $c = (c_1, \ldots, c_m)$ - vector of intensity of premiums incoming for each fields of insurance; $S(t) = (S_1(t), \ldots, S_m(t)$ - total indemnity for each field of insurance by the moment t.

Let us consider the multidimensional index $i = (i_1, \ldots, i_m)$ consisted of 0 and 1, where $i_k = 1$ when claims of the contract k were paid and 0 in other case. Denote by I the set of all possible values of i. Let $N^{(i)}(t), t \geq 0$ be the number of claims happened up to the moment t and having the structure corresponding to the index i. Then

$$S_k(t) = \sum_{i \in I} \sum_{j=1}^{N^{(i)}} X_{j,k}^{(i)} \ ,$$

where $X_{j,k}^{(i)}$ is the claim corresponding to k-th field of insurance.

Let T_k be the moment of ruin in k-th field of insurance.

It is very important for the insurer that each field of insurance is not loss-making. That is why the ruin moment of the insurance company is determined as $T = \min\{T_k, k = \overline{1, m}\}$.

In the report some properties of the vector $(T_1, \ldots, T_m)$ would be investigated and also there would be derived estimations of ruin probability.

References

1. N. L. Ivanova and Yu. S. Khokhlov, Multidimensional collective risk model, *Journal of Mathematical Sciences*, 2007, vol. 146, No. 4, October 2007, p. 6000-6007.

Lower bounds on the convergence rate of the Markov symmetric random search

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The convergence rate of the Markov random search algorithms designed for finding the extremizer of a function is investigated. It is shown that, for a wide class of random search methods that possess a natural symmetry property, the number of evaluations of the objective function needed to find the extremizer accurate to ε cannot grow slower than $|\ln \varepsilon|$.

Let the objective function $f \colon X \mapsto \mathbb{R}$ (where, for instance, $X = \mathbb{R}^d$) take its minimal value at a single point x_*. Consider the problem of finding the global minimizer x_* of this function up to a given accuracy ε (approximation by argument). One way of solving this problem is to use Markov random search algorithms (see [1]–[3]). Such algorithms have long been used for solving difficult optimization problems. Note that the simulated annealing algorithm, which is a well-known stochastic global optimization algorithm, belongs to this class.

In this paper, we consider the case $X = \mathbb{R}^d$ with the Euclidean metric ρ. The closed ball of radius r centered at the point x is denoted by $B_r(x) = \{y \in \mathbb{R}^d$, such that $\rho(x, y) \le r\}$.

Throughout this paper, we assume that the objective function $f\colon \mathbb{R}^d \mapsto \mathbb{R}$ is measurable, and takes its minimum value at a unique point $x_* = \arg\min\{f(x)$, such that $x \in \mathbb{R}^d\}$.

A *random search* is defined as an arbitrary sequence of random variables $\{\xi_n\}_{n\ge 0}$ taking values in $\mathbb{R}^d$. Following [2], we give a general scheme of Markov random search algorithms.

Algorithm 1 (A general scheme of Markov algorithms).

Step 1. Set $\xi_0 = x$ and set iteration number $n = 1$.

Step 2. Obtain a point η_n in $\mathbb{R}^d$ by sampling from a distribution $P_n(\xi_{n-1}, \cdot\,)$. The *transition probability* $P_n(\xi_{n-1}, \cdot\,)$ depends on the iteration number n and on the preceding search point ξ_{n-1}.

Step 3. Set

$$\xi_n = \begin{cases} \eta_n & \text{with probability } Q_n, \\ \xi_{n-1} & \text{with probability } 1 - Q_n. \end{cases}$$

Here Q_n is the *acceptance probability*; this probability may depend on η_n, ξ_{n-1}, $f(\eta_n)$, $f(\xi_{n-1})$.

Step 4. Check a stopping criterion. If the algorithm does not stop, substitute $n + 1$ for n and return to Step 2.

Here, x is the starting point of the search. Different rules of specifying the acceptance probabilities Q_n and the transition probabilities $P_n(x, \cdot\,)$ lead to different variants of the Markov random search algorithms. We will consider the Markov random search whose transition probabilities $P_n(x, \cdot\,)$ have *symmetric densities* $p_n(x, y)$ of the form

$$p_n(x, y) = g_{n,x}\big(\rho(x, y)\big), \tag{1}$$

where ρ is a metric and $g_{n,x}$ are nonincreasing nonnegative functions defined on $(0, +\infty)$. The Markov search defined by Algorithm 1 with the transition probabilities with densities (1) is called the *Markov symmetric random search.*

We use a random search for finding the minimizer x_* with a given accuracy ε (approximation with respect to the argument). In this case, we want the search to hit the ball $B_\varepsilon(x_*)$. Denote by $\tau_\varepsilon = \min\{n \ge 0$, such that $\xi_n \in B_\varepsilon(x_*)\}$ the time when the search first hits the ε-neighborhood of the global minimizer. The distribution of the random variable τ_ε provides sufficient information about the quality of the random search (see [2, p. 127]). Indeed, τ_ε steps of Algorithm 1 require $\tau_\varepsilon + 1$ evaluations of f.

