

Physical modeling of resonance phenomena in the long wave dynamics

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ABSTRACT. – We have prepared two sets of experiments in a wave flume to model effects occurring in nature and to demonstrate resonance phenomena in laboratory conditions. The first set was performed to investigate non-linear wave run-up on the beach caused by harmonic wave maker located at some distance from the shore line. It is revealed that under certain wave excitation frequencies a significant increase in run-up amplification is observed [Ezersky *et al.* 2013]. It is found that this amplification is due to the excitation of resonant mode in the region between the shoreline and wave maker. Frequency and magnitude of the maximum amplification are in good correlation with the numerical calculation results represented in the recently published paper [Stefanakis *et al.* 2011]. The second set of experiments was performed to study resonance effects due to parametric excitation of edge waves. It is known that surface waves propagating toward the shore can excite edge waves propagating along the shore line. Although the edge wave amplitude decreases in an offshore direction they may contain enough energy to be responsible for erosion of the shore and generate so-called cusps [Buchan *et al.* 1995]. We investigate parametric mechanism of such generation when plane surface wave with frequency W excite edge wave with frequency $W/2$. It is show that parametric generation of edge waves can amplify run-up up to two times.

Key-words: edge waves, wave breaking, parametric instability, run-up, turbulent viscosity

Modélisation physique du phénomène de résonance dans la dynamique d'onde longue

Nous avons préparé deux séries d'expériences dans un canal à houle pour modéliser physiquement les effets que se produisent dans la nature et démontrer les phénomènes de résonance dans des conditions en laboratoire. La première série a été réalisée pour étudier le jet de rive (run-up) d'une onde non linéaire générée par un générateur de ondes harmoniques située à une certaine distance de la ligne de rivage. Il est révélé que, pour certaines fréquences d'excitation d'onde, une augmentation significative de l'amplification du run-up est observée [Ezersky *et al.* 2013]. On constate que cette amplification est due à l'excitation du mode de résonance dans la région entre la rive et le générateur d'ondes. La fréquence et l'amplitude de l'amplification maximale sont en bonne corrélation avec les résultats de simulations numériques représentés dans le papier récemment publié [Stefanakis *et al.* 2011]. La deuxième série d'expériences a été réalisée pour étudier les effets de résonance due à une excitation paramétrique des ondes de bord.

On sait que les ondes de surface se propageant vers le rivage peuvent exciter des ondes se propageant le long des bords de la ligne de rivage. Bien que l'amplitude des ondes de bords diminue vers le large, elles peuvent contenir assez d'énergie pour être responsable de l'érosion de la rive et de générer des croissants de plages [Buchan *et al.*, 1995].

Nous étudions le mécanisme paramétrique d'une telle génération lorsque des ondes de surface avec une fréquence W excitent des ondes de bords avec une fréquence $W/2$. Il est de montrer que la génération paramétrique des ondes de bord peut amplifier le run-up jusqu'à deux fois.

Mots-clés : vagues de bords, déferlement, l'instabilité paramétrique, run-up, viscosité turbulente

I. INTRODUCTION

The resonance phenomena plays significant role in the amplification of the long surface waves, especially tsunami waves, in coastal areas leading to the long-time weakly damped water oscillations, late approach of a maximal amplitude wave comparing with leading waves, grouping structure of tsunami waves. Recent huge tsunamis demonstrate the nonlinear behaviour on the coast leading to the strong impact. It was also revealed recently that the number of abnormally large and suddenly appearing waves (rogue waves) observed in the coastal zone is sufficiently larger than Gaussian statistics predicts [Nikolkina & Didenkulova

2011, Nikolkina & Didenkulova 2012]. Analysis of tsunami records showed that reflections due to bottom topography may result in appearance of resonant mode in coastal zone, see for instance Neetu *et al.* [2011]. The study of the tsunami and coastal rogue waves is based on the nonlinear theory of shallow water [Kharif *et al.* 2009; Didenkulova *et al.* 2011; Slunyaev *et al.* 2011]. To characterize the impact of waves on coastal infrastructure, the systematic study of run-up processes is undertaken and a lot of papers summarizing the progress in the analytical solutions of the nonlinear shallow water theory have been published by now (see for instance [Pelinovsky 1982; Synolakis 1987; Pelinovsky *et al.* 1992; Carrier *et al.* 2003].

Recently, [Stefanakis *et al.* 2011; Ezersky *et al.* 2013] on the basis of numerical simulations of the nonlinear shallow-water equations, it was pointed out the existence of resonance effects in the process of the long wave run-up. It should be noted that such resonance effect was predicted in [Antuono *et al.* 2010] in the framework of linear theory. The main result [Stefanakis *et al.* 2011] is that at a certain frequency of the waves there exists a significant increase in the run-up amplitude. According to calculations, the maximal run-up height can be 50 times greater than the free surface oscillation amplitude used as the boundary conditions in the numerical calculations. It was established in [Stefanakis *et al.* 2011] that the wave period for which maximal run-up amplification is observed depends on the slope of the bottom and the depth of water in the place where the waves are excited. This period is much larger than the “natural period” – time needed for perturbations to run from the point of excitation to the shoreline and return back. Results obtained in [Stefanakis *et al.* 2011] pose a lot of questions. That is why, we carried out a physical simulation of this process in the wave flume with an inclined bottom [Ezersky *et al.* 2013].

