Expert Systems with Applications 40 (2013) 6601–6623

Contents lists available at SciVerse ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Review

Formal Concept Analysis in knowledge processing: A survey on models and techniques

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ARTICLE INFO

Keywords:
Formal Concept Analysis (FCA)
Knowledge discovery in databases
Text mining
Data analysis models
Systematic literature overview

ABSTRACT

This is the first part of a large survey paper in which we analyze recent literature on Formal Concept Analysis (FCA) and some closely related disciplines using FCA. We collected 1072 papers published between 2003 and 2011 mentioning terms related to Formal Concept Analysis in the title, abstract and keywords. We developed a knowledge browsing environment to support our literature analysis process. We use the visualization capabilities of FCA to explore the literature, to discover and conceptually represent the main research topics in the FCA community. In this first part, we zoom in on and give an extensive overview of the papers published between 2003 and 2011 on developing FCA-based methods for knowledge processing. We also give an overview of the literature on FCA extensions such as pattern structures, logical concept analysis, relational concept analysis, power context families, fuzzy FCA, rough FCA, temporal and triadic concept analysis and discuss scalability issues.

1. Introduction

Formal Concept Analysis (FCA) was introduced in the early 1980s by Rudolf Wille as a mathematical theory (Wille, 1982) taking its roots in the works of Birkhoff (1973) and Barbut and Monjardet (1970) and others, for the formalization of concepts and conceptual thinking. FCA has been applied in many disciplines such as knowledge discovery, software engineering, and information retrieval during the last 15 years. The mathematical foundation of FCA is described by Ganter and Wille (1999) and introductory courses were written by Wolff (1994) and Wille (1997). An overview of a part of the literature published until the year 2004 on the mathematical and philosophical background of FCA, some of the applications of FCA in the information retrieval and knowledge discovery field and in logic and AI given in Priss (2006). A comparison of algorithms for generating concept lattices is given by Kuznetsov and Obiedkov (2002). An overview of available FCA software is provided by Tilley (2004). Carpineto and Romano (2004a) present an overview of FCA applications in information retrieval. In Tilley and Eklund (2007), an overview of 47 FCA-based software engineering papers is given. The authors categorized these papers according to the 10 categories as defined in the ISO 12207 software engineering standard and visualized them in a concept lattice. In Lakhal and Stumme (2005), a survey on FCA-based association rule mining techniques is given. Poelmans, Elzinga, Viaene, and Dedene (2010b) give a comprehensive overview of FCA-based research in knowledge discovery and data mining. Poelmans, Ignatov, Viaene, Dedene, and Kuznetsov (2012b) survey research on FCA in information retrieval. Doerfel, Jäschke, and Stumme (2012) investigated patterns and communities in co-author and citation contexts obtained from publications in the International Conference on Formal Concept Analysis (ICFCA), International Conference on Conceptual Structures (ICCS) and Concept Lattices and Applications (CLA).

In this paper, we describe how we used FCA to create a visual overview of the existing literature on concept analysis published between the years 2003 and 2011. The core contributions of this paper are as follows. We visually represent the literature on FCA using concept lattices, in which the objects are the scientific papers and the attributes are the relevant terms available in the title, keywords and abstract of the papers. We developed a toolset with a central FCA component that we use to index the papers with a thesaurus containing terms related to FCA research and to generate the lattices. We zoom in on and give an extensive overview of the papers published between 2003 and 2011 on using FCA for knowledge discovery and ontology engineering in various application domains. We also give an overview of the literature on FCA extensions such as fuzzy FCA, logical concept analysis, pattern structures, power context families, relational concept analysis, rough FCA, temporal and triadic concept analysis and discuss scalability issues.

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http://dx.doi.org/10.1016/j.eswa.2013.05.007

The remainder of this paper is composed as follows. In Section 2 we introduce the essentials of FCA theory and closely related disciplines such as description logics and conceptual graphs. In Section 3 we describe the dataset used and discuss the knowledge browsing environment which was developed to support this literature analysis. In Section 4 we visualize the FCA literature on knowledge discovery, using a lattice diagram. In Section 5, we discuss the models and methods developed in FCA-based KDD research. Section 6 concludes the paper.

2. Formal Concept Analysis and related disciplines

Formal Concept Analysis (Ganter et al., 1999; Wille, 1982) is a mathematical framework that underlies many methods of knowledge discovery and data analysis. Over the past 30 years the initial theory has been combined with and enriched by other research domains in mathematics including description logics and conceptual graphs.

2.1. Basic notions of FCA

The initial structure of FCA is a triple of sets $\mathcal{K} = (G,\ M,\ I)$ called a *formal context*, where $I \subseteq G \times M$ is a binary relation. This triple can be represented by a cross table consisting of set of rows G (called *objects*), columns M (called *attributes*) and crosses representing *incidence relation I*. An example of a cross table is displayed in Table 1 where objects are papers, attributes are terms and the incidence relation shows how terms occur in papers. In what follows, scientific papers (i.e. the objects) are related (i.e. the crosses) to a number of terms (i.e. the attributes); here a paper is related to a term if the title or abstract of the paper contains this term. The dataset in Table 1 is an excerpt of the one we used in our research. Given a formal context, we then derive concepts and order them according to a subconcept-superconcept relation. This order makes a lattice which can be visualized by a line diagram.

The notion of a *formal concept* is central to FCA. The way FCA looks at concepts is in line with the international standard ISO 704, that formulates the following definition: "A concept is considered to be a unit of thought constituted of two parts: its extent and its intent." The *extent* consists of all objects belonging to the concept, while the *intent* comprises all attributes shared by those objects. Let us illustrate the notion of concept of a formal context using the data in Table 1. For a set of objects $O \subseteq G$, the set of common attributes can be defined by:

$$A = O' = \{m \in M | (o, m) \in I \quad \text{for all } o \in O\}$$

Take the attributes that describe paper 4 in Table 1, for instance. By collecting all papers of this context that share these attributes, we get to a set $O \subseteq G$ consisting of papers 1 and 4. This set O of objects is related to the set A consisting of the attributes "browsing", "software" and "FCA."

$$O = A' = \{o \in G | (o, m) \in I \quad \text{for all } m \in \ A\}$$

That is, O is the set of all objects sharing all attributes of A, and A is the set of all attributes that are valid descriptions for all the objects contained in O. Each such pair (O,A) is called a formal concept (or concept) of the given context. The set $A = O'$ is called the intent, while $O = A'$ is called the extent of the concept (O,A).

There is a natural hierarchical ordering relation between the concepts of a given context that is called the *subconcept-superconcept relation*.

$$(O_1, A_1) \leqslant (O_2, A_2) \Longleftrightarrow (O_1 \subseteq O_2 \Longleftrightarrow A_2 \subseteq A_1)$$

A concept $C_1 = (O_1,A_1)$ is called a *subconcept* of a concept $C_2 = (O_2,A_2)$ (or equivalently, C_2 is called a *superconcept* of a concept C_1) if the extent of C_1 is a subset of the extent of C_2 (or equivalently, if the intent of C_1 is a superset of the intent of C_2). For example, the concept with intent "browsing", "software", "mining" and "FCA" is a subconcept of a concept with intent "browsing", "software" and "FCA." With reference to Table 1, the extent of the latter is composed of papers 1 and 4, while the extent of the former is composed of paper 1.

The set of all concepts of a formal context ordered by the subconcept-superconcept relation $\leqslant$ makes a complete lattice (called the *concept lattice* of the context), i.e. every subset of concepts has infimum (meet) and supremum (join) w.r.t. $\leqslant$. Concept lattices, like every ordered sets, can be visualized by *line diagrams*, where nodes stay for concepts and edges connect pairs of neighboring concept nodes, i.e. those that do not have any concept nodes between them. The line diagram in Fig. 1, for example, is a compact representation of the concept lattice of the formal context generated from Table 1. The circles or nodes in this line diagram represent the formal concepts. The shaded boxes (upward) linked to a node represent the attributes used to name the concept. The non-shaded boxes (downward) linked to the node represent the objects used to name the concept. The information contained in the formal context of Table 1 can be derived from the line diagram in Fig. 1 by applying the following reading rule: An object "g" is described by an attribute "m" if and only if there is an ascending path from the node named by "g" to the node named by "m." For example, paper 1 is described by the attributes "browsing", "software", "mining" and "FCA.".

Another important notion of FCA is that of *attribute implication*(see also Section 5.1.1). For subsets $A,B \subseteq M$ one has $A \rightarrow B$ if $A' \subseteq B'$. Implications obey Armstrong rules, which are valid for functional dependencies in relational databases (Maier, 1983). Moreover, there is a two-way reduction between implications and functional dependencies. A minimal subset of implications from which all the rest can be deduced by means of Armstrong rules is called an *implication base*. Most well-known bases are *Duquenne-Guigues* or *stem* base (Guigues & Duquenne, 1986), which is cardinality minimal, and *proper premise* base (Ganter & Wille, 1999). Implications can be read from the lattice diagram too, implications with singleton premises of the form $a \ \rightarrow B$ are easily seen: the concept (a',a'') lies below the concept (B',B''). The first and one of the most well-known knowledge discovery FCA-based procedure is *Attribute Exploration* (Ganter & Wille, 1999), at each step of which an implication base for the current context representing domain data is generated and offered to a domain expert, who either accepts all implications of the base (in which case the procedure terminates) or gives a counterexample to the implication he finds wrong. Data can be given in a more general way by

Table 1
Example of a formal context.

	Browsing	Mining	Software	Web services	FCA	information retrieval
Paper 1	X	X	X		X	
Paper 2			X		X	X
Paper 3		X		X	X	
Paper 4	X		X		X	
Paper 5				X	X	X

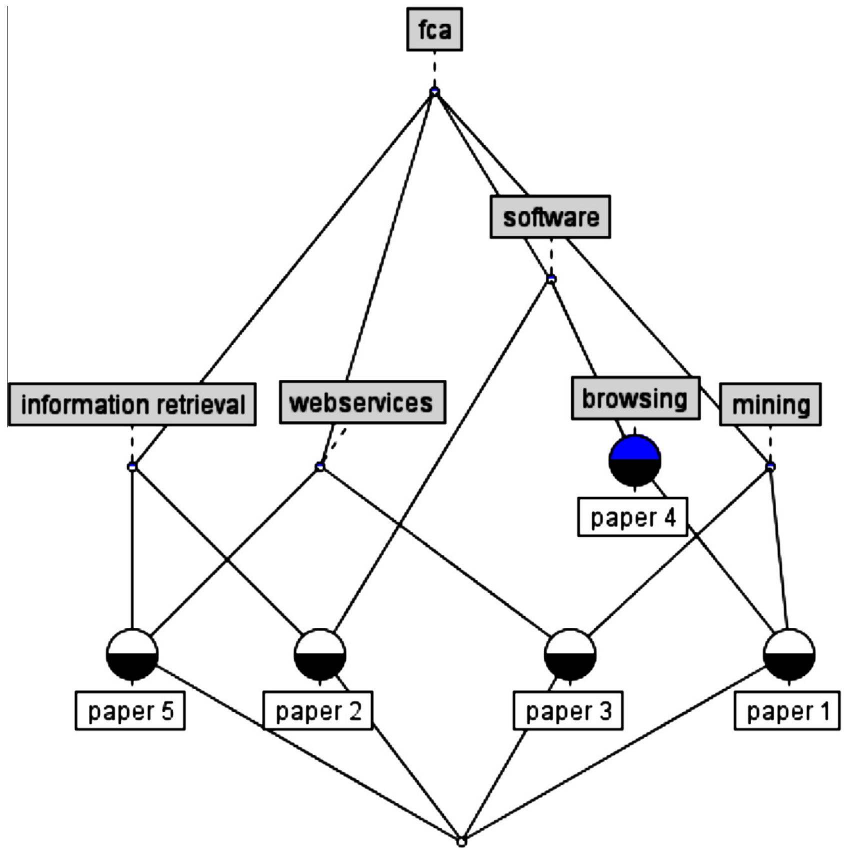

Fig. 1. Line diagram corresponding to the context from Table 1.

many-valued contexts (relational tables), in which case they are reduced to binary contexts by means of *conceptual scaling* (Ganter & Wille, 1999). Each type of scaling (nominal, ordinal, interordinal, dichotomic, etc.) is given by a scaling context.

Retrieving the extent of a formal concept from a line diagram such as the one in Fig. 1 implies collecting all objects on all paths leading down from the corresponding node. To retrieve the intent of a formal concept one traces all paths leading up from the corresponding node in order to collect all attributes. The top and bottom concepts in the lattice are special. The top concept contains all objects in its extent. The bottom concept contains all attributes in its intent. A concept is a subconcept of all concepts that can be reached by traveling upward. This concept will inherit all attributes associated with these superconcepts.

2.2. Description logics

Description logics (DLs) provide a structured language for representing knowledge in a certain domain (Baader, Calvanese, McGuinness, Nardi, & Patel-Schneider, 2003). *Concept descriptions*, i.e. expressions built from unary and binary predicates using constructors provided by the DL language, are used to formally express important notions in the domain. Concept descriptions are used in combination with a *terminology box*, defining the terminology of the domain, and an *assertion box*, stating facts about a specific world. A terminology box can consist of e.g. definitions $N \equiv C$ assigning name N to description C. An *interpretation R* is a pair (domain Δ, interpretation function $\mathfrak{f}$) which maps each concept in the terminology box to a subset of the domain, every role to a binary relation on the domain, and each name in the assertion box to an element of the domain. Checking of subsumption $C \sqsubseteq D$, where C and D are concept descriptions, means checking if concept description D is more general than concept description C.

