Default Risk Estimation in Reinsurance Contracts on the Base of Simulation Model

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Introduction

Insurance plays an important role in the economic activity. It helps business development increasing the confidence that the future plans won't be destroyed by accidental events. This confidence stimulates companies to create and develop business in such areas of human activity, where a risk of big losses of random character is present. Nowadays insurance is an integral part of the world's financial system. Bankruptcies of insurance companies harm seriously both individual business and the economy in whole.

The estimation of the default risk is an important point for an insurance company. There are a number of mechanisms used in insurance which influences the default risk. One of them is reinsurance, where part of risk is taken on charge by the reinsurance company [1, 3, 5]. We consider excess-of-loss reinsurance, which means that each insurance contract is reinsured individually. The insurance company chooses a so-called retention limit r, such that if the losses X in an insurance contract are greater than r then the insurance company pays only r and all the losses above it ($X - r$) are paid by the reinsurance company [2]. On the one hand the less is the chosen retention limit r (the limit of the losses retained by the insurance company), the less is the dispersion of the insurance company payments. On the other hand the less is the limit r, the greater is the price of such reinsurance and the expected value of the insurance company payments. The main goal of this paper is to analyze the dependency of the default risk from the chosen retention limit. As a main result we discover the new phenomena of a possible jump of the reliability function, which has an important practical influence on the default risk estimation.

The presence of the retention limit causes a discontinuity in the point r of a single claim payment distribution. This discontinuity makes the classical methods of probability theory (such as using of the Fourier transformation) inapplicable for the calculation of the total claim payments distribution [6]. And this distribution is needed to calculate the default risk. Traditionally the default risk is estimated by means of the normal approximation applied to the total claim payments distribution. This method gives an approximation to the default risk value and it is widely used in the insurance practice [7,8]. In the current paper we use a different approach. More precisely we use the methods of simulation modeling (Monte-Carlo method) for estimating the insurance company reliability (no-default probability, complement to the default risk). With the help of Monte-Carlo method we have found out the cases when the reliability function (the dependency of the no-default probability from **r**) has a considerable jump. It means that there exist such parameter values for which the reliability fails down sharply, and the insurance company can find itself on the verge of bankruptcy. Note, that this jump of the reliability can't be found by the traditional methods because the normal or another traditionally used approximation is a continuously differentiable function which smoothes all the jumps.

Problem description

The paper is devoted to analyzing of the dependency of the insurance company reliability from the retention limit value, chosen in the excess-of-loss reinsurance contract. Here and after this dependency will be called "reliability function". It is shown that reliability function can have a considerable jump for some parameter values.

The main difficulty in the reliability calculation is the discontinuity of a single payment distribution $G(x)$, introduced by excess-of-loss reinsurance. On the following picture (Fig.1) this discontinuity is shown for the insurance and reinsurance companies.

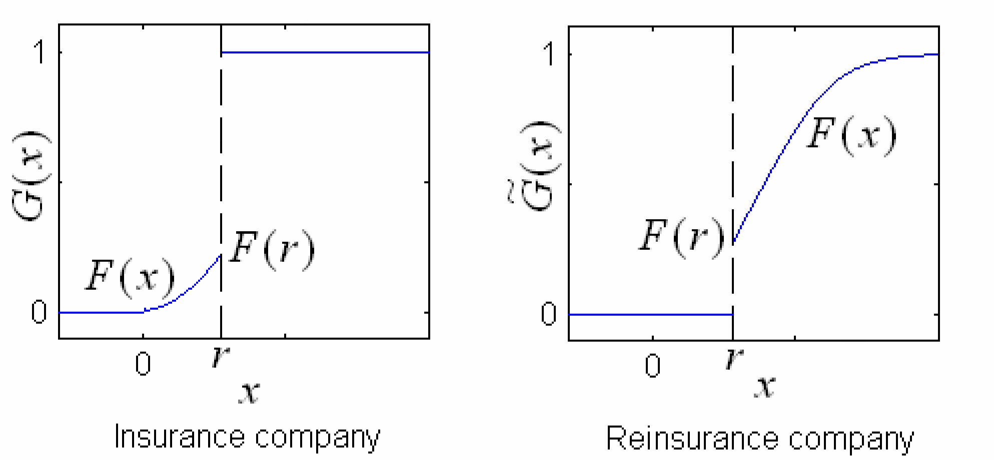

Fig.1. Discontinuity of a single payment distribution

We overcome this difficulty by means of simulation methods. For generating of random values we use Matlab generators with good random properties. For example Marsaglia's ziggurat method is used for generating normally distributed random numbers.

