Kinetic temperature of dust particle motion in gas-discharge plasma

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A system of equations describing motion of dust particles in gas discharge plasma is formulated. This system is developed for a monolayer of dust particles with an account of dust particle charge fluctuations and features of the discharge near-electrode layer. Molecular dynamics simulation of the dust particles system is performed. A mechanism of dust particle average kinetic energy increase is suggested on the basis of theoretical analysis of the simulation results. It is shown that heating of dust particles' vertical motion is initiated by forced oscillations caused by the dust particles' charge fluctuations. The process of energy transfer from vertical to horizontal motion is based on the phenomenon of the parametric resonance. The combination of parametric and forced resonances explains the abnormally high values of the dust particles' kinetic energy. Estimates of frequency, amplitude, and kinetic energy of dust particles are close to the experimental values.

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I. INTRODUCTION

The phenomenon of warming of dust particle motion in gas discharge plasma to an abnormally large kinetic energy is of great interest [1–3]. Dust particles under certain conditions can acquire kinetic energy of the order of 10 eV and higher, far above the temperature of gas and temperatures of ions and electrons in the discharge. The evidence of this phenomenon is found in laboratory experiments [4–14]. Despite such kinetic energy, dust particles can form a crystalline structure [7–9,15].

The anomalous warming of dust particle motion in plasma has been known since 1996. There have been several attempts to find a mechanism of the phenomenon [10–13,16–27]. For example, the influence of the electric field fluctuations [10,11,16–18] leads to the heating of the dust particle motion to the kinetic temperature above the ambient gas, but two or more orders of magnitude smaller than the experimental values. However, the absence of correlation between the kinetic temperature of dust particles and electric field fluctuations has been shown in experiment [11]. The impact of dust particles' charge fluctuations on the kinetic temperature is examined in Refs. [10,19–24], but the evaluation of the kinetic temperature appears to be lower than the experimental values. Mechanisms, based on the finite time of particle charging [12,13,24] and the spatial variation of dust particles' charge [24,25], give the estimates of the kinetic temperature by several orders of magnitude below the experimental value. Influence of ion wake [26] and mode-coupling instability [27] can also heat dust particle motion, but there are no estimates of kinetic temperature due to these effects. Thus, the cause of the heating of dust particle motion to the abnormally high kinetic energy remains unclear. The main difficulties of solving this problem are the impossibility of analyzing the impact of various stochastic and nonlinear effects on the system of dust particles using analytical methods. The authors of the previous studies have considered only individual factors rather than their aggregate.

In this paper we take into account most of the factors, including nonlinear and stochastic, by using the molecular dynamics method [17,22,28–31]. Varying the parameters of the equations of dust particle motion allows us to avoid the problem of the lack of exact values of some parameters of the plasma-dust system. In addition the variation of the parameters allows us to find the dependence of the average kinetic energy and other characteristics of the system on its parameters. The features of gas discharge near-electrode layer are taken from papers [25,27,32–36]. The effects of vibrational motion of dust particles and potential instabilities are accounted for by means of the theory of forced oscillation and parametric resonance [37,38]. We study the process of energy transfer from discharge to dust particle motion and mechanism of the anomalous warming of dust particle motion using the considered approach.

Forces and phenomena that determine the motion of dust particles are introduced in Sec. II, where a system of equations describing the motion of dust particles in near-electrode layer in gas discharge is formulated as well. Dependence of system parameters on position and time is discussed in Sec. III. The initial conditions and results of numerical simulation of dust particles system are presented in Sec. IV. The validity of using the term "temperature" for the dust particle average kinetic energy is discussed in Sec. V. Analysis of acquired results and heating mechanism of vertical and horizontal motion of the particles are presented consequently in Secs. VI and VII. The scheme of energy transfer is discussed in Sec. VIII. Comparison with experimental data is carried out in Sec. IX.

II. EQUATIONS OF DUST PARTICLE MOTION

The three-dimensional motion of N dust particles, forming a horizontal layer in the gas discharge near-electrode layer is considered. The dust particle is a ball with mass m. Dust particles in gas-discharge plasma acquire a significant negative charge $Q = Ze$, where Z is a charge number and e is electron charge. Dust hangs in the near-electrode layer at an altitude of z, where electric field $\mathbf{E}(z)$ is strong enough to compensate for gravity. Magnitude of the gravitational force is given by $\mathbf{F}^{\text{grav}} = m\mathbf{g}$, where g is acceleration of free fall. The starting

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point of z is the point where gravity is balanced by the electric force $\mathbf{F}^{\text{el}} = Q\mathbf{E}(z)$:

$$QE(0) = mg. \tag{1}$$

The force of friction acts on the moving dust particle from the ambient gas. The main contribution to the friction force is given by the neutral particles in the case of weakly ionized plasma. Thus, gas effects on dust particles can be simulated by the Langevin thermostat in the case of low pressure. Then the force acting on a particle from the neutral gas can be written as

$$\mathbf{F}_i^{\text{fr}} = -2m\gamma\dot{\mathbf{r}}_i + \sqrt{2m\gamma k_B T/dt}\,\mathbf{h}(t), \tag{2}$$

where γ is a friction coefficient, k_B is the Boltzmann constant, $\mathbf{r}_i = \{x,y,z\}$ is a three-dimensional position vector of the ith dust particle, $\dot{\mathbf{r}}_i$ is a dust particle velocity, T_n is the temperature of the neutral gas, dt is integration step, and $h(t)$ is a normally distributed random variable.

The dust particles interaction potential is chosen in the form of the screened Coulomb potential:

$$U_{ij}(\mathbf{r}_i - \mathbf{r}_j) = Q_i Q_j e^{-\kappa|\mathbf{r}_i - \mathbf{r}_j|}/|\mathbf{r}_i - \mathbf{r}_j|, \tag{3}$$

where κ is a screening parameter. The force of the interaction of charged dust particles is $\mathbf{F}_i^{\text{inter}}(\mathbf{r}_i - \mathbf{r}_j) = -\sum_j \frac{\partial U_{ij}(\mathbf{r}_i - \mathbf{r}_j)}{\partial \mathbf{r}_i}$. The trap potential that holds charged dust particles from spreading horizontally is a parabolic $U_{\text{trap}} = \varepsilon(x^2 + y^2)$, where ε is a trap-potential parameter.

Dust particles of size from 1 to 50 μm are considered in this work. These grains form a horizontal layer and may gain high average kinetic energy. The discharge is maintained at room temperature and low pressures. The viscous friction force prevails over the force of the ion drive and the thermophoretic force [33] in this formulation of the problem, so the influence of these forces on the motion of dust particles is neglected.

Thus, the motion of dust particles in gas-discharge plasma is described by the equation

$$m\ddot{\mathbf{r}}_i = \mathbf{F}_i^{\text{inter}} + \mathbf{F}_i^{\text{trap}} + \mathbf{F}_i^{\text{fr}} + \mathbf{F}_i^{\text{grav}} + \mathbf{F}_i^{\text{el}}. \tag{4}$$

III. PARAMETERS OF THE FORCES ACTING ON DUST PARTICLES

A. Electric field and particle charge

The electric field E is strongly dependent on the vertical coordinate z in the near-electrode layer of gas discharge. The electric field depends linearly on the vertical coordinate [17,25] for the gas pressure considered:

$$E(z) = E_0(1 + e'z), \tag{5}$$

where e' is a normalized vertical gradient electric field component in the near-electrode layer.

Dust particle charge is not a fixed value; it is determined by local plasma parameters near the particle. When the local plasma parameters are perturbed, the charge also experiences the disturbance. Near-electrode layer electron density and the concentration of ions vary considerably in height. It leads to the dependence of the time-averaged particle charge on the height [17,25]:

$$Q_{\text{eq}}(z) = Q_0(1 + q_z' z + \cdots), \tag{6}$$

where Q_0 is an average charge of a dust particle and q_z' is a normalized vertical gradient of the dust particle charge, caused by changes in the concentrations of electrons and ions in the electrode layer. Quadratic and other terms of the expansions can be neglected due to the fact that relative change in charge due to the vertical displacement is not large for a monolayer of dust particles.

