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Numerical Analysis of the Effect of Hysteresis in the Access Control of Wireless Broadband Network on the Functional Efficiency

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Abstract—The effect of the mutual positions of hysteresises in the access control of the wireless broadband network with different thresholds with respect to access enable and disable to the network resources with allowance for the service class is numerically analyzed. Three variants of the mutual positions of hysteresises in the access control of three flows with different requirements on the level of service are considered. The efficiencies of the additional processing of the low-class requests with unnormalized losses are calculated with allowance for diurnal variations in the network load.

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1. INTRODUCTION

We numerically study the characteristics of the three-flow model from [5] with different positions of the hysteresises of access control of the network resources for the requests of the second and third service classes.

The simplified model from [5] is used for the processing of requests of three service classes and involves birth-and-death processes with hysteresis. We consider the available resource of the basic transmission units (BTUs) with volume V and three input request flows that correspond to different user service classes. The flows are stationary Poisson processes with intensities λ_1, λ_2, and λ_3. Request processing times are exponential with parameters a_i. All of the resources (V) are available for the first flow, and k_2 and $k_3 < V$ resources are available for the second and third flows, respectively. Quantity k_i characterizes the load (i.e., the number of resources allocated to each service class). The request is rejected if all of k_i resources are busy at the moment of the query of the ith service class. For each service class, we use the volume of busy resources $f_i < k_i$ at which the access of requests of the ith flow to network resources is enabled. Thus, a hysteresis for the requests of the ith service class emerges. The hysteresis is determined by the upper-bound limit when the access to network is disabled and the lower-bound limit when the access is enabled.

We introduce four subsets of the state of the system: S^{++} (all of the requests of the three classes are processed), S^{+-} (the requests of the lower service class are not processed), S^{--} (only the requests of the first service class are processes), and S^{-+} (only the requests of the first and third classes are processed).

After such simplifications, we characterize the state of process using parameter v (volume of the resource being used). With allowance for the above subsets, we use quantities v^{++}, v^{+-}, v^{-+}, and v^{--}. The corresponding stationary probabilities are p_v^{++}, p_v^{+-}, p_v^{-+}, and p_v^{--}.

A detailed formulation of the problem can be found in [5].

In the numerical analysis, we calculate the probability of the request lost for each service class. To characterize the allowed access control, we assume that the loss probabilities for the requests of the first and second service classes are no greater than 0.5 and 5%, respectively. The calculations are performed at different specific loads per one BTU, and we differ between the total load and the load related to the requests of the first and second classes. In addition, we set different limits for hysteresises and consider the three possible variants of the mutual positions.

We consider the loss probability for the first-class requests as the output characteristic:

$$\pi_1 = \sum_{v = V+1-a_1}^{V} (p_v^{++} + p_v^{+-} + p_v^{-+} + p_v^{--}). \tag{1}$$

Here, a_1 is the number of BTUs that are used by first-class requests and p_v^{++}, p_v^{+-}, p_v^{-+}, and p_v^{--} are stationary probabilities that are calculated using the formulas from [5]. We use similar formulas to calculate the loss probabilities for the second- and third-class requests:

$$\pi_2 = \sum_{v=0}^{V} (p_v^{-+} + p_v^{--}), \quad (2)$$

and

$$\pi_3 = \sum_{v=0}^{V} (p_v^{+-} + p_v^{--}). \quad (3)$$

The number of input requests that takes into account the possible repeated requests is an important parameter. Thus, we calculate the rate of requests using the method from section 3 in [5]:

$$\Lambda_i = \frac{\lambda_i}{1 - \pi_i H_i}. \quad (4)$$

Here, λ_i is the rate of requests of the ith service class and H_i is the probability of the repeated request of the ith service class in the case when the primary request is rejected.

Loss probability p_a that does not affect the repeated requests (user A) is introduced in [1–3] and the analysis of its effect on the network characteristics is presented. This parameter is interpreted as the probability of the event in which the user is switched to an alternative network due to unsatisfactory service in the network under study [2]. Evidently, parameter p_a depends on the user service class, so that quantities p_{ai} must be considered. In this case, expression (4) is written as

$$\Lambda_i = \frac{\lambda_i}{1 - \pi_i(1 - p_{ai})H_i}. \quad (5)$$

Formula (5) shows that parameters p_{ai} can be interpreted as the parameters that affect only user persistence H in the above formulation of the problem.

We characterize the working efficiency of the resources of the network unit using the mean number of busy BTUs

$$m = a_1\Lambda_1(1 - \pi_1) + a_2\Lambda_2(1 - \pi_2) + a_3\Lambda_3(1 - \pi_3), \quad (6)$$

where a_i is the number of BTUs that are used by the requests of the ith service class (in the calculations, we assume that $a_1 = 2$, $a_2 = 3$, and $a_3 = 1$).

We numerically study the control properties for the nonintersecting hysteresises at various loads per BTU and consider overloaded and weakly loaded units. Different limits of hysteresises are considered at different loads. We choose the hysteresis limits for the second-class requests to minimize the limitations on such queries and to provide the predetermined number of processings for the first-class requests. The hysteresis limits for the third-class requests are varied in wide ranges to analyze the possibility of a wide hysteresis that allows the rejection of the third-class requests at the interval at which the load determined by the first- and second-class requests oscillates in the vicinity of peak values. The constant values of the remaining parameters can be found in the table captions.

In Section 2, we numerically analyze the access control using the nonintersecting hysteresises (see section 4 in [5]).

In Section 3, we numerically study the access control using the intersecting hysteresises (see section 5 in [5]).

In sSection 4, we numerically investigate the access control in the case when a hysteresis is located inside another hysteresis (see section 6 in [5]).

In Section 5, we introduce cost functional P that makes it possible to take into account the service volume with allowance for the BTU cost, which naturally depends on the service class, since different service levels are available for different requests. In addition, we introduce functional R that takes into account the network costs related to setup of connections (identical costs for the three classes). Such functionals were introduced in [4] in the analysis of two flows with control hysteresis for the second-flow requests. Here, we consider a model of the unit of a wireless broadband network (WBN) with three flows and two hysteresises.

In Section 6, we numerically study the properties of functionals P and R in the existence of three flows and two hysteresises.

The results from Section 6 show that the presence of the additional flow of requests of the third service class can be economically unfavorable in terms of the criteria depending on functionals P and R provided that the total load is extremely high. On the other hand, the profile of the diurnal load related to the requests of one class normally exhibits intervals with high and low loads. Note the possibility of noncoincidence of the high-load intervals for requests of different service classes, since the sources of load exhibit different time dependences. Thus, the periods with undesired third-class requests are followed by relatively long time intervals in which such requests provide additional income. Therefore, it is expedient to analyze the effect of the diurnal oscillations of load on the processing efficiency of the third-class requests. The results of the numerical study can be found in Section 7.

