Model of "Chain of Coupled Resonators"-Type Slow-Wave Structure's Cell Based on Equivalent Systems Method

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***Abstract*—Based on equivalent systems method model of "chain of coupled resonators"-type slow-wave structure's cell is developed. Rectangular slow-wave structures of "winding waveguide"-type are considered. Model of the cell represents ramified circuit, composed of quadripoles. Assessment of model's adequacy was conducted by comparison of dynamic characteristics calculated by using a model and HFSS.**

Keywords*— resonator slow-wave structures; model of slow-wave structure's cell; equivalent systems method; ramified circuit from quadripoles

I. Introduction

The all-metal resonator slow-wave structures are used in development of high and average power microwave amplifiers in order to provide the necessary heat sink. This slow-wave structures (SWSs) are three-dimensional and the modeling of devices based on them by means of strict electrodynamic programs demands high computational burden. Therefore, the development of simple and accurate models of resonator SWSs used in device modeling is an actual problem.

II. Model of "Winding Waveguide"-Type Slow-Wave Structure Taking into Account Transit Channel

In order to construct the cell's model in the absence of the exiting current the equivalent systems method is used and the rectangular slow-wave structures of "winding waveguide"-type are used, which are the most simply described with this method (Fig. 1).

There D is the period of SWS.

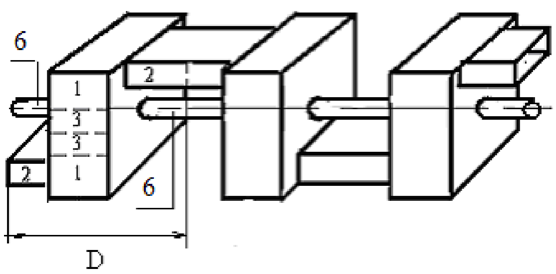

Fig. 1. Slow-wave structure of "winding waveguide"-type

When constructing the model the original SWS is divided into partial areas by planes perpendicular to the direction of propagation of the microwave energy inside it. Selected partial areas are replaced by segments of waveguide channels with a rectangular and circular cross-section, for which an analytical solution of the inner electrodynamic problem is known. In order to describe partial areas of the system the equivalent areas $U_{eq}, I_{eq}, Z_{eq}, \gamma_{eq}$ are introduced, which allows to move with their description to the equivalent transmission lines and uniformly describe them by quadripoles transmission matrices. Based on the analysis of fields matching conditions at the boundaries of the partial areas the quadripoles are combined into equivalent circuit of the analyzed SWS.

The central channel is a periodic structure composed of segments of circular waveguides and interaction gaps. It is assumed that only wave travels in circular waveguides. For this wave parameters U_{eq}, I_{eq}, Z_{eq} of the equivalent transmission line defined by the transverse components of the fields E^{τ}, H^{τ} as follows:

$$U_{eq} = -\int_1^2 E^{\tau} dl, \qquad I_{eq} = \oint_1 H^{\tau} dl, \qquad Z_{eq} = \frac{U_{eq}}{I_{eq}},$$

$$\gamma_{eq} = \frac{2\pi}{\lambda}\sqrt{1-\frac{\lambda^2}{\lambda_{cr}^2}}$$

There λ is a wavelength, λ_{cr} is a critical wavelength.

This line is described by the transmission matrix

$$CAj(\lambda) := \begin{pmatrix} \cosh(\psi j(\lambda)) & -zj(\lambda)\cdot\sinh(\psi j(\lambda)) \\ \frac{-\sinh(\psi j(\lambda))}{zj(\lambda)} & \cosh(\psi j(\lambda)) \end{pmatrix}$$

where

$$\mathrm{zj}(\lambda) := \frac{-\sqrt{\frac{\mu 0}{\varepsilon 0}} \cdot \mathrm{J0}(0) \cdot \sqrt{\left(1 - \frac{p^2 \cdot \lambda^2}{r1^2 \cdot 4 \cdot \pi^2}\right)}}{2 \cdot \pi \cdot p \cdot \frac{d}{dp}\mathrm{J0}(p)},$$

$$\psi\mathrm{j}(\lambda) := \frac{2 \cdot i \cdot \pi \cdot 15}{\lambda} \cdot \sqrt{\left(1 - \frac{p^2 \cdot \lambda^2}{r1^2 \cdot 4 \cdot \pi^2}\right)},$$

$J0(p)$ is Bessel function of the first kind of zero order, p is the first root of the Bessel function of zero order, r1 is radius of the transit channel.