Simulation

The simulation is realized on the basis of the individual risks model widely used in insurance practice. Modeling is performed according to the scheme shown on the picture below (Fig.2), where L is the responsibility limit set by both the insurance and reinsurance companies, r is the retention limit set in the excess-of-loss reinsurance contract between the insurance and reinsurance companies, ϕ is the franchise level set in the insurance contract.

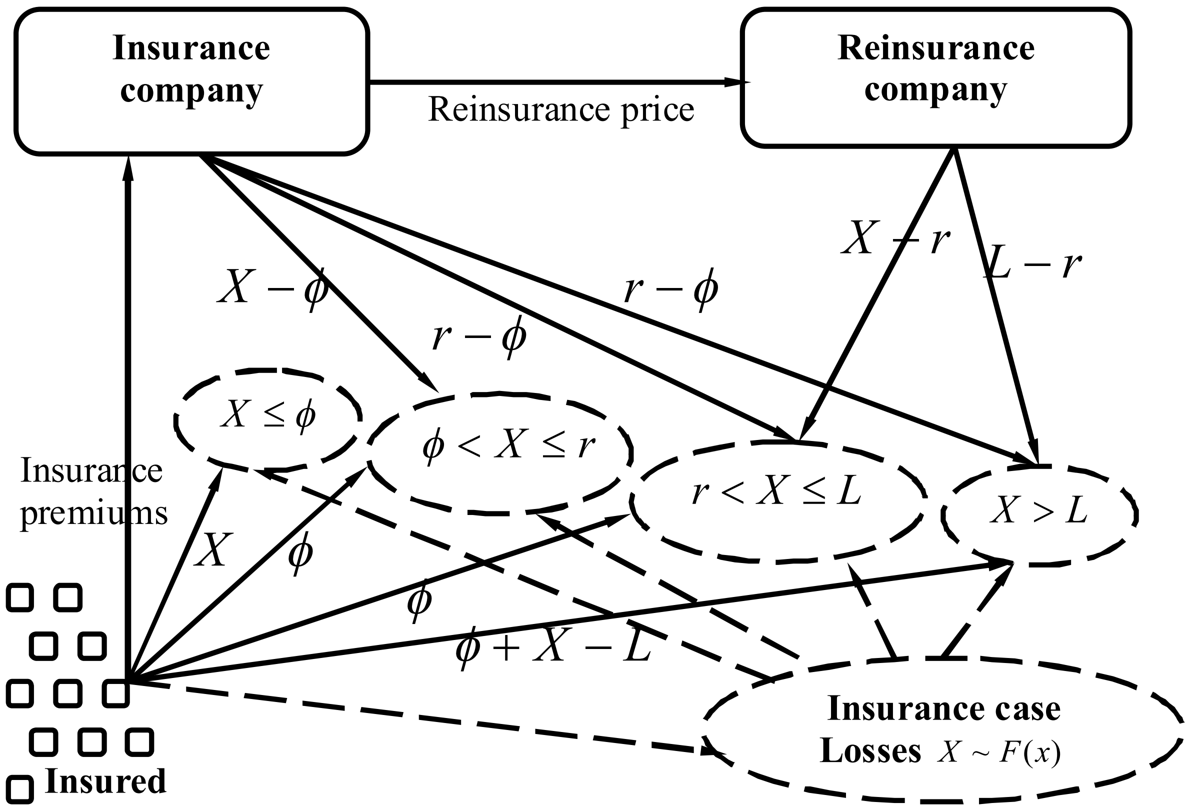

Fig.2. Simulation scheme

First, all the insurance cases are generated for the specified portfolio. Then in each of these cases losses value is generated. The losses are divided among the insurance company, reinsurance company and insured according to the retention limit and responsibility limit. For performing all the calculations a special software package is written in Matlab. All the parameters, losses distribution and the number of simulation iterations can be specified in the programs. The insurance company reliability is calculated for all the possible values of the retention limit and the graph of the resulting reliability function is shown.