Two main research directions have emerged from attempts to combine DL and FCA (Sertkaya, 2010):

1. Enriching FCA by using constructors from DL languages
   - Prediger and Stumme (1999) have introduced theory driven scaling by combining DLs with attribute exploration. DLs are used to indicate restrictions on co-occurrence of FCA attributes, a DL reasoner is used during the attribute exploration to confirm or to provide a counterexample if the implication is valid or not respectively.
   - Prediger (2000a), Prediger (2000b) enriched FCA with logical constructors including relations, existential and universal quantifiers, and negation.
   - Rouane-Hacene, Huchard, Napoli, and Valtchev (2007) extended FCA with inter object and concept links encoded by description logics resulting in Relational Concept Analysis (see also Section 5.3.3).
2. Using FCA for improving the potential of DLs in knowledge representation
   - Baader (1995); Baader and Sertkaya (2004) used attribute exploration for computing the subsumption hierarchy of all conjunctions of a set of DL concepts.
   - Stumme (1996) extended the subsumption hierarchy of Baader (1995) with disjunctions of DL concepts and used distributive concept exploration instead of attribute exploration.
   - Baader and Molitor (2000) used FCA for bottom-up construction of DL knowledge bases, i.e. the concepts of the domain are not defined, rather examples of a concept are given and the system composes a concept description.

- Rudolph (2006) proposed relational exploration, where DLs are used to specify FCA attributes, and FCA to refine DL knowledge bases.
- Baader and Distel (2008, 2009) and Distel (2010a) Distel (2010b) replaced the atomic attributes in FCA by complex formulae in a DL language.

2.3. Conceptual graphs

Sowa (1976) wrote the first paper (followed by the book Sowa (1984)) introducing the notion of *Conceptual Graphs* (CGs). Conceptual graphs were used as intermediate language for mapping natural language questions and assertions to a relational database. Many types of CGs have been proposed since the initial papers of John Sowa, cf. the books Dau (2003) and Chein and Mugnier (2009). For a simple CG, the knowledge base structure consists of two entities: the *vocabulary* which encodes hierarchies of types and the conceptual graph itself which represents entities and relations between them. A vocabulary is a triple

$$\mathcal{V} = \left(T_C, T_R = \left(T_R^1, \ldots, T_R^K\right), I\right)$$

where T_C is a partially ordered set of *concept types*, each is a partially ordered set of *relation types* of arity i, and I is a set of individual markers. These sets are pairwise disjoint and all partial orders are denoted by $\leqslant$. Other features may also appear in a vocabulary (Baget, Croitoru, Gutierrez, Leclere, & Mugnier, 2010; Baget & Fortin, 2010). A *conjunctive concept type* over a vocabulary $\mathcal{V}$ is a set $T = \{t_1, \ldots, t_p\}$ of concept types. If $T = \{t_1, \ldots, t_p\}$ and $T' = \left\{t'_1, \ldots, t'_q\right\}$ are two conjunctive concept types, then we also note

$$T \leqslant T' \iff \forall t'_I \in T', \quad \exists t_j \in T$$

such that $t_j \leqslant t'_i$. The *signature* σ maps each relation type of arity k to a k-tuple of conjunctive concept types, that encodes the maximal type of its arguments. The signature has the covariance property, meaning that if $r_2 \leqslant r_1$, then the ith argument of r_2 is a specialization of the ith argument of r_1.

A basic CG on a vocabulary $\mathcal{V} = (T_C, T_R)$ is a bipartite graph. The sets C and R contain a respectively concept and relation nodes. A *concept node* $c \in C$ is labeled by a pair (type (c), marker (c)) where type (c) is a conjunctive concept type built on $T_C$and marker (c) is either an *individual marker* of $\mathcal{I}$, in which case c is called individual, or the *generic marker**, in which case c is called generic. A relation node $r \in R$ is labeled by type $(r) \in T_R$ and is linked to k concept nodes, where k is the arity of type (r).

Concept nodes with the same marker refer to the same entity. A CG is *normal* when no two distinct concept nodes denote the same entity. A CG can be transformed into a normal graph by merging nodes representing the same entity. CGs can be translated into first order logic. Conceptual graphs are visualized as a combination of rectangles representing concepts, circles representing names of conceptual relations and arcs representing relations. An arc pointing from a rectangle to a circle indicates that the corresponding concept is the first argument of the relation, and an arc pointing from a circle to a rectangle indicates the corresponding concept is the last argument. The relation between concept graphs and FCA was studied in numerous papers, see Dau (2003, 2003b) and Dau et al. (2005).

3. Dataset

This Systematic Literature Review (SLR) has been carried out by considering a total of 1072 papers related to FCA published between 2003 and 2011 in the literature and extracted from the most relevant scientific sources. The sources that were used in the search for primary studies contain the work published in those journals, conferences and workshops which are of recognized quality within the research community. These sources are:

- IEEE Xplore Digital Library
- ACM Digital Library
- Sciencedirect
- Springerlink
- EBSCOhost
- Google Scholar
- Conference repositories: ICFCA, ICCS and CLA conference

Other important sources such as DBLP or CiteSeer were not explicitly included since they were indexed by some of the mentioned sources (e.g. Google Scholar). In the selected sources we used various search strings including "Formal Concept Analysis", "FCA", "concept lattices", "Temporal Concept Analysis". To identify the major categories for the literature survey we also took into account the number of citations of the FCA papers at CiteseerX. Perhaps the major validity issue facing this systematic literature review is whether we have failed to find all relevant studies, although the scope of conferences and journals covered by the review is sufficiently wide to achieve sufficient coverage. We also ensured that papers appearing in multiple sources were taken into account only once, i.e. duplicate papers were removed.

The papers that were downloaded from the World Wide Web were all in PDF format. These PDF-files were converted to ordinary text and the abstract, title and keywords were extracted. We used the knowledge browsing environment "COncept Relation Discovery and Innovation Enabling Technology" (CORDIET, Elzinga, 2011; Poelmans et al., 2012a; Poelmans, Elzinga, Viaene, & Dedene, 2010a) to support our literature analysis process. One of the central components of our text analysis environment is the thesaurus containing the collection of terms describing the different research topics. The initial thesaurus was constructed based on expert prior knowledge and was incrementally improved by analyzing the concept gaps and anomalies in the resulting lattices. The thesaurus is a layered thesaurus containing multiple abstraction levels. The first and finest level of granularity contains the search terms of which most are grouped together based on their semantic meaning to form the term clusters at the second level of granularity. CORDIET was used to index the extracted parts of the papers using the thesaurus. The result was a cross table describing the relationships between the papers and the term clusters or research topics from the thesaurus. This cross table was used as a basis to generate the lattices with Concept Explorer (Yevtushenko, 2000).

4. Literature overview

The 1072 papers are grouped together according to a number of features within the scope of FCA research. We visualized the papers using concept lattices, which facilitate our exploration and analysis of the literature. The lattice diagram in Fig. 2 contains seven categories which were prominent topics in the 1072 FCA papers. Knowledge discovery is the most popular research theme covering 23% of the papers and will be analyzed in detail in this survey. Recently, improving the scalability of FCA to larger and complex datasets emerged as a new research topic covering 9% of the 1072 FCA papers. In particular, we note that more than one third of the papers dedicated to this topic work on issues in the KDD domain. Scalability will be discussed in detail in Section 5.5. Another important research topic in the FCA community is information retrieval covering 13% of the papers. 36 of the papers on information retrieval describe a combination with a KDD approach and in 27 IR papers authors make use of ontologies. 15 IR papers deal with the retrieval of software structures such as software components. The FCA papers on information retrieval are discussed in detail in Poelmans et al. (2012b). In 13% of the FCA papers, FCA is used in combination with ontologies or for ontology engineering. FCA research on ontology engineering will be discussed in part 2 of this survey (Poelmans, Ignatov, Kuznetsov, & Dedene, 2013a). Another important topic is using FCA in software engineering (13%), which is also discussed in part 2 of this survey. Finally in 24% of all papers complex descriptions for FCA were investigated, see Section 5.3 for a detailed overview.

5. Models and tools in FCA-based KDD research

Knowledge Discovery in Databases (KDD) and Data Mining (DM) is a research field in which methodologies are developed for extracting knowledge from data. In the past, the focus was on developing fully automated tools and techniques that extract new knowledge from data. These techniques assumed clear definitions of the concepts available in the underlying data, which is often not the case. They allowed almost no interaction between the human actor and the tool and failed in incorporating valuable expert knowledge into the discovery process (Keim, 2002), which is needed to go beyond uncovering the fool's gold. Visual data exploration (Eidenberger, 2004) and visual analytics (Thomas & Cook, 2006) are especially useful when little is known about the data and exploration goals are vague. Since the user is directly involved in the exploration process, shifting and adjusting the exploration goals is automatically done if necessary.

In *Conceptual Knowledge Processing* (CKP) the focus lies on developing methods for processing information and knowledge which stimulate conscious reflection, discursive argumentation and human communication (Wille, 2006). The word "conceptual" underlines the constitutive role of the thinking, arguing and communicating human being and the term "processing" refers to the process in which something is gained which may be knowledge. An important subfield of CKP is Conceptual Knowledge Discovery (Stumme, 2002, 2003; Stumme, Wille, & Wille, 1998). FCA is particularly suited for exploratory data analysis because of its human-centeredness (Hereth, Stumme, Wille, & Wille, 2003). The generation of knowledge is promoted by the FCA representation that makes the inherent logical structure of the information transparent. The philosophical and mathematical foundations of FCA tools for knowledge discovery have been briefly summarized in Priss (2006). The systems TOSCANA (Vogt & Wille, 1994), Concept Explorer (Yevtushenko, 2000), FcaStone (Priss, 2008), InClose (Andrews, 2009), ToscanaJ (Becker, Hereth, & Stumme, 2002), Galicia (Valtchev, Missaoui, & Godin, 2008) have been used as knowledge discovery tools in various research and commercial projects.

In Section 5.1 we zoom in on the papers in the field of association rule mining. In Section 5.2 the relation of FCA to some standard machine learning techniques is investigated. Section 5.3 discusses the complex descriptions of FCA theory for knowledge discovery. The methods for dealing with uncertain data such as fuzzy and rough FCA, will be discussed in Section 5.4. In Section 5.5 we discuss scalability issues in the KDD studies.

5.1. Association rule mining

Association rule mining (ARM) is one of the main models in Data Mining (Agrawal et al., 1996). An association rule is just a statement about conditional sample probability (called *confidence*) of an event wrt. another one, together with the statement of the joint sample probability of the two events (called *support*), where both events are described in terms of attribute sets. In FCA terms, for two subsets of attributes (called *itemsets* in Data Mining) Y_1 and $Y_2 \subseteq M$ the *association rule* $Y_1 \to Y_2$ has *support*

$$\frac{|(Y_1 \cup Y_2)'|}{|G|}$$

and *confidence*

$$\frac{|(Y_1 \cup Y_2)'|}{|Y_1'|}$$

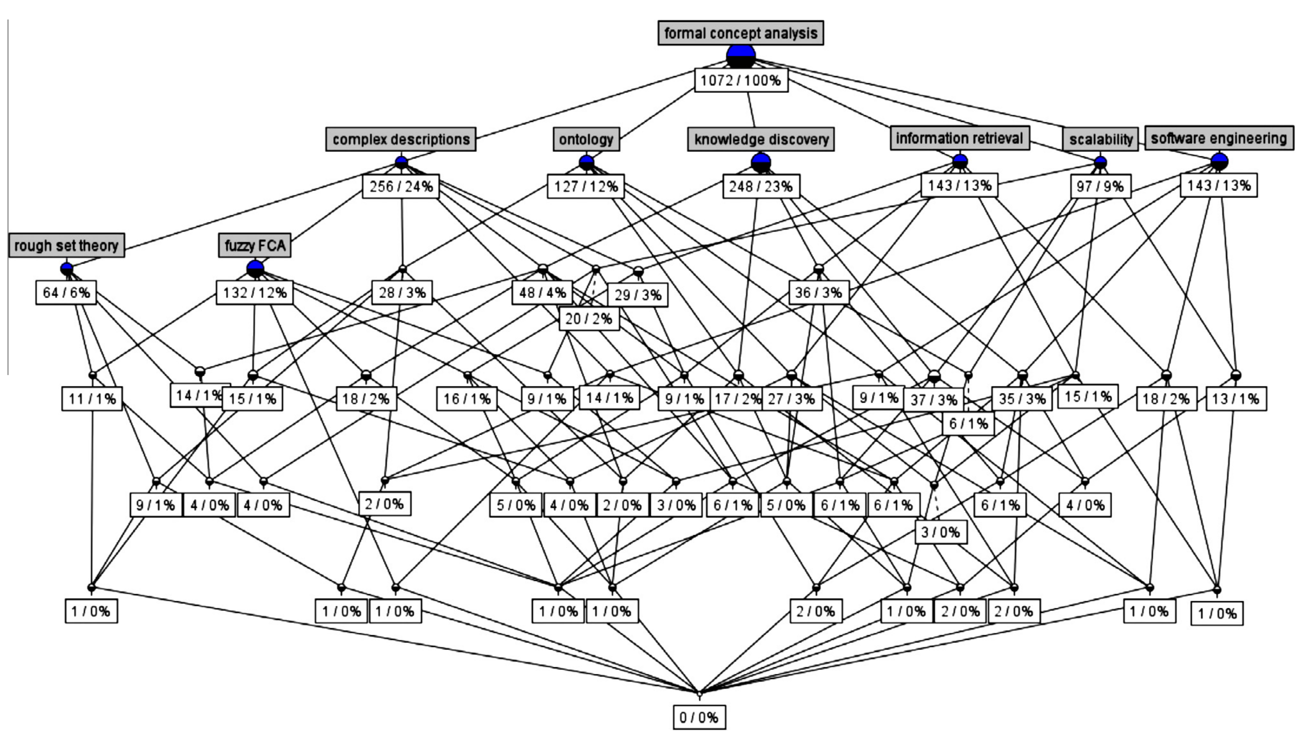

Fig. 2. Lattice diagram containing 1072 papers on FCA.

The rule $Y_1 \rightarrow Y_2$ is called frequent if $supp(Y_1 \rightarrow Y_2) \geqslant minsupp$ for some threshold *minsupp*. In FCA association rules were known under the name of *partial implications*.