It is known that in the coastal zone, waves coming from the open sea can excite so-called edge waves which are localized near the shore [Johnson R.S. 2005; Johnson R.S. 2007]. The edge wave field can not be represented in the dimensional approximation: edge waves propagate along coastal line and their amplitude decreases in off shore direction. Characteristics of linear and nonlinear edge waves were studied in numerous theoretical papers [Akylas 1983; Dubinina *et al.* 2004; Grimshaw 1974; Kurkin *et al.* 2002; Minzoni *et al.* 1977; Pelinovsky *et al.* 2010]. Characteristics of edge wave are also investigated in marine experiments and numerical simulations. These studies focus on investigation of edge waves excitation in coastal zone and correlations between characteristics of edge waves and the spectra of waves propagating toward the shore. The edge waves localized at the shoreline may contain enough energy to be responsible for the erosion of the shore and the generation of the so-called cusps [Blondeaux *et al.* 1995; Carter 2002; Coco 2003]. Therefore, investigation of the generation mechanisms of edge waves and studying of run-up caused by them are important problems of wave - coastal zone interaction.

By now, one mechanism of edge wave generation is investigated very widely. This mechanism is connected with

parametric excitation of standing edge wave with frequency $w/2$ by surface wave with frequency w propagating perpendicular a shore line. Such mechanism was investigated theoretically [Blondeaux *et al.* 1995; Guza *et al.* 1974] and it was identified in marine experiments in coastal zone [Huntley *et al.* 1978]. The laboratory experiments on parametric excitation of edge wave are described in [Buchan *et al.* 1995]. The laboratory study of edge wave parametric excitation in the hydrodynamic flume allowed them to choose plane bottom and explore the simplest spectrum of waves propagating toward to the shoreline. It should be noted that in this laboratory experiments wave breaking was absent meanwhile breaking effects are important in natural conditions. Principal question of the influence of wave breaking on parametric edge wave generation is not investigated yet. Exactly this problem is investigated in our paper. We concentrate on study of influence of wave breaking on characteristics of the edge waves and on run-up amplification occurred in this case.

The paper is organized as follows. Section 1 is devoted to the description of the experimental setup, section 2 presents the results of measurements of resonance phenomena and excitation of edge wave. In section 3, we discuss the experimental data and present a theoretical model to describe the modes of parametric excitation of edge waves and section 4, conclusion.

II. EXPERIMENTAL SETUP

Experiments were performed in the wave flume of the Laboratory of Continental and Coastal Morphodynamics in Caen. This flume has a length of 18 meters, a width of 0.5 m. The flume is equipped with a wave-maker controlled by computer. To simulate an inclined bottom, a PVC plate with thickness of 1 cm is used. The plate is placed at different angles relative to the horizontal bottom of the flume in the vicinity of the wave maker (see fig. 1). Three series of experiments have been performed for water depth h near the wave maker and length L , ($h_0 = 0.245$ m, $L = 1.458$ m, $\tan\alpha = 0.168$) ($h_0 = 0.26$ m, $L = 1.35$ m, $\tan\alpha = 0.192$) and ($h_0 = 0.32$ m, $L = 1.21.5$ m, $\tan\alpha = 0.263$). Three resistive probes (P1, P2, P3, fig. 1) are used to measure a displacement of water surface. First of them, P1 is placed at the distance of 1 cm from the wave maker. The probes P2 and P3 representing thin copper strips are glued on the inclined

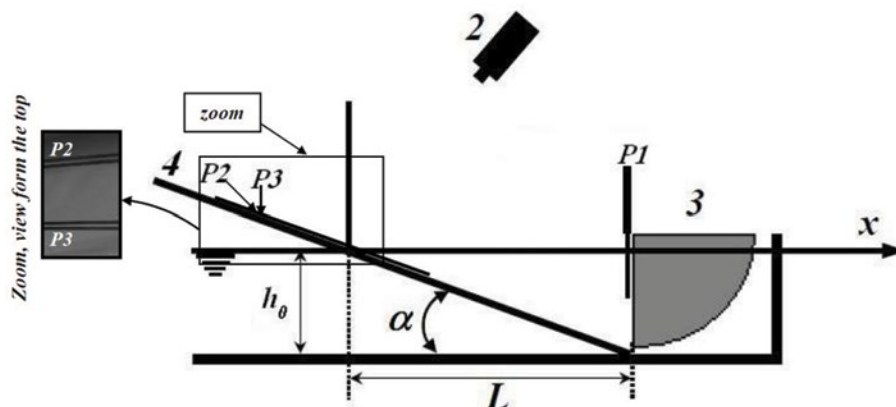


Figure 1: Schema of experiment: resistive probes: vertical probe (P1) and inclined probes (P2, P3), high-speed video camera (2), wave maker (3), inclined bottom (4).

bottom. These probes allow us to record run-up at two different points on the plat. Besides, run-up characteristics are determined by processing a movie which is shot by a high-speed camera mounted as shown in fig. 1. These characteristics are determined within a precision of 2%. Wave-maker allows us to excite harmonic wave of a given frequency and it works in two regimes: displacement-control and force-control. It is not possible to control free surface displacement, as it was done in the numerical experiment. That is why to study the run-up amplification, simultaneous measurements of the amplitude of free surface displacement near the wave maker and maximal run-up are carried out for different frequencies of excitation.

