

## Contribution of sublinear and supralinear dendritic integration to neuronal computations

Alexandra Tran-Van-Minh, Romain Daniel Cazé, Therese Abrahamsson, Laurence Cathala, Boris S. Gutkin and David A. DiGregorio

Journal Name:	Frontiers in Cellular Neuroscience
ISSN:	1662-5102
Article type:	Review Article
Received on:	15 Oct 2014
Accepted on:	13 Feb 2015
Provisional PDF published on:	13 Feb 2015
Frontiers website link:	<a href="http://www.frontiersin.org">www.frontiersin.org</a>
Citation:	Tran-van-minh A, Cazé RD, Abrahamsson T, Cathala L, Gutkin BS and Digregorio DA(2015) Contribution of sublinear and supralinear dendritic integration to neuronal computations. <i>Front. Cell. Neurosci.</i> 9:67. doi:10.3389/fncel.2015.00067
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5 **Contribution of sublinear and supralinear dendritic integration to neuronal**  
6 **computations**

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8 Alexandra Tran-Van-Minh<sup>1</sup>, Romain D. Cazé<sup>3,4</sup>, Thérèse Abrahamsson<sup>1,2</sup>, Laurence Cathala<sup>5</sup>,  
9 Boris S. Gutkin<sup>3,6</sup>, David A. DiGregorio<sup>1\*</sup>

10  
11  
12 <sup>1</sup> Unit of Dynamic Neuronal Imaging, Department of Neuroscience, CNRS UMR 3571,  
13 Institut Pasteur, Paris, France

14 <sup>2</sup> Centre for Research in Neuroscience, Department of Neurology and Neurosurgery, The  
15 Research Institute of the McGill University Health Centre, Montreal General Hospital,  
16 Montreal, QC, Canada

17 <sup>3</sup> Group for Neural Theory, LNC INSERM U960, Institut d'Etude de la Cognition de l'Ecole  
18 normale supérieure, Ecole normale supérieure, Paris, France

19 <sup>4</sup> Department of Bioengineering, Imperial College London, London, UK

20 <sup>5</sup> Sorbonne Universités, UPMC Univ Paris 6, UMR 8256 B2A, Team brain development,  
21 repair and aging, Paris, France

22 <sup>6</sup> Federal Research University Higher School of Economics, Moscow, Russia

23  
24 Correspondence:

25 Dr David A. DiGregorio

26 Department of Neuroscience

27 CNRS UMR 3571

28 Institut Pasteur

29 25, rue du Dr Roux

30 75724 Paris Cedex 15

31 [david.digregorio@pasteur.fr](mailto:david.digregorio@pasteur.fr)

32 **Abstract**

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Nonlinear dendritic integration is thought to increase the computational ability of neurons. Most studies focus on how supralinear summation of excitatory synaptic responses arising from clustered inputs within single dendrites result in the enhancement of neuronal firing, enabling simple computations such as feature detection. Recent reports have shown that sublinear summation is also a prominent dendritic operation, extending the range of subthreshold input-output transformations conferred by dendrites. Like supralinear operations, sublinear dendritic operations also increase the repertoire of neuronal computations, but feature extraction requires different synaptic connectivity strategies for each of these operations. In this article we will review the experimental and theoretical findings describing the biophysical determinants of the three primary classes of dendritic operations: linear, sublinear, and supralinear. We then review a Boolean algebra-based analysis of simplified neuron models, which provides insight into how dendritic operations influence neuronal computations. We highlight how neuronal computations are critically dependent on the interplay of dendritic properties (morphology and voltage-gated channel expression), spiking threshold and distribution of synaptic inputs carrying particular sensory features. Finally, we describe how global (scattered) and local (clustered) integration strategies permit the implementation of similar classes of computations, one example being the object feature binding problem.

## 52 Introduction

53  
54 In order to control behavior, the brain relies on the ability of its neuronal networks to  
55 process information arising from external and internal sources. How single neurons decode  
56 combinations of sensory features and transform them into a spiking output is still unknown,  
57 and represents a subject of intense study. The complexity of the single neuronal coding  
58 problem can be illustrated by the paradoxical finding that neurons exhibiting narrowly tuned  
59 receptive fields often appear to be driven by synaptic inputs that themselves are broadly tuned  
60 (Chadderton et al., 2014). One hypothesis is that nonlinear dendritic transformations are  
61 critical for such neuronal computations. Decades of experimental and modeling studies on  
62 dendrites have led to the consensus that active properties of dendrites are primarily  
63 responsible for nonlinear integration, in particular **supralinear** operations (Johnston and  
64 Narayanan, 2008; Mel, 1994; Spruston and Kath, 2004). Nonetheless other findings indicate  
65 that **sublinear** integration of synaptic inputs is possible in multiple neuron types, and results  
66 from either active (Cash and Yuste, 1998; Hu et al., 2010) or passive dendritic properties  
67 (Abrahamsson et al., 2012; Vervaeke et al., 2012).

68  
69 What is the evidence that nonlinear dendritic properties contribute to neuronal  
70 computations? Numerical simulations suggest that supralinear dendritic operations are  
71 essential for translation-invariant orientation tuning (Mel et al., 1998) and binocular disparity  
72 tuning (Archie and Mel, 2000), while sublinear dendritic operations contribute to coincidence  
73 detection of auditory stimuli (Agmon-Snir et al., 1998). Recently, state-of-the-art *in vivo*  
74 recordings have shown that dendritic supralinearities are associated with various other  
75 neuronal computations: formation of hippocampal place fields (Lee et al., 2012), detection of  
76 multi-modal sensory stimuli (Xu et al., 2012), angular tuning of barrel cortex pyramidal  
77 neurons (Lavzin et al., 2012), and enhancement of orientation tuning (Smith et al., 2013).  
78 Sublinear operations have also been shown to underlie orientation selectivity of binocular  
79 neurons in visual cortex *in vivo* (Longordo et al., 2013).

80  
81 Nevertheless, a direct link between the dendritic transformations and the associated  
82 neuronal computations is still lacking. Analytical methods implementing mathematical  
83 approximations of measured dendritic operations can be used to make estimates of the  
84 possible number and type of neuronal computations. For example, binary neuron models were  
85 used to quantify what was previously shown with biophysical models (Mel, 1994), namely  
86 that nonlinear dendrites support a larger number of neuronal computations (Cazé et al., 2013;  
87 Poirazi and Mel, 2001). Such simplifications can provide analytical insight and make testable  
88 predictions as to which computations are made possible by dendritic operations. Moreover,  
89 analytical methods show under which conditions the expanded computational capacities are  
90 generic, i.e. not tied to the specific example parameters of the biophysical model.

91  
92 Here we review the biophysical determinants of different classes of dendritic  
93 operations (linear, sublinear and supralinear), how they are measured experimentally, and  
94 finally, using a recently published Boolean-based analysis of equivalent dendritic trees (Cazé  
95 et al., 2013; Cazé et al., 2012, 2014), we review how these operations combine with other  
96 cellular properties to determine neuronal computations.

## 97 Dendritic integration

98  
99  
100 Neurons integrate synaptic inputs arriving primarily on dendritic trees carrying  
101 information from presynaptic neurons, by transforming them into synaptic potentials using a

102 variety of cell-specific synaptic and cellular mechanisms. During synaptic transmission, the  
103 activation of neurotransmitter-gated conductances results in either a transient depolarization  
104 or hyperpolarization of the postsynaptic membrane potential. When the net depolarization  
105 resulting from **synaptic integration** of multiple synaptic inputs is greater than the spike  
106 threshold potential, the neuron generates an **action potential** (AP), or **spike**. Synaptic  
107 integration is a critical determinant of **neuronal computations**, the process by which a  
108 postsynaptic neuron transforms presynaptic information (coded in input activation patterns)  
109 into an output signal (encoded in a firing pattern) (Chadderton et al., 2014; Häusser and Mel,  
110 2003; Larkum, 2013; London and Häusser, 2005; Silver, 2010). This review will focus  
111 primarily on the integration of excitatory post-synaptic potentials (EPSPs) mediated by  
112 ionotropic glutamate receptors.

113  
114 Dendritic integration can be quantified by comparing the **observed** depolarization  
115 resulting from the simultaneous activation of the same synaptic inputs (Figure 1B), also called  
116 a compound EPSP, and the arithmetic sum of individual EPSPs (**expected** membrane  
117 depolarization) (Figure 1C). The dendritic subthreshold input-output (sI/O) relationship is  
118 easily described by plotting observed versus expected depolarizations for different numbers of  
119 co-activated synapses (Figure 1). Mathematical functions can be used to describe the  
120 **operation** performed. The sI/O relationships fall into three categories of dendritic operations:  
121 1) **linear**, where the observed depolarization equals the expected depolarization, 2)  
122 **supralinear**, where the observed depolarization exceeds the expected depolarization (thus  
123 above the linear line; Figure 1D, left), and 3) **sublinear**, where the observed depolarization is  
124 less than the expected depolarization (thus below the linear line; Figure 1D, right). Much of  
125 the experimental evidence of nonlinear integration suggests dendrites perform **supralinear**  
126 operations, resulting from the contribution of active dendritic conductances (Johnston and  
127 Narayanan, 2008; Mel, 1994; Spruston, 2008). Recent studies suggest that **sublinear**  
128 operations could be mediated solely by passive properties (Abrahamsson et al., 2012;  
129 Vervaeke et al., 2012), while other studies have shown that activation of potassium channels  
130 can produce sublinear summation (Cash and Yuste, 1999; Hu et al., 2010). The detailed  
131 biophysical mechanisms determining specific dendritic operations are discussed in depth  
132 below.

133  
134 The type of **dendritic operation** strongly contributes to the nature of the resultant  
135 **neuronal computation**. For example, co-activation of synapses within a single electrical  
136 compartment that exhibits supralinear integration will produce dendritic voltage signals that  
137 are larger than expected due to amplification by activation of voltage-sensitive channels. This  
138 large depolarization is thereby more likely to drive the neuron to spike threshold. The  
139 resulting sI/O will reflect a neuronal computation that is **cluster sensitive** (Figure 1E-F, left,  
140  $\theta_1$ ). For a neuron with sublinear dendrites, clustered synaptic activity will be less efficient at  
141 triggering a spike than if the same inputs were distributed in different compartments, thus  
142 promoting computations that are **scatter sensitive** (Figure 1E-F, right,  $\theta_1$ ) (Cazé et al., 2013).  
143 Such neuronal computations enable the discrimination of patterns of synaptic activation with  
144 different levels of spatial and temporal correlations, which could not be otherwise performed  
145 by linear dendrites (Mel, 1992). Nevertheless, it should be noted that the dendritic operation is  
146 insufficient to define the computation, synaptic placement and spike threshold also influence  
147 the final neuronal computation. In Figure 1D-F we show that lowering the spike threshold ( $\theta_2$ )  
148 would restrict the access to only the linear regime of the subthreshold dendritic operation.  
149 Finally, ongoing synaptic activity can occur in the presence of AP firing, and thus constitutes  
150 supra-threshold synaptic integration (Silver, 2010), which we will not address in this review.