According to the individual risk model the insurance company has N contracts. For each contract an insurance case can happen with probability p and not happen with probability $q = 1 - p$. So it is necessary to generate N times the value of the following random variable (we use value-probability sheem):

$$I \; \begin{array}{c|c} 1 & 0 \\ \hline p & q \end{array}$$

For generating of its values the uniform random generator is used. A value of I is calculated from the generated value of a uniformly distributed variate U using the following formula:

$$I = \begin{cases} 1, & if \;\; U < p \\ 0, & else \end{cases}$$

Then if the generated value of I is equal to 1, it means that the insurance case has happened for the current contract, and it is necessary to generate a value of losses with the specified distribution function $F(x)$. For standard distributions such as exponential and normal distributions Matlab generators are used. For a not standard distribution again a uniformly distributed variate U is used. To get a variate X with the distribution function $F(x)$ the following formula is used:

$$X = F^{-1}(U).$$

If the resulting value of the losses is negative, it is taken equal to 0. If it is greater than the chosen retention limit r, then the value of the insurance company payment is taken equal to r, because the remaining losses are paid by the reinsurance company. Thus we get the claim payments of the insurance company for each of the N contracts and then sum it to determine the value of the total payments. Making such a generation sufficiently many times we'll get a set of values of the variate which has the necessary distribution (total claim payments distribution). These values are used to calculate the empirical distribution function. The reliability of the insurance company can be calculated from this distribution function. Calculating the reliability for all the possible values of the retention limit r we get the reliability function $Rl(r)$.

Smoothing

Simulation approach causes the following problem however: the calculated results have an important dispersion. This means that the calculated function $Rl(r)$ turns out to be far from smooth and has essential fluctuations. That's why a smoothing is applied to the calculated values Rl_k according to the following formula (moving average):

$$\overline{Rl}_k = \frac{1}{m} \sum_{j=0}^{m-1} Rl_{k-j}$$

The size m of the "window" depends on the number if points Rl_k. Such a smoothing is applied to the function several times

$$\overline{Rl}_k^{(1)} = \frac{1}{m} \sum_{j=0}^{m-1} Rl_{k-j}, \quad \overline{Rl}_k^{(2)} = \frac{1}{m} \sum_{j=0}^{m-1} \overline{Rl}_{k-j}^{(1)}, \quad \ldots$$

The corresponding results for the reliability function $Rl(r)$ are presented on the picture below (Fig.3). The fluctuating line on the picture is the graph of $Rl(r)$ calculated by means of the simulation approach. The smooth line is the graph of this function but after applying of the smoothing algorithm to it.