Dust particle charge fluctuates due to fluctuations in the local plasma parameters near the particle. Moreover, a dust particle charge undergoes random fluctuations around its equilibrium value even in an isotropic spatially uniform unperturbed plasma. This takes place due to the fact that the ions and electrons are absorbed (emitted) by the surface of dust grains in random times and in random order. Thus, the charge of dust particles in plasma is determined by the flow of plasma components on the surface of dust particle, so its charge fluctuates in time [22] because of fluctuations and discreteness of ions and electrons flow on the surface of dust particles:

$$Q = Q_0[1 + \delta q(t) + \cdots)], \tag{7}$$

where the normalized charge fluctuations $\delta q(t)$ is described by the correlation

$$\langle \delta q(t)\delta q(t')\rangle = \delta q^2 \exp[-\Omega(t - t')],$$

where Ω^{-1} is the characteristic relaxation time of the small deviations of the charge. Normalized amplitude of charge fluctuations δq for a standard laboratory experiment is estimated as $\delta q \approx 0.5/\sqrt{(Q/e)}$ [2,27,32,34].

Dust particles in discharge plasma gain large charge and begin to affect the parameters of the surrounding plasma changing the concentrations of electrons and ions. For that reason, charges of two neighboring dust particles depend on the distance between them:

$$Q = Q_0[1 + q_r'(|r_i - r_j| - \langle |r_i - r_j|\rangle) + \cdots], \tag{8}$$

where q_r' is the normalized gradient of dust particle charge, caused by changes in the concentrations of electrons and ions around a charged dust particle.

Note that the average charge without regard to the latter effect is assumed to be the same for all of the dust particles. Such an assumption is valid as we consider a horizontal layer of dust particles. The effect (8) makes a small difference between the charges of different particles. Combining (6)–(8) results in the final expression for dust particle charge:

$$Q(\mathbf{r},t) = Q_0[1 + q_z' z + q_r'(|r_i - r_j| - \langle |r_i - r_j|\rangle) + \delta q(t)]. \tag{9}$$

B. Dependence of forces on position and time

The electric field (5) determines only the electric force. Dust particle charge is included in most of the forces that determine the motion of dust particles. Below all the forces that are modified by a modified charge are given. The dependence of all forces on any spatial and temporal coordinates is also determined. Electrical power $F^{\rm el}$ is represented as

$$F^{\rm el} = F^{\rm el}(\mathbf{r},t) = Q(\mathbf{r},t)\cdot E(z) = Q_0E_0[1+q'z + q'_r(|r_i-r_j| - \langle|r_i-r_j|\rangle) + \delta q(t)](1+e'z). \quad (10)$$

The force of horizontal trap is expressed as

$$F_x^{\rm trap} = F_x^{\rm trap}(\mathbf{r},t) = -2\varepsilon Q(z,t)x = -2\varepsilon Q_0 \cdot [1+q'_z z + q'_r(|r_i-r_j| - \langle|r_i-r_j|\rangle) + \delta q(t)]x. \quad (11)$$

The interaction of dust particles is described by the screened Coulomb potential:

$$U_{ij}(\mathbf{r}_i-\mathbf{r}_j,t) = Q_i(\mathbf{r}_i,t)Q_j(\mathbf{r}_j,t)\frac{e^{-\kappa|\mathbf{r}_i-\mathbf{r}_j|}}{|\mathbf{r}_i-\mathbf{r}_j|}, \quad (12)$$

$$F^{\rm inter} = F^{\rm inter}(\mathbf{r},t) = -dU_{ij}(\mathbf{r}_i-\mathbf{r}_j,t)/dr. \quad (13)$$

C. Final system of equations

Thus, dust particle motion is described by the system of equations:

$$m\ddot{\mathbf{r}}_i = \sum \mathbf{F}_i^{\rm inter}(\mathbf{r},t) + \mathbf{F}_i^{\rm trap}(\mathbf{r},t) + \mathbf{F}_i^{\rm fr}(\dot{\mathbf{r}}_i,t) + \mathbf{F}_i^{\rm grav} + \mathbf{F}_i^{\rm el}(\mathbf{r}_i,t). \quad (14)$$

This system differs from those examined in other studies by the presence of a number of dependencies: (a) charge dependence on time, (b) charge dependence on the distance from the electrode, (c) charge dependence on the distance to the other charged dust particles, and (d) dependence of the electric fields on the distance from the electrode. These factors lead to the appearance of several different independent fluctuating processes and nonlinear effects. The joint consideration of these factors, the fluctuating processes and nonlinear effects, distinguishes this work from others.

IV. SIMULATION OF DUST PARTICLE MOTION

The system of Eq. (14) describes the motion of dust particles in a monolayer in the gas discharge near-electrode layer. The presence of various stochastic and nonlinear terms does not allow analytical solution of this system of equations. The only simulation is able to take into account the combined influence of all these members.

A. Model

Dust particles are considered as material points with charge and mass. Effect of gas-discharge plasma on a dust particle is simulated by a stochastic term of dust particle charge. The influence of the gas discharge near-electrode layer is simulated by dependence of the electric field and dust particles' charge on the distance from the electrode. The influence of other charged dust particles on ambient plasma and, consequently, on dust particle charge is simulated by the dependence of dust particle charge on the distance to other dust particles.

Dust particle motion in a gas discharge near-electrode layer is simulated by the numerical integration of Eq. (14), just as is done by molecular dynamics. A Verlet scheme is used for numerical integration. Integration step dt is chosen so that it is at least one order of magnitude smaller than the characteristic time of the fastest process in the system simulated. The fastest processes in this model of plasma-dust systems are perturbation and relaxation of small fluctuations of the charge toward its equilibrium value. The characteristic time for this process is above $1\mu s$. The characteristics of the dust particles system do not depend on the integration step if we use this value.

The total time of simulation is chosen so that the system could reach a steady state. The second reason, which leads to large simulation time, is the requirement for a large enough set of statistics for computed characteristics. Sufficient statistics is accumulated through the decomposition of the trajectory into segments and averaging the calculated characteristics on these segments. This approach is justified because of the fact that dust particle motion is randomized along the trajectory segment because of the presence of stochastic terms in the equations for dust particle motion.

The third reason is the duration of laboratory experiment that can be hundreds of seconds. All three reasons lead to the fact that simulation of plasma-dust system on a steady trajectory is more than 10^9 steps. At every step all the forces affecting the dust particles are calculated.

The system of Eq. (14) contains 12 parameters: the number of particles N, dust particle mass m, gravitational acceleration g, coefficient of friction of dust particles on neutral gas γ, dust particle charge Q, normalized gradients of dust particle charge in vertical and radial directions q'_z and q'_r, normalized amplitude of charge fluctuations δq, normalized electric field gradient of near-electrode layer e'_z, inverse relaxation time of small deviations of charge Ω, screening parameter κ, and parameter of trap potential ε.

The values of some of the above-mentioned parameters are known with uncertainty. Therefore, the system of dust particles is simulated for the three core data sets (Table I). The parameter values in the underlying data sets correspond to the parameters of standard laboratory dusty plasma experiments [1]. Variation of system parameters in turn allows us to study the dependence of various characteristics on the system parameters. The system parameters are varied in a large range, which exceeds physical values, for example, for the amplitude of charge fluctuations. It is done to determine the functional dependencies of the characteristics on the system parameters.

The characteristics that are calculated on the stationary part of the trajectory are (a) average kinetic energy of dust particle motion K, (b) kinetic temperature of dust particle motion in horizontal K_h and vertical K_v directions, (c) kinetic temperature of the center of mass of dust particle system $K_{\rm cm}$, (d) average interparticle distance $\langle\Delta r_{i,j}\rangle$, (e) coupling parameter $\Gamma = U/K$, where U is potential energy of the system, (f) vibrational spectrum of the dust particle motion, and (g) vibrational spectrum of the center of mass of the dust particle system. The kinetic temperature K is calculated from the expression $\langle mv^2/2\rangle = k_BKf/2$, where $\langle mv^2/2\rangle$ is average kinetic energy, k_B is the Boltzmann constant, f is the number of degrees of freedom.