III. RESULTS OF MEASUREMENTS

III.1. Resonance phenomena

Frequency dependence of the amplitude of free surface displacement near the wave maker (a), maximal run-up (R) and coefficient of run-up amplification (C , $C = R/a$) are shown in fig. 2 for the slope of the bottom $\tan\alpha = 0.263$. The amplitude of free surface displacement has peaks at frequencies $f_1 = 0.44$ Hz and $f_2 = 0.78$ Hz. It is resonance frequencies of the system. The maximal run-up does have sharp peaks, only small increase in the vicinity of f_1 and f_2 is observed (fig. 2b), but coefficient of run-up amplification (fig. 2c) increases very sharply in the vicinity of $f_3 = 0.28$ Hz and $f_3 = 0.63$ Hz. It is evident that maximal amplification of run-up is observed for frequencies corresponding to the minimal amplitude a . In the vicinity of the wavemaker the amplitude is sufficiently small and the signal is very noisy. That is why the coefficient of run up amplification requires rather delicate measurements of free surface displacement: a band-pass filter was used to measure the amplitude of harmonic corresponding to wave-maker forcing. It is important to note that frequency of maximal amplification does depend on method of wave excitation. Results presented in fig. 2 were obtained for force-controlled regime of wave maker; the same results for coefficient of run-up amplification were obtained for displacement control regime.

Amplification coefficient C was investigated for three bottom inclinations. Frequencies of maximal amplification depend on angle a and to compare results obtained for different angles a , the non-dimensional frequency F was introduced:

$$F = f / f_0, f_0 = K^{-1}(g / h_0) \tan \alpha, K = 5.23$$

where g is for acceleration of gravity, h_0 is for water depth near wave maker.

Results are presented in fig. 3. Non-dimensional frequencies of maximal run-up amplification $F = F_1 = 1$ for different angle a coincide very precisely. The coefficient of maximal amplification, corresponding to the frequency $F_1 = 1$ is approximately the same for different inclinations: $C \approx 20 - 25$. The second pick of run-up amplification coefficient is observed for frequency $F_2 = (2.2 - 2.3)F_1$. Non-dimensional frequency F_2 slightly depends on bottom slope; small pick is observed also for frequency $F_3 \approx 3.5F_1$.

It should be noted that for our experimental conditions, linear run-up is observed for small frequencies of wave excitation $F < 2$, while for higher excitation frequencies $F > 2$ near surface wave becomes strongly nonlinear and run-up occurs after the wave breaking. The wavebreaking does not prevent

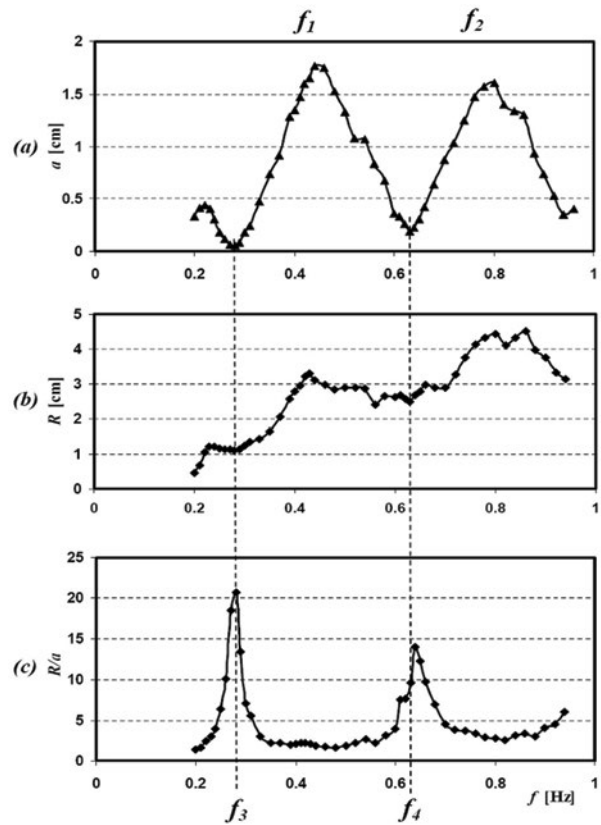


Figure 2: Frequency dependence of amplitude on free surface displacement (resonance curve) (a), maximal run-up (b) and amplification of run-up (ratio of maximal run-up and amplitude of surface wave) (c) for slope $\tan\alpha = 0.263$.

precise determination of maximal run-up position. Excepting high frequencies $F > 3$ the border of the water on slop beach was one dimensional and maximal run-up did not depend on coordinate along direction perpendicular to axis x .