151

## Biophysical mechanisms influencing synaptic integration

### *Effect of passive membrane properties on EPSPs summation*

Because neurons communicate with each other using electrical signals, the analysis of their signaling properties is generally performed using principles of electrical circuits. A **single compartment** equivalent circuit describes well the electrical behavior of a cell without any dendrite or active properties. Four parameters determine the amplitude and time course of the EPSP: a transient synaptic conductance ( $G_{\text{syn}}$ ), the electromotive force of its ion flux (driving force), the membrane resistance (specific membrane resistance;  $R_m$ ), and the specific membrane capacitance ( $C_m$ ). The difference between the membrane potential and the reversal potential for  $G_{\text{syn}}$  sets the driving force ( $V_m - E_{\text{rev}}$ ; Figure 2A, equation (1)), thus as  $G_{\text{syn}}$  increases,  $I_{\text{syn}}$  increases, and  $V_m$  becomes more depolarized. For large conductances,  $V_m$  approaches  $E_{\text{rev}}$  and the driving force is reduced, resulting in decreased current flow for the same  $G_{\text{syn}}$  (Figure 2A). This results in a sublinear relationship between  $G_{\text{syn}}$  and EPSP size. Since quantal synaptic conductances are generally small, it is when multiple synapses activated simultaneously that the driving force decreases sufficiently to produce sublinear integration (Figure 2C). Therefore, for passive single compartment model cells, synaptic summation is already essentially sublinear, which was first demonstrated at the neuromuscular junction (Martin, 1955).

More complex, but also more realistic, equivalent circuit models take into account neuronal morphology, such as dendritic arborizations. Wilfrid Rall pioneered the use of such multi-compartmental equivalent circuit models in order to study synaptic integration in neurons with **passive** dendrites. His primary advance was to consider dendrites as electrical cables (Rall, 1967) that contained an additional parameter, the axial resistance ( $r_a$ ), which electrically couples multiple elementary single compartment models (Figure 2B). Because each elementary compartment will allow current to leak across the membrane, the current injected in the next compartment (across  $r_a$ ) decreases progressively as it travels along the cable or dendrite, which results in an attenuation of the local EPSP amplitude and a slowing of its time course. Such **dendritic filtering** accounts for why local EPSPs in dendrites tend to be larger and faster than those recorded in the soma. It therefore follows that more distal synaptic inputs (for a given  $G_{\text{syn}}$ ) would result in a progressively smaller somatic depolarization and thus a smaller influence on the firing output of a neuron (Magee and Cook, 2000; Rinzel and Rall, 1974; Spruston, 2008). Also in dendrites the **local input resistance ( $R_D$ ) or impedance ( $Z_D$ )**, to account for the effect of capacitance on fast time-varying inputs) increases with increasing distance from the soma due to a diminished shunt effect of the soma and the high resistance of the sealed cable (Rinzel and Rall, 1974). We will henceforth refer to  $Z_D$ , since it is the more general form that accounts for the capacitive current dependence on synaptic conductance time course. It should be noted that at steady state  $Z_D = R_D$ . This distance-dependent increase in  $Z_D$  results progressively larger local EPSPs, which in some morphologies, can combine with an efficient passive propagation of EPSPs to the soma (transfer impedance), thereby counteracting the distance-dependence reduction in the somatic EPSP amplitude due to cable filtering (Jaffe and Carnevale, 1999; Nevian et al., 2007; Schmidt-Hieber et al., 2007). This location independence of EPSP amplitude is also referred to as passive normalization (Jaffe and Carnevale, 1999). Distance-dependent increases in  $Z_D$  are also thought to be important to increase the probability of evoking a local dendritic spike at distal inputs of basal dendrites of pyramidal neurons, which can then propagate to the soma (Rudolph and Destexhe, 2003).

202 Rall provided a simple parameter that describes cable filtering: the space constant ( $\lambda$ ),  
203 derived from the steady state ( $\lambda_{DC}$ ) or frequency-dependent ( $\lambda_{AC}$ ) solution to the cable  
204 equations. It represents the distance along a cable where the membrane potential is 63 % of  
205 the maximal at the site of current injection. Therefore if the dendrite length is longer than  $\lambda$ ,  
206 significant cable filtering can be expected; similarly, if the dendritic length is much shorter  
207 than  $\lambda$  then EPSPs propagating to the soma are filtered very little. A critical morphological  
208 parameter determining  $\lambda$  is the dendritic **diameter**, to which  $\lambda$  is proportional (Figure 2B);  
209 meaning a larger diameter produces a longer  $\lambda$  (Figure 3A, left). For fast synaptic  
210 conductances (rise and decay  $< 2$  ms), the capacitive current acts as a frequency-dependent  
211 shunt and can dramatically alter  $\lambda$ . In cerebellar molecular layer interneurons, for example,  
212 the frequency-dependent length constant ( $\lambda_{AC}$ ) can be over a factor of 5 shorter than  $\lambda_{DC}$ .  
213 Their thin ( $\sim 0.4 \mu\text{m}$  diameter),  $100 \mu\text{m}$  long dendrites are electrically compact for steady-  
214 state depolarizations (with total length 3 times shorter than  $\lambda_{DC}$ ,  $300 \mu\text{m}$ ). But for rapid  
215 synaptic conductances  $\lambda_{AC}$  is  $50 \mu\text{m}$  (half the dendritic length), resulting in significant  
216 dendritic filtering of EPSPs for distances greater than  $20 \mu\text{m}$  (Abrahamsson et al., 2012).  
217 **Dendritic branching** tends to shorten the space constant, since it effectively decreases the  
218 membrane resistance (acting like a shunt for current flow (Figure 3A, right; Abrahamsson et  
219 al., 2012). It is also worth noting that  $\lambda$  also serves as a rough indicator of the size of effective  
220 dendritic compartments. Synapses located within a distance of  $\lambda$  are more likely to interact  
221 than non-neighboring synapses (Figure 2C; Abrahamsson et al., 2012).

222

### 223 *The influence of passive dendrites on sI/Os*

224

225 As described above, sublinear summation of simultaneously occurring EPSPs within  
226 an electrical compartment is a natural consequence of the loss of driving force for synaptic  
227 currents. Dendritic compartments with narrow diameters are particularly sensitive to this due  
228 to a high  $Z_D$ . Therefore when multiple dendritic synapses are activated simultaneously within  
229 a close proximity ( $< \lambda$ ), the local depolarization resulting from the activation of a given  
230 synaptic input will be large, thus decreasing the local driving force, resulting in a sublinear  
231 sI/O (Figure 1D, 2B). As the **diameter** of the passive dendrite decreases,  $Z_D$  will increase and  
232 the local EPSPs will be even larger (Abrahamsson et al., 2012). One can use the equation for  
233  $R_D$  of an infinite cable to appreciate the influence of dendritic diameter (Figure 2B, equation  
234 3). The larger  $Z_D$  causes a larger depolarization, thus the sublinear summation of synaptic  
235 inputs will be more prominent with fewer active inputs (Figure 3A, left; see also Rinzel and  
236 Rall, 1974). If the **distance of the synapse from the soma** increases, the current sink of the  
237 soma, the end effect of the dendrite and/or dendritic tapering will contribute to a distance-  
238 dependent increase in  $Z_D$ , together resulting in more pronounced sublinear sI/O curves  
239 particularly for more distal dendritic compartments (Figure 3A, middle). Finally, the number  
240 of dendritic branch points, despite increasing dendritic filtering, tends to decrease the local  $Z_D$   
241 by adding a current sink, thus favoring a more linear sI/O (Figure 3A, right). Gap junctions  
242 have also been shown to reduce sublinear summation by providing a shunting conductance  
243 (Vervaeke et al., 2012).

244

245 Although passive membrane properties are sufficient to produce sublinear dendritic  
246 operations, experimental evidence of such a mechanism has only recently been described  
247 (Abrahamsson et al., 2012; Vervaeke et al., 2012). The authors concluded that the  
248 combination of thin dendrites and low levels of expression of voltage-gated channels favors  
249 sublinear dendritic operations. In these neurons, sublinear summation is apparent even for as  
250 few as two active synapses (Abrahamsson et al., 2012). Synapses activated on separate  
251 dendrites summed linearly, supporting a **scatter sensitive** neuronal computation

252 (Abrahamsson et al., 2012), that was confirmed in a realistic active model of the same neuron  
253 type (Cazé et al., 2013).

254

255 *The influence of active dendrites on sI/Os*

256

257 The large local synaptic depolarizations produced in dendrites can also recruit the  
258 activation of voltage-dependent channels (NMDARs, Na<sup>+</sup>, Ca<sup>2+</sup>, K<sup>+</sup> and HCN channels, see  
259 Johnston and Narayanan, 2008; Figure 3B). The number of activated synaptic inputs needed  
260 to engage active conductances is determined, in part, by the passive properties of the dendrite,  
261 the amplitude and kinetics of the synaptic conductance, the voltage-dependence of channel  
262 gating, and the channel density and distribution along the somato-dendritic axis. Active  
263 conductances can either enhance (Migliore and Shepherd, 2002; Williams and Stuart, 2000)  
264 or dampen (Cash and Yuste, 1999; Hu et al., 2010) local dendritic depolarizations, depending  
265 on whether the channels mediate inward (depolarizing) or outward (hyperpolarizing) currents,  
266 respectively. Distance-dependent increases in I<sub>h</sub> currents have been shown to compensate for  
267 the temporal slowing caused by dendritic filtering (Magee and Cook, 2000; Williams and  
268 Stuart, 2002). Differential HCN channels expression across mitral cells has also been shown  
269 to increase the membrane noise and lower the rheobase, thus facilitating AP generation  
270 (Angelo and Margrie, 2011). Because of the presence of NMDARs at many glutamatergic  
271 synapses, most studies find that NMDARs activate other voltage-dependent channels by  
272 boosting local synaptic depolarization (Branco and Häusser, 2011; Katona et al., 2011;  
273 Krueppel et al., 2011; Losonczy and Magee, 2006; Makara et al., 2009; Nevian et al., 2007;  
274 Schiller et al., 2000). The resulting dendritic operation is determined by the concurrence of a  
275 passively determined sublinear (Chiovini et al., 2014; Krueppel et al., 2011; Losonczy and  
276 Magee, 2006) or linear operation (Branco and Häusser, 2011), and a supralinear operation.

