A letter to the Editor

Comments on the paper
A selection principle for mappings of bounded variation
by S. A. Belov and V. V. Chistyakov
published in *J. Math. Anal. Appl.* **249**, No. 2 (2000), 351–366.

In this letter we would like to comment on certain parts of our paper.

Above all we mention a paper by W. A. Ślęzak, Concerning continuous selectors for multifunctions with nonconvex values, *Zeszyty Nauk. WSP Bydgoszcz. Probl. Mat.* **9** (1987), 85–104, where the result of our Theorem 3(a) on the existence of Lipschitzian selections, generalizing the corresponding theorem of H. Hermes, was established earlier by a different method. His proof is based on the Arzelà-Ascoli compactness theorem, while ours makes use of a Helly type selection principle. We express our sincere gratitude to Andrzej Nowak (Katowice, Poland) for providing us with the reference.

Reference 6 in our paper, which was to appear at the time of publication, is available now: *Positivity* **5**, No. 4 (2001), 323–358. English translations of references 4 and 5 are also available: *Russian Math. Surveys* **54**, No. 3 (1999), 630–631, and *Pontryagin Conference*, 2, *Nonsmooth Analysis and Optimization* (Moscow, 1998). *J. Math. Sci.* (*New York*) **100**, No. 6 (2000), 2700–2715, respectively.

In step 6 (page 359) of the proof of our Theorem 1 (Helly type selection principle) we have assumed, without loss of generality, that the countable set E consisting of discontinuity points of functions $\{\varphi_n\}_{n=1}^{\infty}$ and φ and points a and b is closed, believing that the remaining details are easily filled in. However, some readers (cf. I. Fleischer and J. E. Porter, Convergence of metric space-valued BV functions, *Real Anal. Exchange* **27**, No. 1, 2001/2002, 315–320) have incorrectly determined that the proof of Theorem 1 is wrong. In order to avoid further misunderstanding of this kind, we exhibit the appropriate details of the proof. Let $\overline{E}$ be the closure of E. The difference $[a,b]\setminus E$ is the disjoint union of two sets $[a,b]\setminus\overline{E}$ and $\overline{E}\setminus E$, only one of which is possibly empty; if the former (resp., latter) set is empty, i.e., $[a,b]=\overline{E}$ (resp., $\overline{E}=E$), one should employ only the arguments of (1) (resp., (2)) below. (1) If $\overline{E}\setminus E$ is nonempty, it is straightforward that the sequence $\{f_n\}_{n=1}^{\infty}$ converges in X pointwise on $\overline{E}\setminus E$ as well. In fact, let $t\in\overline{E}\setminus E$ and $\varepsilon>0$. Since φ is continuous at t and E is dense in $\overline{E}$, choose an $s\in E$ such that $|\varphi(t)-\varphi(s)|\le\varepsilon$. By the pointwise convergence of φ_n to φ, there exists an $n_0(\varepsilon)\in\mathbb{N}$ such that $|\varphi_n(t)-\varphi(t)|\le\varepsilon$ and $|\varphi(s)-\varphi_n(s)|\le\varepsilon$ for all $n\ge n_0(\varepsilon)$. It follows that $d\big(f_n(t),f_n(s)\big)\le|\varphi_n(t)-\varphi_n(s)|\le 3\varepsilon$, $n\ge n_0(\varepsilon)$. Since $\{f_n(s)\}_{n=1}^{\infty}$ is convergent in X, $d\big(f_n(s),f_m(s)\big)\le\varepsilon$ for all $n,m\ge m_0(\varepsilon)$ and some $m_0(\varepsilon)\in\mathbb{N}$. Hence, $d\big(f_n(t),f_m(t)\big)\le 7\varepsilon$ for all $n,m\ge\max\{n_0(\varepsilon),m_0(\varepsilon)\}$, and so, the sequence $\{f_n(t)\}_{n=1}^{\infty}$ is Cauchy in X, and since it is precompact in X by the assumption, it is convergent in X. (2) If the set $[a,b]\setminus\overline{E}$ is nonempty, we develop it as at most a countable union of open intervals (a_k,b_k), $k\in\mathbb{N}$, and apply the diagonal procedure of step 6 to get the diagonal subsequence $\{f_n^n\}_{n=1}^{\infty}$ of $\{f_n\}_{n=1}^{\infty}$ which is convergent pointwise on $\bigcup_{k=1}^{\infty}(a_k, b_k)$ and $\overline{E}$, i.e., on the whole interval $[a, b]$. The rest of the proof is the same as in our paper.

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