B. Vertical temperature

Selection of parameters of Eq. (14) and the subsequent numerical integration allow us to determine the effect of these

TABLE I. The parameters for plasma-dust system simulation.

Parameters	Set 1	Set 2	Set 3
Number of particles N	7	7	9
Integration step dt, s	10^{-6}	10^{-6}	10^{-6}
Particle mass m, 10^{-4} g	10.0	8.0	7.0
Gravity g, cm/s^2	980	980	980
Friction coefficient γ, s^{-1}	1.0	3.0	2.0
Particle charge Q, e	1×10^4	2×10^3	1×10^3
Vertical charge gradient q_z', cm^{-1}	0.10	0.1	0.15
Radial charge gradient q_r', cm^{-1}	−0.1	−0.1	−0.15
Charge fluctuation amplitude δq	0.005	0.011	0.016
Electric field gradient e_z', cm^{-1}	−40.0	−10.0	−20.0
Recharging frequency Ω, s^{-1}	4×10^4	6×10^4	5×10^4
Screening parameter κ, cm^{-1}	30.0	3.0	40.0
Trap parameter ε, CGS units	0.08	0.02	0.03

parameters on system characteristics. The main characteristics to be studied are the coupling parameter, horizontal kinetic temperature, vertical kinetic temperature, kinetic temperature of the center of mass, and average interparticle distance.

The simulation results allow us to divide parameters into two groups: those that affect characteristic under consideration, and those that do not. Let us first consider the effect of system parameters on the kinetic temperature of the vertical motion of dust particles.

The determinants of the average kinetic energy of vertical motion (set 3 is given as an example):

(1) The normalized amplitude of charge fluctuations δq (Fig. 1).

(2) The friction coefficient of a dust particle on the surrounding gas γ (Fig. 2).

(3) The characteristic relaxation frequency of small charge deviations Ω (Fig. 3).

(4) Dust particle mass m (Fig. 4).

(5) Gravity g (Fig. 5).

The factors that do not affect the average kinetic energy of vertical motion (figures are given only for two important factors)

(6) The normalized electric field gradient e_z'.

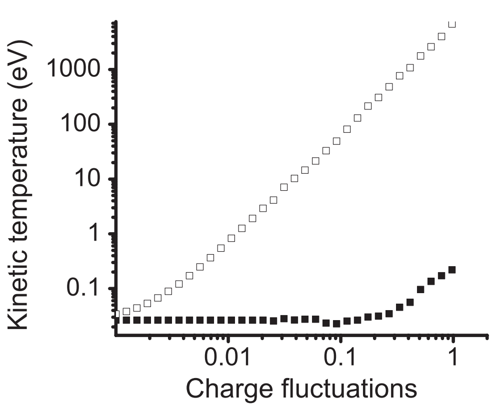

FIG. 1. Dependence of temperatures on the normalized amplitude of charge fluctuations for set 3. Blank squares are the kinetic temperature of the vertical motion of dust particles; filled squares are the horizontal kinetic temperature of dust particles.

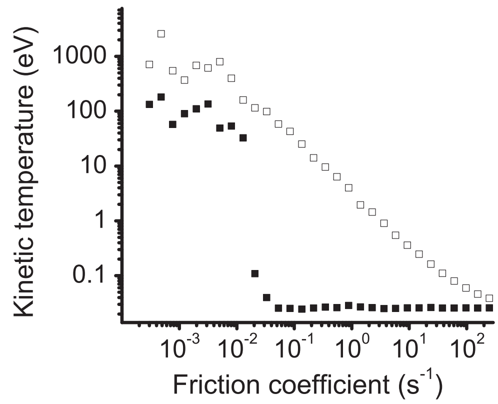

FIG. 2. Dependence of temperatures on the friction coefficient for set 3. Blank squares are the kinetic temperature of the vertical motion of dust particles; filled squares are the horizontal kinetic temperature of dust particles.

(7) Trap-potential parameter ε.

(8) Dust particle charge Q (Fig. 6).

(9) Number of dust particles N (Fig. 7).

(10) The normalized charge gradient in the vertical direction q_z'.

(11) The normalized charge gradient due to the distance between particles q_r'.

(12) The screening parameter κ.

The dependencies of the vertical kinetic temperature on the system parameters can be combined into the formula

$$K_v \approx T_{\text{room}} + (1.0 \pm 0.1)a(N)m(g\delta q)^2/(\gamma\Omega), \qquad (15)$$

where T_{room} is the temperature of neutral gas, which is usually 300 K, and $a(N)$ is a function of number of dust particles. Coefficient $a(N)$ is equal to unity for a large number of particles but may differ from unity for the case of small number of particles. All parameters are in CGS units. Some graphs of kinetic temperature are translated in units of eV for clarity. The dependence of the vertical kinetic temperature on the system parameters can help to determine the mechanism of its increase. Note that this formula with good accuracy is the

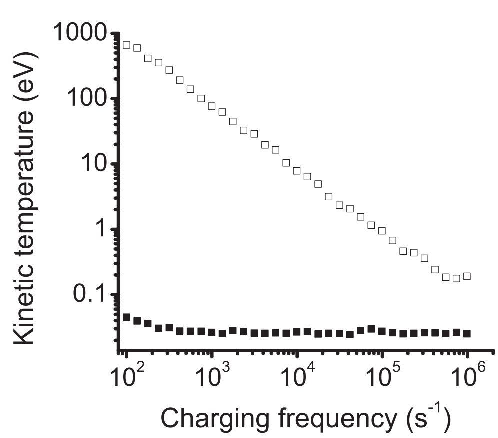

FIG. 3. Dependence of temperatures on the charging frequency for set 3. Blank squares are the kinetic temperature of the vertical motion of dust particles; filled squares are the horizontal kinetic temperature of dust particles.

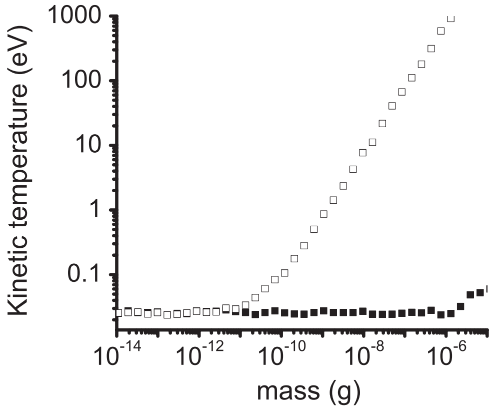

FIG. 4. Dependence of temperatures on the dust particle mass for set 3. Blank squares are the kinetic temperature of the vertical motion of dust particles; filled squares are the horizontal kinetic temperature of dust particles.

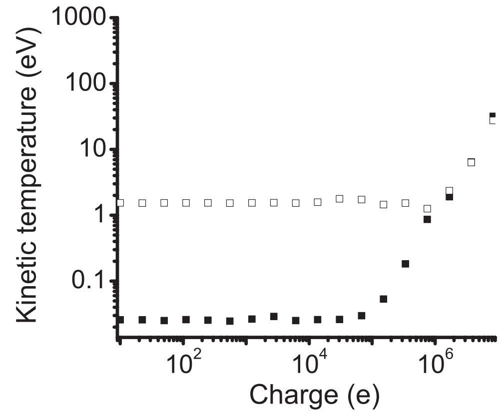

FIG. 6. Dependence of temperatures on dust particle charge for set 3. Blank squares are the kinetic temperature of the vertical motion of dust particles; filled squares are the horizontal kinetic temperature of dust particles.

same for all considered data sets. Similar dependence of the vertical kinetic temperature on the system parameters is previously obtained in some theoretical papers [10,22] in another form.

The resulting formula (15) for kinetic temperature consists of only five parameters: mass m, gravity g, friction coefficient γ, normalized amplitude of charge fluctuations δq, and characteristic frequency of dust particle charge deviations Ω. The value of the vertical temperature depends weakly on the other seven parameters. Two parameters can be identified as the next parameters due to importance: number of particles and dust particle charge. Vertical temperature is nearly constant for the five remaining parameters in the typical laboratory experiment range of parameters.