2. NUMERICAL ANALYSIS OF THE ACCESS CONTROL PROPERTIES FOR NONINTERSECTING HYSTERESISES

Hereafter, we use the following notation in the tables: ρ, specific load per BTU related to the requests of the three classes

$$\rho = \frac{\lambda_1 + \lambda_2 + \lambda_3}{V},$$

and ρ_{12}, specific load per BTU related to the first- and second-class requests

$$\rho = \frac{\lambda_1 + \lambda_2}{V}.$$

Table 1. Results of calculations at $a_1 = 2$, $a_2 = 3$, $a_3 = 1$, $H_1 = 0.9$, $H_2 = H_3 = 0.8$, and $v = 100$

0	1	2	3	4	5	6	7	8	9
λ_1	30	30	30	30	20	20	20	20	20
λ_2	10	10	10	10	10	10	10	10	10
λ_3	30	30	30	30	30	30	30	10	10
ρ	1.2	1.2	1.2	1.2	1.0	1.0	1.0	0.8	0.8
ρ_{12}	0.9	0.9	0.9	0.9	0.7	0.7	0.7	0.7	0.7
k_2	96	93	93	93	93	96	96	96	96
f_2	94	90	90	90	90	93	93	93	93
k_3	70	70	80	89	80	80	90	90	90
f_3	40	40	75	85	75	75	85	85	88
π_1	0.0069	0.0021	0.0023	0.0035	0.0000	0.0001	0.0007	0.0003	0.0001
π_2	0.1456	0.2361	0.2617	0.3902	0.0183	0.0070	0.0795	0.0305	0.0156
π_3	0.9895	0.9907	0.9802	0.9003	0.9557	0.9365	0.9133	0.3744	0.1529
Λ_1	30.18	30.05	30.06	30.09	20.00	20.01	20.00	20.00	20.00
Λ_2	11.26	12.17	12.51	14.49	10.15	10.06	10.68	10.23	10.13
Λ_3	143.45	143.79	138.02	106.70	118.89	119.21	111.04	13.75	11.13
m	89.8	86.0	87.9	90.6	82.0	82.1	90.5	78.3	79.3

We start the numerical analysis and consider strongly overloaded units ($\rho = 1.2$ and $\rho_{12} = 0.9$). The results in the first column in Table 1 show that almost complete rejection of the third-class requests under the given initial conditions does not provide high-quality service for the first and second classes, since the total load related to the first and second classes is 0.9 Erlang per BTU. The results from the remaining columns in Table 1 show that a relatively high processing quality for the first-class requests can be provided using appropriate hysteresis limits for the second-class requests. Note that the first-class requests use two BTUs for the processing, so that an upper-bound limit of 96 for the second-class requests allows a reserve for the processing of only two requests of the first service class. The hysteresis limits for the second class in the second and remaining variants allow the elimination of the negative effect of the remaining flows on the processing quality of the first-class requests provided that the load related to the first-class requests is not excessive. This can be reached using the shift of the hysteresis limits for the second-class requests from [94, 96] to [90, 93]. In practice, the processing probability for the second-class requests decreases by only one line, since three BTUs are needed for the processing.

The data in columns 2–4 are obtained at the same load but different hysteresis limits. Note that the hysteresis limits are fixed and varied for the second- and third-class requests, respectively. The hysteresis intervals gradually become narrower. The hysteresis interval in the second column is relatively wide (the lower-bound limit is 70), so that the third-class requests are almost completely rejected (as in the first column) and the loss parameter π_3 is 0.99.

The comparison of the calculated results with the data from columns 2–4 shows that the third-class queries may affect the processing quality of the priority-class requests even when the requests are rejected with a relatively high probability. Evidently, the requests cannot be processed, since the total load related to the three flows is significantly greater than the capacity of unit. Thus, reasonable processing quality can be provided only for the priority-class requests. The hysteresis position for the third-class requests affects the processing quality of the second- and first-class requests. For example, the comparison of the calculated results from the second and fourth columns shows that the processing quality of the first- and second-class queries decreases by a factor of 1.5 while the processing quality of the third-class requests remains at an extremely low level when the hysteresis for the third-class requests is maximum close to the hysteresis of the second-class requests.

However, the results from columns 5–9 show that the appropriately chosen hysteresis limits for the third-class requests provide the desired limits of the loss probabilities for the second- and first-class requests, so that the effect of the third-class queries is eliminated when a sufficient number of BTUs are used for the processing of the priority requests.

Note that the main disadvantage of the presence of the third-class requests in the case when the unit is overloaded due to such requests is related to a significant increase in the number of the repeated third-class requests, which negatively affects the load of the network signal system. For example, at an initial rate of the third-class requests of 30, the intensity of the corresponding flow of requests with allowance for the repeated events decreases from 143.45 to 106.7 depending on the loss probability of such requests, which is significantly greater than the rate of primary requests of all service classes.

On the other hand, in the presence of the third-class requests, the mean processing rate of the network increases due to better utilization of resources. In columns 1–4, the mean number of the busy BTUs (about 90) corresponds to the utilization of resources at which all of the requests of the first two classes are processed. In columns 5–9, the number of the busy BTUs is significantly greater than 70, which corresponds to the utilization of resources at which all of the requests of the first two classes are processed. However, the results from section 6 show that an increase in the income due to more complete utilization of resources may not lead to an increase in profit when the expenses related to the processing of requests are taken into account.

The data from columns 1–4 show that, in the presence of significant overload, the hysteresis limits for the third-class requests weakly affect parameter m but substantially affect the loss probability of priority queries. Thus, we conclude that the hysteresis limits must be chosen with allowance for the stability of priority users with respect to service quality if the total load of network is significantly greater than the network capacity.

However, it is expedient to decrease the number of repeated requests for the third service class when the desired quality parameters are provided for priority requests.

Columns 5–7 in Table 1 demonstrate the calculated results for the total network load that is equal to the network capacity and a load of 70% of the network capacity for the first- and second-class requests. In this case, the processing quality is provided for the first and second service classes and the loss probability for the second service class is no greater than 1% for the appropriate hysteresis of the second class. When the hysteresis of the third service class is close to the hysteresis of the second service class (column 7) the probability loss for the second service class amounts to 8%. This circumstance indicates that the request hysteresis of the third service class must substantially differ from request hysteresis of the second service class when the total load of all flows is close to or greater than the capacity of unit.

Columns 8 and 9 in Table 1 show the calculated results for a total load of 80% of the network capacity and a load of 70% of the network capacity for the first- and second-class requests. In this case, the processing quality increases for all of the requests when the hysteresises are maximum close to each other. The comparison of the results from columns 8 and 9 shows a two-fold decrease in the losses for all of the requests and an increase in the service load. The overlapping that results from the further shifts of the hysteresises is discussed in the next section.

A decrease in the probability of user persistence is important, since the access control is limited when $\rho \geq 1$. Such an approach allows a decrease in the ineffective load of the signal system of network. A relatively low persistence probability can be reached owing to the predictability of network for users. This means that the stability of access to network resources must be provided at the intervals of peak loads: the rejection levels must be maintained for the third-class requests when the load accidentally decreases. Thus, the user strategy can be formulated: when the requests are rejected, the users who form the requests of the third flow try to avoid the repeated connections over the period of peak loads.

Table 2 presents the calculated results that are similar to the results from Table 1 but the persistence of users who generate the requests of the third service class is decreased by a factor of 2.