The main result of this paper is Theorem 1. It is proved that the computational effort of the Markov symmetric random search needed to guarantee the required accuracy ε of the solution cannot grow slower than $|\ln \varepsilon|$.

Theorem 1. Let the function $f\colon \mathbb{R}^d \mapsto \mathbb{R}$ take its minimum value at a unique point x_*. Consider the Markov symmetric random search $\{\xi_n\}_{n\geq 0}$ defined by Algorithm 1 whose transition probabilities have densities of form (1). Let x be the starting point of the search, $0 < \varepsilon < \delta = \rho(x, x_*)$, and $n \in \mathbb{N}$. Then, it holds that

$$\mathsf{E}\,\tau_\varepsilon \geq \ln(\delta/\varepsilon) + 1, \quad \mathsf{P}(\tau_\varepsilon \leq n) \leq \frac{\varepsilon}{\delta} \sum_{i=0}^{n-1} \frac{\ln^i(\delta/\varepsilon)}{i!}. \tag{2}$$

Let the optimization space $X = \mathbb{R}$ and $f(x) = |x|$. In this case it is easy to construct the Markov symmetric random search with $\mathsf{E}\,\tau_\varepsilon = 2\ln(\delta/\varepsilon) + 2$. This result shows that estimates (2) are accurate estimates of the convergence rate.

References

1. S. M. Ermakov, A. A. Zhigljavsky. *On the Random Search of Global Extremum.* Teor. Veroyatn. Ee Primen., 1983, No. 1, p. 129–136.

2. A. Zhigljavsky, A. Žilinskas. *Stochastic Global Optimization*, Springer, Berlin, 2008.

3. A. Tikhomirov, T. Stojunina, V. Nekrutkin, *Monotonous Random Search on a Torus: Integral Upper Bounds for the Complexity.* Journal of Statistical Planning and Inference, 2007, 137, p. 4031–4047.

4. A. S. Tikhomirov, *Lower Bounds on the Convergence Rate of the Markov Symmetric Random Search.* Computational Mathematics and Mathematical Physics, 2011, vol. 51, No. 9, p. 1524–1538.

Stationary waiting time in queuing system with negative customers, bunker and phase-type service times under Last-LIFO-FIFO discipline

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In this study queue with Poisson flows of ordinary and negative customers is being considered. Ordinary customer upon arrival occupies one place in the queue in the buffer of infinite capacity. Arriving negative customer moves one ordinary customer which is the last one in the queue in the buffer into another queue called bunker (of finite capacity r). Negative customer itself leaves the system. If customer arrives at full bunker it is lost. If at the moment of arrival of negative customer buffer is empty (or system is empty), it leaves the system without causing any impact on it. After service completion first customer in the queue in the buffer goes to server. If upon service completion the queue in the buffer is empty then first customer in the queue in the bunker enters server. Service times of customers from both buffer and bunker have phase-type distribution of order $g < \infty$. This paper continues the research of queueing systems with negative customers and bunker for superseded customers conducted in [1-3]. Stationary waiting time distribution of arbitrary customer is obtained in terms of Laplace-Stieltjes transform.

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References

1. R. Manzo, N. Cascone, R. Razumchik. Exponential queuing system with negative customers and bunker for ousted customers. *Automation and Remote Control*, 2008, vol. 69, No. 9, p. 1542–1551.

2. A. Pechinkin, R. Razumchik. A Method for Calculating a Stationary Queue Distribution in a Queuing System with Flows of Ordinary and Negative Claims and a Bunker for Superseded Claims. *Journal of Communications Technology and Electronics*, 2012, Vol. 57, No. 8, p. 882-891.

3. C. DeNicola, R. Manzo, R. Razumchik, I. Zaryadov. Queuing system with negative customers, bunker and phase-type service time distribution. *International Symposium on Operational Research and Applications (ISORAP2013)* (accepted), 2013.

Multichannel queuing systems with bounded waiting time and regenerative input flow

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We consider queuing systems with r heterogeneous channels.The service time η_n^i of the n-th customer by the i-th server has distribution function $B_i(x)$ with finite mean β_i^{-1}. Let $\beta = \sum_{i=1}^r \beta_i$. Customers are served in order of their arrivals at the system. Service times of customers are independent random variables.