The conditions of the separation of vertical kinetic temperature of dust particles from the neutral gas temperature are estimated by the formula $(\delta q)^2/(\gamma\Omega) \approx T_{\rm room}/(\mathrm{mg}^2)$. The phenomenon may occur at $\gamma < 90\ \mathrm{s}^{-1}$ or $\delta q > 0.0006$, or $\Omega < 3.6 \times 10^6\ \mathrm{s}^{-1}$. The kinetic temperature of dust particle motion in the simulation does not fall below the neutral gas temperature because the model takes into account the effect of neutral gas on dust particle motion with a Langevin thermostat.

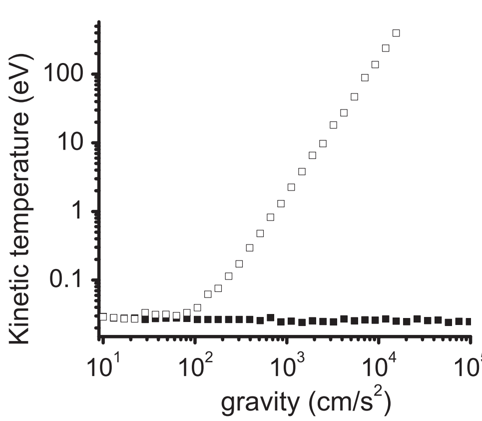

FIG. 5. Dependence of temperatures on gravity for set 3. Blank squares are the kinetic temperature of the vertical motion of dust particles; filled squares are the horizontal kinetic temperature of dust particles.

C. Horizontal temperature

The kinetic temperature of the horizontal motion of dust particles is usually smaller than the kinetic temperature of vertical motion. Numerical simulation shows that the set of parameters that determines the average kinetic energy of horizontal motion significantly differs from the set of parameters that determines the average kinetic energy of vertical motion of dust particles.

The determinants of the average kinetic energy of horizontal motion (*numeration as in Sec*. IV B):

(1) The normalized amplitude of charge fluctuations δq (Fig. 1).

(2) The friction coefficient γ (Fig. 2).

(6) The normalized electric field gradient of near-electrode layer e_z'.

(7) The parameter of trap potential ε.

(8) Dust particle charge Q (Fig. 6).

Note that the average kinetic energy of horizontal motion is determined mostly by the first two factors. The following three factors affect the horizontal kinetic temperature to a lesser degree.

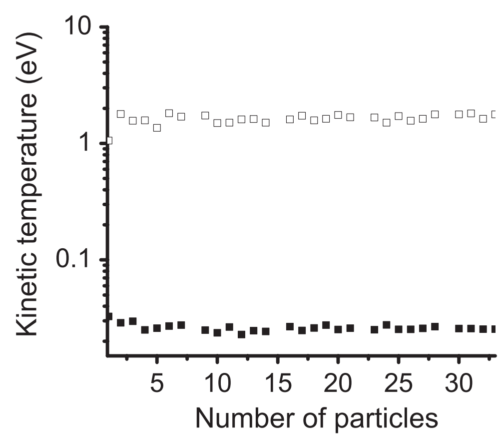

FIG. 7. Dependence of temperatures on the number of particles for set 3. Blank squares are the kinetic temperature of the vertical motion of dust particles; filled squares are the horizontal kinetic temperature of dust particles.

The factors that do not affect the average kinetic energy of horizontal motion:

(3) The characteristic relaxation frequency of small deviations of charge Ω (Fig. 3).
(4) Dust particle mass m (Fig. 4).
(5) Gravity g (Fig. 5).
(9) Number of dust particles N (Fig. 7).
(10) The normalized charge gradient in the vertical direction q'_z.
(11) The normalized charge gradient due to the distance between particles q'_r.
(12) The screening parameter κ.

Another difference between horizontal and vertical kinetic temperature is a threshold dependence of the horizontal temperature on the parameters. Horizontal temperature remains approximately constant (at approximately neutral gas temperature) for a large range of parameters. Horizontal temperature starts to rise sharply near certain values of parameters.

D. Other results

The dependencies of the coupling parameter Γ, the average kinetic energy of the center of mass of dust particles K_{cm}, and the average interparticle distance $\langle \Delta r_{i,j} \rangle$ on the system parameters are obtained by the similar method. Dependence of coupling parameter Γ (Fig. 8) on the system parameters is obtained in the form $\Gamma = 1.0 + A$, where the values of A for three sets of parameters are

$$(13 \pm 1)N^{0.7}\varepsilon^{0.5}Q^{1.5}[\gamma\Omega/m(g\delta q)^2]/(\kappa + 24),$$
$$(2.1 \pm 0.2)N^{0.7}\varepsilon^{0.65}Q^{1.33}[\gamma\Omega/m(g\delta q)^2]/(\kappa + 27),$$
$$(8.4 \pm 0.8)N^{0.8}\varepsilon^{0.53}Q^{1.5}[\gamma\Omega/m(g\delta q)^2]/(\kappa + 40).$$

Exponents for N, ε, and Q, proportionality factors, and κ dependence turn out to be different for each set of parameters. Therefore, dependencies are approximated by a power function in the vicinity of the point corresponding to a set of parameters (Table I). The main reason for differences in the formulas for different sets of data is the complexity of the system: the presence of stochastic and nonlinear terms.

The dependence of the average kinetic energy of the center of mass of a dust particle system on the parameters is very accurately approximated, and it is the same for the three sets of parameters:

$$K_{\text{cm}} \approx (1.0 \pm 0.1)N^{-1}[T_{\text{room}} + m(g\delta q)^2/(\gamma\Omega)]. \quad (16)$$

The average kinetic energy of the center of mass per one particle of dust with good accuracy is equal to the average kinetic energy of vertical motion of dust particles divided by the number of particles.

Dependence of the average interparticle distance on the system parameters is poorly consistent for three sets of parameters (Table I):

$$\langle \Delta r_{i,j} \rangle \approx (0.36 \pm 0.02)N^{-0.22}Q^{0.3}\varepsilon^{-0.3}/(\kappa + 156),$$
$$\langle \Delta r_{i,j} \rangle \approx (0.8 \pm 0.1)Q^{0.17}\varepsilon^{-0.3}/(\kappa + 169), \quad (17)$$
$$\langle \Delta r_{i,j} \rangle \approx (37 \pm 2)Q^{0.2}\varepsilon^{-0.25}/(\kappa + 231).$$

Dependence of the average interparticle distance on the system parameters can be approximated by a power function (Fig. 9) in the vicinity of the point corresponding to a set of parameters (Table I). The main reason for differences in the formulas for different sets of parameters is the complexity of the system.

The units of the parameters in the formulas of this subsection correspond to the CGS electrostatic system of units. Units of numerical coefficient are such that characteristics are in the desired units.

Variation of the sets of parameters may affect the abovementioned dependencies of the characteristics on the parameters. The dependencies could be different in the range of the characteristic parameters of laboratory experiments.

The dependence of the spectrum of dust particle motion on the parameters of the interaction potential is of great interest too. We used the Yukawa potential and the sum of two screened Coulomb potentials [39] in the simulation. It is found that the vibrational spectrum of the dust particles and the average kinetic energy of vertical oscillations are practically independent of the screening parameter in the selected range of parameters. This may be due to the fact that a system with friction damps eigenmodes in the presence of forced oscillations. Thus, the simulation shows that the vibrational spectrum is independent of the interaction potential of dust particles for the case of "warmed up" dust particle motion in the considered range of parameters (see Sec. IX C).

