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Mathematical Models of Corruption

[Abstract:] *The article considers mathematical models of corruption as an economic and sociopolitical phenomenon. They are classified according to the objects of corrupt deals, as well as the structure of corrupt relations.*

1. Model of corruption in tax agencies

P. Chander and L. Wilde have suggested a model of corruption in tax agencies [1]. Its participants are taxpayers who conceal taxes and auditors (tax officials) who are supposed to check the legitimacy of tax payments. The auditors can take bribes from taxpayers to cover up for them in the case of false reporting of income. The model shows that in the case when some of the taxpayers want to give bribes and some of the auditors are willing to accept them, the tax inspection will most likely refrain from any kind of audit, in contrast to the case when no one gives or takes bribes. If any audits are still conducted, the possibility of establishing corrupt relations leads to higher audit rates than

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in the absence of corruption. It is shown in [1] that with the existence of corruption it is possible to maintain an equilibrium in which all incomes undergo an audit. What is more, if some of the auditors take bribes, a situation arises where an increase in the tax rates or the amount of the fine decreases the possible tax collection.

Since the model is of interest due to the timeliness of the situation, we will examine it in greater detail. It expands the GRW game model (Graetz, Reinganum, and Wilde), adding to it possibilities of corruption in the tax inspection.

In the GRW model, there are two possible levels of incomes: I_L and I_H, where $0 < I_L < I_H$. Income is a random quantity independently distributed among taxpayers. Wtih probability q, a given player has a high income; and with probability $(1 - q)$, a low one. A taxpayer's declared income must be equal to either I_L or I_H. Consequently, those whose income is actually I_L declare it as such, while those who have an income equal to I_H may also declare I_L. The tax rates for high and low incomes are equal to T_L and T_H, respectively, where $0 < T_L < T_H$. The fine for concealing one's income is equal to F, where $F > 0$.

If a taxpayer has declared a low income, the tax authorities may conduct an audit. The cost of an audit is c, where $T_H - T_L + F > c > 0$. As a result of an audit, a taxpayer who has concealed income I_H will be found without fail.

Suppose that α is the probability of preserving a high income, and β is the probability of an audit of a declared low income. We will assume that all of the players are neutral to risk. In this case, the taxpayers maximize their expected income, while the tax services maximize their expected profit, minus auditing expenses. If $c > q(T_H - T_L + F)$, then the sole Nash equilibrium in the game will be $\alpha^* = 1$, $\beta^* = 0$. If $c \geq q(T_H - T_L + F)$, then the sole Nash equilibrium in the game will be

$$\alpha^* = (1 - q)c/q(T_H - T_L + F - c),\ \beta^* = (T_H - T_L)/(T_H - T_L + F). \quad (1)$$

Now we will assume that the auditor can hide the results of the audit and thereby protect a taxpayer who has falsely declared his income from paying the difference between the taxes on the different incomes $(T_H - T_L)$, as well as the fine for concealing income F. Say that, with some probability, both of them will be caught, and in that case they will incur additional expenses. We will now include in the model the premise that some auditors will take bribes and others will not, while some taxpayers want to give bribes and others do not. We will let γ be the portion of the sum of the fine and the additional tax $(T_H - T_L + F)$ that goes to the financial inspector as a bribe and K_A be the punishment that an auditor incurs if he is caught taking a bribe (this information is concealed from the other players). We will also let K_T be the punishment received by a taxpayer who tries to conceal his income if the fact that he has given a bribe is discovered (this information is concealed from the other players) and p be the probability that the fact that a bribe has been given will be discovered. We will introduce certain assumptions.

Assumption 1. K_T does not depend on the amount of income concealed or the size of the bribe. This random quantity is distributed in the set of taxpayers according to the distribution function $G_T(\cdot)$, with probability density $g_T(\cdot)$.

Assumption 2. K_A does not depend on the amount of the total tax evasion known to a given auditor or on how many bribes he has received. The punishment for taking a bribe is collected from an auditor once for each known bribe. This random quantity is distributed in the set of financial inspectors according to the distribution function $G_A(\cdot)$, with probability density $g_A(\cdot)$.

Assumption 3. The value of γ is assigned from outside.

Assumption 4. The variable p does not depend on the amount of total tax evasion known to a given auditor or on how many bribes he has already received. This is the risk to which the auditor and the taxpayer who offers a bribe are exposed in each specific case of bribery. It is the same for all auditors.

Assumption 5. The auditors are neutral to risk and maximize their expected income.