Recent approaches for FI mining use the *closed itemset* paradigm to limit the mining effort to the subset of frequent closed itemsets (FCIs). A frequent itemset $Y \subseteq M$ is a frequent closed itemset if and only if $Y'' = Y$, the intent of a concept C is a closed itemset. Several FCA-based algorithms were developed for mining frequent closed itemsets including CLOSE, PASCAL, CHARM, CLOSET and TITANIC (Stumme, Bestride, Taouil, & Lakhal, 2002a) which mines frequent closed itemsets by constructing an iceberg concept lattice. The reader is kindly referred to Lakhal and Stumme (2005) for a detailed overview.

5.1.1. Minimal generators and generic basis

For a context $\mathcal{K}$, a set $H \subseteq M$ is a *minimal generator* of a closed set hence of the concept (Y', Y) if and only if $H$is a minimal subset of Y such that $H'' = Y$. There can be multiple minimal generators for an intent Y. Therefore we define the set-valued function *mingen* which associates to concepts their minimal generator sets:

$$mingen(Y', Y) = \{H \subseteq Y | H'' = Y \text{ and for all } E \subset H,\ E'' \subset Y\}.$$

Minimal generators (Bastide, Pasquier, Taouil, Stumme, & Lakhal, 2000) were introduced in various fields under various names: minimal keys in the database field (Maier, 1983), irreducible gaps (Guigues & Duquenne, 1986), minimal blockers (Pfaltz, 2002) and 0-free itemsets (Boulicaut, Bykowski, & Rigotti, 2003). Minimal generators have some nice properties: they have relatively small size and they form an order ideal (downset) of $(2^M, \leqslant)$. Recall that an order ideal of an ordered set $(P, \leqslant)$ is a subset $Q \subseteq P$ such that if $x \in Q$, $y \leqslant x$ implies $y \in Q$. They were typically extracted to support frequent itemset computation (Bastide et al., 2000), frequent closed itemset computation (Pasquier, Taouil, Bastide, Stumme, & Lakhal, 2005), iceberg lattice construction (Stumme et al., 2002), etc. Other applications of minimal generators include deriving a *generic association rule basis* (Gasmi, Yahia, Nguifo, & Slimani, 2005a,Gasmi et al., 2005b), obtaining concise representations of frequent itemsets (Boulicaut, 2005) etc.

Implications or exact association rules are rules of the form r: $Y_1 \rightarrow (Y_2, Y_1)$ between two frequent itemsets Y_1 and Y_2 whose closures are identical $Y_1'' = Y_2''$, i.e. *confidence* $(r) = 1$. Let *FCI* be the set of frequent closed itemsets extracted from the context K and, for each frequent closed itemset F, let G_F denote the set of generators of F. The generic basis for exact association rules is:

$$GB = \{r : H \rightarrow F \setminus H | F \in FCI \wedge H \in G_f \wedge H \neq F\}.$$

All valid exact association rules, their supports and confidences can be deduced from the rules of the generic basis and their supports. A generic basis is a reduced set of association rules which preserves the most relevant rules without loss of information (Bastide et al., 2000). The generic basis for exact association rules contains only minimal non-redundant rules. Each approximate association rule $Y_1 \rightarrow (Y_2 \backslash Y_1)$ relates two frequent itemsets Y_1 and Y_2 such that the closure of Y_1 is a subset of the closure of $Y_2 : Y_1'' \subset Y_2''$, i.e. *confidence*$(r) < 1$.

A cover of approximate association rules can be defined as follows: let *FCI* be the set of frequent closed itemsets and let *mingen* denote the set of their minimal generators. Then $(F', F) \prec (H', H)$ in the order of the concept lattice. The informative cover for approximate association rules is:

$$IC = \{r : H \rightarrow F \setminus H | F \in FCI \wedge H \in mingen_F \wedge H'' \subset F\}.$$

All valid approximate association rules, their supports and confidences, can be deduced from the rules of the informative cover, their supports and their confidences. In terms of a concept lattice a rule from IC corresponds to a descending path in the lattice diagram, the premise being a mingen of the starting concept of the path, the conclusion being the end point of the path. Confidence of such a rule can be given as a product of confidences of the rules corresponding to the shortest such paths, i.e. edges diagram. This consideration gives a smaller subset of association rules representing a whole set. Bastide et al. (2000) provided algorithms for deriving generic bases of both exact and approximate association rules. In Ben Yahia and Nguifo (2004) inference axioms are provided for deriving association rules from generic bases. Other bases include the representative rules (Kryszkiewicz, 1998), Duquennes–Guiges basis (Guigues & Duquenne, 1986), Luxenburger basis (Luxenburger, 1991), proper basis (Ganter & Wille, 1999), structural basis (Pasquier, Bastide, Taouil, & Lakhal, 1999). For a comparative study of these bases the reader is referred to (Kryszkiewicz, 2002).

5.1.2. Algorithms for computing minimal generators

During recent years FCA researchers have targeted minimal generators as an instrument to improve the performance of ARM algorithms. Table 2 summarizes the most recent evolutions in FCA-based methods for mining minimal generators.

Valtchev, Missaoui, and Godin (2004); Valtchev et al. (2008) start with an overview of the existing FCA-based association rule mining techniques. They also propose two incremental methods for computing the minimal generators of a closure system, namely during factor lattice assembly and upon insertion of new objects in the context. Another important result of this work is the Galicia platform. Rouane-Hacene, Nehme, Valtchev, and Godin (2004) adapt this object insertion maintenance method for iceberg lattices. Nehmé, Valtchev, Rouane, and Godin (2005) propose a novel method for efficiently updating the minimal generator set after expanding the context attribute set. They also adapted this incremental method for iceberg lattice maintenance. Tekaya, Yahia, and Slimani (2005) propose an algorithm called GenAll to build an FCA lattice in which each concept is decorated by its minimal generators with the aim of deriving a generic basis of association rules. The GenAll algorithm is based on the algorithm presented by Nourine and Raynaud (1999). Hamrouni, Yahia, and Slimani (2005a); Hamrouni, Yahia, and Slimani (2005b) propose an algorithm called PRINCE which builds a minimal generator lattice from which the derivation of generic association rules becomes straightforward. Innovative is that the partial order is maintained between minimal generators instead of closed itemsets. Dong, Jiang, Pei, Li, and Wong (2005) introduce the Succinct System of Minimal Generators (SSMG) as a minimal representation of the minimal generators of all concepts and provide an efficient algorithm for mining SSMGs. The SSMGs are used for reducing the size of the representation of all minimal generators. Unfortunately their work was slightly flawed, but the SSMG system was improved by Hamrouni, Ben Yahia, and Mephu Nguifo (2008). Hamrouni, Yahia, and Nguifo (2007) present a new sparseness measure for formal contexts using the framework of SSMGs. Their measure is an aggregation of two complementary measures, namely the succinctness and compactness measures of each equivalence class, induced by the closure operator. This is important for the performance of frequent closed itemset mining algorithms which is closely dependent on the type of handled extraction context, i.e. sparse or dense. Hermann and Sertkaya (2008) investigate the computational complexity of some of the problems related to generators of closed itemsets. The authors also present an incremental polynomial time algorithm that can be used for computing all minimal generators of an implication-closed set. The ZART algorithm (Szathmary, Napoli, & Kuznetsov, 2007) built further on the Pascal algorithm (Bastide, Taouil, Pasquier, Stumme, & Lakhal, 2002) and relies on pattern counting inference. Frequent itemsets can be grouped in equivalence classes and each class contains a minimal generator (which is discovered first by ZART), a frequent closed itemset and all other

Table 2
Research on FCA-related minimal generator computation.

Paper	Main contributions to FCA's minimal generator research subfield
Pasquier et al. (1999) Stumme et al. (2002a) Valtchev et al. (2004, 2008)	• A-Close and Close algorithm mine FCIs like Apriori but replace candidates by generators • Titanic builds further on A-Close and uses advanced properties of generators to efficiently compute iceberg concept lattices • Algorithm Inc-gen for incrementally updating the minimal generators after new objects are inserted in the context • Algorithm for computing minimal generators during factor lattice assembly • Implementation and benchmarking of algorithm Inc-gen with Close and A-Close • Galicia software system
Rouane-Hacene et al. (2004) Frambourg et al. (2005)	• Mingen-related part of lattice maintenance method in Valtchev et al. (2004) is modified for iceberg lattice maintenance • Incremental updating of minimal generators during lattice merging • Implementation and performance evaluation of their minimal generator computation algorithm
Tekaya et al. (2005)	• GenAll algorithm builds FCA lattice which is based on the algorithm in Nourine and Raynaud (1999) • Each concept is decorated by its minimal generators • Generic basis are derived from these concepts and generators
Nehmé et al. (2005)	• Algorithm IncA-Gen for incrementally updating the set of minimal generators upon increases in the attribute set of the context • Adaptation of IncA-Gen for iceberg lattice maintenance resulting in Magalice-A • Magalice-A has better performance on sparse datasets than Titanic
Hamrouni et al. (2005a, 2005b)	• PRINCE algorithm for building a minimal generator lattice • The partial order is built and maintained between mingens instead of CIs • Generic basis is derived from this lattice and the minimal generators • Experimental results on 2 datasets show that the partial order computation does not result in worse performance than Titanic, Close or A-Close
Dong et al. (2005)	• The authors noted the redundancy in the minimal generators associated to a closed itemset • Introduction of two main categories of minimal generators: succinct and redundant • Succinct system of Minimal Generators as a concise representation of minimal generator set and SSMG-Miner algorithm
Hamrouni et al. (2007) Hamrouni et al. (2008)	• New sparseness measure for contexts using the frame work of SSMGs • Corrects flaws in the work of Dong et al. (2005) • The Succinct System of Minimal Generators now forms an exact representation of the minimal generator set (i.e. no loss of information) • The SSMGs associated to a context share the same size • The SSMG makes it possible to remove redundant minimal generators without loss of information • The improved SSMG framework is used to derive a more compact generic basis of association rules
Hermann and Sertkaya (2008)	• Overview of minimal generator-related algorithm complexities • Adaptation of algorithm from relational databases to determine all minimal generators of an implication–closed set • Introduction of "minimal cardinality generators" and algorithms for their generation
Szathmary et al. (2007)	• ZART algorithm for levelwise mining of frequent itemsets • Pattern counting inference improves performance compared Apriori for frequent itemsets generation • Identification of frequent closed itemsets. • Association of minimal generators with frequent closed itemsets
Szathmary et al. (2008, 2009, 2011)	• "TOUCH" method which combines CHARM-based FCI extraction and Talky-G-based frequent generator extraction and then matches the FGs toFCIs • SNOW method which extracts precedence links from FCIs and frequent generators • SNOW-TOUCH method which can be used for iceberg lattice construction • SNOW-TOUCH has high efficiency on dense datasets

elements. ZART generates these itemsets like Apriori in levelwise manner but support only needs to be counted for the generators, since all itemsets in the equivalence class have the same support value which significantly improves efficiency. Then closed itemsets are identified among frequent itemsets and the generators are associated to their closures. From this information minimal non-redundant association rules can easily be derived. Szathmary, Valtchev, Napoli, and Godin (2008) proposes a new method Touch which extracts FCIs using CHARM and frequent generators using Talky-G. These frequent generators are then matched to FCIs. The Snow method (Szathmary, Valtchev, Napoli, & Godin, 2009) extracts precedence links from frequent generators and FCIs and is combined with Touch (Szathmary et al., 2011) to form a novel iceberg lattice construction algorithm. Their method Snow-Touch performs particularly well on dense datasets. Missaoui, Nourine, and Renaud (2010); Missaoui, Nourine, and Renaud (2012) present an approach for exhaustive generation of implications without negation and implications with negation. In their approach, minimal generators of the apposition of a formal context and its complementary context whose attributes are negative are generated first. Then implications are inferred from implications whose premises are these minimal generators.

5.1.3. Other FCA-related ARM research

Richards and Malik (2003a, 2003b) start from the methods of generating Ripple-Down Rules (RDR) (Compton et al., 1991) which is used to acquire classification rules, i.e. association rules with a set of conditions in the premise and a class label in the consequent from cases. These low-level rules are then taken as objects (identified by the rule number and conclusion) and the rule conditions as attributes. From this formal context a concept lattice is created which the authors use to uncover redundancies in the knowledge base and derive higher-level rules. Like in the work of Pasquier (2000) they focus on producing non-redundant classification rather than association rules. Experiments with their method were performed on 13 datasets from chemical pathology. Their method allows for queries at and across different levels of abstraction. Gupta, Kumar, and Bhatnagar (2005) proposed a method for extracting classification rules based on FCA. The incremental CBALattice algorithm constructs for each label a concept lattice from which these rules are derived. Maddouri (2005) proposes a new approach to mine interesting itemsets as the optimal concepts covering a binary table. To speed up the costly calculation of frequent itemsets, the author proposes a heuristic polynomial time algorithm to calculate these itemsets and the concept coverage. Experimental evaluation on five datasets representing biological sequences coding macromolecules shows that this heuristic method outperforms Apriori in terms of calculation time. Maddouri and Kaabi (2006) summarize and compare 15 statistical measures which were introduced for selecting pertinent association rules. Their bibliographic summary is complemented by an experimental study in which rules are extracted from several datasets with various densities.

The pertinence measures are applied to these rule bases and the results of this exercise may help users select the appropriate measure for their data. Maddouri (2004) outlines a new incremental learning approach IPR based on FCA that supports incremental concept formation and applies it to the problem of cancer diagnosis. The incremental approach has the advantage of handling the problem of data addition, data deletion, data update, etc. Meddouri and Maddouri (2009) combine FCA with AdaBoost.M2 to extract classification rules. The authors first extract a subset of the objects and then select the attribute with minimal entropy. Then they apply the closure operators of FCA to obtain a pertinent formal concept. This concept is transformed into a classification rule which has its intent as condition and the majority class of the extent as conclusion. The discovered rule is then used to classify all objects and the whole procedure is repeated several times. Using AdaBoost these different weak classifiers are combined resulting in better error rate and less extracted concepts than IPR. In Quan, Ngo, and Hui (2009), a new cluster-based method is proposed for mining association rules. To improve the computational effort required for ARM the authors first cluster objects based on the similarity of their attributes. For each cluster they create a concept lattice from which the association rules are extracted. In Zarate and Dias (2009), FCA is used to extract and represent knowledge in the form of a non-redundant canonical rule base with minimal implications from a trained Artificial Neural Network (ANN).