277

278 In some cases, the activation of some conductances results not only in boosting, but in  
279 a threshold-dependent, all-or-none regenerative response, often called a dendritic spike. This  
280 regenerative behavior is characterized by a steep change in the sI/O followed by a plateau  
281 (Figure 1D and 3B, Larkum, 2013; Losonczy and Magee, 2006; Polsky et al., 2004). Locally-  
282 generated dendritic spikes can be mediated by either Na<sup>+</sup> channels, Ca<sup>2+</sup> channels or NMDA  
283 receptors (NMDARs). Na<sup>+</sup>-spikes are triggered by high-amplitude local depolarization, are  
284 relatively brief, and can be accompanied by entry of Ca<sup>2+</sup> through VGCC or NMDARs. In  
285 pyramidal cells, these dendritic Na<sup>+</sup> spikes can be generated in most regions of the dendritic  
286 tree, propagate throughout the dendritic tree, albeit with some attenuation, but can still trigger  
287 somatic spiking (Golding and Spruston, 1998; Nevian et al., 2007; Rudolph and Destexhe,  
288 2003). Recent findings have also shown Na<sup>+</sup>-channel dependent spikes in dendrites of dentate  
289 gyrus granule cells (Chiovini et al., 2014). On the other hand, Ca<sup>2+</sup> and NMDA spikes are  
290 longer, plateau-like events, that are thought to be generated in particular regions of the  
291 dendritic tree, and require the synchronous activation of many clustered synapses. The  
292 biophysical mechanisms of the NMDA spikes and their functional consequences have been  
293 described in detail in a recent review (Major et al., 2013). In cortical pyramidal neurons, the  
294 Ca<sup>2+</sup> spike is likely to propagate actively from the primary apical dendrite to the soma,  
295 thereby representing a more global dendritic operation, whereas NMDA spikes are locally  
296 restricted to dendritic compartments such as tufts or basal dendrites (Larkum, 2013). In  
297 contrast, simulations of *in vivo* spontaneous synaptic activity allow glutamate-bound  
298 NMDARs to act as global nonlinearities providing an entirely different computation than  
299 those initiated in single dendrites (Farinella et al., 2014). Nevertheless, several recent *in vivo*  
300 studies have reported the involvement of local NMDA spikes during sensory processing,  
301 across all layers of the cortex (Gambino et al., 2014; Lavzin et al., 2012; Palmer et al., 2014;



302 Smith et al., 2013; Xu et al., 2012). It should also be noted that Polsky et al. (Polsky et al.,  
303 2004) pointed out that a  $\text{Ca}^{2+}$ -spike exhibits saturation of the voltage response and thus can  
304 also be considered sublinear for very high stimulation strengths.

305  
306 In summary, the modus operandi of supralinear dendritic compartments is comprised  
307 of a continuum of voltage-dependent operations from simple boosting of synaptic  
308 depolarization to regenerative spikes, all within single dendritic compartments. Considering  
309 the biophysical underpinnings of this range of operations, it follows that the interplay of the  
310 active and passive properties of dendrites ultimately determines the shape of the sI/O (Figure  
311 3C). For example, sI/Os of thick dendrites, which have a low  $Z_D$ , do not suffer from driving  
312 force losses, thus sum linearly for low numbers of activated synapses, then transition into  
313 supralinear summation (Makara and Magee, 2013). Thin dendrites on the other hand may  
314 exhibit sublinear sI/O relationships for only a few inputs, but then easily engage NMDAR and  
315  $\text{Ca}^{2+}$  channels (Chiovini et al., 2014; Losonczy and Magee, 2006) with fewer synaptic inputs  
316 than in larger dendrites (Figure 3C). Due to tapering of dendritic width, which increases the  
317  $Z_D$  along the dendrite with increasing distance to the soma, the dendritic operations can be  
318 altered as a function of distance from the soma (Branco and Häusser, 2010; Branco and  
319 Häusser, 2011).

320  
321 *The influence of the size, time course and location of the synaptic conductance on sI/Os*

322  
323 The strength of synaptic conductance varies from synapse to synapse across neuron  
324 types, but also within neurons. The **synaptic strength** not only serves to bias the output of a  
325 neuron to particular inputs (Ko et al., 2011), but it can also be tuned to compensate for  
326 dendritic attenuation by passive dendritic properties (Magee, 2000). Synaptic strength  
327 modulates dendritic operations by tuning the gain (slope) and shape of the sI/O by engaging  
328 sub- and supralinear transformations with different numbers of synaptic inputs (Figure 3D).  
329 Larger synaptic conductances will lead to larger dendritic depolarizations, and in turn either a  
330 larger reduction in driving force or increased activation of voltage-gated conductances.  
331 Depending on the intrinsic membrane properties and synaptic conductance amplitude the  
332 “linear regime” may be more or less prominent in the sI/O relationship.

333  
334 The **temporal window** for synaptic interactions depends ultimately on the time course  
335 of local EPSPs, which is itself shaped by the local passive dendritic properties and the time  
336 course of the synaptic conductance (Jonas, 2000). Although the local dendritic EPSPs are  
337 larger than those at the soma, it is important to note that their time course is generally much  
338 faster, due to charge redistribution down the dendrite (Schmidt-Hieber et al., 2007). The  
339 degree to which nonlinear mechanisms are engaged during EPSP summation also depends on  
340 the temporal summation of local EPSPs (Abrahamsson et al., 2012; Losonczy and Magee,  
341 2006; Makara and Magee, 2013). Simultaneous synaptic activation enables the largest degree  
342 of nonlinear summation, which will progressively decrease as the time difference between  
343 synaptic events increases (Figures 2A and 3D). Thus, combined with the synaptic strength,  
344 the temporal coincidence between co-activated synapses within a single dendritic  
345 compartment will determine gain of the dendritic operations (Abrahamsson et al., 2012;  
346 Gomez González et al., 2011; Makara and Magee, 2013).

347  
348 The location of synapses carrying similar information (e.g. a single sensory feature)  
349 determines which dendritic mechanism is recruited. For example, if features of an object are  
350 always clustered on a single dendritic compartment, then nonlinear summation will be the  
351 prominent operation influencing their integration. Below we will use a mathematical

352 formalism to provide insight into how **synaptic placement** and dendritic operations influence  
353 neuronal computations.

354

### 355 **Experimental strategies for studying dendritic integration**

356

357 How do researchers study the biophysical properties of dendrites and their influence  
358 on excitatory synaptic integration? Classical electrophysiology methods such as sharp  
359 electrode- or patch-clamp-based recordings of somatic membrane potential provided insight  
360 into the intrinsic passive electrical properties of neurons by measuring the input resistance and  
361 the membrane time constant ( $\tau = R_m * C_m$ ) (Spruston and Johnston, 1992). When combined  
362 with multi-compartmental dendritic models, with either simplified morphologies (equivalent  
363 cylinder approximation) or full anatomical reconstructions (Clements and Redman, 1989;  
364 Major et al., 1994), the passive electrotonic properties of dendrites can be estimated from  
365 model parameters that predict the membrane potential decay from somatic current injections  
366 (Rall et al., 1992). These constrained models are then used to examine dendritic  
367 transformations of EPSPs as they propagate to the soma.

368

369 Unfortunately, single electrode recordings at the soma do not provide sufficient  
370 information about dendritic properties to constrain complex morphological models. With the  
371 advent of dendritic patch recordings (Stuart et al., 1993), at least for large diameter dendrites  
372 ( $\geq 1 \mu\text{m}$ ), cable model predictions could be directly verified since this powerful  
373 electrophysiological strategy allows estimations of the critical parameters influencing  
374 dendritic filtering, such as internal resistivity ( $R_i$ ; Hu et al., 2010; Nevian et al., 2007; Roth  
375 and Hausser, 2001; Schmidt-Hieber et al., 2007; Stuart and Spruston, 1998; Stuart et al.,  
376 1993),  $R_m$  and voltage-gated channel properties and density along the somato-dendritic axis  
377 (Hu et al., 2010; Magee and Johnston, 1995; Stuart and Spruston, 1998). Dendritic recordings  
378 also enabled the measurement of local EPSPs and EPSCs, which allowed the authors to  
379 conclude that dendritic filtering can be compensated by a distance-dependent increase in  
380 synaptic conductance in certain neuron types (Magee and Cook, 2000).

381

382 More recently, fluorescence imaging techniques have greatly increased the toolkit for  
383 studying dendritic integration, particularly in those dendrites with narrow diameters ( $< 1 \mu\text{m}$ ).  
384  $\text{Ca}^{2+}$  indicators are one of the most popular class of fluorescence probes, which are used to  
385 indirectly study dendritic nonlinearities resulting from activation of voltage-dependent ion  
386 channels, provided at least one type of  $\text{Ca}^{2+}$  conductance was activated (Markram et al., 1995;  
387 Schiller et al., 1995; Schiller et al., 2000; Schiller et al., 1997).  $\text{Ca}^{2+}$  indicators have also been  
388 used to monitor synaptic activity because of the prevalence of NMDAR activation in single  
389 spines and  $\text{Ca}^{2+}$ -permeable AMPARs at synapses in interneurons (Soler-Llavina and Sabatini,  
390 2006). *In vivo* two-photon  $\text{Ca}^{2+}$  imaging experiments provided the first insights into the  
391 spatial and temporal distribution of sensory-evoked synaptic signaling within dendrites (Jia et  
392 al., 2014; Lavzin et al., 2012; Palmer et al., 2014; Smith et al., 2013; Varga et al., 2011). The  
393 contribution of *in vivo*  $\text{Ca}^{2+}$  imaging studies to understanding dendritic function has been  
394 recently reviewed by Grienberger et al (Grienberger et al., 2015). However, a limitation of  
395 using  $\text{Ca}^{2+}$  imaging to study synaptic integration is that it does not report the true dendritic  
396 voltage, a parameter critically influencing dendritic operations. Also, the slow nature of the  
397 whole-cell averaged  $[\text{Ca}^{2+}]$  and the use of high affinity  $\text{Ca}^{2+}$  indicators limits the temporal  
398 resolution of this method (Farinella et al., 2014; Fernandez-Alfonso et al., 2014). Voltage-  
399 sensitive dyes are, in principle, an ideal alternative for direct measurement of dendritic  
400 integration. Whereas voltage-sensitive dye recordings have provided unprecedented optical  
401 reports of the spatial and temporal distribution of APs in axons (Foust et al., 2010; Popovic et

402 al., 2011) and dendrites (Acker and Antic, 2009; Casale and McCormick, 2011), their use to  
403 monitor EPSPs in dendrites has been less successful due to poor signal-to-noise ratio,  
404 typically requiring hundreds of trials of averaging (Canepari et al., 2008; Palmer and Stuart,  
405 2009). However inhibitory post-synaptic potentials (IPSPs) have been detected (Canepari et  
406 al., 2008) and a recent study reports good signal-to-noise ratios sufficient to detect spine  
407 EPSPs (Popovic et al., 2014). The advances in genetically-encoded voltage indicators are also  
408 rapidly maturing (Hochbaum et al., 2014; St-Pierre et al., 2014; Zou et al., 2014), and could  
409 eventually provide a powerful tool for studying dendritic integration *in vivo*.