This periodic function is a sum of spatial harmonics:

$$E_0(r,\varphi,z) = \sum_{n=-\infty}^{n=\infty} c_n e^{-i\frac{2\pi n}{D}z}$$

In a known manner the equivalent current is introduced, and in the center of the gap the resistance, which relates the voltage across the gap and the longitudinal current, defined by the azimuthal magnetic field, is determined.

$$Z = \frac{E_0 d}{I_{eq}}$$

where E_0 is constant field in the gap of interaction, d is the width of the gap of interaction. The magnitude of the resistance is determined by expressions

$$Z = \frac{1}{Y},$$

where

$$Y = \frac{-\pi 2ir1\left[\sum_{k=1}^{10}\left[\frac{4\pi \sin\left(k\pi\frac{d}{D}\right)\cos(k\pi)^2 \bullet}{k\pi\lambda\sqrt{-\left[\frac{2\pi k}{D}\right]^2+\left[\frac{2\pi}{\lambda}\right]^2}\bullet}\right.\right.}{\sqrt{\frac{\mu 0}{\varepsilon 0}}d}$$

$$\left.\left.\frac{I1\left[r1\sqrt{-\left[\frac{2\pi k}{D}\right]^2+\left[\frac{2\pi}{\lambda}\right]^2}\right]}{I0\left[r1\sqrt{-\left[\frac{2\pi k}{D}\right]^2+\left[\frac{2\pi}{\lambda}\right]^2}\right]}\right] + \frac{dJ1\left(\frac{2\pi r1}{\lambda}\right)}{DJ0\left(\frac{2\pi r1}{\lambda}\right)}\right]$$

There I0(x), I1(x) are modified Bessel functions.

Cell of "winding waveguide" is divided into partial areas by planes perpendicular to the direction of propagation of the microwave energy, which are modeled by segments of rectangular waveguides, in which the wave H_{10} propagates.

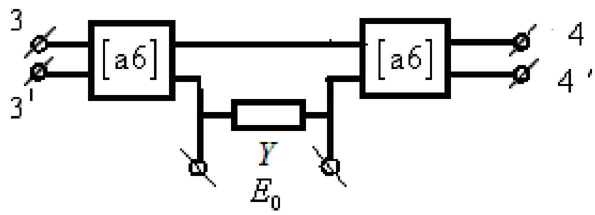

Fig. 2. Equivalent circuit of transit channel

After determination of $U_{eq}, I_{eq}, Z_{eq}, \gamma_{eq}$, waveguide segments are modeled by quadripoles connected in cascade and sequentially [1] (Fig. 3).

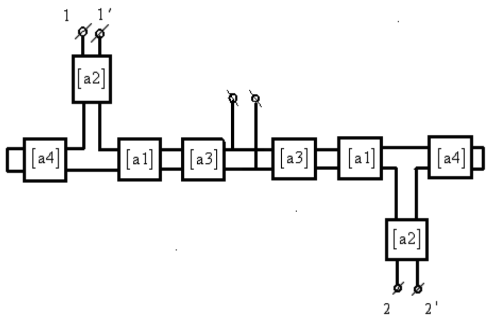

Fig. 3. Equivalent circuit of "winding waveguide"

Transmission matrices, which models segments of waveguide channels, in this case are

$$\mathrm{CAj}(\lambda) := \begin{pmatrix} \cosh(\psi\mathrm{j}(\lambda)) & -\mathrm{zj}(\lambda)\cdot\sinh(\psi\mathrm{i}(\lambda)) \\ \frac{-\sinh(\psi\mathrm{j}(\lambda))}{\mathrm{zj}(\lambda)} & \cosh(\psi\mathrm{j}(\lambda)) \end{pmatrix}$$

$$\mathrm{zj}(\lambda) := \frac{b2 \cdot \sqrt{\frac{\mu 0}{\varepsilon 0}}}{a2 \cdot \sqrt{1 - \frac{\lambda^2}{4 \cdot a2^2}}},$$

$$\psi\mathrm{j}(\lambda) := \frac{2 \cdot i \cdot \pi \cdot lj \cdot \sqrt{1 - \left(\frac{\lambda}{2 \cdot a2}\right)^2}}{\lambda}$$

where $b2$ is girth, $a2$ is width and l_j is length of rectangular waveguide segment.

The exciting current or other effects can be connected to the central terminals of this circuit. The resulting equivalent circuit channels (Fig. 2,3) of the microwave power transmission in the considered SWS are combined into a single scheme, taking into account the fact that there is a connection for both voltage and current (magnetic and electric field) between them [2]. The equivalent circuit takes the following form (Fig. 4).