5.2. FCA-based learning models

A relatively small subset of the surveyed papers about KDD investigate the relationships between FCA and other machine learning techniques. Machine learning models start from input data consisting of positive and negative examples of a target attribute w and try to construct a generalization of the positive examples that would not cover any negative example. Important machine learning tasks where concept lattices were used include the generation of implication bases, bases of association rules and closed itemsets. Multiple authors tried to formulate several machine learning methods in FCA terms. Ganter and Kuznetsov (2003) express version space learning in terms of FCA. In Kuznetsov (2004a) the author described version space and decision tree learning in the language of FCA. The first approach for formulating learning models from positive and negative examples in concept lattices makes use of standard FCA (Ganter & Kuznetsov, 2000; Kuznetsov, 2004a, 2004b). In this case the input data consists of positive (+)-examples, which are objects that are known to have the target attribute and negative (-)-examples which are known not to have the target attribute. The results for learning are rules for classification of undetermined (u)-examples. Given a positive context $\mathcal{K}_+ = (G_+,\ M,\ I_+)$, a negative context $\mathcal{K}_- = (G_-, M, I_-)$ and an undetermined context $\mathcal{K}_u = (G_u, M, I_u)$, where G_+ is a set of positive examples, G_- a set of negative examples and G_u a set of undetermined examples, M is a set of attributes and $I_\varepsilon \subseteq G_\varepsilon \times M, \varepsilon \in \{+, -, u\}$. A learning context is defined as

$$\mathcal{K}_\pm = (G_+ \cup G_-,\ M \cup \{w\}, I_+ \cup I_- \cup G_+ \times \{w\}).$$

A classification context is defined as

$$\mathcal{K}_c = (G_+ \cup G_- \cup G_u,\ M \cup \{w\}, I_+ \cup I_- \cup I_u \cup G_+ \times \{w\}).$$

The derivation operators in these contexts are denoted by $(.)^+$, $(.)^-$, $(.)^u$, $(.)^\pm$, respectively. If a pair (A_+, B_+) is a concept of the context $\mathcal{K}_+$, then it is called a positive (+)-concept and the sets A_+ and B_+ are called positive (+)-intent and extent. If $(B_+ \not\subset \{g_-\}^-)$ for any $g_- \in G_-$, then it is called a positive (+)-hypothesis with respect to the target attribute w. A (+)-intent B_+ is called falsified if $B_+ \subseteq \{g_-\}^-$ for some negative example g_-. Negative (-)-intents and hypotheses are defined similarly. These hypotheses can be used for classifying undetermined examples from G_u. If $g_u \in G_u$ contains a positive hypothesis H_+, i.e. $\{g_u\}^u \supseteq H_+$, we say that H_+ is a positive classification of g. If there is a hypothesis for the positive classification of g_u and no hypothesis for the negative classification of g_u, then g_u is classified positively. If $\{g_u\}^u$ does not contain any negative or positive hypothesis, then no classification is made. If $\{g_u\}^u$ contains both positive and negative hypothesis then the classification is said to be contradictory. A positive hypothesis H_+ is a minimal positive hypothesis if no $H \subseteq H_+$ is a positive hypothesis.

In the second case the data is more complex and instead of having attributes, the objects satisfy certain logical formulas or are described by labeled graphs. In this case pattern structures (see Section 5.3.1) have been proposed to describe these learning models (Kuznetsov, 2004a).

Fu, Fu, Njiwoua, and Nguifo (2004a) performed a benchmarking study of FCA-based classification algorithms including GRAND (Oosthuizen, 1994), LEGAL (Liquiere & Nguifo, 1990), GALOIS (Carpineto et al., 1993), RULEARNER (Sahami, 1995), CIBLe (Njiwoua & Mephu Nguifo, 1999), and CLNN & CLNB (Xie, Hsu, Liu, & Lee, 2002). Nguifo and Njiwoua (2001) proposed IGLUE, an algorithm combining lattice-based and instance-based learning techniques. Ricordeau (2003) and Ricordeau and Liquiere (2007) used FCA to generalize policies in reinforcement learning by grouping similar states using their descriptions. A policy represents the probability for an agent to select action a being in state s. Aoun-Allah and Mineau (2006) propose a method for distributed data mining which first mines the datasets in a distributed manner and then uses FCA to gather the results as a set of rules to form a meta-classifier. Rudolph (2007) proposes to use FCA to design a neural network architecture in case some partial information about the networks desired behavior is already known and can be stated in the form of implications on the feature set. The author first represents a closure operator as a formal context and then translate this context into a 3-layered feedforward network which calculates the closure A'', where $A \subseteq M$. Tsopze, Mephu Nguifo, and Tindo (2007) propose the CLANN algorithm which uses concept lattices to build the architecture of a neural network. First they build a join semi-lattice of formal concepts by applying constraints to select relevant concepts. Second they translate the join semi-lattice into a topology for the neural network, and set the initial connection weights. Third, they train a network on a supervised dataset using this topology. Nguifo, Tsopze, and Tindo (2008) later on extended their approach to multiclass datasets resulting in the M-CLANN algorithm.

Belohlavek, De Baets, Outrata, and Vychodil (2009) uses FCA to induce decision trees from data tables. In a first step, the categorical attributes are scaled to logical attributes. Then a modified version of the NextNeighbor algorithm (Lindig, 2000) is used to build a reduced concept lattice. From this lattice, trees of concepts can be selected. A tree of concepts can then be transformed into a decision tree. Outrata (2010) used FCA as a data preprocessing technique. The authors used boolean factor analysis to transform the attribute space to improve the results of machine learning and in particular decision tree induction. Boolean factor analysis is a method which decomposes an $n \times m$ binary matrix I into a Boolean product $A \circ B$ of an $n \times k$ binary matrix A and a $k \times m$ binary matrix B with k as small as possible. To compute the decomposition, the algorithms in Belohlavek and Vychodil (2009) and Belohlavek and Vychodil (2010) were used with modified optimality criterion of the computed factors. In a first variant, the set F consisting of k factors is added to the collection of m attributes used as input for the machine learning method. In the second variant, the original m attributes are replaced by k factors. The authors validated the second data preprocessing variant by means of decision trees and an instance based learning method on UCI machine learning repository datasets. The models obtained on average better performance on test datasets.

Visani, Bertet, and Ogier (2011) proposed Navigala, a navigation-based approach for supervised classification, and applied it to noisy symbol recognition. The majority of papers on using Galois lattices for classification first select the concepts which encode relevant information. Another approach is based on navigation, classification is performed by navigating through the complete lattice (similar to the navigation in a classification tree), without applying any selection operation. Their article also includes a comparison between possible selection and navigation strategies. (See Fig. 3).

5.3. Complex descriptions for KDD

In practice, data is generally more complex than those described by attributes within formal contexts. A first approach to handle complex data in FCA is conceptual scaling (Ganter & Wille, 1989), which is a process that takes complex data as an input, and outputs a standard formal context, called the scaled context. Nested line diagrams is a technique that was initially introduced by Wille (1984). The line diagrams for practical applications typically get very large, then the set of attributes may be split into two parts, a line diagram is drawn for each part and then these line diagrams are nested into each other. The result is a simplified diagram which can be "zoomed" to see the "fine" structure. Nested line diagrams are, supported in TOSCANA for instance (Vogt & Wille, 1994).

In the last years, multiple theoretical extensions and applications have been introduced into the literature that further improve the applicability of traditional FCA theory to knowledge discovery problems. In this section we describe a selection of some notorious extensions of traditional FCA theory which can be found in Fig. 4.

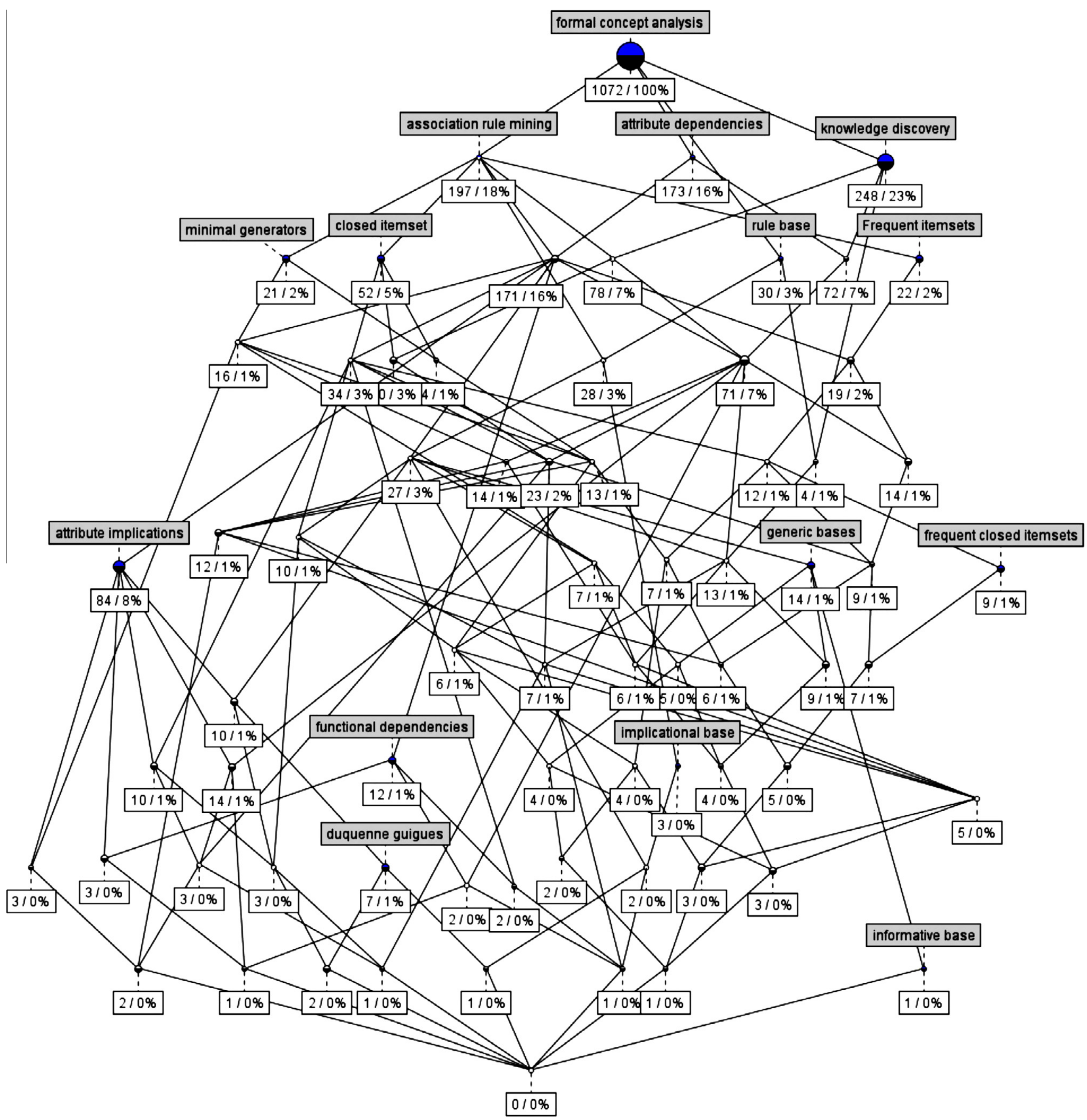

Fig. 3. Lattice diagram containing 1072 papers on FCA and association rule mining topics.

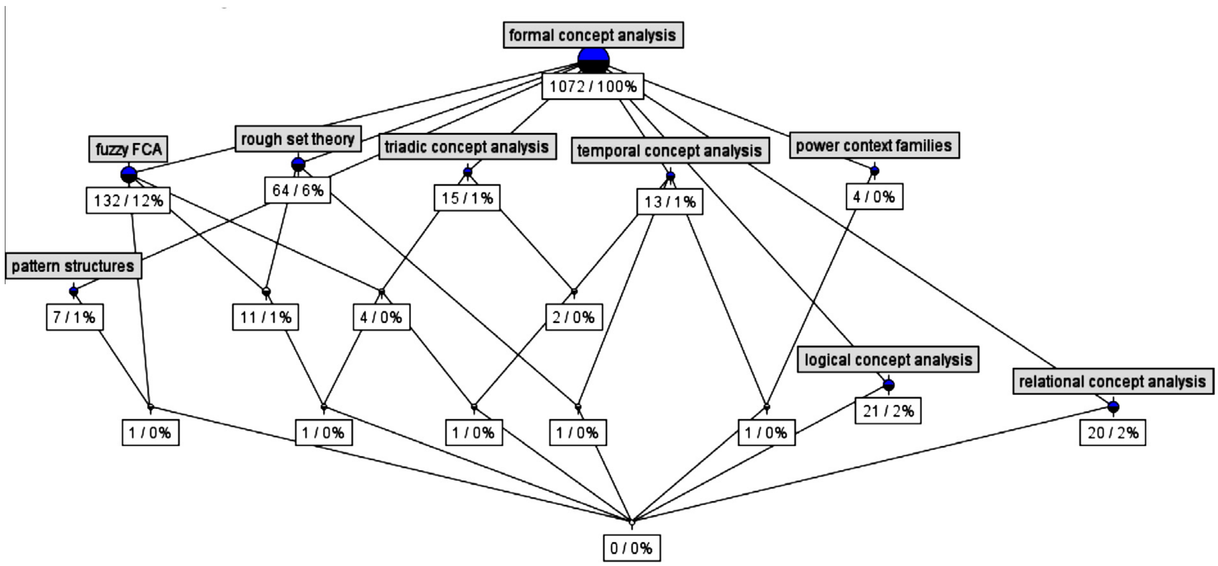

Fig. 4. Lattice diagram with FCA extensions.

