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XXXI International Seminar on Stability Problems for Stochastic Models

and

VII International Workshop "Applied Problems in Theory of Probabilities and Mathematical Statistics Related to Modeling of Information Systems"

and

International Workshop "Applied Probability Theory and Theoretical Informatics"

Book of Abstracts

2013

Faculty of Computational Mathematics and Cybernetics
Moscow State University
(CMC MSU)

Institute of Informatics Problems
Russian Academy of Sciences
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В сборник включены тезисы докладов, представленных на XXXI Международный семинар по проблемам устойчивости стохастических моделей (ISSPSM'2013), VII Международный рабочий семинар "Прикладные задачи теории вероятностей и математической статистики, связанные с моделированием информационных систем" (APTP + MS'2013) (весенняя сессия) и Международный рабочий семинар "Прикладная теория вероятностей и теоретическая информатика".

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Developing a new approach to the problem of optimal control in the open dynamical model of a three-sector economy

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In some works of V. A. Kolemaev it was developed and analyzed a dynamic model of three-sector economy. The zero sector produces job objects, the first- means of labor, the second-consumables. In particular, in [1] there was considered an open version of the model, taking into account the impact of foreign trade.

Here is a list of the main indicators of this model:

Y_j - the volume of output in the j-th sector;

K_j - the main production funds (capital) in the j-th sector;

L_j - the number of employed in the j-th sector;

I_j - the volume of investments in the j-th sector;

X_j - imports of goods-sector j (j = 1, 2);

Z_0 - the export volume of materials.

v - the growth rate of employment;

q_0 - the world price of exported materials;

q_1^+, q_2^+ - world prices of imported consumer goods and investment;

μ_j - wear out factor of MPF j-th sector

$\lambda_j = \mu_j + v$ - the coefficient of reduction of assets through depreciation of physical capital and the increase in the number of employed j-th sector.

An analytical study of dynamic model of three-sector economy will be produced in the unit settings. This small Latin letters are indicated by the appropriate amount related to the volume of labour resources in the sector and small Latin letters with cover are related to the total. (In addition $\theta_j = \frac{L_j}{L}$ to the j-th-share sector in the allocation of labor resources).

The basic dynamic and balance sheet ratios describing the open model of three-sector economy are given in [1].

New mathematical optimal control problem

New statement of a problem of optimal control may be developed. In order to do this, we introduce some additional assumptions in the original model.

Problem management is considered in the specified target timescale $[0, T]$.

State of the system is described by a dynamic set of parameters specific to sectors of the capital. Control settings are specific investments $i_1(t)$ and specific volume of imports $x_1(t)$ in the first sector.

We introduce several additional assumptions regarding key ratios of the models:

2.1. The distribution of investments: $I_0 = \rho[Y_1 + X_1 - I_1]$,

$I_2 = (1-\rho)[Y_1 + X_1 - I_1]$, where $\rho, 0 < \rho < 1$ - the specified number.

2.2. Specific import volumes in the first and second sector are in a constant ratio: $x_2 = wx_1$, where w - given a positive number.

2.3. Limit on the share of export: Suppose that $z_0 \le z_0^*$, where z_0^* - the maximum share of exports. Then from the assumption 2.2. it should be: $x_1 \le \frac{q_0\theta_0 z_0^*}{q_1^+\theta_1 + q_2^+\theta_2 w}$.

2.4. Restrictions on specific investments in the establishing fund sector: $i_{\min} \le i_1 \le \varepsilon_1(y_1 + x_1)$.

As a criterion of optimality is considered a mixed target functionality consisting of integral and terminal components. Integral component is the discounted consumption at a given time interval. Terminal element characterizes the effect on the efficiency of the system parameter values of capital at the end of the process of the management .

We can obtain the differential equations on the functions of system states $k_0(t), k_1(t), k_2(t)$, that will play the role of differential due to the optimal control problem.

It is expected that the initial values of parameters of system states are set.

Restrictions on management are based on assumptions 2.3., 2.4.

As a result, we get a new setting of optimal control problem in canonical form:

1. $\int\limits_0^T e^{-\delta t}\theta_2\left(A_2k_2^{\alpha_2} + wx_1\right)dt + \psi\left(k_0(t), k_1(t), k_2(t)\right) \to \max$
2. Differential association:

$$\begin{cases} \dot{k_0} &= -\lambda_0 k_0 + \rho l_{1,0}\left(x_1 + A_1k_1^{\alpha_1} - i_1\right) \\ \dot{k_1} &= -\lambda_1 k_1 + i_1 \\ \dot{k_2} &= -\lambda_2 k_2 + (1-\rho)\, l_{1,2}\left(x_1 + A_1k_1^{\alpha_1} - i_1\right) \end{cases}$$

3. Initial conditions: $k_0(0) = k_{0,0}, k_1(0) = k_{1,0}, k_2(0) = k_{2,0}$
4. Restrictions on management:

$$\begin{cases} i_{\min} \le i_1 \le \varepsilon_1(y_1 + x_1) \\ 0 \le x_1 \le \min\left(\gamma_1 A_1 k_1^{\alpha_1}, \frac{q_0\theta_0 z_0^*}{q_1^+\theta_1 + q_2^+\theta_2 w}\right) \end{cases}$$

The subject of our future study will be the task of optimal control. This issue is examined through the principle of Pontryagin's maximum. From the condition of maximum Pontryagin's function defines the structure of optimal control. Further we study adjoint equations and differential association.

References

1. V. A. Kolemaev. *Optimal balanced growth of open a three-sector economy*, 2008, Applied econometrics, No. 3.

Trajectory analysis of control process for optimal control of investments in the model of a three-sector economy

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In this research, we consider the optimal control problem for an economic system whose behavior is described by the dynamical model of a three-sector economy. The results are summarized in [1,2]. For the state parameters of the system we take the capital-labor ratio functions (specific capital) in each sector and for the control parameter we take the amount of specific investments in the capital generating sector. The solution of the optimal control problem under consideration is based on the Pontryagin maximum principle. We find the optimal control structure depending on some auxiliary function, which is expressed in terms of conjugate variables. Analytic solutions of the systems of differential equations for the state variables and the conjugate variables are obtained. The system of differential equations (differential association), describing the dynamics of the system states has the form:

$$\begin{cases} \dot{k}_0(t) = -\lambda_0 k_0(t) + l_0^{(1)} \rho (A_1 k_1^{\alpha_1}(t) - i_1(t)), \\ \dot{k}_1(t) = -\lambda_1 k_1(t) + i_1(t), \\ \dot{k}_2(t) = -\lambda_2 k_2(t) + l_2^{(1)} (1-\rho)(A_1 k_1^{\alpha_1}(t) - i_1(t)) \end{cases}$$

In previous papers [1, 2] there were obtained optimal control structure, and solutions of differential equations and conjugate parameters. Four basic modes of optimal control were analyzed on the time interval $[0, T]$. For each of the options issued to differential equation $k_0(t)$, $k_1(t)$, $k_2(t)$ for relative depending on the control structure.

Further it is shown how $k_0(t)$, $k_1(t)$, $k_2(t)$ behave at different optimal control regimes. Below we state stationary solutions of a differential system association for the first option of the structure of optimal control with additional

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