III.2. Parametric excitation of edge waves

Wave-maker allows us to excite harmonic wave propagating towards the shore with controlled amplitude and frequency and to study the characteristics of edge waves using

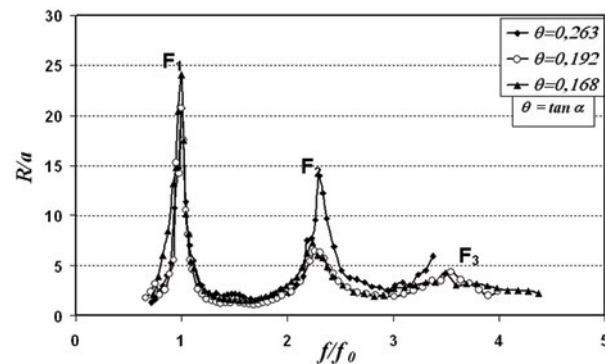


Figure 3: Dependence of run-up amplification on normalized frequency for different bottom slopes. Frequency. $f_0 = 5.23\sqrt{g / H \tan \alpha}$.

simultaneous measurements of the amplitude of free surface displacement near the wave maker and run-up height.

Sub-harmonic instability is investigated in the flume for different amplitudes and frequencies of wave maker. To understand if instability occurs or not, signals at probes P2 and P3 are analysed. Signals from P2 and P3 are presented in fig. 4. At the beginning of perturbation evolution time series is presented in fig. 4, left block. One can observe that signals at two probes have the same frequency as the one of the wave maker ($f = 1.06 \text{ Hz}$) and are in phase. On the background of oscillations with frequency of wave maker, sub harmonic oscillations (with frequency $f = 0.53 \text{ Hz}$) appear. Filtering of signal shows that amplitude of sub harmonic at initial stage grows exponentially from very small value and asymptotically approaches a constant value at large time. For developed parametric instability, the period of oscillations is two times more than period of wave maker. The phase difference between the two signals measured by probes P2 and P3 is equal π (fig. 4, right block). It means that sub-harmonic generation of edge mode (standing wave) is observed in the flume. The motion of shoreline due to edge wave excitation is registered by camera. Analysis of movies shows that sub-harmonic oscillations represent the first mode: maxima of horizontal displacement (anti nodes) are near the lateral walls of the flume, zero displacement (node) is at the center of the channel. Instability begins with the exponential growth of small perturbations. In some cases noise perturbations were so small that instability cannot be observed for reasonable time. In this case we introduced small perturbations artificially by an oscillating plate for several seconds in the vicinity of coastline. Instability occurs if the frequency of excitation is close to double frequency of the first edge mode. Example of instability is shown in fig. 4.

To describe the instability in the system, partition of plane (a_L, f) into regions with different stability is performed. Results are presented in fig. 5 (a_L is for amplitude of free

surface displacement near the wave maker, f is for wave maker frequency).

Amplitude of edge waves bifurcating from the zero value grows continuously with the amplitude surface wave a_L near the wave maker, if it exceeds the critical value. The growth of wave amplitude leads to appearance of wave breaking in surface waves propagating along the flume. When wave breaking is developed sub harmonic instability is suppressed.

The run-up amplitudes before and after the development of parametric instability are also measured. Results are presented in fig. 6. This figure demonstrates dependence of run-up amplitude due to edge waves on amplitude of run-up of surface waves, exciting parametric instability. One can see that for small amplitude of parametric excitation, amplitude of run-up increases in two times. Amplification decreases for large amplitude of excitation when wave breaking appears.

IV. DISCUSSION OF EXPERIMENTAL RESULTS

IV.1. Resonance phenomena

To study frequency dependence of run-up amplification more precisely, the spatial structures of the free surface oscillations occurring at frequencies corresponding to the resonant frequencies of the system (f_1, f_2) and at frequencies of maximum run-up amplification (f_3, f_4) have been investigated. The results are shown in fig. 7 for bottom slope $\tan\alpha = 0.168$. Amplitude and phase of free surface displacement are shown by rhombs and circles. Experimental data are compared with well know analytical solution for free surface displacement η ,

$$\eta = J_0 \left(\sqrt{\frac{4\omega^2 x}{g \tan \alpha}} \right) \cos(\omega t) \quad (1)$$