Since the 1980s many authors attempted to generalize its definitions to more complex data representations, such as graphs, intervals, logical formulas, etc. Pattern structures (Ganter & Kuznetsov, 2001) were introduced to analyze data which cannot be directly described by object-attribute matrices. Three extensions offer the possibility to analyze arbitrary relations between objects: Power Context Families (Wille, 1997a), Relational Concept Analysis (Huchard et al., 2002) and Logical Concept Analysis (Ferré & Ridoux, 2000). Triadic Concept Analysis was introduced by Lehmann and Wille (1995) and Wille (1995) to analyze three dimensional data. Wolff (2000) described Temporal Concept Analysis for investigating temporal relations in data with FCA. Finally, fuzzy FCA and rough FCA were developed to work with uncertain data. The extensions are summarized in Tables 3 and 4.

5.3.1. Pattern structures

Pattern structures generalize previous models extending FCA definitions to graphs and numeric intervals. Let G be a set of objects, let $(D,\sqcap)$ be a meet-semilattice (of potential object descriptions) and let $\delta{:}G \to D$ be a mapping. Then $(G,\underline{D},\delta)$ with $\underline{D} = (D,\sqcap)$ is called a pattern structure, and the set $\delta(G){:=}\{\delta(g)|g \in G\}$ generates a complete subsemilattice $(D_\delta, \sqcap)$, of $(D,\sqcap)$. Thus each $X \subseteq \delta(G)$ has an infimum $(\sqcap X)$ in $(D,\sqcap)$ and $(D_\delta,\sqcap)$ is the set of these infima. Each $(D_\delta, \sqcap)$ has both lower and upper bounds, respectively 0 and 1. Elements of D are called patterns and are ordered by subsumption relation $\sqsubseteq$: given c, dD one has $c \sqsubseteq d \Leftrightarrow c \sqcap d = c$. A pattern structure $G,\underline{D},\delta$ gives rise to the following derivation operators (.):

$$A^{\square} = \sqcap_{g\in A}\delta(g) \quad \text{for } A \subseteq G, g \in A$$
$$d^{\square} = \{g \in G | d \sqsubseteq \delta(g)\} \quad \text{for } d \subseteq D$$

These operators form a Galois connection between the powerset of G and (D,$\sqsubseteq$). Pattern concepts of $G,\underline{D},\delta$ are pairs of the form (A,d), $A \subseteq G, d \in D$ such that $A^{\square} = d$ and $A = d^{\square}$. For a pattern concept (A,d) the component d is called a pattern intent, it is a description of the set of all objects in A, called pattern extent. If (A, d) is a pattern concept, adding any element to A changes d through $(.)^{\square}$ and equivalently taking $e \supset d$ changes A. Like in case of formal contexts, for a pattern structure $G,\underline{D},\delta$ a pattern $d \in D$ is called closed if $d^{\square\square} = d$ and a set of objects $A \subseteq G$ is called closed if $A^{\square\square} = A$. Obviously, pattern extents and intents are closed. A seeming limitation of pattern structures to descriptions in semi-lattice is easily waived: one can start from descriptions in an arbitrary order $(P,\leqslant)$ taking as $(D,\sqcap)$, the lattice of all order ideals of $(P,\leqslant)$.

Since 1980s pattern structures on sets of graphs (Kuznetsov, 1999) and vectors of intervals (Kuznetsov, 1991; Kaytoue, Kuznetsov, Macko, & Meira, 2011a) were used in the study of the Structural-Activity Relation problem (for more details, see Section 5 in part 2 of this survey). Pattern structures were used to express some traditional machine learning methods such as version spaces in FCA terms (Ganter & Kuznetsov, 2001; Kuznetsov, 2004a). Ganter, Grigoriev, Kuznetsov, and Samokhin (2004) and Kuznetsov and Samokhin (2005) summarize applications of pattern structures in chemistry and show the relation to methods based on closed graphs. Kuznetsov (2009) discusses the analysis of complex data with pattern structures, stressing that pattern structures are computationally more efficient and provide better visualization of results than FCA methods based on scaling. Kaytoue, Duplessis, Kuznetsov, and Napoli (2009); Kaytoue et al. (2011a) applied pattern structures to mine gene expression data (for more details, see Section 5 in part 2 of this survey). This gene expression data was available as a many-valued context and therefore had to be scaled to a binary context. The authors compared traditional FCA with interordinal scaling and pattern structures which were extended to deal with interval data. The authors found the interval pattern structures offering a more concise representation, better scalability and better readability of the pattern concept lattice. Assaghir, Kaytoue, Meira, and Villerd (2011) did an exploratory study on the possibilities of decision tree induction from numerical data by using interval pattern structures. Interval pattern structures are used to extract sets of minimal positive and negative hypotheses. They then propose an algorithm to extract decision trees from these minimal hypotheses, where minimal hypotheses correspond to optimal splits in decision trees.

5.3.2. Logical concept analysis

Logical Concept Analysis (LCA) started as an extension of FCA to description based on first order formulas (Ferré & Ridoux, 2004). Later on the notion of logic in this model was generalized, which made it similar to pattern structures. Ferré and Ridoux (2000) give an introduction to Logical Information Systems (LIS) which are based on the theory of LCA. The main characteristics and aims of an LIS are:

Table 3
Extensions of Formal Concept Analysis and their applications.

Extension name	Landmark paper	Notorious further research
Pattern structures	Ganter and Kuznetsov (2001)	• Ganter and Kuznetsov (2003): Complex version spaces for which classifiers form a complete semilattice, are described using pattern structures • Kuznetsov (2004a): Machine learning models from positive and negative examples can in certain cases be described using pattern structures • Ganter et al. (2004) and Kuznetsov and Samokhin (2005): Approximate labeled graphs from chemistry through pattern structures • Kaytoue et al. (2009, 2011a) Mining gene expression data with interval pattern structures compared to FCA with interordinal scaling • Kuznetsov (2009): Pattern structures allow for directly processing complex data and are a computationally more efficient method than scaling • Assaghir et al. (2010): Pattern structures are used to organize the result of merging numerical information from several sources • Assaghir et al. (2011): Pattern structures are used to extract positive and negative hypotheses from numerical data from which decision trees can be built
Logical Concept Analysis	Ferré and Ridoux (2000)	• Ferré and King (2004):Introduction to Logical Information Systems based on LCA • Ferré and Ridoux (2004):The novel data browsing system for bioinformatics databases, "BLID", is based on LCA • Pecheanu et al. (2004, 2006):Conceptual modeling of training domain knowledge presented to the learner in computer-assisted instructional environment • Ferré et al. (2005):Detailed description on how to model relations between objects in LCA • Morin and Debar (2005) and Morin et al. (2009):Support alarm fusion and correlation to confirm or invalidate alerts raised by intrusion detection systems • Ferré (2006):Represent incomplete knowledge, negation and opposition in LCA • Cellier et al. (2007, 2008a):FCA with a taxonomy is defined in LCA terms and an algorithm for exploring concepts in such a context is provided • Falleri et al. (2007) and Ferré (2009):CAMELIS tool for management of digital photo collections • Bedel et al. (2007a, 2007b):Exploration of individual geographical objects without taking into account their mutual organization • Bedel et al. (2008):Spatial relations are included in the geographical data exploration • Ducassé and Ferré (2008):Decision process in committee meetings is made more rational with LCA • Foret and Ferré (2010):Connection of categorical grammars to LIS • Allard et al. (2010): LCA is used to browse a lattice of cubes extracted from association rules • Ducassé et al. (2011): Case study describing how a publication strategy is determined by six researchers with LIS
Relational Concept Analysis	Huchard et al. (2002)	• Dao et al. (2004): Static UML class model engineering • Huchard et al. (2007): Formal description and state of the art of the RCA method • Rouane-Hacene et al. (2007): Concepts and relations extracted with RCA are represented in description logics • Falleri et al. (2007): Generic encoder for transforming software structures in RCF and decoder for the obtained lattices • Rouane-Hacene et al. (2008): Semi-automated construction of ontologies • Bendaoud et al. (2008): Interactive ontology design method • Moha et al. (2008): Correcting design defects in software engineering • Falleri et al. (2008): Generic method for improving class models by discarding redundancy and adding relevant abstractions • Chen et al. (2009): Identification of service candidates in legacy systems • Hamdouni et al. (2010): Identifying architectural components from object-oriented system code • Dolques et al. (2010b): Generating transformation rules from examples of transformed models • Dolques et al. (2010a, 2012): Refactoring use case diagrams. • Rouane-Hacene et al. (2010): Designing concept lattices that can be reengineered as ontology design patterns • Azmeh et al. (2011): Web service selection based on QoS properties and composebility of services • Azmeh et al. (2011): Relational queries for navigating a concept lattice family
Power Context Families	Wille (1997)	• Wille (1998): Triadic concept graphs and triadic power context families • Prediger (1998): Logical approach to represent concept graphs with a semantic in power context families • Prediger and Stumme (1999): Relational scaling to transform a many-valued context into a power context family • Wille (2001): Boolean judgment logic to define negation of formal judgments represented as concept graphs of power context families • Groh and Eklund (1999) and Groh (2002): Algorithms and implementations for creating power context families from conceptual graphs • Hereth (2002): Power context family resulting from canonical translations of a relational database is used to create a formal context of functional dependencies • Wille (2004): Implications can be seen as specific concept graphs of power context families • Rudolph (2004): Binary power context families are used to assign to a concept with DL descriptions in the intent, all entities in the universe which fulfill this description • Varga and Janosi-Rancz (2008): Software tool which constructs formal context of functional dependencies using power context families.
Triadic FCA	Wille et al. (1995), Lehmann and Wille (1995)	• Biedermann (1998): Powerset trilattice theory • Voutsadakis (2002): Generalization of triadic concept analysis to n-dimensional contexts • Biedermann (1997), Ganter et al. (2004a): Implications in triadic formal context. • Jaschke et al. (2006): TRIAS algorithm for mining iceberg tri-lattices • Jäschke et al. (2008): Mining folksonomies with TRIAS algorithm. • Belohlavek and Osicka (2010): Triadic concept analysis with fuzzy attributes • Glodeanu (2010) and Belohlavek and Vychodil (2010): Factorization of triadic data. • Konecny and Osicka (2010): General concept-forming operators

(*continued on next page*)

Table 3 (*continued*)

Extension name	Landmark paper	Notorious further research
		• Ignatov et al. (2011): Triclustering extension of biclustering • Glodeanu (2011): Fuzzy valued triadic implications • Kaytoue et al. (2011b): Derive biclusters from gene expression data using triadic FCA • Missaoui and Kwuida (2011): Mining triadic association rules
Temporal FCA	Wolff et al. (2000)	• Wolff (2002a): Introduction of transitions in Conceptual Time Systems • Wolff (2002b): Conceptual Time System with Actual Objects and a Time relation • Wolff (2004), Wolff (2006) and Wolff (2007): Temporal Conceptual Semantic systems to represent distributed objects. • Wolff (2005) and Wolff (2011): Overview of state of Temporal Concept Analysis theory, applications and SIENA tool • Wolff (2009a, 2009b): Relational Semantic Systems • Wolff (2010): Temporal Relational Semantic Systems • Poelmans et al. (2011b): Profiling human trafficking suspects extracted from unstructured texts • Wollbold et al. (2011): Application of Temporal Conceptual Semantic Systems to study disease processes of arthritic patients • Elzinga et al. (2012): Analyzing the evolution of pedophile chat conversations with Temporal Concept Analysis

Table 4
Combining Formal Concept Analysis with fuzzy and rough set theory.

Extension name	Landmark paper	Notorious further research
Fuzzy FCA	Burusco and Fuentes-Gonzalez (1994)	• Belohlavek (1998) and Pollandt (1997): Developed the basic notions of FCA with fuzzy attributes independently • Georgescu and Popescu (2002): Defined the notion of a fuzzy concept lattice associated with fuzzy logic with a non-commutative conjunction • BenYahia and Jaoua (2001) and Krajci (2003): Introduced the one sided fuzzy concept lattices • Krajci (2005a): Defined generalized concept lattices, which use three sets of truth degrees, i.e. for the objects, for the attributes and for the degrees to which objects have attributes • Belohlavek and Vychodil (2005a): Survey of the different fuzzy concept lattice approaches • Belohlavek and Vychodil (2006a): Attribute implications in a fuzzy setting • Belohlavek et al. (2007a, 2011): Identify dependencies between demographic data and degree of physical activity • Belohlavek and Konecny (2007): Scaling and granulation for FCA with fuzzy attributes • Medina et al. (2007, 2009): Multi-adjoint concept lattices were introduced which have adjoint triples, a.k.a. biresiduated structures, at their core • Pankratieva and Kuznetsov (2010, 2012): Mapping between crisply-generated concepts, proto-fuzzy concepts and pattern structures
Rough FCA	Kent (1996)	• Yao and Lin (1997): Generalized formulation of RST using binary relation on two universes instead of equivalence relation • Saquer and Deogun (1999): Approximation of a set of objects, a set of properties and a pair of a set of objects and a set of properties is performed using concepts of a concept lattice • Hu et al. (2001): Definition of partial order on the set of objects instead of equivalence relation as in Kent (1996) • Gediga and Duntsch (2002): Introduction of lattice constructed on approximate operators • Duntsch and Gediga (2003): Introduction of modal-style operators based on a binary relation and the property-oriented concept lattice • Yao (2004): Used modal-style operators to introduce object-oriented concept lattice • Yao and Chen (2006, 2005): Combination of FCA and RST based on definability • Deogun and Jiang (2005): Evaluation of three different approaches for context approximation • Ganter (2008): Lattices obtained from generalized approximation operators forming a kernel-closure pair are described in the language of FCA • Meschke (2009): Robust elements in rough concept lattices • Meschke (2010): Approximations in concept lattices • Ganter and Meschke (2009, 2011): Analysis of large rough datatables

1. combine querying and navigation
2. be reasonably efficient on large collections of objects
3. make use of an expressive language for object descriptions and queries.
4. be generic with respect to the set of objects and language

A logical context $\mathcal{K}$ is a triple $(\mathcal{O}, \mathcal{L}, d)$ where $\mathcal{O}$ is a set of objects, $\mathcal{L}$ is a logic (e.g. proposition calculus) and d is a mapping from $\mathcal{O}$ to $\mathcal{L}$ that describes each object by a formula. A logic is a 6-tuple $\mathcal{L} = (L, \sqsubseteq, \sqcap, \sqcup, \top, \bot)$ where.