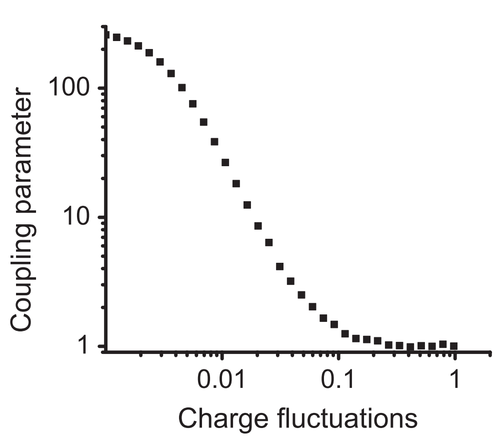

FIG. 8. Dependence of coupling parameter on the amplitude of charge fluctuations for set 3.

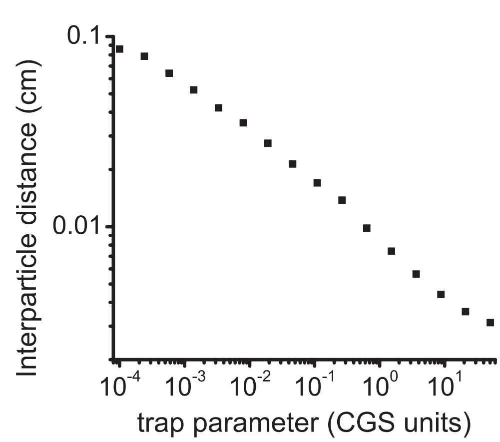

FIG. 9. Dependence of interparticle distance on parameter of trap potential for set 3.

V. VALIDITY OF THE TERM "TEMPERATURE"

During the last 15 years different scientists have regularly and carefully tried to name the average kinetic energy of dust particle motion as the kinetic temperature of the dust component. Note that the term "temperature" here means the average kinetic energy of dust particle motion, rather than the temperature of dust particles material. By definition the temperature is a physical quantity characterizing the thermodynamic equilibrium state of macroscopic systems. The openness of dust particle system does not allow us to say that the system of dust particles is in thermodynamic equilibrium. It is unclear under what conditions the system becomes equilibrium and the concept of temperature becomes valid. The kinetic temperature of the dust particles can exceed tens of eV, and this causes confusion in the physics community, since it does not coincide with the intuitive notion of temperature of condensed matter. In addition, in laboratory experiments under conditions of strong gravity the considering system has a preferred direction, and kinetic energy of horizontal motion and vertical motion can vary greatly [10,11,13,40–42], which also suggests inconsistency of the kinetic temperature of the three-dimensional motion.

It has been found that the velocity of dust particles in laboratory experiments has a Maxwellian distribution [10,11,13,40–45]. A monolayer of dust particles in the near-electrode layer of RF discharge is considered in Ref. [42]. Dust particles' velocities are measured for three different degrees of heating of a dust particle system, from gaseous to crystalline state. The velocity distribution is Maxwellian in all cases, and for some cases the distribution of vertical velocities differs from the distribution of horizontal velocities. Note that both distributions are Maxwellian, but the parameters of the distributions differ from each other. Thus, if we want to introduce the concept of kinetic temperature as a characteristic of velocity distribution, then it is necessary to introduce the concepts of vertical and horizontal kinetic temperature of dust particles.

It is found in simulation that the system of dust particles approach a partial equilibrium, and velocity distribution becomes Maxwellian in a period of time smaller than the period time of several dust particle oscillations (the time required for the collection of statistics and verification of the distribution). Maxwellization of the velocity distribution is explained by the influence of two stochastic processes on the system: a stochastic Langevin force and stochastic fluctuations of dust particle charge. Stochastic fluctuations of charge dominate for considered conditions, since these fluctuations lead to a Maxwellian distribution of dust particles' velocities faster. Particular attention should be paid to the fact that the distribution of the vertical velocity differs from the distribution of horizontal velocity under certain conditions in the simulation, but both distributions are Maxwellian. Results for the equilibrium state for one and the same set of parameters are shown in Figs. 10 and 11.

Thus, the system of dust particles is divided into two subsystems that are in partial equilibrium. Accordingly, the concept of temperature can be introduced for each subsystem. The situation with separated temperatures for two subsystems occurs frequently in nonequilibrium systems. For example, translational, vibrational, rotational, and other temperatures are often marked out in a nonequilibrium low-temperature plasma. Horizontal and vertical motions are equalized for certain parameters, the velocity distribution are also beginning to match, and then it becomes possible to talk about the temperature of the dust system without any division into additional subsystems. It is possible to divide the dynamics of dust particles kinetic temperature on the three basic modes: (1) vertical and horizontal temperatures coincide and are equal to the ambient gas, (2) vertical temperature is much greater than the horizontal temperature and is higher than the temperature of the ambient gas, and (3) vertical and horizontal temperatures coincide and are much more than the temperature of the ambient gas.

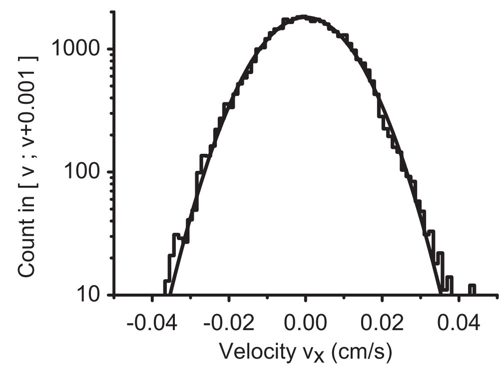

FIG. 10. Distribution of horizontal velocities. Histogram is the result of simulation; line is the approximation by Maxwell distribution. Distribution corresponds to the kinetic temperature 0.026 eV.

A striking example of the existence of such regimes is the dependence of vertical and horizontal temperature on the coefficient of friction of dust particles on the neutral gas (Fig. 2). If we look at this chart from right to left, we see first mode number 1, then mode number 2, and finally mode number 3. Another striking example of these regimes is the dependence of vertical and horizontal temperature on dust particle charge (Fig. 6).

It is still necessary to confirm that the system is macroscopic.The requirement of macroscopicity is satisfied for systems with many particles. This requirement can be met in simulation by averaging over the ensemble taken at different

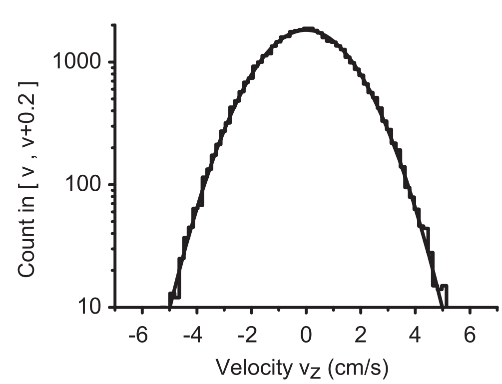

FIG. 11. Distribution of vertical velocities. Histogram is the result of simulation; line is the approximation by Maxwell distribution. Distribution corresponds to the kinetic temperature 538 eV.

times for systems with small number of particles. This is possible due to the ergodicity of the system.

VI. THEORETICAL ANALYSIS: VERTICAL OSCILLATIONS

The system of Eqs. (14) can be solved only numerically. Now we proceed with the theoretical analysis of the system (14) to give the qualitative explanations of the simulation results. The presence of various stochastic and nonlinear terms does not allow us to solve this system of equations analytically, so we divide the system into parts and reduce equations to the forms that are available for the analysis. The analysis of a simplified system of equations of dust particle motion leads to the identification of key physical factors and phenomena that can be a reason for the results of simulation, such as abnormally high kinetic energy. We try to consider the combined influence of different factors in contrast to other articles on this theme [17,22,28–30], where these combinations are generally not treated.

The first approximation is the consideration of the vertical motion of a single dust particle. An estimate for the frequency of oscillations and the vertical kinetic temperature of dust particles and also possible reasons for the differences with the formula derived from simulations can be given.