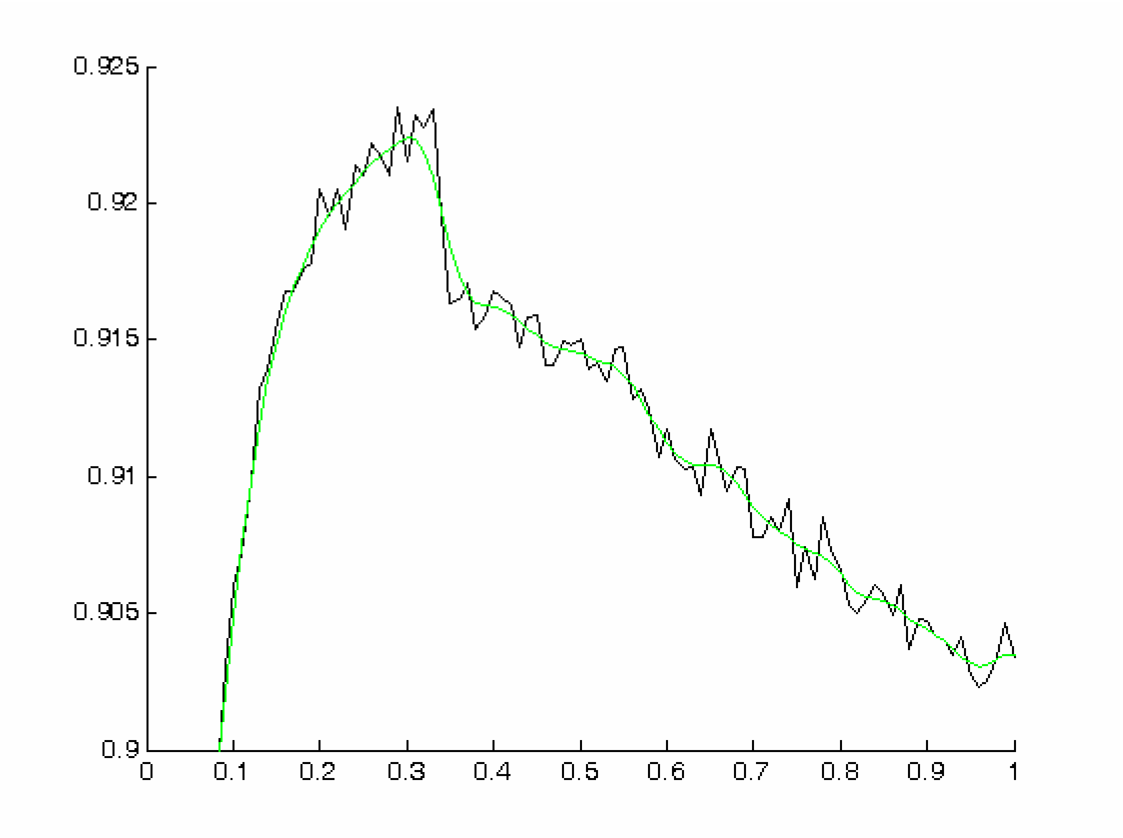

Fig.3. Smoothing of the calculated reliability function

The precision of the results calculated by means of simulation is increasing with increasing of the iterations number. The necessary precision is reached when the difference between the function got after *m* iterations and the function got for *2m* iterations becomes less than the precision value.

Results

With the help of the simulation programs implemented in Matlab the reliability function of the insurance company has been calculated for many possible values of the parameters. Different losses distribution functions have been considered including uniform, exponential and normal distributions. For each of the considered distributions we have managed to find the values of the parameters which make the reliability function to have a jump at some point. This means that the found reliability jump is a common phenomenon existing in many cases. This phenomenon is new in the default risk estimation theory. Note that it is quite possible that some of the insurance companies' bankruptcies are caused by this very jump of the reliability.

Below is an example of the reliability jump (Fig.4) observed for an insurance portfolio of N = 500 contracts with the probability of an insurance incident p = 0.01 and the exponential distribution of losses. The number of iterations here is rather big – 100000, and it gives rather precise results for the reliability (precision is about 0.01%). The jump of the reliability occurs when the retention limit is equal to 0.12 approximately. The value of this jump is about 2% – it can have a serious influence on the welfare of the company.

The second example (Fig.5) is presented for an insurance portfolio of 1000 contracts with the same probability of an insurance incident and exponential distribution of losses. The number of iterations is also 100000 and the precision is about 0.01%. No jumps of the reliability are observed for these values of parameters. Many examples show that the more is the expected number N*p of insurance incidents, the less is the value of the possible reliability jump.