- L is the language of formulas
- $\sqsubseteq$ is the subsumption relation
- $\sqcap$ and $\sqcup$ are respectively conjunction and disjunction
- $\top$ and $\bot$ are respectively tautology and contradiction

The extent of a logical formula f is the set of objects in $\mathcal{O}$ whose description is subsumed by f: $\forall f \in \mathcal{L}$ ext $(f) = (o \in \mathcal{O} | d(o) \sqsubseteq f)$.

The intent of a set of objects O is the most precise formula that subsumes all descriptions of objects in O: $\forall O \subseteq \mathcal{O}$ int $(O) = \sqcup_{o \in O} d(o)$.

A logical concept is a pair c=(O,f) where $O \subseteq \mathcal{O}$ and $f \in \mathcal{L}$ such that int $(O) = f$ and ext $(f) = O$. O is called the extent of the concept c, i.e. ext (c) and$f$ is the intent, i.e. int (c). The set of all logical concepts is ordered and forms a lattice: let c and c' be two concepts, $c \leqslant c'$ iff ext $(c) \subseteq$ ext (c'). Note also that $c \leqslant c'$ iff int $(c) \sqsubseteq$ int (c'). c is called a subconcept of c'.

Here is an overview of some notorious applications. Ferré et al. (2004b) developed a database system for bio-informatics research based on LIS. The objects are segments of DNA in chromosomes of some organism, attributes are diverse in nature. The authors integrated data from different sources and build a query language specifically suited for browsing through bioinformatics data. In Ferré (2007); Ferré (2009) the author introduces CAMELIS, an implementation of LIS with multiple applications of which they chose the management of a photo collection as being the most convincing one. Photos are being organized in a lattice using the metadata attached to them. By entering a query, the user can retrieve the extent of a concept in the lattice (which consists of photos) and navigate up or down in the lattice by expanding or refining the entered query respectively. Ducassé and Ferré (2008) apply Camelis in combination with Concept Explorer in the setting of committee meetings to support more rational and comprehensible decision-making (such as choosing among job applicants). Morin and Debar (2005, 2009) developed an LCA based system for alarm fusion and correlation in intrusion detection systems to reduce the number of alerts. LCA is used to reason over the alerts and their relation to other alerts which may help confirm or invalidate them. Pecheanu, Dumitriu, Stefanescu, and Segal (2004); Pecheanu, Dumitriu, Stefanescu, and Segal (2006) used LCA to model training domain knowledge in a computer-assisted instructional environment. Each learner has its own cognitive style and conceptually organizing the available knowledge may help improve the interaction with the user. Bedel, Ferré, Ridoux, and Quesseveur (2007a, 2007b) used LIS to query and browse a collection of individual geographical objects using spatial properties such as position, shape and date. These attributes were used to group these objects. Bedel, Ferré, and Ridoux (2008) increase the complexity of their system by including spatial relations between objects such as distance relations. They show how LCA can be used to navigate through these data and how these spatial relations can be exploited. Cellier, Ferré, Ridoux, and Ducassé (2007); Cellier, Ferré, Ridoux, and Ducassé (2008a) relate FCA with a taxonomy to LCA and describe how the attributes of a context are ordered and can be considered as a logic where the taxonomy represents the order relation. The goal of their research was to learn interesting association rules from such contexts. Ferré (2009) presented an approach for decomposing contexts into simpler and specialized components named logical context functors. Allard, Ferré, and Ridoux (2010) proposed a method based on Online Analytical processing for presenting rules (e.g. association rules) as a set of cubes. The authors build a lattice which has cubes as concepts and the links between concepts represent possible navigation paths. LIS is used to browse this cube lattice. In Ducassé, Ferré, and Cellier (2011) six researchers used LIS to jointly select appropriate conferences for submitting their papers.

The main stream of research about combining logic and FCA focused on LCA, however some other authors did similar research. Chaudron and Maille (1998); Chaudron, Maille, and Boyer (2003) used "cubes" which are existentially quantified conjunctions of first-order literals as intents of formal concepts, resulting in a lattice of cubes and a definition of first-order FCA. Another related approach was developed by Bain (2003, 2004) who combined FCA with first-order logic to learn ontological rules.

5.3.3. Relational concept analysis

Relational concept analysis (RCA) was introduced to take into account relational attributes. Huchard et al. (2002) introduced the relational context family, which is a collection of contexts and inter-context relations, the latter being binary relations between pairs of object sets lying in two different contexts. The objective was to build a set of lattices whose concepts are related by relational attributes.

A relational context family $\Re$ is a pair ($\boldsymbol{K}$,$\boldsymbol{R}$), where $\boldsymbol{K}$ is a set of contexts $\mathcal{K}_i = (O_i, A_i, I_i)$, $\boldsymbol{R}$ is a set of relations $r_k \subseteq O_i \times O_j$, where O_i and O_j are the object sets of the formal contexts $\mathcal{K}_i$ and $\mathcal{K}_j$. A relation $r_k \subseteq O_i \times O_j$ has a domain and a range, where.

- $\boldsymbol{O} = \{O_i | \mathcal{K}_i = (O_i, A_i, I_i),\ \mathcal{K}_i \in \boldsymbol{K}\}$
- $r_k: O_i \rightarrow 2^{O_j}$
- $dom: \boldsymbol{R} \rightarrow \boldsymbol{O}$ and $dom\ (r_k) = O_i$
- $ran: \boldsymbol{R} \rightarrow \boldsymbol{O}$ and $ran\ (r_k) = O_j$
- $rel: \boldsymbol{K} \rightarrow 2^R$ and $rel\ (\mathcal{K}_i) = \{r_k | dom(r_k) = O_i\}$

The instances of relation r_k, $r_k(o_i, o_j)$, where $o_i \in O$ and $o_j \in O_j$, are called links. The links can be " scaled" to be included as binary attributes of the form r:c through relational scaling. Given a relation $r \in rel(\mathcal{K}_i)$ and a lattice $\mathfrak{L}_j$ on $\mathcal{K}_j$ with $O_j = ran(r)$, the narrow scaling operator $sc_x^{(r,\mathfrak{L}j)}: \boldsymbol{K} \rightarrow \boldsymbol{K}$ is defined as follows:

$$sc_x^{(r,\mathfrak{L}j)}(\mathcal{K}_i) = \left(O_i^{(r,\mathfrak{L}j)}, A_i^{(r,Lj)}, I_i^{(r,\mathfrak{L}j)}\right),$$

where

$$O_i^{(r,\mathfrak{L}j)} = O_i, A_i^{(r,\mathfrak{L}j)} = A_i \cup \{r : c | c \in \mathfrak{L}_j, r(o) \neq \emptyset, r(o) \subseteq \text{ extent } (c)\}$$

Applications of RCA theory are mainly in the fields of software (re) engineering and ontology design. The RCA approach is implemented and freely available in the Galicia platform (Huchard, Rouane-Hacene, Cyril Roume, & Valtchev, 2007). The first motivation for developing the RCA theory was the engineering of a static UML class model as described in Dao, Huchard, Rouane-Hacene, Roume, and Valtchev (2004). When applying RCA to software analysis, a returning and tedious task is encoding the model or program as a relational context family and after analysis, decoding the obtained concept lattice family into the initial language. Falleri, Arévalo, Huchard, M, and Nebut (2007) developed a generic encoder/decoder to ease this task in the future. Further research of the author focused on the development of a generic method for improving class models, by removing redundancy and suggesting relevant abstractions. Moha, Rouane-Hacene, Valtchev, and Guéhéneuc (2008) used RCA to automatically propose the user software hierarchy refactorings to correct some specific design defects. In particular, the authors focused on how to split up a Blob class, a very large and computationally heavy collection of methods (Brown, Malveau, McCormick, & Mowbray, 1998) into multiple cohesive sets of class members that can be used to improve the hierarchy. Another approach that uses RCA is proposed by Dolques, Huchard, Nebut, and Reitz (2010b). The authors use RCA to semi-automatically generate transformation rules between different meta-models (i.e. syntaxes) which can be of interest to software developers using Model Driven Engineering. Instead of writing a transformation program, which is common practice their method learns transformation rules from transformation examples. Hamdouni, Seriai, and Huchard (2010) use RCA to recover software components from legacy code. These components can be used to build a high-level architecture which can be used to improve the quality of the system. Chen, Zhang, Li, Kang, and Yang (2009) reengineer existing legacy code to service-oriented software by identifying service candidates using RCA. Azmeh et al. (2011) use RCA to organize web services (FCA objects) based on amongst others quality of service properties (FCA attributes) and composebility of services chosen for neighboring tasks in a workflow (inter-object links). Azmeh et al. (2011) introduce relational queries for navigating such a concept lattice family. Dolques, Nebut, Huchard, and Reitz (2010a, 2012) use RCA to refactor use case diagrams specified in UML.

The second popular application domain of RCA is ontology engineering. Rouane-Hacene, Napoli, Valtchev, Toussaint, and Bendaoud (2008) propose an approach for semi-automated construction of ontologies using RCA. Text analysis is used to transform a document collection into a set of data tables, or contexts and inter-context relations. RCA then turns these into a set of concept lattices with inter-related concepts. Core ontology is derived from the lattice in a semi-automated manner by translating relevant elements into ontological concepts and relations. Rouane-Hacene, Fennouh, Nkambou, and Valtchev (2010) build further on this work by using FCA and RCA for Ontology Design Pattern (ODP) development. ODPs can be reused to improve the quality of ontologies and are in essence similar to design patterns from software engineering.

5.3.4. Power context families

Wille (1997) proposed a method to transform conceptual graphs into a family of formal contexts, called a *Power Context Family* (PCF). A PCF $\mathcal{K} = (\mathcal{K}_n)_{n \in \mathbb{N}0}$ is a family of formal contexts $\mathcal{K}_k = (G_k, M_k, I_k)$ such that $G_k \subseteq (G_o)^k$ for k = 1,2, … The formal contexts $\mathcal{K}_k$ with $k \geqslant 1$ are called relational contexts. The power context family $\mathcal{K}$ is said to be limited of type $n \in \mathbb{N}_0$ if $\mathcal{K} = (\mathcal{K}_0,\ \mathcal{K}_1,\ \mathcal{K}_{2,\dots,}\ \mathcal{K}_n)$, otherwise, it is called unlimited. In Wille (1998), triadic concept graphs and triadic PCFs were proposed. Prediger (1998) presented a logic for defining sample concept graphs as syntactical constructs with a semantics in PCFs. In Prediger (2000a) this research was extended for defining nested concept graphs as situation-based judgments in triadic PCFs. Prediger and Stumme (1999) introduced relational scaling to transform a many-valued context into a PCF. The concept graphs of a PCF were shown to form a lattice which can be described as a subdirect product of specific intervals of the concept lattices of the PCF. Groh and Eklund (1999) discussed various algorithms for creating relational power context families from conceptual graphs. Groh (2002) implemented an algorithm which can transform a simple CG into a PCF and can deal with positive nested CGs. Wille (2001) proposed "Boolean Judgment Logic" and the "negating inversion" to define the negation of formal judgments (defined as valid propositions) represented by concept graphs of PCFs. Wille (2004) proves that the implicational theory of implicational concept graphs of PCFs is equivalent to the theory of attribute implications of formal contexts. In other words implications can be seen as special judgments, namely as specific concept graphs of PCFs. Rudolph (2004) uses binary power context families to define an extensional semantic for concept descriptions expressed in $\mathcal{FLE}$. The authors show how a special kind of contexts can be constructed using a binary power context family from a set of DL-formulae.