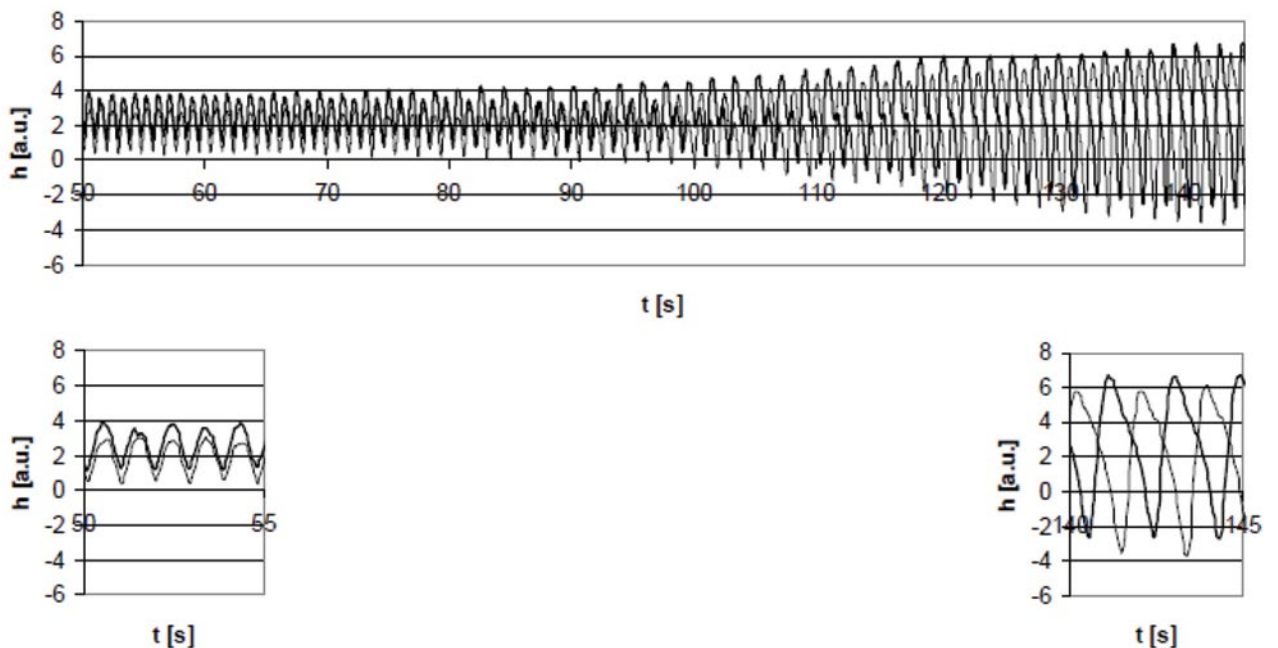


Figure 4: Temporal evolution of small initial perturbations. $f=1.06\text{Hz}$, $a_L=0.43 \text{ cm}$: time series recorded at time interval $50 \text{ s} < t < 145 \text{ s}$. At the beginning of perturbation evolution ($50 \text{ s} < t < 55 \text{ s}$) time series recorded by probes P2 (thin curve) and P3 (thick curve) have the same phase (left block), for developed parametric instability ($140 \text{ s} < t < 145 \text{ s}$) period doubling is observed and phase difference between signals recorded by probes P2 and P3 is π (right block).

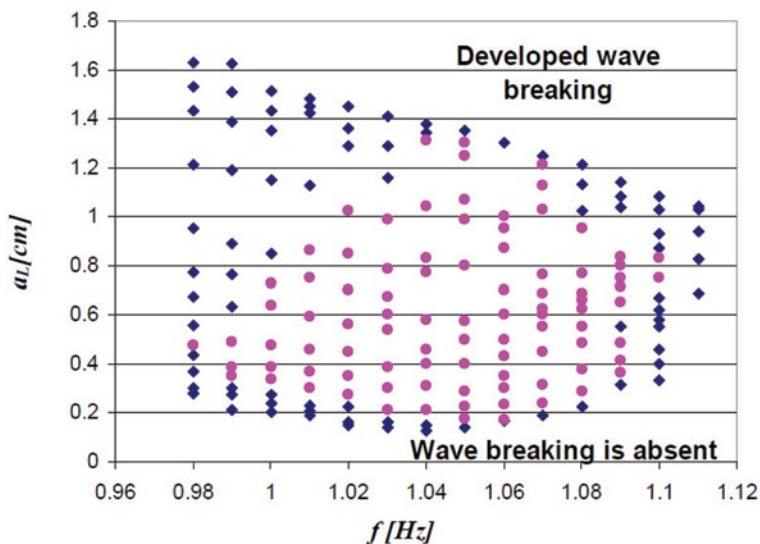


Figure 5: Diagram of stability/instability regimes. Circles correspond to parametric instability, diamonds correspond to stability regimes.

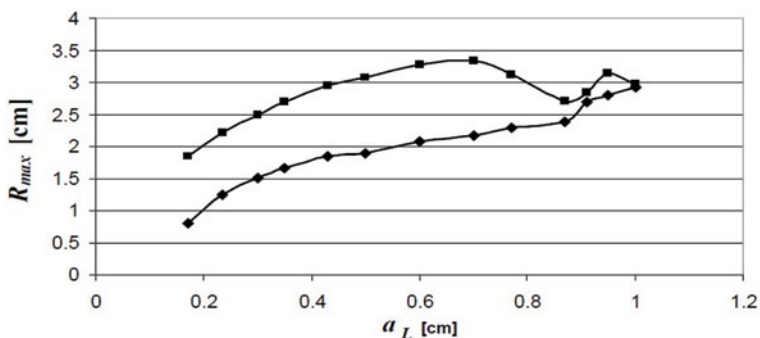


Figure 6: Dependences of run-up amplitude on wave amplitude near wave maker ($f = 1.06$ Hz) without parametric excitation of edge waves (diamantes) and with parametric excitation of edge waves (squares).