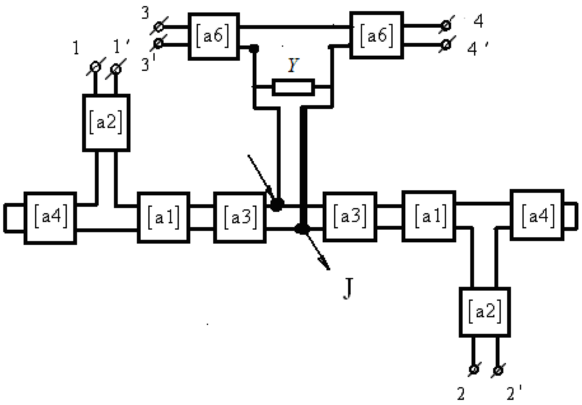

Fig. 4. Equivalent circuit of SWS cell

In the absence of the exciting current, the octopole models selected SWS cell.

Octopole is described by the transmission matrix, the coefficients of which are determined by the coefficients of transmission matrices of quadripoles, which composes the branched equivalent circuit.

The Z-matrix of octopole was used as the initial matrix. Its coefficients were determined from idling experiments, which were carried out using the equivalent circuit of the cell, composed of quadripoles. As a result of these experiments, six power transmission channels have been allocated (Figure 5), composed of cells of quadripoles equivalent circuit.

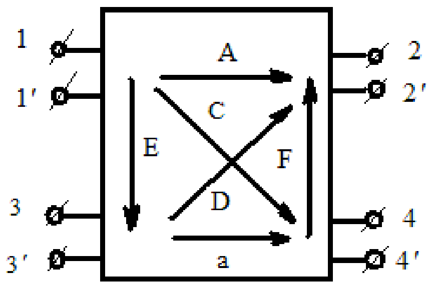

Fig. 5. Power transmission channels

When calculating the dispersion and characteristic impedance of channels the standard algorithms for determining the eigenvalues and eigenvectors of the transmission matrix are used. Based on this model algorithm and program for calculating the dispersion and the characteristic impedances have been developed. The program is implemented in Mathcad.

III. Assessment of Model's Adequacy

In order to assess the adequacy of the developed model the results of calculation of electrodynamic characteristics using strict electrodynamic program HFSS with the results obtained on the developed program were compared. The algorithm of this program is similar to the given above, but uses Z-matrix calculated by 3D simulation results.

The calculations were performed for different radii of the transit channel and interaction gaps and cavities of the same size.

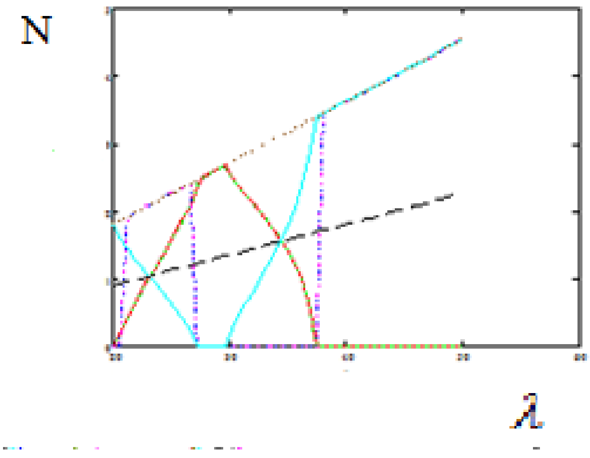

Fig. 6a. Dependence of slowing factor N, calculated by developed model, from wavelength (millimeters)

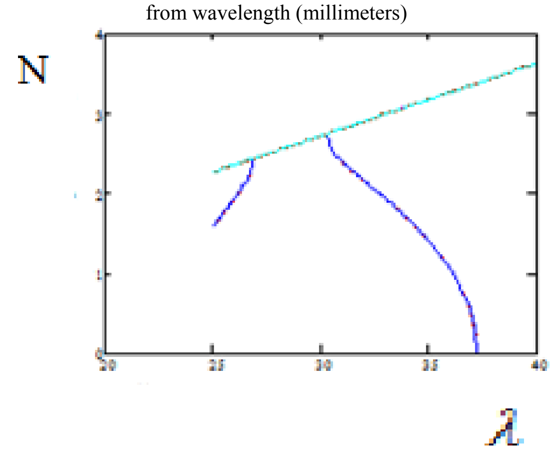

Fig. 7b. Dependence of slowing factor N, calculated by HFSS, from wavelength (millimeters)