410

411 Another widely-used *in vitro* technique to characterize the integration properties of  
412 dendrites is to directly activate postsynaptic neurotransmitter receptors using photolysis of  
413 caged-neurotransmitter (i.e. caged-glutamate) within the diffraction-limited focal volume of  
414 the microscope (Gasparini and Magee, 2006; Losonczy and Magee, 2006). Using  
415 galvanometer-driven mirrors, regularly used in scanning confocal microscopy, the focal  
416 illumination volume can be rapidly moved and positioned at multiple locations, generally  
417 within 0.1-1 ms. The uncaging light pulse is then rapidly gated at each location to focally  
418 release glutamate. This allows for the near simultaneous activation of many postsynaptic sites.  
419 The somatic depolarization is then recorded using standard whole-cell patch-clamp methods.  
420 The observed response to uncaging at multiple synaptic locations (typically within 1 ms) is  
421 compared to the arithmetic sum of the uncaging-evoked responses at individual sites. The  
422 resulting plot is identical to the s/I/O plots described in Figure 1 and 3, provided that the  
423 uncaging responses are similar to synaptic activation. Using light, rather than presynaptic  
424 vesicular release, to activate neurotransmitter receptors provides a more flexible strategy to  
425 systematically vary the number, pattern, and timing of synapse activation. Electrical  
426 stimulation does not permit a precise identification of the synapses being activated, nor  
427 precise control of the number of synapses activated. Holographic illumination provides an  
428 alternative strategy for true simultaneous glutamate uncaging at multiple sites within the  
429 dendrites and is more amenable to multibranch activation (Lutz et al., 2008; Yang et al., 2014;  
430 Yang et al., 2011). The only potential drawback of uncaging is the difficulty in some  
431 preparations to accurately reproduce very fast synaptic conductances due to the large volume  
432 of diffraction-limited focal spots relative to the point source nature of neurotransmitter release  
433 from synaptic vesicles (DiGregorio et al., 2007), as well as a tendency to partially block  
434 GABARs (Fino et al., 2009). Nevertheless, neurotransmitter uncaging is an essential tool for  
435 quantifying the biophysical properties underlying dendritic operations.

436

### 437 Linking dendritic operations to neuronal computations using models

438

439 Because experimental evidence of a direct link between the dendritic operations and  
440 the associated neuronal computations is still lacking, a parallel strategy is to use analytical  
441 models to make testable predictions (Cazé et al., 2013; Legenstein and Maass, 2011; Poirazi  
442 and Mel, 2001). These methods take advantage of mathematical approximations of measured  
443 dendritic operations to make estimates of the possible number and type of neuronal  
444 computations. Biophysical models, in contrast, although explicit, do not easily provide insight  
445 into the classes of possible computations because of the large parameter space. There is no  
446 doubt that such models have provided deep insights into neuronal computations that involve  
447 nonlinear dendritic operations. They have been used to show that neurons with supralinear  
448 dendrites are **cluster-sensitive** (Mel, 1993) whereas neurons with sublinear dendrites are  
449 **scatter-sensitive** (Cazé et al., 2013; Koch et al., 1983). Yet it was not clear whether either  
450 type of nonlinearity provides similar computational advantages. To examine the difference  
451 between supralinear and sublinear operations of binary neuron models Cazé et al. (2013) used

452 a Boolean-based analysis. Here, we review how this Boolean framework can be used to argue  
453 that either supralinear or sublinear summation is sufficient to endow neurons with a new class  
454 of computations.

455

456 Within this analytical framework, neurons are modeled as having binary inputs ( $x_i$ ),  
457 which can be weighted and integrated, resulting in binary outputs ( $y$ ). In this context the  
458 input-output relation is described by a unique truth table, corresponding to a Boolean  
459 function. In Figure 4A, the truth table describes three simple Boolean functions: OR, AND  
460 and XOR. This well-known mathematical framework (Crama and Hammer, 2011; Wegener,  
461 1987), which deals with binary classifications of binary words, allows us to analytically  
462 determine what type of classifications are possible with nonlinear dendrites and which are  
463 otherwise impossible.

464

465 The simplest binary neuron model is called the threshold linear unit, also known as the  
466 point neuron model as described first by McCulloch and Pitts (Figure 4B; McCulloch and  
467 Pitts, 1943). Synapses are assigned a binary value of 0 or 1 for inactive or active states  
468 respectively, which is then multiplied by a positive synaptic weight for excitatory synapses.  
469 The sum of the active weighted inputs is then compared to a somatic spike threshold  $\Theta$ . If this  
470 weighted sum is greater than the threshold, the output is assigned a value of 1, and otherwise  
471 zero. If one considers a neuron with linearly summing excitatory inputs, adjustment of the  
472 threshold allows it to either perform a Boolean AND or OR (Figure 4B). However, it is not  
473 possible to find a threshold value and positive synaptic weight to allow the computation of the  
474 XOR, the function corresponding to a binary neuron that would fire only when one synapse is  
475 active, but not when none or two synapses are active. This illustrates well the fact that the  
476 threshold linear unit can only perform functions that are linearly separable, i.e. there is a set of  
477 weights and a spike threshold that categorizes the inputs into two distinct groups, which differ  
478 by their output values (Figure 4A). The XOR does not meet this criterion and is therefore a  
479 part of the class of functions that are linearly non-separable. To solve this problem we must  
480 either invoke a non-monotone function to combine synaptic values (Zador et al., 1992) or  
481 consider synaptic inhibition by using negative weights (Cazé et al., 2014; Mel, 1994).  
482 Because the former has not been described experimentally, and the latter requires specific  
483 wiring within the network, we will focus here on linearly non-separable functions that can be  
484 implemented with only excitatory synapses and monotone dendritic operations. These  
485 functions are known as positive Boolean functions (Cazé et al., 2013).

486

487 Linearly-separable functions represent only a small fraction of all the possible  
488 computations (Cazé et al., 2013). However, a neuron with nonlinear dendritic compartments  
489 can implement the set of linearly non-separable functions, which encompasses a much larger  
490 fraction of all computations (Cazé et al., 2013). Thus both supralinear and sublinear  
491 compartments unlock the access to all the possible computations (Mel, 1991; Mel and Koch,  
492 1990). This formal result is true for an infinite number of dendritic compartments (Poirazi and  
493 Mel 2001). This is clearly impossible in practice. So what can a neuron compute with a finite  
494 number of dendritic compartments?

495

496 To address this question we can construct a two-layer binary model with nonlinear  
497 dendritic compartments. We first approximated the dendritic sI/O with functions each having  
498 a characteristic dendritic threshold  $\theta$ , which represents the threshold of the dendritic  
499 nonlinearity, and  $h$ , which represents the maximal value of the dendritic nonlinearity. To  
500 approximate supralinear compartments we used a Heaviside function, and for sublinear  
501 functions we used a piecewise linear saturating function (Figure 4C). The output of the

502 dendritic compartments is then linearly summed and compared with spike threshold (Figure  
503 4D). If we vary synaptic weights, the thresholds, and the nonlinear dendritic operations, we  
504 can use Boolean analysis to examine the different functions this model can implement. A  
505 functionally salient neuronal computation that requires dendritic nonlinearities is the  
506 association (or binding) of two features of an object (for example, their shape and color). This  
507 is known as the feature binding problem (the FBP). If we suppose that that different features  
508 of objects are encoded by different groups of pre-synaptic neurons impinging on the same  
509 post-synaptic neuron, then it is obvious that by allowing the features of an object to target the  
510 same supralinear dendrite, the coincidence of those features can be easily detected when co-  
511 active (i.e. “red” + “apple shape”; Figure 4E). Importantly, the number of required subunits to  
512 implement object detection is linearly proportional to the number of objects (Cazé et al.,  
513 2012), meaning that binding features of more than two objects will require a model with extra  
514 nonlinear compartments. It can also be shown that a sublinear operation can bind features if  
515 the inputs that encode object features are distributed onto different dendritic subunits (and the  
516 spiking threshold increased). From these simple binary models it is again clear that  
517 supralinear operations favor cluster sensitivity and sublinear operations favor scatter  
518 sensitivity. However, a keen eye may notice that the sublinear model will also produce a spike  
519 if the apple shape and banana shape are both activated. This of course should not be the case  
520 if we only want to bind exclusively red-apples and yellow-bananas as objects. This therefore  
521 constitutes a partial feature binding problem. Below we will describe a neuron model with  
522 equivalent dendrites that can implement the complete feature binding problem.

523  
524 Because neurons are known to have both linear and nonlinear compartments, we  
525 considered how more realistic dendritic trees could be represented using our simple binary  
526 model, by creating a neuron model with equivalent dendrites (Figure 5A, B). All linear  
527 regions of the dendritic tree (typically, the perisomatic compartment or the large diameter  
528 primary dendrites) were collapsed to a single equivalent “linear” compartment (black regions  
529 of schematic neuron and left branch of the model neuron). The nonlinear dendritic  
530 compartments receiving more than one synaptic input were represented as a second equivalent  
531 dendritic branch. This then generalizes to an equivalent dendritic branch for each nonlinear  
532 electrical compartment (Figure 4D). The presence of a linear compartment is important, since  
533 inputs synapsing on two separate nonlinear dendrites will sum linearly (Figure 5A). Indeed,  
534 even inputs synapsing on the same nonlinear dendrite, provided they not in the same electrical  
535 compartment, will sum linearly.