The results from Table 2 show that a decrease in the persistence upon the generation of the third-class queries leads to a significant decrease in the number of repeated requests and an increase in the efficiency of network. Note better parameters for both first- and second-class requests. Indeed, the rate of the third-class requests decreases by almost a factor of 3 for the overloaded units (columns 1–7 in Tables 1 and 2) and the loss probability for the second-class requests decreases by 10–15%. The total service load slightly decreases due to a relatively large number of rejected requests| of the third service class. The above properties are clearly manifested when the cost functionals are employed and the processing cost is taken into account (sections 6 and 7).

Thus, the following conclusions can be drawn.

(i) When the unit load is greater than or close to the capacity of unit, it is expedient to employ the hysteresises of the second and third-class requests that are substantially spaced apart.

(ii) The hysteresis for the second-class requests must be sufficiently narrow if the total load related to the first and second service classes is no greater than 0.9 Erlang per BTU.

(iii) The hysteresis for the processing of the third-class requests must be sufficiently broad to provide the maintenance of the access mode in the presence of the load oscillations in the vicinity of peak values.

(iv) When the total load can be processed at a relatively low loss probability, the hysteresises for the second- and third-class requests must be close to each other.

(v) A decrease in the persistence probability of users who generate the third-class load leads to an increase in the efficiency of the network resources when the parameter that characterizes the specific total load of all service classes per BTU is $\rho < 1$.

Table 2. Results of calculations at $a_1 = 2$, $a_2 = 3$, $a_3 = 1$, $H_1 = 0.9$, $H_2 = 0.8$, $H_3 = 0.4$, and $v = 100$

0	1	2	3	4	5	6	7	8	9
λ_1	30	30	30	30	20	20	20	20	20
λ_2	10	10	10	10	10	10	10	10	10
λ_3	30	30	30	30	30	30	30	10	10
ρ	1.2	1.2	1.2	1.2	1.0	1.0	1.0	0.8	0.8
ρ_{12}	0.9	0.9	0.9	0.9	0.7	0.7	0.7	0.7	0.7
k_2	96	93	93	93	93	96	96	96	96
f_2	94	90	90	90	90	93	93	93	93
k_3	70	70	80	89	80	80	90	90	90
f_3	40	40	75	85	75	75	85	85	88
π_1	0.0058	0.0020	0.0031	0.0022	0.0000	0.0001	0.0006	0.0002	0.0001
π_2	0.1248	0.2191	0.2418	0.2419	0.0163	0.0063	0.0659	0.0244	0.0128
π_3	0.9724	0.9781	0.8141	0.9526	0.8376	0.8387	0.7668	0.3022	0.1270
Λ_1	30.12	30.05	30.08	30.05	20.00	20.00	20.01	20.00	20.00
Λ_2	10.80	11.83	13.55	12.13	10.13	10.05	10.53	10.17	10.01
Λ_3	48.35	49.07	44.23	48.25	44.96	44.99	42.93	11.17	10.10
m	85.2	85.1	89.9	87.4	81.2	81.3	88.5	77.5	78.3

Table 3. Results of calculations at $a_1 = 2$, $a_2 = 3$, $a_3 = 1$, $H_1 = 0.9$, $H_2 = 0.8$, $H_3 = 0.8$, and $v = 100$

0	1	2	3	4	5	6	7
λ_1	20	20	20	10	10	10	10
λ_2	10	10	10	10	10	10	10
λ_3	10	10	10	30	30	40	40
ρ	0.8	0.8	0.8	0.8	0.8	0.9	0.9
ρ_{12}	0.7	0.7	0.7	0.5	0.5	0.5	0.5
k_2	96	96	96	96	96	96	96
f_2	93	93	93	90	90	93	93
k_3	90	95	95	95	91	95	91
f_3	88	88	80	80	80	80	80
π_1	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
π_2	0.0156	0.0139	0.0111	0.0090	0.0016	0.0414	0.0058
π_3	0.1529	0.0511	0.1015	0.0723	0.1164	0.3308	0.4222
Λ_1	20.00	20.00	20.00	10.00	10.00	10.00	10.00
Λ_2	10.13	10.01	10.01	10.06	10.01	10.33	10.05
Λ_3	11.13	10.04	10.10	31.44	32.31	53.5	59.64
m	78.3	79.1	78.6	79.1	78.5	85.5	84.4

3. NUMERICAL ANALYSIS OF THE ACCESS CONTROL FOR OVERLAPPED HYSTERESISES

We consider the calculated results for the models with overlapped hysteresises at relatively low loads ($\rho \leq 0.8$), since the results from the previous section indicate the inexpediency of the closely positioned hysteresises in the presence of high load.

For convenience, the first column presents the results from the last column in Table 1. The comparison of the results in first and second columns in Table 3 proves the above assumption that an increase in the upper-bound limit of the hysteresis for the third-class requests causes an increase in the working efficiency of the network resources with maintaining of the limitations on the processing quality of the first-class requests.

The comparison of the results from the second and third columns in Table 3 shows that an increase in the width of the hysteresis for the third-class requests due to the shift of the lower bound limit leads to a minor decrease in the network efficiency although the loss probability for the third-class requests evidently increases. Note a simultaneous decrease in the efficiency of the resources of unit m.

We numerically study the network characteristics at a significantly higher rate of the third-class requests. We compare two upper-bound limits of the hysteresis for the third-class requests. The results show that a further increase in the processing quality of the third-class requests is impossible due to the presence of the limitations on the hysteresis boundaries that is determined by the upper-bound limit of hysteresis for the second-class requests. The boundary cannot be higher than such a limit, since the assumption regarding the resources allocated to the second-class requests involves the simultaneous usage of three BTUs. On the other hand, the service quality for the first two service classes is such that the spare resources can be available for the third-class requests. However, in this case, the upper-bound limit of the hysteresis for the third-class requests is above the upper-bound limit of the hysteresis for the second-class requests. Such hysteresises are considered in the next section.

The remaining columns show the calculated results for the scenarios in which the rate of the third-class requests is even higher. Note that the closer positions of the upper-bound limits of hysteresises for the second- and third-class requests lead to a significant increase in the loss probability for the second-class requests owing to the absence of spare resources available for the processing of the requests of the first two classes at a total load of 0.9 Erlang.

Thus, the following conclusions can be drawn.

(i) When the total load of all flows is no greater than 0.8 Erlang per BTU and the fraction of load related to the third-class requests is significant, it is expedient to fix the upper-bound limit for the hysteresis of the third-class requests at a level that is higher than the upper-bound limit of the hysteresis for the second-class requests.

(ii) When the total load of all flows is no greater than 0.8 Erlang per BTU and the fraction of load related to the third-class requests is relatively small, it is expedient to employ the overlapped hysteresises and the hysteresis for the third-class requests that is sufficiently broad for maintaining of the access mode in the presence of load oscillations in the vicinity of peak levels.

4. NUMERICAL STUDY OF THE ACCESS CONTROL IN THE CASE WHEN A HYSTERESIS CONTAINS ANOTHER HYSTERESIS

We consider the calculated model results for the system in which a hysteresis contains another hysteresis. The previous calculations show that such a scenario is appropriate only in the range of relatively small loads, so that we use $\rho = 0.8$ and $\rho_{12} = 0.7$ or 0.5.