The vertical motion of a single dust particle in gas discharge near-electrode layer is described by the equation:

$$m\ddot{z} = F_z^{\text{inter}} + F_z^{\text{fr}} + F_z^{\text{grav}} + F_z^{\text{el}}, \tag{18}$$

which is the projection of Eq. (4) on the vertical axis. Electrical force (10) and the gravitational force acting on the single dust particle in the vertical direction with an account of the spatial and temporal fluctuations of the charge (8) and features of the near-electrode layer (5) can be written as

$$\begin{aligned} F^{\text{el}} + F^{\text{grav}} &= Q(z,t)E(z) - mg \\ &\approx mg[(q_z' + e_z')z + q_z'e_z'z^2 + \delta q(t) + \delta q(t)e_z'z]. \end{aligned} \tag{19}$$

When Eq. (19) is substituted in Eq. (18) the following equation is obtained:

$$\ddot{z} \approx -2\gamma\dot{z} + g(q_z' + e_z')z + gq_z'e_z'z^2 + g\delta q(t) + g\delta q(t)e_z'z, \tag{20}$$

where the stochastic term of (2) is neglected, because of its relative smallness and $F_z^{\text{inter}} = 0$ for a single particle.

Consider the right-hand side of Eq. (20) in detail. The first two terms on the right-hand side of the equation form a linear oscillator with friction, the third term represents a set of nonlinear terms, the fourth term is analogous to the external random force, and the fifth term is the term random for time and linear for coordinate.

The second term of Eq. (20) allows us to find the first approximation of dust particles' vibrational frequency:

$$\omega_z = \sqrt{-g(e_z' + q_z')}. \tag{21}$$

This expression is only a first approximation, because the value of the frequency will also be affected by other members of the nonlinear equations.

A. Forced oscillations

The next step is simplification of Eq. (20) to the form for which it is possible to evaluate the effect of a fourth term analytically:

$$\ddot{z} \approx -2\gamma\dot{z} + g(q_z' + e_z')z + g\delta q(t). \tag{22}$$

The first two terms on the right-hand side of the equation form a linear oscillator with friction. The term $g\delta q(t)$ is proportional to the fluctuating component of dust particle charge and depends only on time. Dust charge fluctuations are characterized by a broad spectrum. When the frequency of the driving force is close to the eigenfrequency (21) of the system, there is a resonance leading to an increase in the average kinetic energy of the system [37,38]. The eigenfrequency of a dust particle system falls into the spectral range of the dust charge fluctuations, which leads to the possibility of resonance. Consider the effect of the stochastic force, which is independent of the coordinates, on the classical oscillator with friction:

$$\ddot{z} + 2\gamma\dot{z} + \omega_z^2 z \approx g\delta q(t). \tag{23}$$

Stochastic equation (23) allows to evaluate the kinetic temperature of the system in the assumption of the normal distribution of $\delta q(t)$:

$$K_z = \langle mv^2/2\rangle \approx m(g\delta q)^2/(\gamma\Omega). \tag{24}$$

The resulting formula (24) practically coincides with the formula (15), obtained from simulation. However, the approximation formula is not valid for the whole range of parameters. The influence of those terms of Eq. (18), which are not included in Eq. (23), is the main reason for this difference.

The term $g\delta q(t)$ has the same form as the stochastic term in a Langevin thermostat (2), so the mechanism of particle heating may be similar to the corresponding mechanism in a Langevin thermostat.

B. Parametric resonance

The fifth term on the right-hand side of Eq. (20), $e_z'\delta q(t)z$ under certain conditions, leads to the occurrence of parametric resonance. We simplify Eq. (20) to acceptable form for the analysis

$$\ddot{z} = -2\gamma\dot{z} + z[g(q_z' + e_z') + ge_z'\delta q(t)]. \tag{25}$$

The combination in the square brackets has the similar form as Mathieu equation for parametric resonance [37,38]. The condition for the parametric resonance excitation is

$$\gamma\langle(\delta q e_z' g)^2/(4\Omega). \tag{26}$$

Parametric resonance can occur at sufficiently low pressures of the order 1 Pa or lower for the conditions of standard laboratory experiments. Parametric resonance can be related to the results of the simulation of dust particles at low pressures. It leads to the increase of kinetic temperature of more than one order of magnitude for all sets of parameters (Table I).

C. Nonlinear effects

The system of equations (14) contains a large number of nonlinear terms of different nature. As an example consider the following equation:

$$\ddot{z} \approx -2\gamma\dot{z} + g[(q_z' + e_z')z + gq_z'e_z'z^2]. \quad (27)$$

Analysis of the influence of the nonlinear term allows us to understand what effects may arise due to the nonlinearity. The term $gq_z'e_z'z^2$ gives a correction to the eigenfrequency of the oscillations, results in the oscillations at the combination frequencies, and changes oscillations amplitude. In particular, oscillations appear with multiple frequencies $\omega_z, 2\omega_z, \ldots$ due to these nonlinear terms.

D. Joint effects

The combined influence of the nonlinear term $gq_z'e_z'z^2$ and the stochastic force $g\delta q(t)$ can lead to stochastic resonance in the linear oscillator. It is worth mentioning the possibility of interaction of resonance phenomena. All these phenomena cannot be treated by the theoretical analysis, but the numerical simulation allows us to consider all these factors together.

VII. THEORETICAL ANALYSIS: RELATIONSHIP OF VERTICAL AND HORIZONTAL OSCILLATIONS

The phenomena discussed above cannot explain the abnormally high kinetic temperature of the horizontal motion of dust particles in a gas discharge plasma. So we return to the system of Eqs. (14) and look at them from the other side and consider all the forces acting on the system of dust particles in the aggregate. To describe the three-dimensional motion of a dust particle we represent the system of equations as follows:

$$\begin{aligned} m\ddot{x}_i &= \sum F^{\text{inter}} + F^{\text{fr}} + F^{\text{trap}}, \\ m\ddot{y}_i &= \sum F^{\text{inter}} + F^{\text{fr}} + F^{\text{trap}}, \\ m\ddot{z}_i &= \sum F^{\text{inter}} + F^{\text{fr}} + F^{\text{el}} + F^{\text{grav}}. \end{aligned} \quad (28)$$

A. Simplification of the system of equations

The symmetry of two horizontal axes allows us to simplify the system by neglecting one of the axes. In this case, only two equations are left in the system (28) as follows:

$$\begin{aligned} m\ddot{x}_i &= \sum F^{\text{inter}} + F^{\text{fr}} + F^{\text{trap}} \\ m\ddot{z}_i &= \sum F^{\text{inter}} + F^{\text{fr}} + F^{\text{el}} + F^{\text{grav}}. \end{aligned} \quad (29)$$

Interaction of a particle with other dust particles is simulated by the interaction of the particle with two fixed charged points on the same horizontal line. The Yukawa potential (3) is taken to assess the interaction potential. The trap is described by a parabolic potential. The remaining forces acting on the particle are described above. We expand the forces acting on dust particles in a Taylor series based on the smallness of dust particle oscillations as is done in Ref. [23]:

$$\begin{aligned} \ddot{x}_i &= -a_1x + a_2xz + a_3x^3 + a_4xz^2 + \cdots - 2\gamma\dot{x}_i \\ \ddot{z}_i &= -b_1z + b_2z^2 + b_3z^3 + b_4zx^2 + \cdots - 2\gamma\dot{z}_i + g\delta q(t), \end{aligned}$$

where a_i, b_i are the coefficients of the expansion of the forces in a Taylor series. Term $g\delta q(t)$ is the force that leads to the heating of vertical oscillations. The effect of this term is already examined in the Sec. VI, so this term is omitted in this section. It is found that the largest terms can be identified for the typical parameters of the laboratory experiment. If we leave only relevant terms in the expansion, we receive the following system of equations:

$$\begin{aligned} \ddot{x}_i + a_1x &= a_4xz^2 - 2\gamma\dot{x}_i \\ \ddot{z}_i + b_1z &= b_4zx^2 - 2\gamma\dot{z}_i. \end{aligned} \quad (30)$$