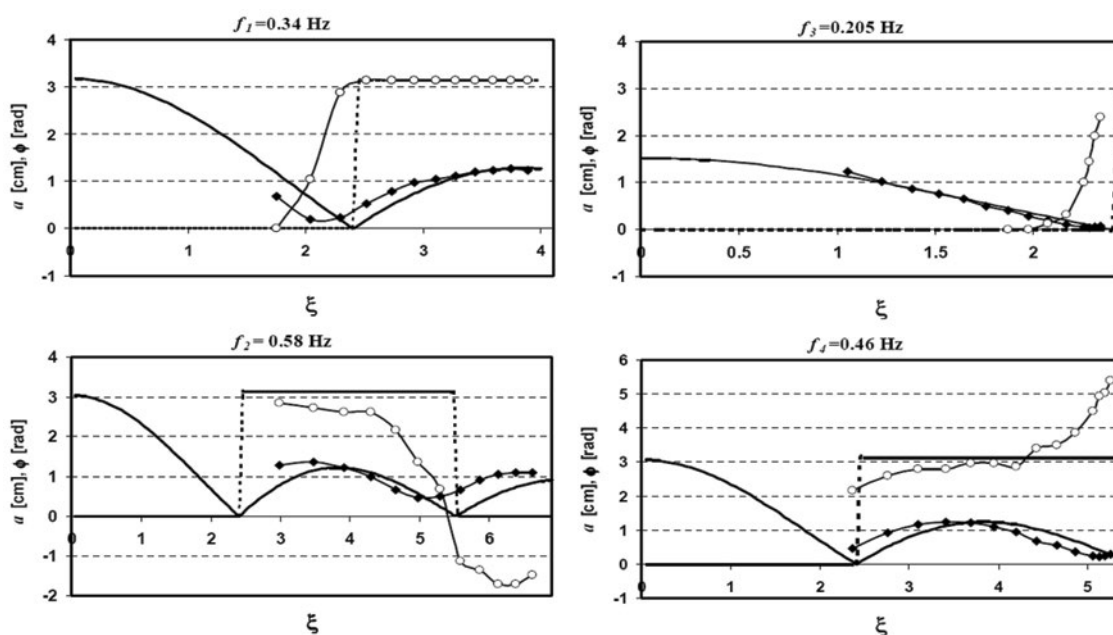


Figure 7: Comparison of the experimental values of amplitude (diamonds) and phase (circles) with theoretical values of amplitude (thick solid lines) and phase (thick dashed lines) obtained from the equation $\xi = (4\omega^2x/g.tan\alpha)^{1/2}$, $\tan\alpha = 0.168$; ends of horizontal axes correspond the positions of the wave-maker edge.

This solution was obtained in a shallow water with linear increasing of water depth h_0 : $h_0 = \tan \alpha x$. Theoretical dependences are shown in fig. 7 by thick lines. The amplitude is chosen as $a = |J_0|$, and $\phi = 0$ if $J_0 > 0$ and $\phi = \pi$ if $J_0 < 0$. One can find in fig. 7 that in the experiment the amplitude does not go to zero and phase changes smoothly for all frequencies. Note that frequencies of maximal run-up amplification ($f_3 = 0.205$ Hz, $f_4 = 0.46$ Hz) correspond to spatial modes having minimal amplitudes near the wave maker; resonance frequencies (f_1, f_2) have maximum amplitude of free surface displacement near the wave maker. It should be noted that according solution (1), frequencies of maximal run-up amplification correspond to the spatial modes with boundary condition $\eta|_{x=L} = 0$, and resonant frequencies correspond to mode with boundary conditions:

$$\frac{\partial \eta}{\partial x} \Big|_{x=L} = 0.$$

In other words, if one uses linear solution (1), the coefficient of run-up amplification in this approximation would be infinite: $a = 0$ at $x = L$. In experiment amplitude is small, but finite. Comparison of curves presented in fig. 7 shows that difference between theoretical solution and experimental data increases with frequency of excitation. For example, these differences much more for f_2 than for f_3 .