Note that the loss of the first-class requests is almost absent in the given examples. However, the results from column 3 in Table 4 show that the loss of the second-class requests can be significant if a relatively high upper-bound limit of the hysteresis for the third-class requests facilitates the access of such requests to the network resources. On the other hand, the lower-bound limit of the hysteresis for the third-class requests weakly affects the network parameters. Parameter m is relatively stable, since almost all of the requests are processed under the given conditions (the processing of repeated requests is possible) and the flow of repeated requests is insignificant.

A weak dependence of the parameters on the lower-bound limit of hysteresis is due to a relatively small probability of reaching the upper-bound limit (i.e., the probability of limitations in the network). In the presence of limitations, the process that determines the volume of the active resource rapidly approaches the lower-bound limit of hysteresis if it is significantly higher than the input rate of the remaining flows, which was always valid in the above examples.

5. COST FUNCTIONALS OF THE WBN UNIT WITH THREE FLOWS AND HYSTERESISES

Based on the simplified three-flow model of the processing of the WBN requests from section 3 in [5], we consider the working efficiencies of various variants of the hysteresis positions in the access control of the requests of three service classes with allowance for additional parameters (usage fare for one BTU per unit time for the request of the ith service class c_i and processing cost of one request α with disregard for the service availability).

Using the parameters of the request processing, we calculate the cost functional, which shows the service cost:

$$R = \Lambda_1(1-\pi_1)c_1a_1 + \Lambda_2(1-\pi_2)c_2a_2 + \Lambda_3(1-\pi_3)c_3a_3, \quad (7)$$

and the income resulting from the rendered services with allowance for the communication costs

$$p = R - \alpha(\Lambda_1 + \Lambda_1 + \Lambda_3). \quad (8)$$

The economic efficiency of network is reached when the maximum possible load increases. Thus, we employ the following assumption for the selection of the initial data. The network is designed for processing of requests of the first two classes with the needed quality over busy hours. However, the equipment is partially inactive due to nonuniform load, so that the processing of the third-class requests is allowed with unwarranted quality of service (i.e., the loss is not specified for the third-class requests). The last circumstance allows the analysis of various scenarios in which

Table 4. Results of calculations at $a_1 = 2$, $a_2 = 3$, $a_3 = 1$, $H_1 = 0.9$, $H_2 = 0.8$, $H_3 = 0.8$, and $v = 100$

0	1	2	3	4	5
λ_1	20	20	20	10	10
λ_2	10	10	10	10	10
λ_3	10	10	10	30	30
ρ	0.8	0.8	0.8	0.8	0.8
ρ_{12}	0.7	0.7	0.7	0.5	0.5
k_2	96	96	96	96	96
f_2	93	93	93	93	93
k_3	99	97	99	99	97
f_3	90	90	80	90	90
π_1	0.0004	0.0002	0.0002	0.0001	0.0000
π_2	0.0351	0.0265	0.0661	0.0305	0.0189
π_3	0.0221	0.0229	0.0164	0.0168	0.0203
Λ_1	20.00	20.00	20.00	10.00	10.00
Λ_2	10.03	10.02	10.05	10.01	10.02
Λ_3	10.02	10.02	10.02	30.04	30.05
m	79.5	79.0	78.6	79.4	79.2

the loss of the third-class requests can be relatively high and the rate of the third-class requests is controlled only by the expediency of the network usage with high loss and relatively low service cost. Under such assumptions, the difference between R and P can be significant and the corresponding qualitative changes to the scenario may affect the selection of hysteresis in the access control of the third-class requests that provides the maximum income from the network operation.

To assess the efficiency of the additional service for the third-class requests, we calculate reference parameters R_0 and P_0, which represent parameters R and P, respectively, in the system in which all of the requests are processed:

$$R_0 = \lambda_1 c_1 a_1 + \lambda_2 c_2 a_2 + \lambda_3 c_3 a_3, \tag{9}$$

$$P_0 = R_0 - \alpha(\lambda_1 + \lambda_2 + \lambda_3), \tag{10}$$

and the network parameters for the processing of only first two classes in the absence of loss R_0^* and P_0^*:

$$R_0^* = \lambda_1 c_1 a_1 + \lambda_2 c_2 a_2, \tag{11}$$

$$P_0^* = R_0^* - \alpha(\lambda_1 + \lambda_2). \tag{12}$$

Hence, the differences $R_0 - R$ and $P_0 - P$ yield the losses related to the partial rejection of requests and the differences $R - R^*$ and $P - P^*$ indicate the additional income related to the processing of the third-class requests.

In many cases, the users who generate the first-class requests use unlimited contracts. Thus, the network income is independent of the rendered services and depends only on the number of such users. In this case, we consider only the quality parameter (loss probability) and employ the value $c_1 = 0$. However, such a variation in the cost functional is insignificant with respect to the analysis of the differences $R - R^*$ and $P - P^*$, since the loss probability for the first service class can be neglected in all of the calculated variants.

6. NUMERICAL ANALYSIS OF THE PROPERTIES OF COST FUNCTIONALS

We use the initial data from the previous sections and calculate the cost functionals. The calculations show that the type of functional, which is optimized using the selection of hysteresises in the request flow control, substantially affects the types of hysteresises.

Columns 1–4 in Table 5 show the calculated results for the WBN unit that is overloaded by the total request flow. The results from the corresponding columns in Table 1 show that the loss for the third-class requests ranges from 90 to 99%, the loss for the first-class queries are in agreement with the desired values (except for the first column), and the loss for the second-class requests ranges from 15 to 39% (significantly higher than the allowed level), since the capacity of unit does not allow the processing of the requests of the first two classes with the desired quality.

Parameter R reaches maximum in the fourth column similarly to parameter m in Table 1. Note that parameter R for the first time becomes greater than parameter R_0^*, which takes into account the requests of the first two classes. Parameter P, which takes into account the processing cost of each request, is always less than parameter P_0^*. Thus, we conclude that, for $\rho > 1$, the presence of the third-class requests leads to a decrease in parameter P, which characterizes the network income from the processing of all requests. Hence, such working intervals must be relatively short and the corresponding lost incomes must be compensated for by intervals over which the unit is not overloaded.

Columns 5–7 in Table 5 show the calculated results for the WBN unit that is strongly overloaded by the total request flow. However, the request flows of the first two classes can be processed with the desired quality. In this case we use the set of hysteresis positions that provide the desired loss parameters for the requests of the first two service classes to select the positions that allow the minimum loss for the third-class requests. The results from the corresponding columns in Table 1 show that the loss for the third-class requests ranges from 90 to 95% and the losses for the first- and second-class requests are in agreement with the desired levels (except for column 7, where the loss for the second-class request is almost 8%). The maximum parameters R and P are observed in column 7 due to greater access for the third-class requests. Note that parameter R is greater than parameter R_0^* and

Table 5. Results of calculations at $a_1 = 2$, $a_2 = 3$, $a_3 = 1$, $c_1 = 5$, $c_2 = 3$, $c_3 = 1$, $H_1 = 0.9$, $H_2 = H_3 = 0.8$, and $v = 100$