536  
537 Legenstein and colleagues demonstrated that a model neuron with supralinear  
538 dendritic integration is capable of learning and computing the FBP (Legenstein and Maass,  
539 2011). This function detects any correct combination of features for an object, but not  
540 incorrect combinations. In the Boolean framework this would be the truth table corresponding  
541 to (“red” + “apple shape”) or (“yellow” + “banana shape”). In Figure 4 we showed that two  
542 supralinear dendrites are sufficient to solve the FBP for two objects made of two features  
543 each. In Figure 5, a neuron displaying at least one supralinear compartment and a linear  
544 compartment can also solve the FBP for four inputs. In this case, inputs encoding the features  
545 of one object are clustered on a supralinear compartment, and the features corresponding to  
546 the other object are clustered on the linear compartment (Figure 5C). Because the features of  
547 the object must cluster on the same compartment we refer to this model as having a **local**  
548 **strategy** of computation. Interestingly, it is also possible to implement the same computation  
549 using a **global strategy**, meaning that the features corresponding to one object need to be  
550 scattered onto both the nonlinear and the linear compartment (Figure 5D), provided that  
551 appropriate changes in the synaptic weights and threshold values are also implemented. As

552 shown by Cazé et al. (2013), a model with a linear and sublinear compartment requires the  
553 global strategy to perform the FBP. The synaptic weights and threshold will also be different  
554 than in the case of a model neuron with a supralinear compartment (Figure 5E). The fact that  
555 the FBP can be implemented using a global strategy contrasts with the notion that recognition  
556 of an object required the clustering of the inputs carrying its features onto a same dendritic  
557 branch (Legenstein and Maass, 2011), and the assumption that two-layer integration models  
558 require independent branch-specific operations (Behabadi and Mel, 2013). Using a  
559 biophysical model with a model stellate cell morphology, Cazé and colleagues showed the  
560 predictions are robust, since only passive thin dendrites were necessary to convey a scatter  
561 sensitivity of output firing, even in the presence of synaptic noise (Cazé et al., 2013).

562 How might simplified Boolean models be modified for more features and/or more  
563 objects? For objects represented by more than two features, clustered strategies would simply  
564 require more synaptic inputs, such that the number of the number of inputs per subunit  
565 (dendritic compartment) equals the number features. A change in threshold would also be  
566 required. The requirements for neuronal computations using sublinear dendrites, however,  
567 depend on the type of computation and are less straightforward to determine explicitly. The  
568 necessary number of nonlinear subunits also varies given the implementation strategy, the  
569 number of objects, the type of nonlinear subunits and the number of features. To solve the  
570 FBP with more objects using supralinear operations, each object will require at least one  
571 subunit (Cazé et al., 2012). For computations with sublinear operations, Cazé and colleagues  
572 showed that using binary weights, the FBP requires a maximum of  $2^n$  subunits (Cazé et al.,  
573 2012). Considering non-binary weights then reduces the number of subunits needed, but this  
574 number is still higher than the number of necessary supralinear subunits ( $n_{\text{subunits}}=n_{\text{objects}}$ ).

575  
576  
577 In summary, neurons with sublinear dendrites are capable of solving linearly non-  
578 separable functions, but require using a distributed strategy regarding the placement of  
579 synaptic inputs (Figure 5E). These neurons will be scatter sensitive. On the other hand,  
580 neurons with supralinear dendrites can also access the same class of computations either by  
581 using this strategy (Figure 5C) or by clustering functionally relevant inputs onto the same  
582 compartment (Figure 5C). Hence they can be either scatter or cluster sensitive. Thus, the final  
583 neuronal computations depend not only on the type of dendritic operation and the dendritic  
584 and spike thresholds, but also the *global* mapping of input features throughout the dendritic  
585 tree.

## 586 587 **Open questions** 588

589 To understand how a neuron integrates its synaptic input we need precise knowledge  
590 of the morphology, ion channel distribution along the tree, strength and time course of  
591 synaptic conductances carrying particular information features, the threshold and the spatio-  
592 temporal pattern of activation of the synapses carrying these features. Although we can  
593 determine most of these parameters, as we reviewed above, the most challenging experiments  
594 are those designed to estimate the spatio-temporal distribution of all synapses carrying  
595 relevant sensory features (i.e. a functional connectivity map). Strategies using injection of  
596 viral-based retrograde tracers (Marshall et al., 2010) are powerful for the identification of  
597 connected presynaptic cells, but these methods lack functional information about the precise  
598 information or features encoded in afferents. Using *in vivo*  $\text{Ca}^{2+}$  imaging, researchers have  
599 begun the herculean task of estimating how sensory features are mapped onto dendritic trees  
600 by examining how single synapses and dendrites respond to behavioral stimuli. It is not clear  
601 whether such feature mapping can be performed on the entire dendritic tree, but initial results

602 provide hints as to whether there may be general mapping rules. Some studies argue that  
603 features are clustered in single dendrites within the somato-sensory cortex (Takahashi et al.,  
604 2012), consistent with a **local computation strategy**, while other studies have shown that  
605 neighboring synapses onto layer 2/3 pyramidal neurons of the visual and auditory cortex  
606 respond maximally for activation of inputs carrying different sensory features (Chen et al.,  
607 2011; Jia et al., 2010), consistent with a **global strategy**. In light of the conclusions described  
608 here, both neurons could still be capable of performing similar linearly non-separable  
609 computations, provided the dendrites exhibit nonlinear operations.

610

611 Why might neurons use different dendritic operations and wiring strategies? It is  
612 conceivable that differences in timing of sensory development or optimal local circuit wiring  
613 may constrain wiring strategies for particular neurons. Thus to perform the same computation,  
614 different wiring and dendritic strategies are used. Global wiring strategies are more amenable  
615 to “random wiring,” in contrast to the specific connectivity required for engaging local  
616 strategies. We speculate that different dendritic operations may be implemented by neurons  
617 given certain biological constraints, such as limitations in the number and location of  
618 synapses carrying a particular feature, or spike threshold. For example when both principal  
619 neurons and interneurons receive a common set of input features along relatively fixed axonal  
620 projections, but are required to perform different computations, they may engage different  
621 dendritic operations. In the cerebellum interneurons have been shown to exhibit sublinear  
622 dendritic operations (Abrahamsson et al., 2012; Vervaeke et al., 2012) on their parallel fiber  
623 inputs, while Purkinje cells are thought to receive the same or similar features from the same  
624 set of input fibers, yet display supralinear dendritic operations (Rancz and Hausser, 2006).  
625 One could speculate that the different nonlinearities and synaptic placement strategies of  
626 Purkinje neurons and interneurons may enable them to implement complementary (i.e.  
627 opposing) computations, which ultimately could result in a microcircuit that is highly  
628 selective for specific input patterns.

629

630 What are the wiring rules? Three possible wiring strategies are 1) predetermined  
631 connectivity (genetically encoded), 2) random connectivity, and 3) activity dependent pruning  
632 and stabilization of connections. Although the exact contribution of each mechanism is yet to  
633 be determined, synaptic plasticity has been shown to modify and ultimately determine the  
634 functional connectivity. For example computational modelling showed that a local wiring  
635 strategy in which synapses carrying features of objects are clustered can be learned using  
636 simple plasticity rules (Legenstein and Maass, 2011). Experimental evidence supports this  
637 theoretical work, suggesting that activity-dependent, branch-specific plasticity strengthens  
638 clustered synaptic inputs and their compartmentalization (Makara et al., 2009; Makino and  
639 Malinow, 2011; Takahashi et al., 2012). On the other hand, synaptic plasticity could also  
640 reinforce global computational strategies. In cerebellar stellate cells, high-frequency firing of  
641 clustered inputs has been described to induce profound presynaptic short- and long-term  
642 synaptic depression (Beierlein and Regehr, 2006; Soler-Llavina and Sabatini, 2006). Such  
643 plasticity mechanisms would reinforce the neuron’s **scatter sensitivity**, and thus tends to  
644 optimize the output firing for specific spatially and temporally sparse synaptic activity  
645 patterns (Abrahamsson et al., 2012; Cazé et al., 2013).

646

647 Synchronized neuronal activity is known to cause oscillations of the dendritic voltage,  
648 which would inevitably reinforce electrical interactions between dendrites and thus alter the  
649 effective number of isolated dendritic subunits that contribute to the neuronal computation.  
650 For example, Remme et al. (Remme et al., 2009) showed theoretically that input-dependent  
651 synchronization of intrinsic dendritic voltage oscillations can facilitate a global voltage

652 propagation, even throughout highly distributed dendritic trees. It will be important to  
653 examine how local and global dendritic integration strategies might be influenced by brain  
654 oscillations, thus ultimately altering neuronal and even circuit computations.  
655

656 Since many types of interneurons are known to contact specific locations within the  
657 dendritic tree, inhibition will undoubtedly influence integration properties and information  
658 processing by neuronal circuits (as reviewed by (Palmer et al., 2012)). Nevertheless the  
659 experimental challenge is to determine not only the timing and location of inhibition within  
660 the dendrite, in order to determine their alteration of dendritic operations, but also whether  
661 particular features are conveyed similarly or differently by excitatory and inhibitory inputs.  
662 Although complex, the problem is critical to understanding brain function as the balance of  
663 excitation and inhibition is well known to be tightly regulated, with alterations being  
664 implicated in disease (Yizhar et al., 2011). Using the Boolean analysis of equivalent dendrites,  
665 one can deduce that negative weight associated with inhibition is capable of performing the  
666 Boolean NOT function. Such a function would enable a simple implementation of XOR  
667 computations, further expanding the number of computable linearly non-separable functions.  
668

## 669 **Summary**

670

671 In this review we described categories of biophysical and cellular mechanisms that  
672 influence dendritic operations: passive and active membrane properties of the dendritic tree,  
673 the time course and amplitude of synaptic activation, and finally the location and pattern of  
674 the activation of synaptic inputs. We showed how each of these parameters shapes and tunes  
675 the s/I/O. We briefly discussed techniques for the characterization of dendritic operations,  
676 including electrode-based methods to stimulate and/or record from dendrites, optical  
677 techniques to image dendritic activity or uncage neurotransmitter, and biophysical modeling.  
678 In order to link the major classes of dendritic operations (linear, sublinear and supralinear) to  
679 neuronal computations, we reviewed the use of binary models associated with Boolean  
680 analysis. This analysis provides insight into the types of computable neuronal functions, such  
681 as the object feature binding problem. We also reviewed how such functions can be  
682 implemented with either supralinear or sublinear dendrites depending on the spatial mapping  
683 of those features within the dendritic tree. Because the synaptic activity pattern ultimately  
684 determines the neuronal computations, we propose that the elemental computational unit is  
685 the neuron rather than the dendrite (Cazé et al., 2014). Although there are cases (local  
686 strategies) where dendritic operations can dictate the neuronal computation, dendritic  
687 operations must be studied and understood in the context of the knowledge of the wiring of  
688 specific features onto the dendritic tree.  
689



690 **Acknowledgements**

691 We thank members of the DiGregorio lab for helpful discussions and comments on the  
692 manuscript. ATVM and DD were supported by the French National Agency for Research  
693 (ANR-2010-BLANC-1411 and ANR-13-BSV4-0016) and the Fondation de Recherche  
694 Medicale (Team grant). ATVM was also supported by the Ecole des Neurosciences de Paris,  
695 a EMBO Long-Term Fellowship 1582-2011, a Roux-Howard-Cantarini post-doctoral  
696 fellowship and a Marie Curie Individual Fellowship 301362 within the 7th European  
697 Community Framework Program (FP7-PEOPLE-2011-IEF).