B. Parametric resonance

The presence of nonlinear terms and the dimensionality of the system of equations leads to a broadening of the eigenfrequencies range in horizontal and vertical directions. Numerical evaluation of expansion coefficients shows that linear terms a_1x, b_1z have the most significant impact on the system, and the next leading terms are a_4xz^2 and b_4zx^2, where the coefficients a_4 and b_4 are equal to each other and depend mostly on the parameters of the interaction potential. It is possible to mark out the main components that determine the magnitude of these coefficients for the considered parameters:

$$\begin{aligned} a_4 = b_4 \approx{}& Q_0^2 e^{-\kappa\langle\Delta r_{i,j}\rangle}[12 + 12\kappa\langle\Delta r_{i,j}\rangle \\ &+ 5(\kappa\langle\Delta r_{i,j}\rangle)^2 + (\kappa\langle\Delta r_{i,j}\rangle)^3]/m\langle\Delta r_{i,j}\rangle^5. \end{aligned}$$

The predominance of these terms and the intersection of the frequency ranges of vertical and horizontal oscillations leads to the possibility of parametric resonance [37,38], which can pump energy from vertical to horizontal oscillations. Moreover, the horizontal oscillations are heated primarily on multiple frequencies of vertical oscillations $2\omega_z, \omega_z, \omega_z/2, \ldots$ Thus, the amplitude of horizontal oscillations increases due to vertical oscillations. The condition of parametric resonance can be represented as

$$\gamma < a_4 A_z^2 / 8\omega_z. \quad (31)$$

C. Discussion

An estimation of the resonance condition accurately coincides with the condition of resonance, obtained by analysis of the simulation results (Fig. 2). The resulting agreement allows us to assume that this mechanism can be realized in a real system of dust particles in plasma. The condition of the resonance can be converted to

$$\gamma < a_4 g\delta q^2 / \left(8\gamma\Omega {e'_z}^{0.5}\right).$$

The sudden change in the horizontal temperature at $\gamma \approx 2 \times 10^{-2}\ \text{s}^{-1}$ confirms this formula.

Note that when the amplitude of the horizontal oscillations approaches the value of the amplitude of the vertical oscillations a second parametric resonance can occur. This resonance can pump energy in the opposite direction from the horizontal oscillations to the vertical oscillations. This phenomenon can stop heating of the horizontal oscillations and brings the system to equilibrium. The condition of equilibrium of the energy flows reveals relationship between the amplitudes of vertical and horizontal oscillations. The equilibrium of energy fluxes gives the relationship between the amplitudes of vertical and horizontal oscillations:

$$A_z^2 - A_x^2 \approx \gamma\omega_z/b_4 > 0. \quad (32)$$

This relationship between the amplitudes is valid only when this mechanism operates the exchange of energy between vertical and horizontal oscillations. The simulation results are in satisfactory agreement with the given formula.

VIII. ENERGY TRANSFER IN PLASMA-DUST SYSTEM

The general scheme of the heating is shown in Fig. 12.

External power supports gas discharge plasma, which in turn provides dust particles' charge fluctuations. Charge fluctuations give rise to the effectively stochastic driving force. The driving force pumps energy in the vertical motion of dust particles due to the intercrossing of frequency ranges of driving force and vertical oscillations of dust particles. The same process takes place for the horizontal motion, but it is of minor importance. The frequency of vertical oscillations is mainly determined by the parameters of the near-electrode layer.

Large amplitude of the dust particles' oscillations increases the importance of the nonlinear terms in the expansion of the forces acting on the particles. Short-range order of dust particles, nonlinear effects, and overlapping of ranges of vertical and horizontal oscillation frequencies lead to the appearance of the parametric resonances, which support exchange of energy between the horizontal and vertical oscillations. Pumping of horizontal oscillations occurs mainly at the double frequency of vertical oscillations and at the frequency of vertical oscillations.

The loss of kinetic energy of dust particles happens due to collisions with particles of the ambient gas. Collisions of the dust particles with gas particles do not cause heating of the gas, because of the small value of the gas pressure. Therefore particles of gas lose their excess energy in collisions with the wall. The balance of the energy loss of dust particles due to the friction with neutral gas and the flow of energy due to resonance phenomena determines the amplitude and the kinetic energy of the dust particle motion.

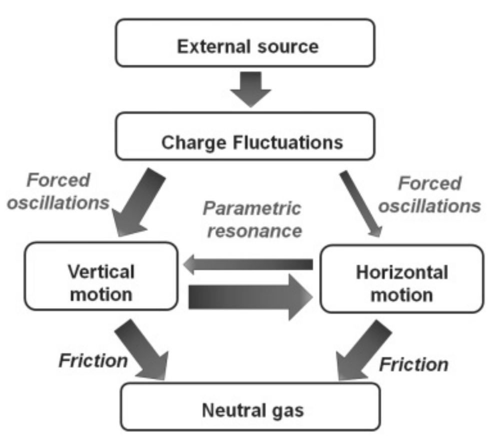

FIG. 12. Scheme of energy transfer in the plasma-dust system in gas discharge.

IX. COMPARISON WITH EXPERIMENT

A. Amplitudes of dust particle oscillations

The value can be estimated on the basis of (15):

$$A_z \approx v_z/\omega_z \approx \sqrt{2g(\delta q)^2/[\gamma\Omega(-e_z')]}. \quad (33)$$

The dependence of the amplitude on the neutral gas pressure p is determined by the pressure dependence of the parameters in the expression for the amplitude. The friction coefficient γ, the charging frequency $\Omega \approx \omega_{pi} a/\lambda_{Di}$, and the normalized electric field gradient e_z' are proportional to the pressure. Substitution of $\gamma(p),\Omega(p),e_z'(p)$ in Eq. (34) reveals a power dependence of the amplitude of dust particles oscillations on neutral gas pressure $A_z \propto p^{-1.5}$

The dependence of the amplitude of the dust particles oscillations on neutral gas pressure and power W of gas discharge is measured in Ref. [13]. Approximation of the experimental data [13] confirms the dependence $A_z \propto p^{-1.5}$ (Fig. 13). The experimental points for the lowest discharge power show that dependence is slightly different from that predicted. The authors suggest that the dependencies of the parameters of plasma-dust system on the neutral gas pressure may change at low discharge power.

Approximation of the experimental data [13] gives the dependence of the oscillation amplitude on the discharge power and pressure

$$A_z(p,W)_{[\mathrm{mm}]} \approx \left(-0.9 + 145 W_{[W]}^{-1.0}\right) p_{[\mathrm{Pa}]}^{-1.5}. \quad (34)$$

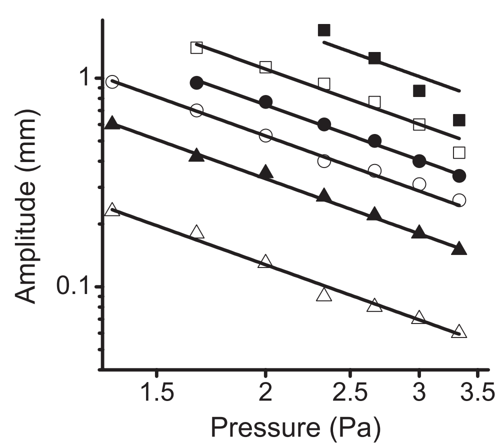

FIG. 13. The amplitude of vertical motion of dust particles at different pressures p of neutral gas and different discharge power W. Points are experimental data. The experimental data at different discharge power: filled squares, 20 W; blank squares, 35 W; filled circles, 50 W; blank circles, 65 W; filled triangles, 80 W; blank triangles, 100 W. Solid lines are the approximations of experimental data found on the theoretical dependence. $A_z(p,W)_{[\mathbf{mm}]} = (-0.9 + 145 W_{[\mathbf{W}]}^{-1.0}) p_{[\mathbf{Pa}]}^{-1.5}$.

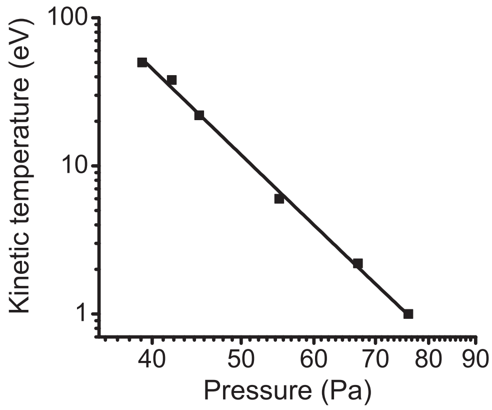

FIG. 14. Dependence of the horizontal kinetic temperature on neutral gas pressure. Filled squares are the experimental data [26]; solid line is the approximation of experimental data by a power law $T_h = 0.026 + (1.8 \pm 0.1)10^{11} p_{[\mathrm{Pa}]}^{-6.0}$.