0	1	2	3	4	5	6	7	8	9	10
λ_1	30	30	30	30	20	20	20	20	20	20
λ_2	10	10	10	10	10	10	10	10	10	10
λ_3	30	30	30	30	30	30	30	10	10	10
ρ	1.2	1.2	1.2	1.2	1.0	1.0	1.0	0.8	0.8	0.8
ρ_{12}	0.9	0.9	0.9	0.9	0.7	0.7	0.7	0.7	0.7	0.7
k_2	96	96	93	93	93	96	96	96	96	96
f_2	93	93	90	90	90	93	93	93	93	93
k_3	70	70	80	89	80	80	90	90	90	95
f_3	40	60	75	85	75	75	85	85	88	88
R	237.0	235.5	237.4	242.6	187.4	187.5	188.4	188.0	189.0	188.6
P	218.6	216.6	219.3	227.4	172.5	172.5	174.2	183.6	184.9	184.6
R_0	270.0	270.0	270.0	270.0	210.0	210.0	210.0	190.0	190.0	190.0
R_0^*	240.0	240.0	240.0	240.0	180.0	180.0	180.0	180.0	180.0	180.0
P_0	263.0	263.0	263.0	263.0	204.0	204.0	204.0	186.0	186.0	186.0
P_0^*	236.0	236.0	236.0	236.0	177.0	177.0	177.0	177.0	177.0	177.0

parameter P is close to parameter P_0^* in all of the columns.

Also note that cost parameters weakly depend on loss levels: the parameters in column 7 are greater than parameters in column 6 but, for the data from column 6, the loss for the second-class requests is less than the loss in column 7 by a factor of greater than 10 and is in agreement with the desired loss level.

Columns 8–10 in Table 5 demonstrate the calculated results for a significant decrease in the flow rate for the third service class. The results from columns 8 and 9 in Table 1 and column 2 in Table 3 show that the loss for the third-class requests ranges from 5 to 37%, the losses for the first- and second-class requests are in agreement with the desired levels, and the loss for the third-class requests corresponding to column 9 is greater than the loss corresponding to column 10 by a factor of 3 although the cost parameters are slightly higher. Parameters R and P are always greater than parameters R_0^* and P_0^*, respectively. Note that parameters R and P are close to cost parameters R_0 and P_0, respectively, since almost all of the requests are processed. For such input flows, the presence of the third-class requests allows more effective operation and provides additional income related to the processing of such requests.

The difference between the scenarios corresponding to columns 8–10 and 5–7 in Table 5 lies only in a decrease in the flow rate of the third-class requests. This circumstance naturally causes an increase in parameter P, since the total flow decreases, and an increase in parameter R owing to a decrease in losses for requests of the first two service classes.

Based on the above results, we draw the following conclusions.

(i) Cost functionals are weakly sensitive to loss probabilities especially of priority requests.

(ii) For the overloaded network ($\rho > 1$), the presence of the third-class requests leads to a decrease in the cost parameters, especially, parameter P, which shows the network income from the processing of all requests and decreases due to an increase in the flow rate of the repeated third-class requests.

(iii) The working efficiency of the strongly loaded network ($\rho > 0.8$) increases due to the presence of the third-class requests but functional P is sensitive to hysteresis limits for the third-class requests.

(iv) In the weakly loaded network ($\rho \leq 0.7$), the working efficiency increases when the processing of the requests of the first two classes is supplemented with the processing of the third-class requests.

7. NUMERICAL STUDY OF THE PROPERTIES OF MATHEMATICAL MODEL OF WBN UNIT WITH ALLOWANCE FOR DIURNAL OSCILLATIONS OF LOAD

The results from the previous section show that the presence of the third-class requests does not necessarily lead to an increase in the network income (i.e., provides economic efficiency). On the other hand the diurnal load pattern normally exhibits intervals with relatively low load and the presence of the third-class

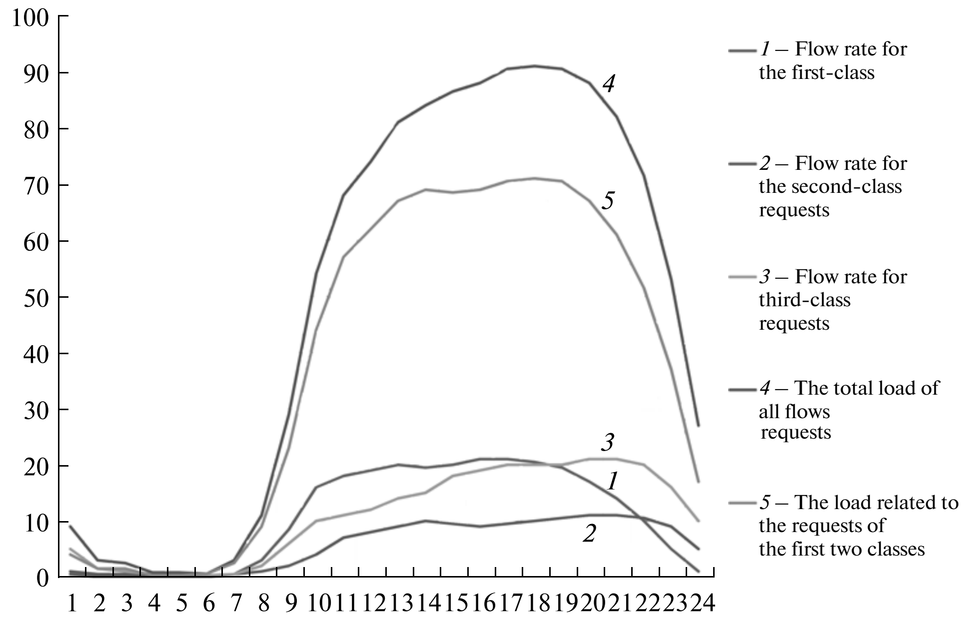

Fig. 1. Diurnal load patterns for low partial load related to the third-class requests.

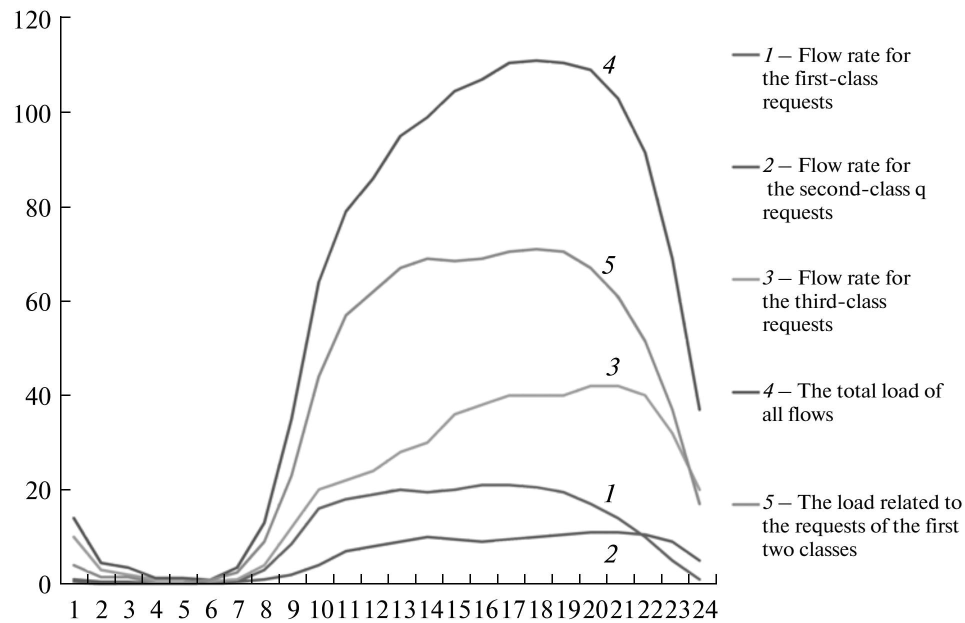

Fig. 2. Diurnal load patterns for high partial load related to the third-class requests.

requests leads to additional income. Therefore, it is expedient to analyze the effect of the diurnal load oscillations on the processing efficiency of the third-class requests. In this section, we present the corresponding results.