Let us compare the experimental results with numerical simulations [Stefanakis *et al.* 2011]. In the experiment, unlike the numerical calculations, it is not possible to introduce the waves with fixed free surface displacement at definite coordinate. In our opinion this point is not principal. Instead it, the simultaneous measurements of the free surface displacement and maximal run-up have been performed. In our experiment the frequencies of maximal run-up amplification are very close to those that were obtained in the numerical calculation. We estimated the frequencies of the first pick as: $f_3 = K^{-1} (g / h_0)^{1/2} \tan \alpha$; $K \cong 5.23$; in [Stefanakis *et al.* 2011] coefficient is estimated as $K \cong 5.1$. Second pick f_4 in the experimental frequency dependence of run-up is more visible than in numerical simulation [Stefanakis *et al.* 2011]. Authors [Stefanakis *et al.* 2011] did not give any estimations of second pick frequency, but if one use their data it is possible to conclude that frequency of the second pick is 2.5 – 2.7 times more frequency of the first one. In our experiments frequency of the second pick exceeds the frequency of first one in 2.2 – 2.3 times. Experimental values of frequencies $f_{3,4}$ practically coincide with frequencies of modes having nodes near the wave maker; numerical values exceed this frequency by 2.5% for all bottom inclinations. The reason of such differences is not clear yet. Non linearity, wave dispersion, viscous dissipation influence the frequency of these picks, but simple estimations for linear waves in shallow water with zero viscosity provide values which are close to experimental data. Authors [Stefanakis *et al.* 2011] do not mention any dissipation of energy neither non-linear parameter, which they use in numerical simulations. As for the coefficient of run-up amplification, the maximal value that was observed in experiment is $C = 20 - 25$, whereas in [Stefanakis *et al.* 2011] this value reaches $C = 50 - 60$. The difference is apparently due to viscous dissipation, which is essential in our experiments.

IV.2. Parametric excitation of edge waves

Our experiments have shown that the generation of edge waves is significantly affected by wave breaking of the

surface wave propagating toward the shore. To discuss this effect, we primarily consider how to explain the generation of sub-harmonic in the absence of wave breaking. Temporal evolution of complex amplitude b of parametrically excited edge wave modes is described by equation [Yang 1995]:

$$\frac{\partial b}{\partial t} = -\gamma b + h b^* + i \Delta b + (i\sigma - \rho) b |b|^2 \quad (2)$$

Here γ is a wave decrement due to viscous dissipation, $h = a_0 \omega^3 S(\alpha) / 4g \tan^2 \alpha$, $S(\alpha)$ is a function of inclination angle α determined in [Minzoni *et al.* 1977], a_0 is for surface wave amplitude at $x = 0$, * means complex conjugation, $\Delta = \Omega - \omega/2$ is for detuning between wave frequency and frequency of external parametric forcing, s is a non-linear frequency shift, ρ is a non-linear damping coefficient.

To estimate the decrement γ of harmonic edge wave, we investigate the time evolution of amplitude of edge wave after the stopping the parametric excitation. Edge waves decay exponentially and we measured the wave decrement $\gamma \sim 0.13$ s⁻¹ within a precision of 2%.

The frequency of basic edge mode W [Yang 1995]:

$$\Omega = \sqrt{g \pi \tan \alpha} / L \approx 3.41 \text{ rad/s}, \quad (3)$$

or $f_0 = \Omega / 2\pi \approx 0.54$ Hz.

For the threshold of parametric excitation h_{th} from eq.(2), we have: $h_{th} = (\gamma^2 + \Delta^2)^{1/2}$. For the resonance $\Delta = 0$, parametric instability appears at the minimum external forcing $h_{th, min} = \gamma$. It allows us to calculate theoretical critical amplitude of wave $|a_{0, th}|$:

$$|a_{0, th}| = \gamma (4g \tan^2 \alpha) / (\omega^3 S(\alpha)) \approx 0.71 \text{ cm} \quad (4)$$

In our experiment the amplitude of surface wave displacement is measured at $x = L$. To compare theoretical and experimental values of instability threshold, we used correlation between amplitude a_0 and a_L , see [Yang 1995]. Comparison of instability thresholds is presented in fig. 8.

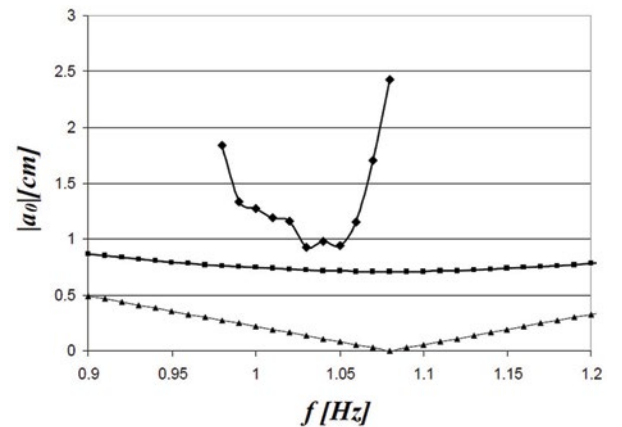


Figure 8: Experimental tang of instability (diamonds) and theoretical curves: triangles correspond to the model without dissipation: $h_{th} = |\Delta|$, curve indicated by squares is obtained for experimentally measured decrement of edge waves: $h_{th} = (\gamma^2 + \Delta^2)^{1/2}$.