698

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966 **Figure legends**

967

968 **Figure 1. Dendritic operations and their influence on neuronal firing**

969 (A) Schematic diagram of a subthreshold synaptic input-output experiment in a neuron with  
970 supralinear dendritic compartments (*left*, supralinear compartments in green, linear  
971 compartments in black) or in a neuron with sublinear dendritic compartments (*right*, sublinear  
972 compartments in blue). The red spots are sites of synaptic activation or sites of glutamate  
973 uncaging.

974 (B) Arithmetic sum of individual responses to synaptic activation or uncaging.

975 (C) Somatic voltage responses evoked by simultaneous synaptic activation or uncaging.  
976 Green curves are responses evoked with increasing number of synapses activated within a  
977 supralinear dendrites. Blue curves are similarly obtained within a sublinear dendrite.

978 (D) Subthreshold input/output relationships (sI/O) used to quantify dendritic operations. The  
979 dashed line represents a linear relationship. Two horizontal dotted lines indicate two example  
980 somatic spike thresholds ( $\theta_1$  and  $\theta_2$ ).

981 (E, F) Example of synaptic integration of three synaptic inputs distributed across the dendritic  
982 tree (E) or clustered on a single dendritic branch (F) of a neuron with supralinear dendritic  
983 compartments (*left*) or sublinear compartments (*right*). The output spike train, and hence  
984 neuronal computation, differs depending on the threshold. The more depolarized threshold  
985 value ( $\theta_1$ ) allows the neuron with supralinear dendrites to exhibit a cluster-sensitive neuronal  
986 computation (fires only when three inputs are activated in the same compartment). The  $\theta_1$   
987 threshold also allows a neuron with sublinear dendrites to exhibit scatter-sensitive neuronal  
988 computations. The lower threshold ( $\theta_2$ ) imparts a different neuronal computation based on  
989 simple linear summation and is not sensitive to activated synapse location.

990

991 **Figure 2. Theoretical basis for sublinear summation within passive dendrites**

992 (A) Equation (1) describes the different current components underlying an EPSP in a single  
993 electrical compartment. Integration of this equation describes the variation of the membrane  
994 voltage over time. The transient local dendritic EPSP ( $V_m$ , *bottom trace*) sets the amplitude  
995 and time window over which the driving force ( $\Delta V = V_m - E_{syn}$ ) is altered. At the peak of the  
996 EPSP (solid blue arrow) the driving force is maximally reduced, and then recovers back to  
997 that at resting membrane potentials during the EPSP decay (dotted blue arrow). The reduced  
998 driving force decreases the synaptic current, and hence the net depolarization, creating a  
999 sublinear relationship between EPSP and its underlying conductance.

1000 (B) Equivalent circuit for dendritic cables, where  $g_m$  and  $c_m$  are the membrane conductance  
1001 and capacitance, respectively, and  $r_a$  is the axial resistance of a unit of cable. A synapse is  
1002 represented in the circuit ( $G_{syn}$  and  $E_{syn}$ ). For an infinite cable, the spatio-temporal distribution  
1003 of voltage is described by the relation (2), where  $\tau$  is the membrane time constant, and  $\lambda$  is  
1004 the length constant. The length constant relationships are derived from solving the cable  
1005 equation (2) for step changes in membrane voltage ( $\lambda_{DC}$ ) or for a sinusoidal membrane  
1006 potential change ( $\lambda_{AC}$ ). The latter is helpful to understand the dendritic filtering of transient  
1007 EPSPs. Equation (3) expresses the input resistance  $R_D$  for an infinite cable.

1008 (C) *Top*, ball-and-stick model of a neuron with colored arrows indicating the location of three  
1009 synapses. The graph above the diagram represents the peak amplitude of a dendritic EPSP as  
1010 a function of distance. *Bottom*, the two graphs describe respectively the dendritic and somatic  
1011 depolarizations in response to individual (colored lines) or combined synaptic inputs (black  
1012 lines). Concomitant activation of two neighboring synaptic inputs (within  $\sim \lambda_{AC}$ ) will  
1013 therefore mutually reduce their driving force and sum sublinearly (for example synapses 1 and  
1014 2, solid black trace for the EPSP observed in response to their simultaneous activation, dashed



1015 black trace for the arithmetic sum of the individual EPSPs). Distant synapses will, however,  
1016 sum more linearly (synapses 1 and 3, gray trace).

1017

### 1018 **Figure 3. Contribution of dendritic and synaptic properties to EPSP summation**

1019 (A) Influence of morphological parameters dendritic diameter (*left*) location along the  
1020 dendrite (*middle*) and dendritic branching (*right*) on the dendritic sI/O. The inserts illustrate  
1021 the effect of morphology on somatic EPSPs under the different conditions. Synapse location  
1022 and traces are color coded. Dashed line shows a linear I/O for reference.

1023 (B) The role of ion channels on the shape of the sI/O, illustrated by adding to a passive  
1024 dendrites (*blue curve*) either  $K^+$  channels, (*orange curve*),  $Na^+$  channels (*green curve*),  
1025 VGCC (*pink curve*), or NMDA receptors (*sky blue curve*).

1026 (C) Example of sI/O in three realistic combinations: thick ( $>2\mu m$ ) dendrites with active  
1027 conductances (blue curve, as in Branco et al., 2011), thinner dendrites with active  
1028 conductances (brown curve,  $<1\mu m$ , (Losonczy and Magee, 2006)), or thin dendrites with only  
1029 passive properties (green curve, (Abrahamsson et al., 2012)).

1030 (D) Influence of synaptic properties on the sI/O. An increase in synaptic strength makes the  
1031 sI/O steeper (*left*), whereas increasing the interval or the distance between synaptic inputs  
1032 tends to linearize the curve (*right*).

1033

### 1034 **Figure 4. Using Boolean algebra to analyze binary neuron models with dendritic 1035 nonlinearities**

1036 (A) Truth tables for the Boolean functions AND, OR and XOR for two synaptic inputs ( $x_1$  and  
1037  $x_2$ ). The two colored horizontal lines illustrate how the AND and OR functions are linearly  
1038 separable, i.e. a single line divides all inputs between two groups, one group having an output  
1039 of 0 and the other group having an output of 1. Neuron output binary value is denoted as  $y$ .

1040 (B) Threshold linear unit model neuron with two inputs. The weight of each input is  
1041 represented by the area of the black disc drawn between the input and the model neuron. Here  
1042 all weights are equal to 1. A spiking threshold ( $\theta$ ) of 2 allows the model neuron to compute  
1043 the AND function (*left*), whereas if  $\theta = 1$  the neuron computes the OR function (*right*).

1044 (C) *Top*, Simplified representations of a supralinear sI/O (*left*) and its mathematical  
1045 approximation by a Heaviside function (*right*) with a height  $h$  and a threshold  $\theta$ . *Bottom*,  
1046 simplified representation of a sublinear sI/O and its mathematical approximation by a  
1047 piecewise linear then saturating function.

1048 (D) Generalized diagram representing a two-layer integration model neuron with several  
1049 compartments and  $n$  inputs. Each branch represents a dendritic compartment, and the  
1050 integration operation performed by this compartment is represented by the box on the branch.  
1051 This box contains the values of  $\theta$  and  $h$  if the operation is nonlinear. The result from the  
1052 integration from each branch is then linearly summed and compared to the somatic spike  
1053 threshold  $\Theta$ .

1054 (E) Implementation of the (partial) feature binding problem (pFBP) by binary neurons with  
1055 two compartments  $D_1$  and  $D_2$ , either supralinear or sublinear. *Top*, truth table describing  
1056 various input feature combinations, the response of each dendritic compartment,  $D$   
1057 (0:inactive/1:active), and the final neuronal output,  $y$ . Columns with green shading are the  
1058 outputs of dendrites exhibiting supralinear operations, while columns shaded in blue contain  
1059 outputs of dendrites that exhibit sublinear operations. *Bottom*, Model neuron with equivalent  
1060 dendrite representation that can implement the pFBP using supralinear (*left*) or sublinear  
1061 dendritic compartments (*right*). The threshold and the height of the nonlinearity are indicated  
1062 within the box. If dendritic integration is supralinear, two groups of inputs are needed to  
1063 activate a compartment, and a single compartment can trigger a spike. If dendritic integration

1064 is sublinear, a single input can activate the dendritic compartment and the two compartments  
1065 must be active to trigger a spike.

1066

1067 **Figure 5. Computing a linearly non-separable function (full FBP) with supralinear and**  
1068 **sublinear dendrites and using local vs. global synaptic wiring strategies**

1069 (A) *Left*, model neuron with equivalent dendrite representation of two compartments, linear  
1070 (black) and supralinear (green), and a clustered distribution of object features (local strategy).  
1071 *Right*, schematic representations of synaptic placements equivalent to the model on the left.

1072 (B) *Left*, model neuron with equivalent dendrite representation of two compartments, linear  
1073 (black) and sublinear (blue), and a distributed placement of inputs carrying object features.

1074 (C-E) Implementation of the full FBP ( $y=1$ ; “apple shape and red” or “banana shape and  
1075 yellow”).

1076 (C), implementation of the full FBP using a model with a supralinear compartment and a local  
1077 wiring strategy. Inactive inputs are represented in light gray and the corresponding feature in  
1078 lighter color.

1079 (D) Implementation of the full FBP ( $y=1$ ) using a model with a supralinear compartment and  
1080 a global wiring strategy. The area of the disc adjacent to a compartment next to each object  
1081 feature represents the relative weight of this feature. Here the relative weights used are of 1  
1082 and 2.

1083 (E) Implementation of the full FBP ( $y=1$ ) using a model with sublinear compartment and a  
1084 global wiring strategy.