We consider two variants of the load, which differ by the partial load related to the third-class requests. The load due to the requests of the first two classes is chosen in such a way that the network is able to process all of the requests with the desired quality during busy hours: the loss probabilities for the first- and second-class requests are no greater than 0.5 and 5%, respectively. The loss probability is not specified for the third-class requests. In the first case, the maximum load of network with respect to the requests of all classes is no greater than 0.9 Erlang per BTU during busy hours (Fig. 1). In the second case, we consider a two-times greater partial load related to the third-class requests and a total load of greater than 1 Erlang per

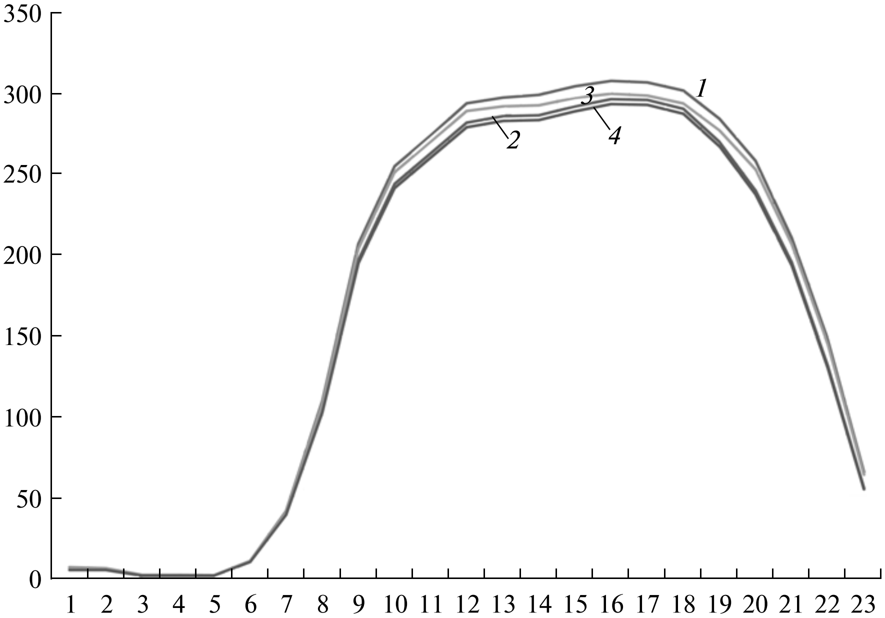

Fig. 3. Plots of the cost functionals at low partial load related to the third-class requests.

BTU during busy hours (Fig. 2). In the first and second cases, we consider low and high partial loads related to the third-class requests, respectively.

Figure 1 demonstrates the diurnal load pattern. We assume that the load related to the first-class requests is almost constant during daytime (curve *1* in Fig. 1). For the second service class, we consider the evening busy hour and a relatively high flow rate during daytime (curves *2* in Figs. 1 and 2). For the third-class requests, we assume the evening busy hour that is shifted to later hours relative to the busy hour of the second service class and a slower increase in the load during daytime (curves *3* in Figs 1 and 2). As in the previous calculations, we assume that different numbers of BTUs are used for the requests of different classes: the first-, second-, and third-class requests employ two, three, and one BTUs, respectively. Figures 1 and 2 demonstrate the total loads related to all of the requests with allowance for BTU usage (curves *4*) and the requests of the first two service classes with allowance for the corresponding BTU usage (curves *5*).

Thus, a relatively high load is generated by the queries of the first two classes over the time interval from 10 a.m. to 9 p.m. Note that the network resources are sufficient for the processing of the mean load related to these classes.

The total load due to all of the requests is peaked at the interval from 4 p.m. to 8 p.m., when the load level does not allow the acceptable processing quality for all of the requests and the limitations on the access to network must be employed. This time interval also corresponds to a relatively high partial load related to the third-class requests, so that a decrease in the activity of the sources of such load must be stimulated.

When the load due to the third-class requests is two times greater (Fig. 2), the system is strongly overloaded at the interval from noon to 10 p.m. The results of the calculations in the previous sections show that the processing of the third-class requests is economically expedient when the persistency upon rejection of signal during busy hours decreases (i.e., parameter p_a decreases) and the activity of such users decreases. The hysteresis limits for the third-class requests are chosen in such a way that the requests are not processed at the above time interval and we assume that the users employ alternative networks.

To take into account such a behavior of users, we consider two types of persistence of the third-class users and use the values 0.8 and 0.4. The second value corresponds to the scenario in which the user abandons the network after the second rejection.

When the hysteresis interval for the third-class queries is relatively narrow, such requests can be occasionally processed at the intervals with high load and the repeated requests can be stimulated. In particular, we study the effect of the third-class user persistence on the cost parameters of the network operation.

In the calculations, we always use the following parameters: $a_1 = 2$, $a_2 = 3$, $a_3 = 1$, $c_1 = 5$, $c_2 = 3$, $c_3 = 1$, $H_1 = 0.9$, $H_2 = 0.8$, and $v = 100$. The hysteresis limits are fixed during the whole day: $k_2 = 95$, $f_2 = 90$, $k_3 = 93$, and $f_3 = 70$. Figures 3–8 demonstrate the plots of cost functionals R, R_0^*, P, and P_0^* (curves *1–4*, respectively).

Figure 3 shows the curves of cost functional at low partial load related to the third-class requests and persistence $H_3 = 0.8$. In this case, the total values of

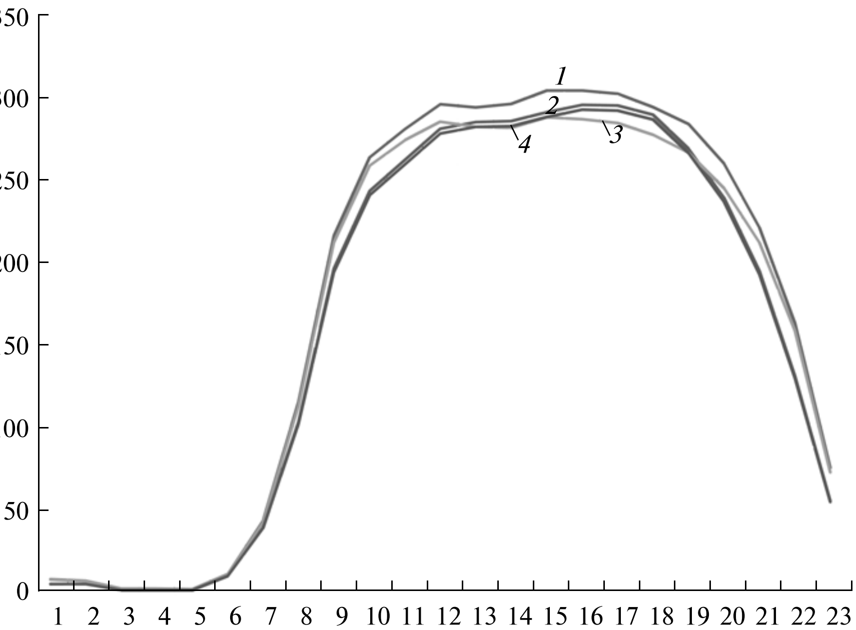

Fig. 4. Plots of the cost functionals at high partial load related to the third-class requests.