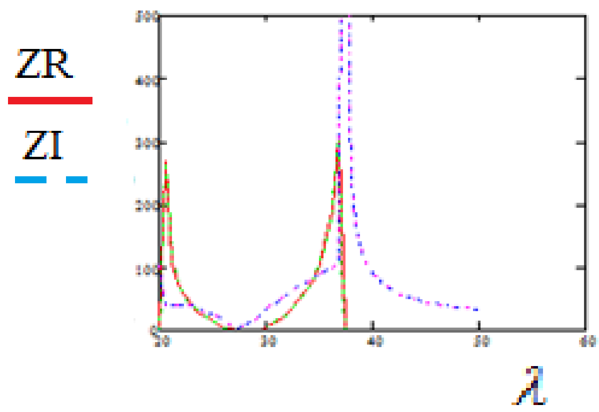

Fig. 7a. Dependence of real and imaginary parts of characteristic impedance Z (Ohms), calculated by developed model, from wavelength (millimeters)

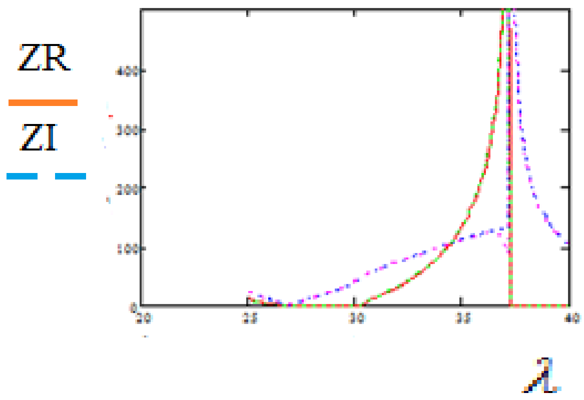

Fig. 7b. Dependence of real and imaginary parts of characteristic impedance Z (Ohms), calculated by HFSS, from wavelength (millimeters)

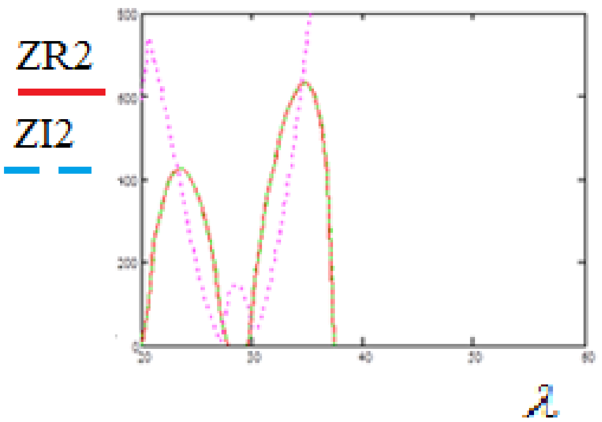

Fig. 8a. Dependence of real and imaginary parts of characteristic impedance of transit channel Z2 (Ohms), calculated by developed model, from wavelength (millimeters)

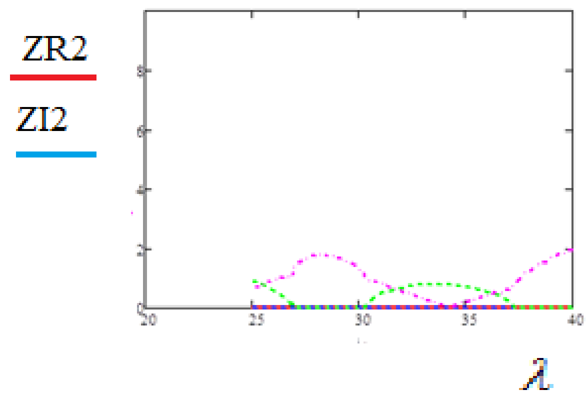

Fig. 8b. Dependence of real and imaginary parts of characteristic impedance of transit channel Z2 (Ohms), calculated by HFSS, from wavelength (millimeters)

On Fig. 6,7,8 (a) the characteristics of the SWS calculated by the developed model are shown.

Analysis of the results shows that in the considered octopole there are four modes corresponding to the two pairs of complex conjugate roots. For a small radii transit channel dispersion characteristics calculated by HFSS (Fig. 6,7,8 (b)) and the developed model are practically identical. With increasing radius of the transit channel the nature of the bandwidth change is preserved in both cases, but there is a slight difference in the passband position.

Thus, it can be argued that the developed model qualitatively reflects the nature of the physical processes occurring in the real slow-wave structure.

IV. Conclusion

The characteristic channel impedances calculated by HFSS and developed models are complex. For the main microwave power transmission channel, they are the same in a wide range of transit channel radii. For transit channel the characteristic impedances differ in size, but the nature of the changes is the same.

References

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[2] Mukhin S.V., Software package “VEGA” for modeling of resonator slow-wave structures and interaction processes in TWT based on them, Journal of Radioelectronics (http://jre.cplire.ru), No.12, 2002.