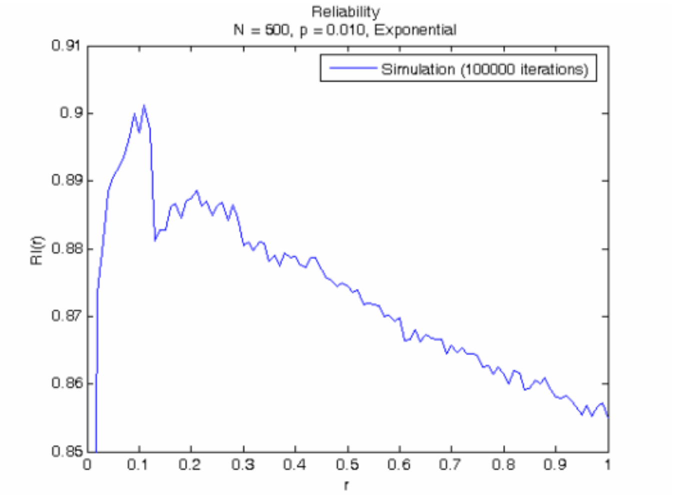

Fig.4. Reliability jump, N=500, exponential distribution

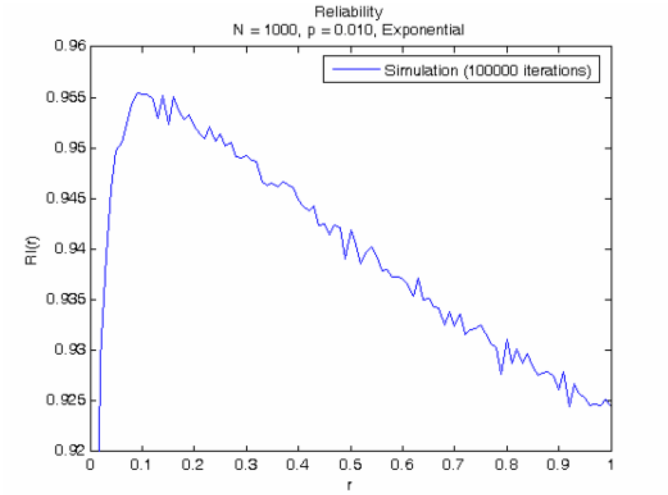

Fig.5. Reliability, N=1000, exponential distribution

On the next two pictures (Fig.6, Fig.7) big reliability jumps are shown for normal and uniform distribution of losses correspondingly. Such big jumps occur for portfolios with a small number of contracts N = 500. The number of iterations is also 100000. The size of the jump reaches 2% and will definitely result in the default of the insurance company. The jump of the reliability occurs here when the retention limit is equal to approximately 0.3 for normal losses distribution and 0.24 for uniform.

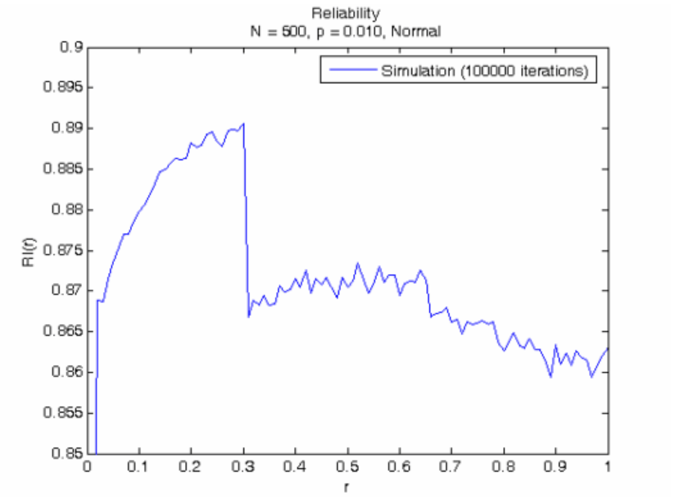

Fig.6. Reliability jump, N=500, normal distribution

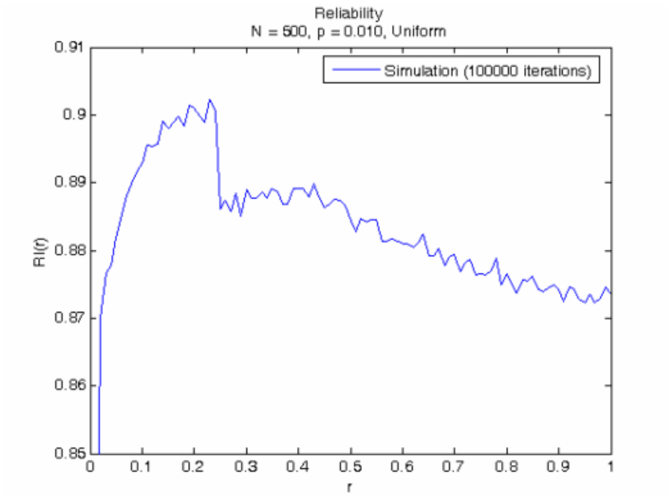

Fig.7. Reliability jump, N=500, uniform distribution

The obtained results for the reliability function have also been compared with the results which can be obtained using the traditional approaches based on the normal and gamma approximations of the total claim payments distribution. The comparison has shown an important fact: the point of the maximum of the reliability function calculated by means of an approximation is usually very close to the jump of the reliability. Thus if the insurance company is trying to optimize its reinsurance contract (which is usually the case) and takes this optimal point from the approximated reliability function there is a high probability that it will coincide with the point of the jump. Also taking into account that the normally approximated reliability in all the points lies higher than the reliability obtained by means of simulation approach, we can conclude that the insurance company usually has smaller reliability than it considers.

On the left figure below (Fig.8) this difference with normal approximation reaches the value of 1.5%. The gamma approximation gives better results, but also ignores the jump of the reliability function observed with the help of simulation. The difference reaches the value of 0.75% for the gamma approximation. The distribution of losses is exponential here. The jump here occurs for the value of the retention limit equal to 0.12 approximately. The number of iterations is 100000.

On the right figure (Fig.9) the corresponding graphs for the normal distribution of losses are shown. The difference between the calculated reliability functions for both approximations reaches the value of 1.0%. The jump here occurs for the value of the retention limit equal to 0.3 approximately.