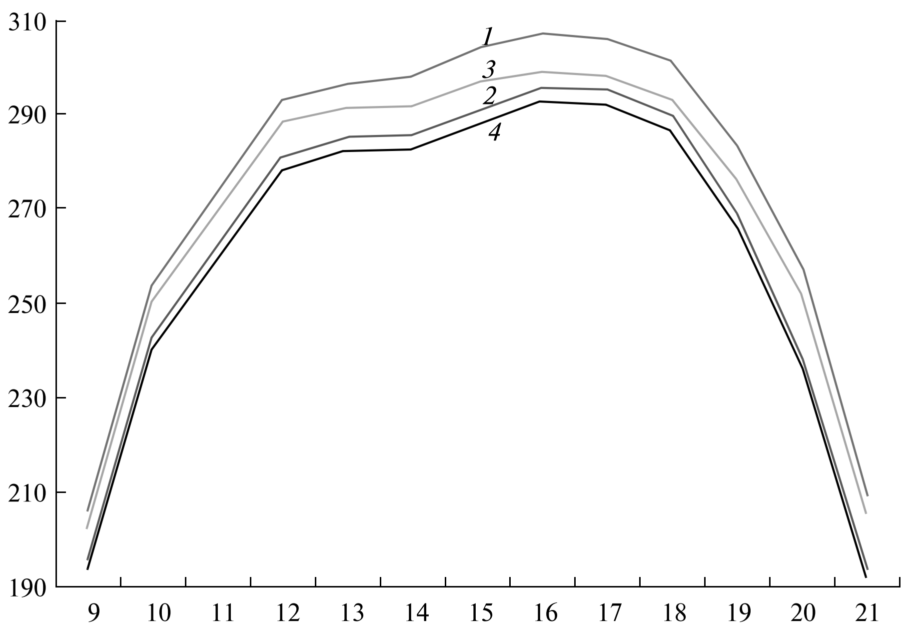

Fig. 5. Plots of the cost functionals at low partial load related to the third-class requests and $H_3 = 0.8$ for daytime and evening.

parameters R and P per day are 3922.8 and 3712.7, respectively. The level of load due to the first- and second-class requests is such that the requests of the first two classes can be processed with zero losses in the absence of the third-class requests. Thus, the total values of parameters R_0^* and P_0^* per day are 3840.34 and 3674.27, respectively. An additional increase in the rendered service owing to the processing of the third-class requests is 5.7%, and an additional increase in income is 4.5%. The second parameter is smaller due to a relatively large number of the rejected third-class requests at the interval with a high total load.

Thus, the application of parameter R (P) upon an increase in the flow rate of the third-class requests causes an increase (decrease) in this parameter. Hence, we must appropriately choose the efficiency parameter for the access control of network resources.

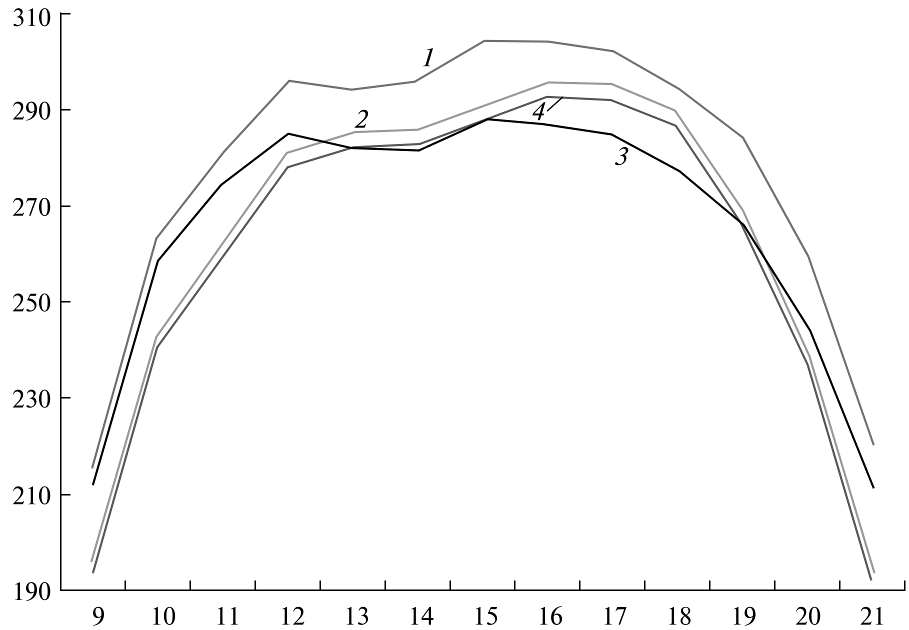

Fig. 6. Plots of the cost functionals at high partial load related to the third-class requests and $H_3 = 0.8$ for daytime and evening.

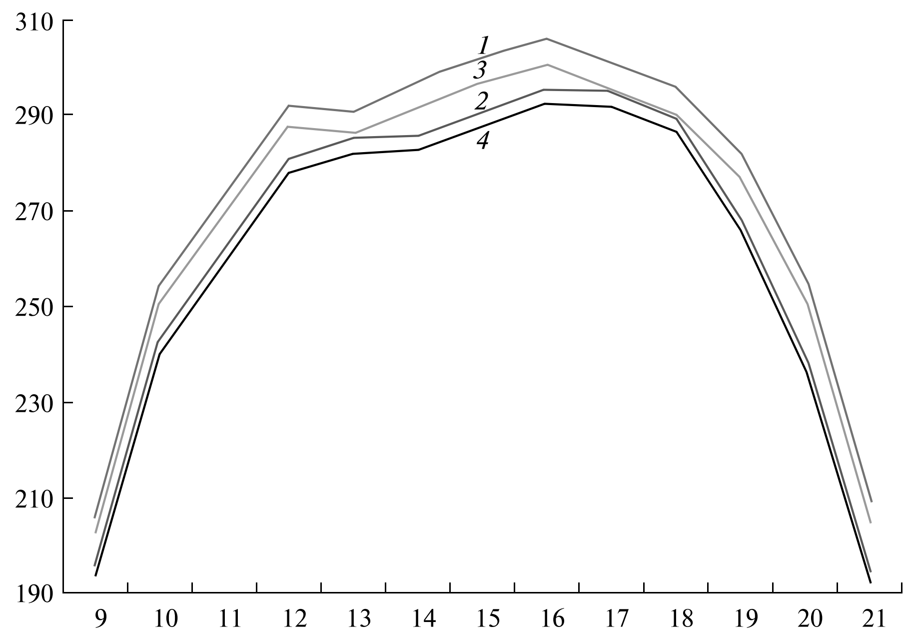

Fig. 7. Plots of the cost functionals at low partial load related to the third-class requests and $H_3 = 0.4$ for daytime and evening.