5.3.5. Triadic FCA

Recently the focus shifted from developing 2-dimensional to 3-dimensional data analysis techniques. This evolution is driven by the increase in popularity of social resource sharing systems called folksonomies where users can assign tags to resources. *Triadic Concept Analysis* (Triadic CA), started by Lehmann and Wille (1995) and Wille (1995), can provide a solid FCA background for dealing with three-way data. A triadic context is defined as a quadruple (*G*, *M*, *B*, *Y*) where *G* (objects), *M* (attributes) and *B* (conditions) are sets and *Y* is a ternary relation between *G*, *M* and $B{:}Y \subseteq G \times M \times B$. A triple $(g,m,b) \in Y$ means object *g* has attribute *m* under condition *b*. A triadic concept is a triple (A_1,A_2,A_3) with $A_1 \subseteq G$(extent), $A_2 \subseteq M$(intent) and $A_3 \subseteq B$(modus) such that for $X_1 \subseteq G, X_2 \subseteq M$ and $X_3 \subseteq B$ with $X_1 \times X_2 \times X_3 \subseteq Y$; the containments $A_1 \subseteq X_1, A_2 \subseteq X_2$ and $A_3 \subseteq X_3$ always imply $(A_1,A_2,A_3) = (X_1,X_2,X_3)$. Triadic concepts consist of three sets, namely objects, attributes and conditions under which objects may possess certain attributes. Stumme (2005) discusses how traditional line diagrams of standard dyadic concept lattices can be used for exploring triadic data. The author showcases how it can be used for navigating through the Federal Office for Information Security IT baseline Protection Manual. In Jaschke et al. (2006) the author introduced the TRIAS algorithm for mining all frequent closed itemsets from 3-dimensional data and applied it to the popular bibsonomy (users-tags-papers) dataset in Jäschke, Hotho, Schmitz, Ganter, and Stumme (2008). TRIAS basically relies on extraction of closed 2-sets from two binary relations. Belohlavek and Osicka (2010) extended Triadic CA to deal with fuzzy attributes instead of traditional boolean data tables. Alternatives to TRIAS include the algorithms RSM and Cube-Miner which were introduced by Ji, Tan, and Tung (2006) for mining closed 3-sets from ternary relation. In contrast to RSM and TRIAS, Cube-Miner works directly on the ternary relation. The authors use cubes, which have the particularity that none of their tuples are in relation and hereby generalize the closedness checking of Besson, Robardet, Boulicaut, and Rome (2005) to the triadic case. Both TRIAS and CubeMiner have some issues: TRIAS's performance suffers if the first set of objects in the ternary relation is too large. CubeMiner needs for each candidate several checks to ensure its closedness and each candidate can be generated several times. Another approach was proposed in Zhao and Zaki (2005) for mining temporal gene expression data. The authors extracted maximal triclusters satisfying certain homogenity criteria from these gene expression tables. Mining triadic implications from Boolean triadic contexts was discussed in Biedermann (1997) and Ganter et al. (2004). Glodeanu (2011) generalized these triadic implications to a fuzzy setting. Missaoui and Kwuida (2011) discuss the different types of methods for mining triadic association rules from ternary data. Ignatov, Kuznetsov, Magizov, and Zhukov (2011) proposed concept-based triclustering as an extension of concept-based biclustering. The authors use a notion of box-operator instead of triadic concept forming operators and obtain as a result a relaxation of the conventional formal concept (tricluster). Kaytoue, Kuznetsov, Napoli, and Duplessis (2011b) apply the methods of Triadic CA for analysing gene expression data. They scale this data into a triadic context and derive maximal biclusters. Several authors generalized bi-and triclustering even further to the n-dimensional case. Voutsadakis (2002) extended triadic concept analysis to n-dimensional contexts.

5.3.6. Temporal FCA

Temporal Concept Analysis (Temporal CA) addresses the problem of conceptually representing discrete temporal phenomena (Wolff, 2001). Wolff (2005) and Wolff (2011) give an overview of the developments in Temporal CA. The program SIENA can be used for graphical representation of temporal systems. In Wolff (2000), the author introduced the notions of a Conceptual Time System (CTS), a situation and a state of a CTS. A conceptual time system (Wolff & Yameogo, 2003) contains the observations of objects at several points of time. Abstraction is made of the duration of an observation and a point of time, which is called time granule. The starting many-valued context consists of an event part which is scaled to

$$\boldsymbol{C} = ((G,E,V,I),\ (\boldsymbol{S}_e|e \in E))$$

and a time part which is scaled to

$$\boldsymbol{T} = ((G,M,W,I_T),(\boldsymbol{S}_m|m \in M)),$$

both on the same object set **G**. The attributes observed at each time granule are described in the event part of the data table. The pair (***T***, ***C***) is called a CTS on the set ***G*** of time granules. The derived context of T is denoted by $\mathcal{K}_T$, the derived context of **C** is denoted by $\mathcal{K}_C$ and the apposition of $\mathcal{K}_T$ and $\mathcal{K}_C$is denoted by $\mathcal{K}_{TC} = \mathcal{K}_T|\mathcal{K}_C$. The object concepts of this context are called situations, the object concepts of *C* are called states and the object concepts of *T* are called time states. The sets of situations, states and time states are called the situation space, the state space and the time state space of (*T*, *C*), respectively. Wolff (2002) introduced a CTS with a time relation (***T***, **C**, *R*), where $R \subseteq G \times G$ are transitions in a CTS denoted as pairs of time objects $(g,h) \in R$. Let *X* be a set and $f{:}G \rightarrow X$, then *f* induces the mapping

$$f_R : R \to \{(f(g), f(h)) | (g, h) \in R\}$$

where $f_R((g,h)) = (f(g),f(h))$. The element $(g(h),(f(g), f(h))) \in f_R$ is called the f-induced R-transition on X leading from the startpoint $(g,f(g))$ to the endpoint $(h,f(h))$. A transition can intuitively be considered as a "step from one point to another" for an object. Transitions between situations, states, time states, and phases can then be induced easily. This leads to temporal representations of processes. Each arrow in a Temporal CA lattice represents a "transition of the object" and corresponds to an element of R. The transitions form a life track of the object f in X, i.e. the set $f = \{(g,f(g)) | g \in G\}$. In the visualization of the data, the "natural temporal ordering" of the observations is expressed using this time relation R introduced on the set G of time granules of a conceptual time system.

Wolff (2002b) relates the notions of states and transitions in automata theory to the conceptual description of states, situations and transitions in Temporal CA. A CTS with Actual Objects and a Time Relation was introduced in Wolff (2004) to have a mathematical notation for temporal systems in which many objects are moving, e.g., physical particles and waves. Waves and wave packets are "distributed objects" in the sense that they may appear simultaneously at several places. Wolff (2006) investigate how to represent the state of such a distributed object in Temporal CA theory. Wolff and Yameogo (2005) investigate the connection between the theory of computation and Temporal CA. Wolff (2009a); Wolff, (2009b) introduces the relational extension of Temporal CA theory called Temporal Relational Semantic Systems. The applications of Temporal CA theory are discussed in part 2 of this survey (Poelmans et al., 2013a).

5.3.7. Other emerging extensions of FCA

Multiple authors emphasized the need for taking into account *background knowledge* in FCA. Belohlavek and Vychodil (2009c) present an approach for modeling background knowledge that represents user's priorities regarding attributes and their relative importance. Only those concepts that are compatible with user's priorities are considered relevant and are extracted from the data. In Pogel and Ozonoff (2008), FCA is used in combination with a tag context to formally incorporate important kinds of background knowledge. The results are Generalized Contingency Structures and Tagged Contingency Structures which can be used for data summarization in epidemiology. In Cellier et al. (2008a) an algorithm is proposed to learn concept-based rules in the presence of a taxonomy. In its classical form FCA considers attributes as a non-ordered set. When attributes of the context are partially ordered to form a taxonomy, conceptual scaling allows the taxonomy to be taken into account by producing a context completed with all attributes deduced from the taxonomy.

In Besson, Robardet, and Boulicaut (2006), FCA is extended to cope with faults and to improve formal concepts towards fault tolerance. Pfaltz (2007) extends FCA to deal with numerical values, by deriving logical implications containing ordinal inequalities as atoms, such as $y \leqslant 11$. This extension employs the fact that orderings are antimatroid closure spaces. Valverde-Albacete and Pelaez-Moreno (2006) introduced a generalization of FCA for data mining applications called $\mathcal{K}$-Formal Concept Analysis where incidence values are elements of a semiring instead of 0 or 1. This idea was further developed in Valverde-Albacete and Pelaez-Moreno (2007) where the lattice structure for such generalized contexts was introduced. This research topic was further investigated in Valverde-Albacete and Pelaez-Moreno (2008, 2011). González Calabozo, Peláez-Moreno, and Valverde-Albacete (2011) applied $\mathcal{K}$-FCA to the analysis of gene expression data. In Deogun, Jiang, Xie, and Raghavan (2003), FCA is complemented with Bacchus probability logic, which makes use of statistical and propositional probability inference. The authors introduce a new type of concept called "previously unknown and potentially useful" and formalize KDD as a process to find such concepts.

5.4. Mining uncertain data with FCA

In this section we give a concise overview of the developments in fuzzy and rough FCA theory. We chose to mention in Table 4 only a selected number of publications which gives an overview of the main research streams in these areas. The reader is kindly referred to Poelmans, Ignatov, Kuznetsov, and Dedene (2013b) for a full overview of the development of this theory.

5.4.1. Fuzzy FCA

Fuzzy concept lattices were first introduced by Burusco and Fuentes-Gonzalez (1994), however their theory had its limitations since they did not use residuated lattices and merely introduced some basic notions. Belohlavek and Pollandt further developed the basic notions of FCA with fuzzy attributes independently, so, there are two seminal works Belohlavek (1998) and Pollandt (1997) on this topic. The paper Belohlavek (1998) focuses on studying similarity in fuzzy concept lattices rather than introducing basic notions. The structure of truth degrees L endowed by logical connectives is represented by a complete resituated lattice $\boldsymbol{L} = \langle L, \wedge, \vee, \otimes, \to, 0, 1\rangle$ such that:

- $\langle L, \wedge, \vee, 0, 1\rangle$ is a complete lattice with 0 and 1 being the least and greatest element of L respectively
- $(L, \otimes, 1)$ is a commutative monoid, i.e. $\otimes$ is commutative, associative and $a \otimes 1 = 1 \otimes a = a$ for each $a \in L$
- $\otimes$ and L satisfy the adjointness property $a \otimes b \leqslant c \Leftrightarrow a \leqslant b \to c$; for each $a,b,c \in L$
- $\otimes$ and $\to$ are (truth functions of) "fuzzy conjunction" and "fuzzy implication"
- By $\boldsymbol{L}^U$ (or L^U) we denote the collection of all fuzzy sets in a universe U, i.e. mappings A of U to L. For $A \in L^U$ and $a \in L$, a set ${}^aA = \{u \in U | A(u) \geqslant a\}$ is called an a-cut of A.

In Belohlavek (2001b) the author studies *fuzzy closure operators*, which are, along with fuzzy Galois connections, the basic structures behind fuzzy concept lattices. These results have been later used many times in different papers on the topic. In Belohlavek (2001a) the author also shows a relationship between fuzzy Galois connections and ordinary Galois connections. He uses this result to show how a fuzzy concept lattice may be regarded as an ordinary concept lattice. Belohlavek and Vychodil (2005a) present a survey and comparison of the different approaches to fuzzy concept lattices which were elaborated till that time. Belohlavek and Vychodil (2006a) give an overview of recent developments concerning attribute implications in a fuzzy setting. The recent computational issues concerning algorithms for finding fuzzy concepts are discussed in the paper Belohlavek et al. (2010).

The following definition of a formal fuzzy concept lattice was introduced in Belohlavek (1999, 2004a) and Belohlavek and Vychodil (2005a). A fuzzy formal context (L-context) is a triple (X,Y,I), where X is the set of objects, Y is the set of attributes and $I: X \times Y \to L$ is a fuzzy relation (L-relation) between X and Y. A truth degree $I(x,y) \in L$ is assigned to each pair (x,y), where $x \in X, y \in Y$ and L is the set of values of a complete residuated lattice $\mathbf{L}$. The element $I(x,y)$ is interpreted as the degree to which attribute y applies to object x. Fuzzy sets $A \in L^X$ and $B \in L^Y$ are mapped to fuzzy sets $A^{\uparrow} \in L^Y, B^{\downarrow} \in L^X$ according to Belohlavek (1999).

$$A^{\uparrow}(y) = \Lambda_{x \in X}(A(x) \to I(x,y))$$
$$B^{\downarrow}(x) = \Lambda_{y \in Y}(B(y) \to I(x,y))$$

for $y \in Y$ and $x \in X$. A *formal fuzzy concept* $\langle A, B\rangle$ consists of a fuzzy set A of objects (extent of the concept) and a fuzzy set B of attributes (intent of the concept) such that $A^{\uparrow} = B$ and $B^{\downarrow} = A$. The set of all formal fuzzy concepts is $\mathfrak{B}(X,Y,I) = \{\langle A,B\rangle | A^{\uparrow} = B, B^{\downarrow} = A\}$ We also define $\leqslant$ which models the subconcept-superconcept hierarchy in $\mathfrak{B}(X, Y, I)$: $\langle A_1, B_1\rangle \leqslant \langle A_2, B_2\rangle \Leftrightarrow A_1 \subseteq A_2 \Leftrightarrow B_2 \subseteq B_1$ for $\langle A_1, B_1\rangle, \langle A_2, B_2\rangle \in B(X,Y,I)$. $\langle \mathfrak{B}(X,Y,I), \leqslant\rangle$, i.e. $\mathfrak{B}(X,Y,I)$ equipped with relation $\leqslant$ is a complete lattice w.r.t. $\leqslant$. For a more complete overview of these developments, the reader is referred to Poelmans et al. (2013b), where other related formalisms, such as one-sided fuzzy concept lattices (BenYahia & Jaoua, 2001;Krajci, 2003) proto-fuzzy concepts (Kridlo & Krajci, 2008), multi-adjoint concept lattices (Medina, Ojeda-Aciego, & Ruiz-Calvino, 2009) and their relations to fuzzy setting are discussed.

5.4.2. Rough FCA

Rough Set Theory (RST) was introduced by Pawlak (1982, 1985, 1992) and is a mathematical technique to deal with uncertainty and imperfect knowledge. In rough set theory, the data for analysis consists of universe U. Objects characterized by the same attribute values are called indiscernible (similar) in view of the available information about them. By modeling indiscernibility as an equivalence relation, $E \subseteq U \times U$ one can partition a finite universe of objects into pair wise disjoint subsets denoted by U/E. The partition provides a granulated view of the universe. An equivalence class is considered as a whole, instead of many individuals. For an object $x \in U$, the equivalence class containing x is given by $[x]_E = \{y \in U | xEy\}$. The empty set, equivalence classes and unions of equivalence classes form a system of definable subsets under discernibility. It is a σ-algebra $\sigma(U/E) \subseteq 2^U$ with basis U/E where 2^U is the power set of U. All subsets not in the system are consequently approximated through definable sets. Various definitions of rough set approximations have been proposed including the subsystem-based, granule-based and element-based formulation (Yao & Chen, 2005). In this section we introduce the subsystem-based formulation. In an *approximation space apr*=(U,E), a pair of approximation operators $\overline{(\cdot)}, \underline{(\cdot)} : 2^U \to 2^U$, is defined. The *lower approximation*

$$\underline{A} \in \sigma(U/E) = \cup\{X | X \in \sigma(U/E), X \subseteq A\}$$

is the greatest definable set contained in A, and the *upper approximation*

$$\overline{A} \in \sigma(U/E) = \cap\{X | X \in \sigma(U/E), A \subseteq X\}$$

is the largest definable set containing A. Any set of all indiscernible (similar) objects is called an elementary set (neighborhood) and forms a basic granule (atom) of knowledge about the universe. Any union of elementary sets is a crisp (precise) set, otherwise the set is rough (imprecise, vague). Each rough set has boundary-line cases, i.e. objects which cannot be classified with certainty as either members of the set or its complement. Crisp sets have no boundary-line elements at all. Boundary-line cases cannot be properly classified by employing the available knowledge.