The phenomenon that the vertical oscillations amplitude is larger than the horizontal oscillations amplitude [13] is consistent with the simulation results (Figs. 1–5) for the kinetic temperatures. The results [13] confirm the formula (32), obtained through a theoretical analysis of the equations of dust particle motion in discharge plasma.

B. Pressure dependence of horizontal kinetic temperature

The dependence of horizontal kinetic temperature on neutral gas pressure has been measured in Ref. [26]. Approximation of the experimental data allows us to obtain the dependence $T_h = T_n + Bp^{-6}$ (Fig. 14). Simulation gives the same dependence of horizontal kinetic temperature on friction coefficient (Fig. 15), which is proportional to p. Horizontal temperature has such dependence when it is changed from room temperature to the vertical temperature of dust particles.

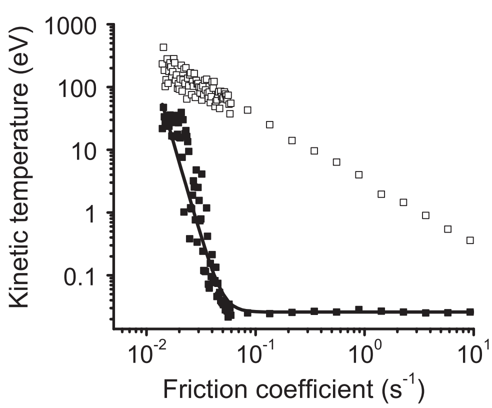

FIG. 15. Dependence of temperatures on friction coefficient for set 3. Blank squares are the kinetic temperature of the vertical motion of dust particles; filled squares are the horizontal kinetic temperature of dust particles; line is the approximation of the vertical temperature $T_v = 0.026 + (4.0 \pm 0.3)10^{-10}\gamma^{-6}$.

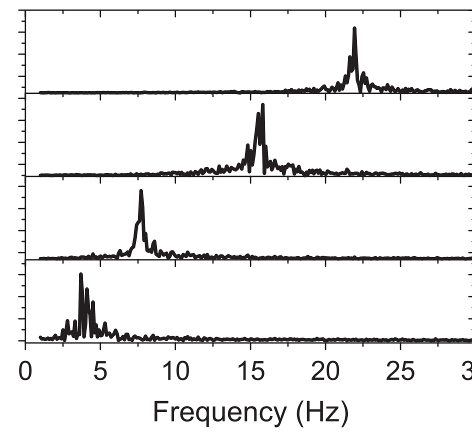

FIG. 16. Vibrational spectrum of simulation results for set 1 for four values of electric field gradient e_z'. From top to bottom: 0.64 cm^{-1}, 2.27 cm^{-1}, 9.67 cm^{-1}, 19.48 cm^{-1}.

C. Frequency of dust particle oscillations

The formula for the frequency (21) includes the gravity g, which can be considered the same in all terrestrial laboratory experiments. A method of accurate measurement of the parameter $e_z' + q_z'$ is still unknown for the present authors. A theory of near-electrode layer of gas discharge allows us only to make a rough estimate of the parameter $e_z' + q_z'$. The range of oscillation frequency values $\omega_z \approx 20$–170rad/s $\approx$ 2–30 Hz is obtained for the considered sets of parameters. The range of frequency can be reduced by more detailed analysis of the near-electrode layer for a specific gas discharge. Simulation gives the values of frequencies in the same range (Fig. 16). Experimental data on the frequency of dust particles oscillations belong to the same range [4].

D. Kinetic temperature

The laboratory experiments, where kinetic temperature of dust particles is measured, can be separated into two parts: experiments with a single-layer structure of the dust [10,14] and experiments with a multilayer structure [4,26]. The effects arising in single-layer structure are the main object of this article, so the comparison must be first carried out with experiments for single-layer structures.

The resulting formula (15) for the kinetic temperature has five parameters. Three of them can be determined by experimental methods, and method of precise determination of two remaining parameters (the amplitude of charge fluctuations δq and the charging frequency Ω) is unknown for authors. The estimates of these quantities allow us to offer a range of possible values.

Comparison of the numerical evaluation of the kinetic temperature according to the formula (15) with the experimental data allows us to estimate the value of a factor $\delta q^2/\Omega$ (Table II). Comparison is carried out for experiments [4,10,14,26], which are performed in the RF discharge with the frequency of 13.56 MHz. The values of the kinetic temperature of dust particles and the parameter $\delta q^2/\Omega$ obtained from the experiment are compared with ranges of theoretical estimates of these quantities (Table II). Values of the kinetic temperature and $\delta q^2/\Omega$ are obtained theoretically within the range of possible values in the case of a single-layer structure [10,14].

TABLE II. Comparison of experimental and theoretical kinetic temperature.

References	[14]	[10]	[26]	[4]
Discharge power, W	3	3	12	12
Gas discharge	Ar	Kr	He	He
Particle radius, mkm	9.55	9.4	9.4	4.7
Gas pressure, Pa	4	10.4	39	50
Kinetic temperature (experiment), eV	6	9	48	20
$\delta q^2/\Omega$, 10^{-9} c for $T_{\text{theory}} = T_{\text{exper}}$	3.2	18	85	707
$\delta q^2/\Omega$, 10^{-9} c (independent estimate)	0.008–52	0.008–53	0.008–53	0.0015–106
Kinetic temperature (independent estimate), eV	0.014–100	0.0038–26	0.0043–30	0.00043–3

The comparison with experiments for the multilayer structures is carried out mainly to detect the presence of additional unaccounted effects arising in such structures. The kinetic temperature of particles in the multilayer structure reaches values of the order of tens of eV at much higher pressures of the neutral gas. The authors of Refs. [4,7,9,14] proposed a mechanism that may explain this phenomenon, but the quantitative agreement of the mechanism with the experiments is not demonstrated. The discrepancy of estimating $\delta q^2/\Omega$ by the formula (15) and the range of possible values obtained by the theoretical estimates shows the existence of differences between the mechanism discussed in this paper and the mechanism realized in the multilayer structure.

The phenomenon of "heating" of dust particle motion can also be explained by an increase in the amplitude of charge fluctuations in a multilayer dust structure due to collective effects [2]. The experimental values of the kinetic temperature and $\delta q^2/\Omega$, obtained by the formula (15) fall into the range of possible values if the impact of these collective effects [2] on the amplitude of charge fluctuations is taken into account.

X. CONCLUSIONS

The system of equations of the dust particle motion in gas discharge are formulated with an account of the charge fluctuations and structural features of the near-electrode layer. Simulation of the system of dust particles is carried out. The theoretical analysis of simulation results is performed. The following results are obtained:

(1) The mechanism of energy transfer from the gas discharge to the vertical motion of dust particles, then to the horizontal motion, and further into the ambient gas is formulated.

(2) The validity of usage of the term "temperature" to describe the average kinetic energy of dust particle motion is discussed. Also the necessity of the difference of the concepts of vertical and horizontal temperature for a certain range of parameters is shown.

(3) Five parameters that determine the vertical kinetic temperature and five parameters that determine the horizontal kinetic temperature are identified.

(4) A formula describing the vertical kinetic temperature is derived for almost all of the parameters under consideration.

(5) Ranges of parameters where the horizontal kinetic temperature coincides with the vertical one and ranges where they differ from each other are determined.

(6) The formulas relating the coupling parameter Γ of the dust subsystem and the average interparticle distance to the dust particle charge, parameter of trap potential, screening parameter, and some others are obtained

(7) The independence of dynamic properties of the dust particle system of the interaction potential for the case of "heated" dust particle motion is shown.

The estimates of frequency, amplitude, and kinetic temperature fluctuations of the dust particle system are in satisfactory agreement with experimental data.

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