Figure 4 demonstrates the cost functionals for a high partial load related to the third-class requests and persistence $H_3 = 0.8$. In this case the total daily parameters R and P are 4052.8 and 3789.2, respectively. Evidently, parameters R_0^* and P_0^* remain unchanged in comparison with the previous case. An additional increase in the rendered service due to the processing of the third-class requests is 7.0%, and an additional increase in income is 3.5%. The first parameter increases owing to an increase in the load at the time interval when the total load is relatively low, and the second parameter decreases due to an increase in the number of rejected third-class requests that arrive at the time interval with a relatively high total load and an increase in the duration of this interval.

Obviously, the most interesting processes take place at the time interval when the total load of unit is

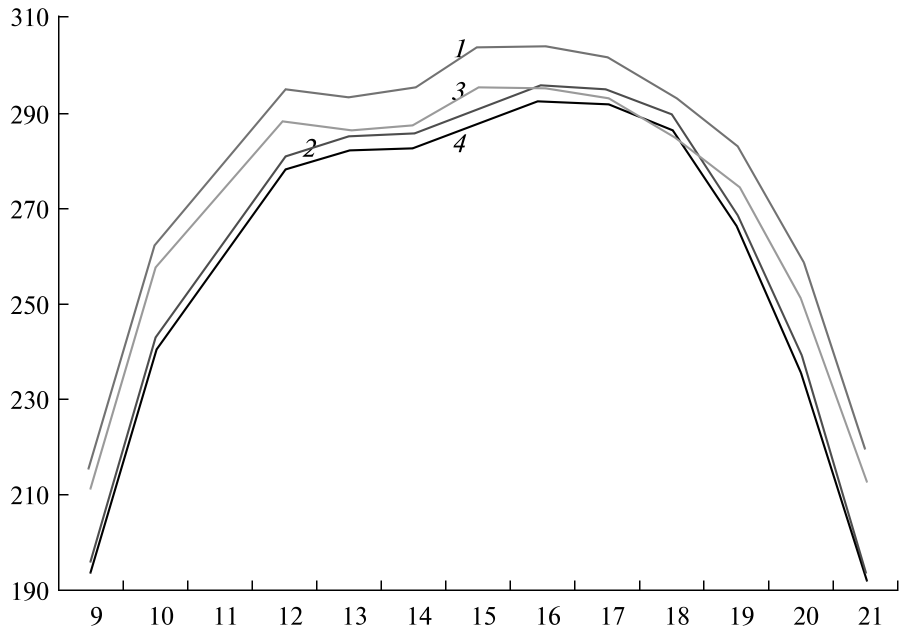

Fig. 8. Plots of the cost functionals at high partial load related to the third-class requests and $H_3 = 0.4$ for daytime and evening.

relatively high. Thus, we consider the time interval from 10 a.m. to 9 p.m.

Figure 5 shows the plots at the low partial load related to the third-class requests and $H_3 = 0.8$. It is seen that the values of cost functional R are always greater than R_0^* and the values of P are greater than P_0^* (i.e., the processing of the third-class requests is always expedient).

Figure 6 demonstrates the plots for the high partial load related to the third-class requests and $H_3 = 0.8$.

It is seen that the values of cost functional R are always greater than R_0^* and the values of P are less than P_0^* at the time interval from noon to 6 p.m. (i.e., the processing of the third-class requests is unprofitable). However, the total daily profit is 3.5%.

Below, we present the calculated results for the same parameters of model and the persistence of the third-class requests that is two times smaller.

Figure 7 shows the plots at the low partial load related to the third-class requests and $H_3 = 0.4$. It is seen that, similarly to the plots in Fig. 5, the values of cost functional R are always greater than R_0^* and the values of P are greater than P_0^*. Note that the R and P curves are closer to each other, since the number of the repeated third-class requests is significantly smaller. An additional increase in the rendered service due to the processing of the third-class requests is 5.1%, and an additional increase in income is 4.4%.

Figure 8 demonstrates the plots at the high partial load related to the third-class requests. It is seen that, similarly to the plots in Fig. 5, the values of cost functional R are always greater than R_0^* and the values of P are greater than P_0^* in contrast to the plots in Fig. 6. An additional increase in the rendered service due to the processing of the third-class requests is 6.7%, and an additional increase in income is 5.0%. The comparison of the results obtained for the persistences $H_3 = 0.4$ and 0.8 shows that an increase in the flow rate of the third-class requests leads to an increase in both functionals R and P.

Thus, we conclude that a decrease in the persistence probability for the users who generate the third-class requests allows an increase in the network efficiency. With respect to the diurnal oscillations of load, the most effective operation involves the processing of the third-class requests that leads to the switching to alternative networks at the intervals with a relatively high load related to the requests of the first two service classes.

CONCLUSIONS

Based on the results of the numerical analysis, we draw the following conclusions.

(i) When the load of unit is greater than or close to the unit capacity, it is expedient to employ the hysteresises for the second- and third-class requests that are substantially spaced apart.

(ii) The hysteresis for the second-class requests must be sufficiently narrow when the total load related to the first- and second-class requests is no greater than 0.7 Erlang per BTU.

(iii) The hysteresis for the processing of the third-class requests must be sufficiently broad to provide the maintenance of the access mode in the presence of the load oscillations in the vicinity of the peak values.

(iv) If the total load of all flows is no greater than 0.8 Erlang per BTU, it is expedient that the upper-bound limit of the hysteresis for the third-class requests is higher than the upper-bound limit of the hysteresis for the second-class requests when the partial load related to the third-class requests is relatively high.

(v) If the total load of all flows is no greater than 0.8 Erlang per BTU and the partial load related to the third-class requests is relatively low, it is expedient to employ overlapped hysteresises such that the hysteresis for the third-class requests is sufficiently broad to maintain the access mode in the presence of the load oscillations in the vicinity of the peak values.

(vi) The cost functionals are weakly sensitive to the loss probability of requests (especially, the loss probability of priority requests). Therefore, the cost functionals must be optimized with allowance for the limitations on the processing of the first- and second-class requests.

(vii) When the network is overloaded ($\rho > 1$), the presence of the third-class requests leads to a decrease in the cost parameters, especially, parameter P, which shows the network income from the processing of queries of all service classes.

(viii) In the heavily loaded network ($\rho > 0.8$), the network efficiency increases owing to the processing of the third-class requests but functional P is sensitive to the hysteresis limits of the third-class requests.

(ix) In a weakly loaded network ($\rho < 0.7$), the network efficiency increases when the processing of the first- and second-class requests is supplemented with the processing of the third-class requests.

(x) A decrease in the persistence of users who generate the third-class requests allows an increase in the network efficiency.

(xi) With respect to the diurnal load oscillations, it is expedient to process the third-class requests in such a way that the flow rate of the third-class requests decreases due to the switching to alternative networks at the intervals with a relatively high total load related to the requests of the first two service classes.

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