A small shift between experimentally obtained frequency $f \sim 1.04$ Hz and theoretical value $f = 2f_0 \sim 1.08$ Hz is observed. The similar shift was observed also in [Ezersky *et al.* 2013]. In our opinion such difference may be connected with influence of wettability of plate on eigen frequency. Besides it, experimentally found region of instability is narrower than theoretical prediction (see fig. 8). This effect may be connected with additional dissipation of energy appearing with increasing of wave amplitude surface wave.

We found that wave breaking leads to decreasing of edge wave amplitude and finally suppresses parametric instability. How to explain this effect using eq.(2) and what is the physical mechanism that is responsible for such suppression? Evidently, there are two possible mechanisms. First, the wave breaking leads to appearance of non-regularity in surface wave: amplitude and phase of the wave vary chaotically. Second, the wave breaking leads also to appearance of water-air bubbles mixture and small-scale turbulence in the near shore zone increasing the wave damping. We discuss the impact of these two physical mechanisms to the suppression of parametric instability.

It should be noted that parametric excitation of waves by non-regular oscillating field was investigated in [Petrelis *et al.* 2005]. It was shown that chaotic amplitude and chaotic phase of external field caused increasing of threshold of parametric excitation and decreasing of the amplitude of parametrically excited oscillations. We verify the applicability of these results for explanation of decreasing of edge wave when breaking surface waves appears. For this purpose using Hilbert transformation we calculated amplitude a and phase Φ of external surface wave $a_0 = a \exp(i\Phi)$ for a regime of developed wave breaking. Root mean square fluctuations of amplitude and phase for $f = 1.06$ Hz, $a_L = 1.3$ cm are determined: $\sqrt{\langle \Phi^2 \rangle} \approx 0.11$, $\sqrt{\langle (a - \langle a \rangle)^2 \rangle} / \langle a \rangle \approx 0.1$.

If we suppose that wave breaking leads to Gaussian noise, decreasing of parametric forcing may be estimated as [29]: $1 - \exp(-\langle \Phi^2 \rangle / 2) \approx 0.005$. The small decreasing of effective external forcing cannot explain suppression of parametric excitation for wave breaking regime. Consequently the first mechanism is apparently ineffective.

The second mechanism seems more realistic because the influence of turbulence is more important. The wave breaking causes generation of turbulence; characteristic turbulent velocity u is proportional to the wave height H [Longo *et al.* 2002]: $u \propto H(g/hT)^{1/3}$, where h is the local water depth and T is wave period. The turbulence leads to the appearance of eddy viscosity ν_{ed} . The eddy viscosity is proportional to kinetic energy of turbulence and for wave breaking case it is possible to consider that $\nu_{ed} \propto a_0^2$. In this case, the wave decrement γ has the following form: $\gamma = \gamma_0 + \gamma_1 |a_0|^2$, where γ_0 is exponential decaying of edge wave in the absence of wave breaking, and γ_1 is responsible for dissipation of energy due to eddy viscosity. The parametric instability occurs if:

$$h > \sqrt{(\gamma_0 + \gamma_1 |a_0|^2)^2 + \Delta^2}, \quad (5)$$

where $h = a_0 \omega^3 S(\alpha) / 4g \tan^2 \alpha$.

Since the external forcing h grows linearly with surface wave amplitude a_0 and dissipation grows as the amplitude squared, the parametric instability is suppressed for large surface waves. Exactly this effect is observed in experiment when developed wave breaking occurs.

V. CONCLUSIONS

On the basis of the experiments, we can conclude firstly, that the value of amplification coefficients and frequencies at which run-up amplification maxima are observed correlate with results of numerical simulations [Stefanakis 2011]. The most important conclusion is that the existence of an abnormally large increase of the coefficient C is due to resonator modes; this coefficient becomes very large because for its determination the amplitude at the mode node is taken as the amplitude of free surface displacement. This effect is very important for the prediction of tsunami run-up using the tide-gauge data. It is not sufficient to know the amplitude of free surface displacement in the near-shore zone; to provide the correct predictions of run-up, it is necessary to know if this value corresponds to the amplitude A of a propagating wave or to the amplitude a of a standing wave at a fixed point. Therefore, it is necessary to install several gauges in the coastal zone.

Secondly, the parametric generation of edge waves was investigated for different regimes of surface wave propagation. It is shown that the threshold amplitude of parametric excitation is very close to the theoretically calculated value. It is found experimentally that increasing of the surface wave amplitude leads to the appearance of wave breaking. The wave-breaking regime does not prevent parametric generation of edge wave; only developed wave breaking can suppress parametric excitation of edge waves. We compared two mechanisms of suppression and found that the most probable mechanism is the increasing of threshold of parametric excitation due to generation of turbulence.

It was found that parametric generation of edge wave could amplify run-up process. Maximal run-up heights in this case exceed maximal run-up heights in the absence of parametric excitation in two or more times. Such amplification is significant in the absence of wave breaking.

VI. ACKNOWLEDGMENTS AND THANKS

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