The input flow $X(t)$ is assumed to be regenerative. Let θ_i be the i-th regeneration point of $X(t)$, $\tau_i = \theta_i - \theta_{i-1}$, $\xi_i = X(\theta_i) - X(\theta_{i-1})$ ($i = 1, 2, \dots; \theta_0 = 0$). Then τ_i is the regeneration period, ξ_i is the number of customers arrived during the i-th regeneration period. Assume that $a = E\xi_i < \infty$, $\tau = E\tau_i < \infty$, and $\lambda = \lim\limits_{t\to\infty} \frac{X(t)}{t} = a\tau^{-1}$ *a.s.*.

Let $\{v_n\}_{n=1}^\infty$ be the sequence of independent identical distributed random variables and it does not depend on the input flow and service times. The random variable v_n can be an improper one, i.e. $\alpha = P\{v_n = \infty\} \geq 0$. Denote $C(x) = P\{v_n \leq x | v_n < \infty\}$. Moreover v_n bounds the waiting time of the nth customer in the system, i.e. if the nth customer does not start it's service during the time v_n then it leaves the system without service at all. Let $q(t)$ be a number of customers in the system at time t. Under some additional assumptions $q(t)$ is a regenerative process and θ_i is it's point of regeneration if $q(\theta_i - 0) = 0$.

Theorem 1. The process $q(t)$ is ergodic iff $\rho = \alpha\lambda\beta^{-1} < 1$.

The proof is based on the lemma about stochastic boundedness and ergodicity of the regenerative process proved in [Afanasyeva, Tkachenko, 2013] and construction of majorizing process. Then results for regenerative process with finite mean of the period of regeneration [Thorisson, 1987] are applied.

First we give the following result concerning so called super-heavy traffic situation ($\rho \geq 1$).

Theorem 2. If $\rho > 1$ ($\rho = 1$) and for some $\delta > 0$

$$E\tau_1^{2+\delta} < \infty,\ E\xi_1^{2+\delta} < \infty,\ E(\eta_1^i)^{2+\delta} < \infty,\ i = \overline{1,r}, \qquad (\star)$$

then the normalized process $\hat{q}_n(t) = \frac{q(nt) - \beta(\rho-1)nt}{\hat{\sigma}\sqrt{n}}$ weakly converges on any finite interval $[0, t]$ to Brownian motion (absolute value of Brownian motion) as $n \to \infty$. Here

$$\hat{\sigma}^2 = \sigma_X^2 + \sigma_\beta^2,\ \sigma_X = \frac{\alpha\sigma_\xi^2}{\tau} + \frac{(\alpha a)^2\sigma_\tau^2}{\tau^3} - \frac{2a\alpha^2 cov(\xi_1, \tau_1)}{\tau^2},$$

$$\sigma_\beta^2 = \sum_{i=1}^r \sigma_i^2\beta_i^3,\ \sigma_\tau^2 = Var(\tau_1),\ \sigma_\xi^2 = Var(\xi_1),\ \sigma_i^2 = Var(\eta_1^i),\ i = \overline{1,r}.$$

In order to prove this theorem we use Brownian approximation for modified multichannel systems [Iglehart, Whitt, 1970] and construct two majorizing systems.

Second we focus on the process $q(t)$ in the heavy-traffic situation ($\rho \uparrow 1$). We consider time-compression asymptotic. Namely the input flow is given by the relation

$$X_n(t) = X\left(\rho^{-1}\left(1 - \frac{1}{\sqrt{n}}\right)t\right)$$

so that the traffic coefficient depends on the parameter n and $\rho_n \uparrow 1$ as $n \to \infty$. Let $q_n(t)$ be the process $q(t)$ for the system with input flow $X_n(t)$.

Theorem 3. Under conditions ($\star$) the normalized process $\tilde{q}_n(t) = \frac{q_n(nt)}{\sqrt{n}}$ weakly converges on any finite interval $[0, t]$ as $n \to \infty$ to the diffusion process with reflecting at the origin and coefficients $(-\beta, \tilde{\sigma}^2)$, where $\tilde{\sigma}^2 = \sigma_\beta^2 + \frac{\sigma_X^2}{\rho}$.

The proof is based on the construction of the functional limit of the fluid process [Whitt, 2001] and some estimates for number of customers in the system.

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References

1. *Afanasyeva L., Tkachenko A.,* Multichannel Queuing Systems with Regenerative Input Flow, Theory of Probability and Application, 2013 (to be published).

2. *Iglehart D., Whitt W.,* Multiple Channel in Heavy Traffic I, Adv. Appl. Prob., 2, 1970, pp. 150-177.

3. *Thorisson H.,* A Complete Coupling Proof of Blackwell's Renewal Theorem, Stochasic Processes and its Application, vol. 26, 1987, pp. 87-97.

4. *Whitt W.,* Stochastic-Process Limits, AT&T Labs Research, New Jersey, 2001.

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