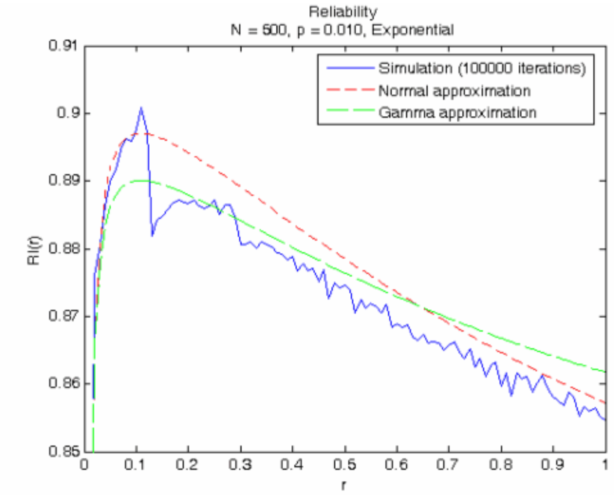

Fig.8. Approximations, N=500, exponential distribution

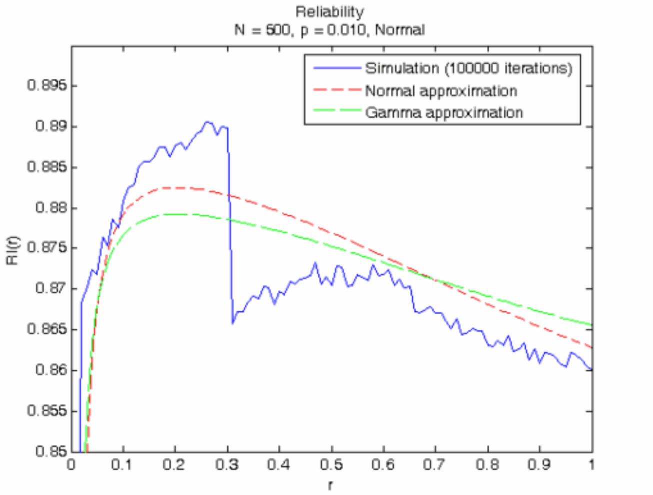

Fig.9. Approximations, N=500, normal distribution

The graphs for the uniform distribution are shown on the picture below (Fig.10). The difference between the calculated reliability functions reaches the value of 1.5% for the normal approximation and 0.5% for the gamma. The jump here occurs for the value of the retention limit equal to 0.24 approximately.

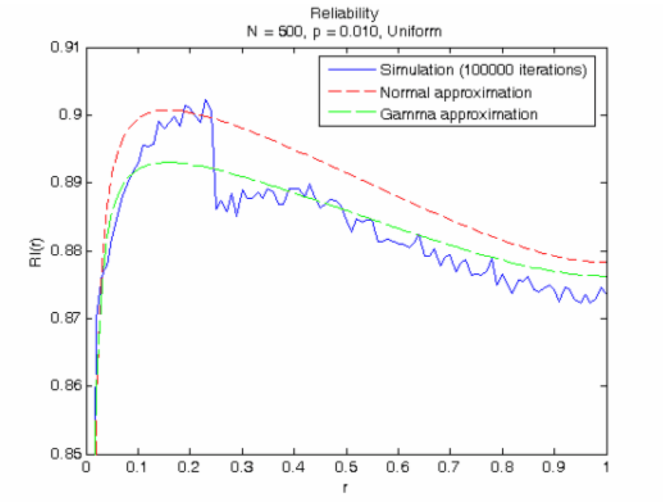

Fig.10. Approximations, N=500, uniform distribution

The results show that the found jump of the insurance company reliability is a common phenomenon, which can take place when an excess-of-loss reinsurance is used. So this phenomenon needs further investigation and analysis.

Conclusion

The simulation approach applied in the paper to the problem of the default risk estimation allows to find out the phenomenon of the default risk jump observed for a wide class of loss distributions. This jump plays an important role in the practical evaluation of the default risk for the insurance company. It is shown that the traditional approach based on the normal approximation is not able to determine this jump and thus cannot be used for the calculation of the no-default probability in the described cases. The nature of the observed jump is not clear yet and is an interesting problem to be investigated in the future. This work was supported by the project 61.1-2010 of the TAPRADESS Laboratory HSE NN

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