Vague concepts (in contrast to precise concepts) cannot be characterized in terms of information about their elements. In other words, given an arbitrary subset $A \subseteq U$ of the universe of objects, it may not be the extent of a formal concept. This subset can be seen as an undefinable set of objects and can be approximated by definable sets of objects, namely extents of formal concepts. Any vague concept is replaced by a pair of precise concepts, called the lower and upper approximation of the vague concept. The lower approximation consists of all objects which surely belong to the concept and the upper approximation contains all objects which possibly belong to the concept. The difference between the lower and upper approximation constitutes the boundary region of the vague concept.

Many efforts have been made to combine FCA and rough set theory (Yao, 2004a). This combination is typically referred to as *Rough Formal Concept Analysis* (RFCA). In RFCA $\sigma(U/E)$ is replaced by lattice $\boldsymbol{L}$ and definable sets of objects by extents of formal concepts. The extents of the resulting two concepts are the approximations of A. For a set of objects $A \subseteq U$ its lower approximation is defined as

$$\underline{l}(A) = (\cup\{X | (X,Y) \in L, X \subseteq A\})''$$

and its upper approximation is defined by

$$\overline{l}(A) = \cap\{X | (X,Y) \in L, A \subseteq X\}$$

The lower approximation of a set of objects A is the extent of $(\underline{l}(A), (\underline{l}(A))')$ and the upper approximation is the extent of the formal concept $(\overline{l}(A), (\overline{l}(A))')$. The concept $(\underline{l}(A), (\underline{l}(A))')$, is the supremum of concepts where extents are subsets of A and is the infimum of those concepts where extents are superset of A. For the other RST formulations a similar description can be given for the combination of FCA and RST. Ganter (2008) use a generalization of the indiscernibility relation which is considered as a quasi-order of equivalence given by $g \leqslant h \Leftrightarrow g' \subseteq h'$ for $g, h \in G$. This generalization allows for defining version spaces and hypotheses in terms of RST. Ganter (2008) generalized the classical rough set approach by replacing lower and upper approximations with arbitrary kernel and closure operators respectively. Lattices of rough set abstractions were described as P-products. Meschke (2009) further investigated the role of robust elements, the possible existence of suitable negation operators and the structure of corresponding lattices. Meschke (2010) builds further on this work by restricting the view for large contexts to a subcontext without losing implicational knowledge about the selected objects and attributes. Ganter and Meschke (2009, 2011) use a slightly different approach based on FCA to mine the Infobright dataset containing so-called rough tables where objects are combined together in data packs (Infobright, 2012). Their approach makes FCA useful for analyzing extremely large data. For a detailed overview of the development of the theory, the reader is referred to Poelmans et al. (2013b).

5.5. Scalability

At the international Conference on Formal Concept Analysis in Dresden (ICFCA, 2006) an open problem of "handling large contexts" was pointed out. Since then, several studies have focused on the scalability of FCA for efficiently handling large and complex datasets. Scalability is a real issue for FCA, since the number of formal concepts can be exponential in the input context and counting them is #P-complete (Kuznetsov, 2001), however, all concepts can be constructed with polynomial delay, see an overview of many FCA-based algorithms in (Kuznetsov, 2002). The sizes of implication bases, even the size of the minimal (stem or Duquenne-Guigues) implication base can also be exponential, counting the size of the stem base being #P-hard (Kuznetsov, 2004c), see more results on algorithmic complexity in (Distel, 2010a; Babin & Kuznetsov, 2010). That is why efficiently handling large and complex datasets became a challenge for FCA-based research. Besides designing more efficient practical algorithms (see, e.g., Andrews, 2009; Berry, Bordat, & Sigayret, 2007; Fu & Nguifo, 2004b; Kengue, Valtchev, & Djamégni, 2005; Krajca & Vychodil, 2009; van der Merwe, Obiedkov, & Kourie, 2004), the following approaches to reducing complexity are popular: nested line diagrams for zooming in and out of the data, conceptual scaling for transforming many-valued contexts into single-valued contexts, interestingness measures including concept stability, size of concept extent, and other pruning strategies to reduce the size of the concept lattice, etc.

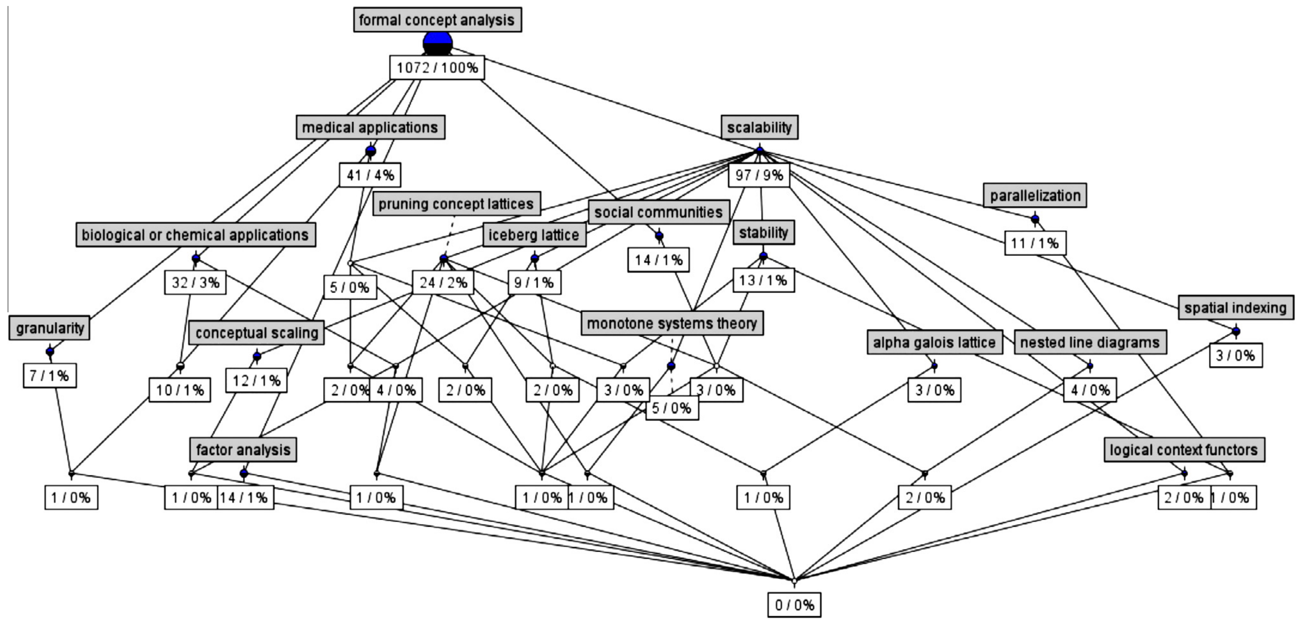

Fig. 5. Lattice containing papers on FCA and scalability.

Popular research topics for improving scalability of FCA are shown in Fig. 5. Nine of these papers use iceberg lattices. Thirteen of these papers apply a pruning strategy based on concept stability to reduce the size of concept lattices. Techniques for handling complex data include logical context functors (see Section 5.3.2) and nested line diagrams. Finally we have the topics parallelization and the combination with binary decision diagrams and spatial indexing for improving the scalability of FCA-based algorithms. In the remainder of this section we focus on the methods developed for selecting interesting concepts.

When we want to apply FCA to formal contexts which contain hundreds of thousands of formal objects, *iceberg lattices* with up to a few thousand formal concepts are one way to deal with this situation. Generating subsets of concepts using thresholds on sizes of extent and intent is a very natural idea coming from pattern recognition and machine learning: it was proposed to control the set of generated concepts by Kuznetsov (1989) and Kuznetsov and Finn (1996). This idea became popular in works on iceberg lattices (Stumme et al., 2002a) which impose an order filter on a concept lattice consisting of concepts with "large" extents (and "small" intents). In terms of data mining, iceberg lattices consist of frequent closed itemsets. They have been used from time to time in knowledge discovery applications. For example in Jay, Kohler, and Napoli (2008a), an experiment is performed in which iceberg lattices are used to discover and represent flows of patients within a healthcare network. In Stumme (2004) it was investigated how iceberg query lattices can be used as condensed representations of frequent datalog queries. Ventos and Soldano (2005) show that iceberg lattices make a special class of the alpha-Galois lattices which they introduced. The algorithmic innovations for mining frequent closed itemsets are discussed in Section 5.1. The extension of iceberg lattices to three dimensional data is discussed in Section 5.3.5.

Kuznetsov (2007b) introduced *concept stability* (Kuznetsov, 1990) as an interest measure for concepts. For a context $\mathcal{K} = (G, M, I)$ and a concept $C = (A, B)$ several stability indices can be defined:

$$\langle c\rangle_j = \{Y \subset A \| y| = j, Y' = B\}, \langle c\rangle_\Sigma = U_{j=2}^{n-1}\langle c\rangle_j,$$
$$\gamma_j(c) = |\langle c\rangle_j|, \gamma_\Sigma(c) = |\langle c\rangle_\Sigma|, n = (A),$$

the stability index $J_j(c)$ of the j-th lvel ($2 \leqslant j \leqslant n$-1):

$$J_j(c) = \frac{\gamma j(c)}{\binom{n}{j}},$$

the integral stability index

$$J_\Sigma(c) = \frac{\gamma_\Sigma(c)}{2^n}.$$

Stability has been used for pruning concept lattices, e.g. in the field of social networks (Kuznetsov, Obiedkov, & Roth, 2007a; Roth, Obiedkov, & Kourie, 2008a) and linguistics (Falk & Gardent, 2011). In Jay, Kohler, and Napoli (2008b), concept stability and support measures were used to reduce the size of large concept lattices. These lattices were used to discover and represent special groups of individuals called social communities. Roth, Obiedkov, and Kourie (2008b) suggest a combined approach of using a pruning strategy based on stability indices of concepts and apply it on its own and in combination with nested line diagrams for representing knowledge communities (a community of embryologists was taken as a case study). *Concept independence* (Klimushkin, Obiedkov, & Roth, 2010) is a similar index showing the sparseness of objects and attributes outside the concept extent and intent that have same attributes and objects, respectively. *Concept probability* (Klimushkin et al., 2010) is the sample probability of a concept in a randomly generated context with the same number of units. Babin and Kuznetsov (2012) proposed an algorithm for approximating the concept stability index calculation, which has a better time complexity.

Hashemi, De Agostino, Westgeest, and Talburt (2004) propose a method for efficiently creating a new lattice from an already existing one when the data granularity is changed. Torim and Lindroos (2008) present a method based on the theory of monotone systems for presenting information in the lattice in a more compressed form. The result of the method is a sequence of concepts sorted by "goodness" thus enabling the user to select a subset and build a corresponding sub-lattice of desired size. This is achieved by defining a weight function that is monotone. Krajca, Outrata, and Vychodil (2010); Krajca, Outrata, and Vychodil (2012) present an algorithm which applies dynamic context reduction and sorting of attributes while computing the formal concepts of a context.

The number of formal concepts computed multiple times by their algorithm is substantially lower than after applying other related algorithms from the Close by One (CbO) family.

Another FCA scalability research topic is attribute reduction. Dias and Vieira (2010) presented the JBOS approach for replacing groups of similar objects by prototypical ones. Ganter and Kuznetsov (2008) describe how scaled many-valued contexts of FCA may make feature selection easier. Snásel, Polovincak, Abdulla, and Horak (2008) study the reduction of concept lattices and implication bases based on matrix reduction. Other authors who investigated attribute reduction in the context of FCA are Wu, Leung, and Mi (2009) and Kumar and Srinivas (2010).

6. Conclusions

Since its introduction in 1982 as a mathematical technique, FCA became a well-known instrument in computer science. Over 1000 papers have been published over the past 9 years on FCA and many of them contained case studies showing the method's usefulness in real-life practice. This paper showcased the possibilities of FCA as a meta technique for categorizing the literature on concept analysis. The intuitive visual interface of the concept lattices allowed us to do an in-depth exploration of the main topics in FCA research. In particular, its combination with text mining methods resulted in a powerful synergy of automated text analysis and human control over the discovery process.

One of the most notorious research topics covering 20% of the FCA papers is KDD. In this paper we surveyed the papers on FCA based methods for data mining. A large amount of research was dedicated to association rule mining. Minimal generators were found to play a crucial role in many of the FCA-related incremental and efficient itemset mining algorithms. Complex descriptions for FCA was a second major research field we surveyed. These complex descriptions include Logical, Relational, Temporal and Triadic Concept Analysis, fuzzy and rough FCA, power context families and pattern structures. Finally, expressing machine learning models such as version space learning in FCA terms and scalability issues were investigated too.

In 18% of the papers, traditional concept lattices were extended to deal with uncertain, three-dimensional and temporal data. In particular, combining FCA with fuzzy and rough set theory received considerable attention in the literature. Temporal and Triadic Concept Analysis received only minor attention. In the future, we will host the references and links to the articles on a public interface and hope that this compendium may serve to guide both practitioners and researchers to new and improved avenues for FCA.

Acknowledgments

We are grateful to Paul Elzinga for his help in processing the papers and generating the context files on which the lattices are based.

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