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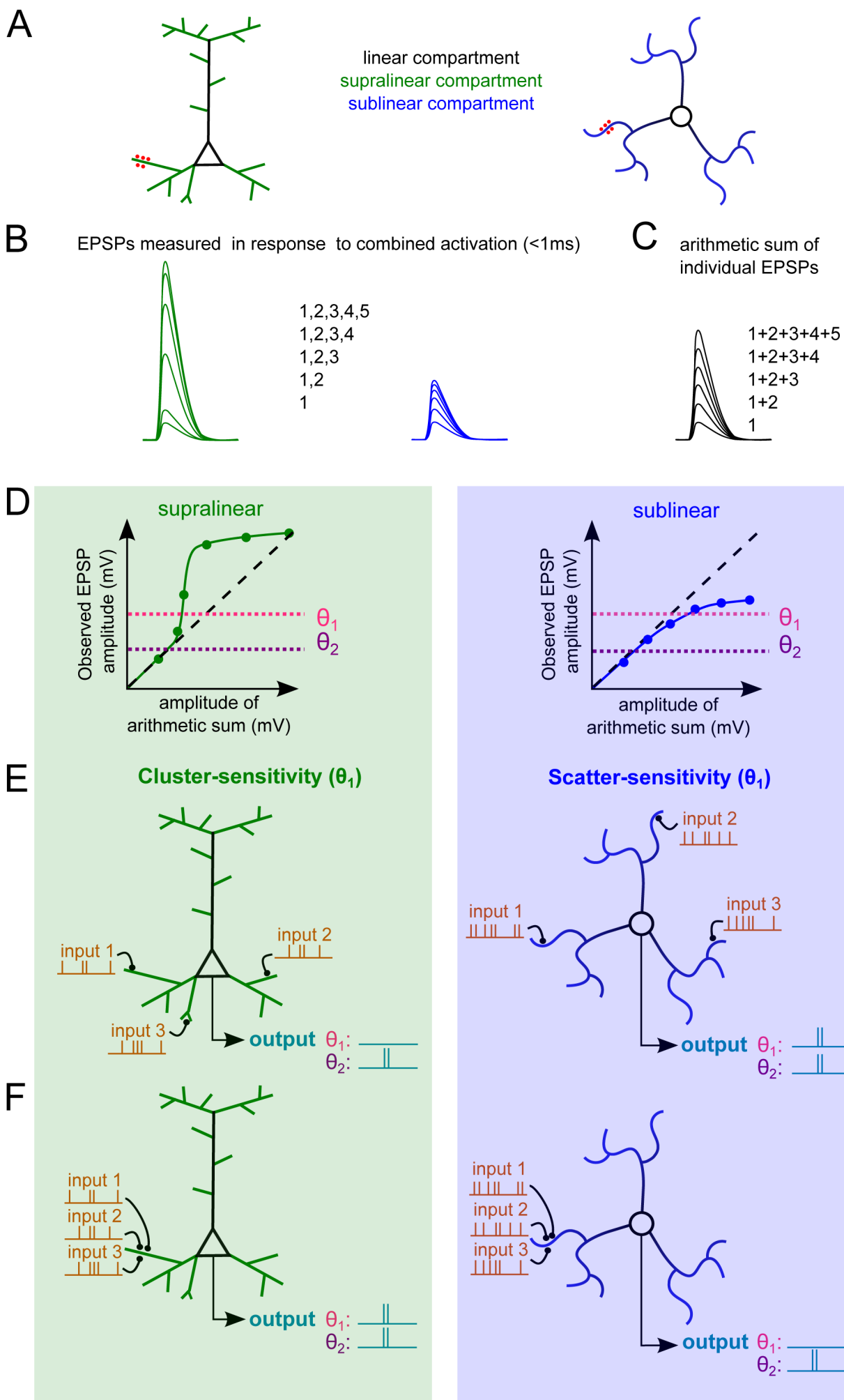
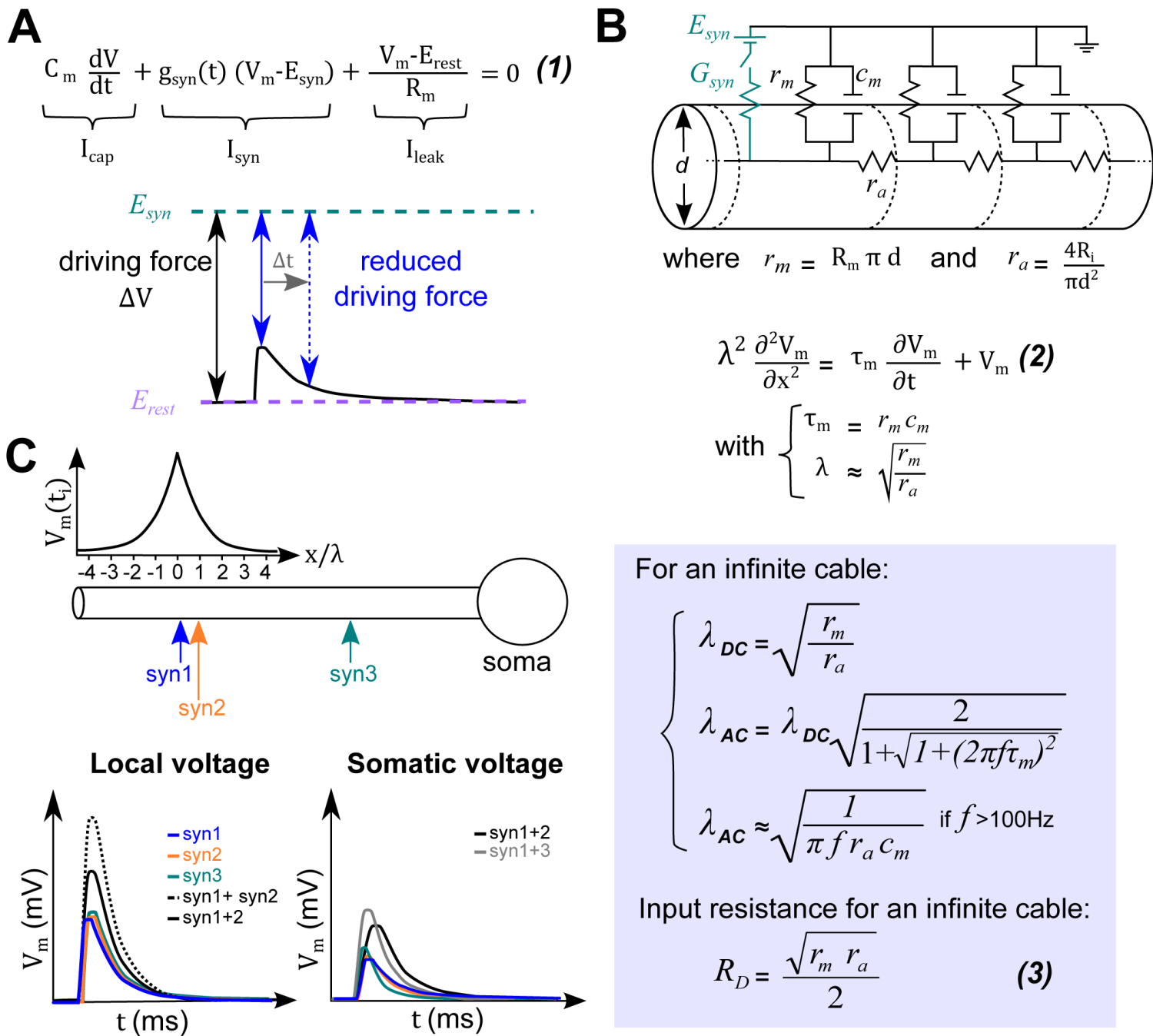


Fig. 2



For an infinite cable:

$$\begin{cases} \lambda_{DC} = \sqrt{\frac{r_m}{r_a}} \\ \lambda_{AC} = \lambda_{DC} \sqrt{\frac{2}{1 + \sqrt{1 + (2\pi f \tau_m)^2}}} \\ \lambda_{AC} \approx \sqrt{\frac{1}{\pi f r_a C_m}} \text{ if } f > 100\text{Hz} \end{cases}$$

Input resistance for an infinite cable:

$$R_D = \frac{\sqrt{r_m r_a}}{2} \quad (3)$$

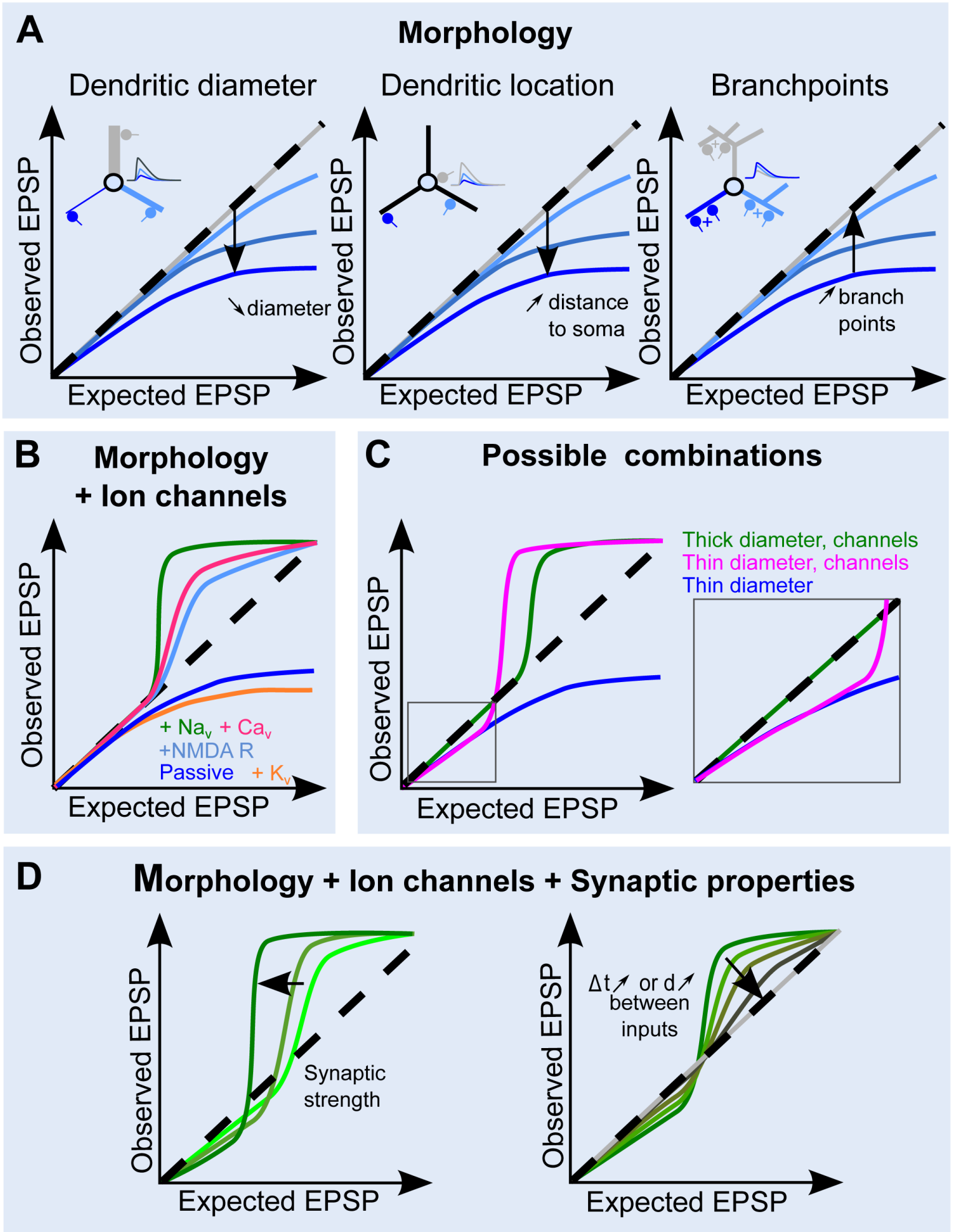
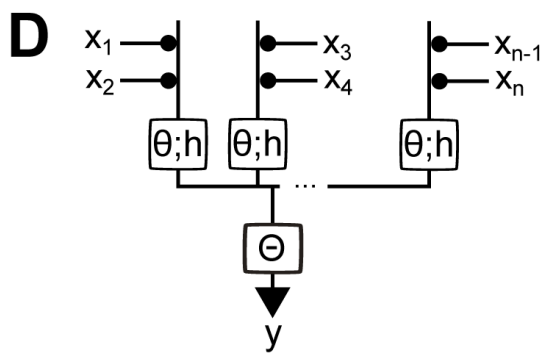
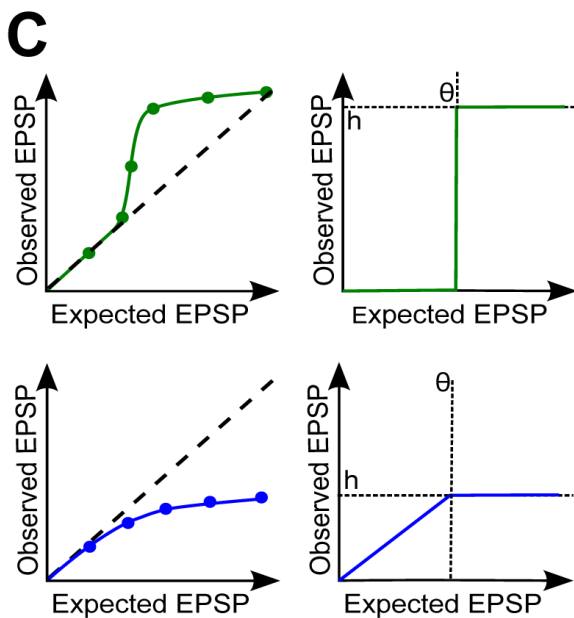
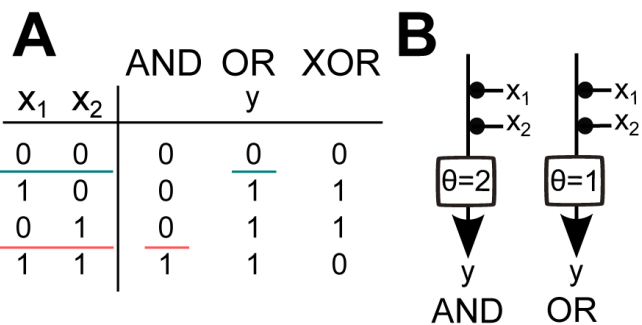


Figure 4.TIF

Fig. 4



**E**

					$D_1$	$D_2$	$D_1$	$D_2$	$y$
	1	1	0	0	1	0	1	1	1
	0	0	1	1	0	1	1	1	1
	0	1	1	0	0	0	1	0	0
	1	0	0	1	0	0	0	1	0

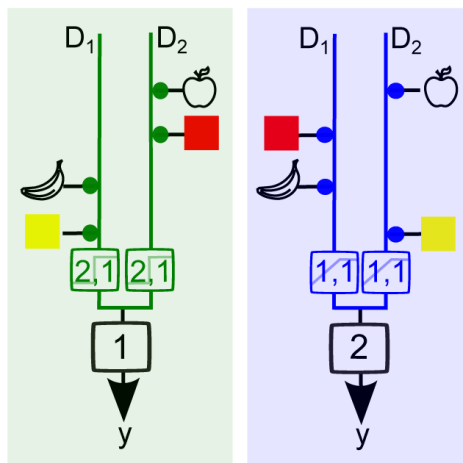
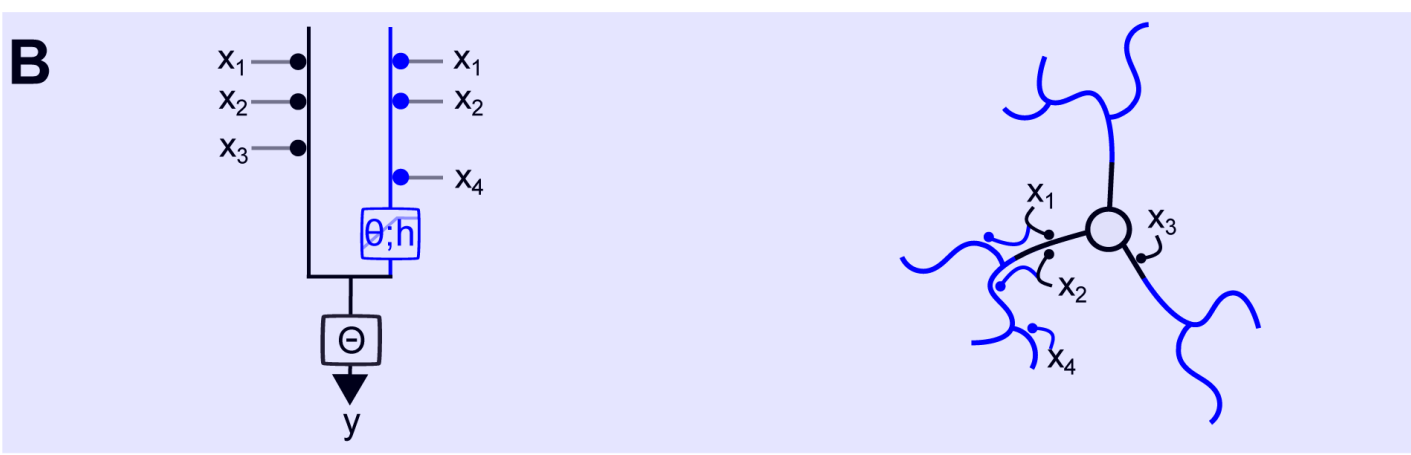
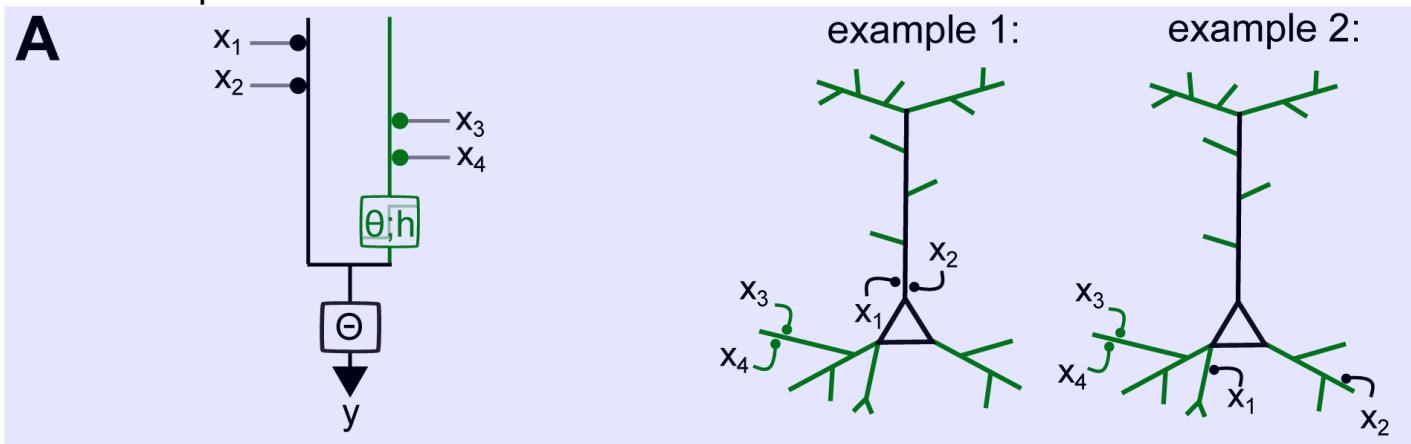


Fig. 5

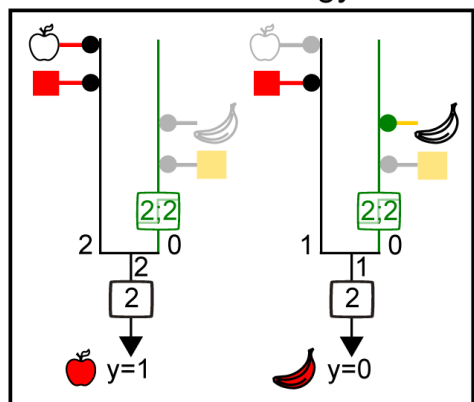
Model neuron with equivalent dendrites

Schematic representation of neurons and inputs

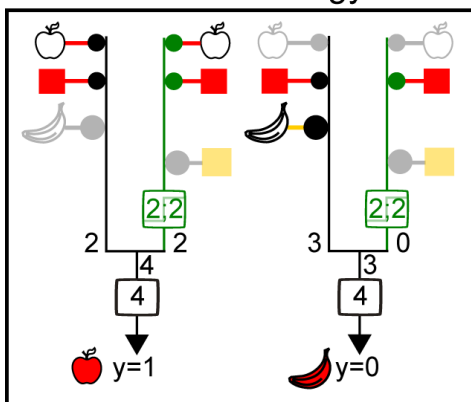


Implementations of (🍏 AND 🍓) OR (🍌 AND 🍌)

**C** Supra-linear compartment, Local strategy



**D** Supra-linear compartment, Global strategy



**E** Sublinear compartment, Global strategy