Thus, an auditor can either completely conceal the fact of tax evasion or turn in a negligent taxpayer. If the financial inspector shuts his eyes to the false report, then he receives $\gamma(T_H - T_L + F)$, provided that he himself is not caught taking a bribe, or he loses K_A if the bribe is discovered. Consequently, an auditor, for whom the punishment for taking a bribe is K_A, will agree to it only if

$$\gamma(T_H - T_L + F)(1 - p) > pK_A. \tag{2}$$

In other words, an auditor will take a bribe only if $K_A < K_A^*$, where

$$K_A^* = \gamma\Delta(1 - p)/p, \text{ where } \Delta = T_H - T_L + F. \tag{3}$$

Assumption 6. The taxpayers are neutral to risk. They minimize their expected expenses connected with paying taxes. These expenses include the taxes themselves, bribes, and punishment for tax evasion.

We will assume that the punishment that a given taxpayer incurs for concealing income is K_T. He received a high income, but reported a low one. If he is audited by a tax inspector, for whom the punishment for taking a bribe is K_A, with $K_A < K_A^*$, that is, in principle, he will take bribes, but the taxpayer does not offer him a bribe, then the taxpayer will have to pay $T_H + F$. If the taxpayer decides to give a bribe, then his expected expenses will be $T_L + (1 - p)\gamma\Delta + p(\Delta + K_T)$. Consequently, the taxpayer offers a bribe only if

$$T_H + F > T_L + (1 - p)\gamma\Delta + p(\Delta + K_T). \tag{4}$$

In other words, the taxpayer offers a **bribe only if** $K_T < K_T^*$, **where**

$$K_T^* = (1 - \gamma)\Delta(1 - p)/p. \tag{5}$$

Definition 1. *We will call a taxpayer honest if* $K_T \geq K_T^*$, *and dishonest if* $K_T < K_T^*$.

We will let α^H represent the probability that an honest taxpayer will decide to evade taxes; and $\alpha^D(K_T)$, the probability that a dishonest taxpayer, for whom the punishment for bribery is K_T, will conceal his high income. Since all taxpayers are equal *a priori* for the tax inspection, β (the probability of auditing a low income) is the same for all of them. Let $C^H(\alpha^H, \beta)$ be a function of the total expenses that apply to an honest taxpayer who conceals his income and $C^D[\alpha^D(K_T), \beta; K_T]$ be a function of the expenses of a dishonest taxpayer who evades paying his taxes, for whom the punishment for bribery is K_T. Then,

$$C^H(\alpha^H,\beta)=\alpha^H[\beta(T_H+F)+(1-\beta)T_L]+(1-\alpha^H)T_H=\alpha^H[\beta\Delta-(T_H-T_L)]+T_H, \qquad (6)$$

$$C^D[\alpha^D(K_T),\beta;K_T]=\alpha^D(K_T)[\beta\{G_A^*[T_L+(1-p)\gamma\Delta+p(\Delta+K_T)+$$
$$+(1-G_A^*)(T_H+F)\}+(1-\beta)T_L]+[1-\alpha^D(K_T)T_H=\alpha^D(K_T)[\beta\Delta-(T_H-T_L)]+$$
$$+T_H-\alpha^D(K_T)\beta G_A^*[\Delta(1-p)(1-\gamma)-pK_T], \qquad (7)$$

where $G_A^* \equiv G_A(K_A^*)$. (8)

Definition 2. *The best answer for an honest tax defaulter* $\varphi^H(\beta)$ *is the value of* α^H *that minimizes* $C^H(\alpha^H, \beta)$. *The best answer for a dishonest defaulter* $\varphi^D(\beta)$ *is the value of* $\alpha^D(\beta)$ *that minimizes the function* $C^D[\alpha^D(K_T), \beta; K_T]$.

From the linearity of C^H with respect to α^H and C^D with respect to $\alpha^D(K_T)$, it follows that:

Proposition 1. *The best answer for an honest tax defaulter is*

$$\varphi_H(\beta)=\begin{cases}0, & if \;\; \beta\Delta > T_H - T_L,\\ \in[0,1], & if \;\; \beta\Delta = T_H - T_L,\\ 1, & if \;\; \beta\Delta < T_H - T_L.\end{cases} \qquad (9)$$

The best answer for a dishonest defaulter will be

$$\varphi_H(\beta;K_T)=\begin{cases}0, & \text{if} \;\; \beta\Delta > T_H-T_L+\beta G_A^*[\Delta(1-p)(1-\gamma)-pK_T],\\ \in[0,1], & \text{if} \;\; \beta\Delta = T_H-T_L+\beta G_A^*[\Delta(1-p)(1-\gamma)-pK_T],\\ 1, & \text{if} \;\; \beta\Delta < \mathrm{T_H}-T_L+\beta G_A^*[\Delta(1-p)(1-\gamma)-pK_T].\end{cases} \qquad (10)$$

For the subsequent analysis, it is convenient to consider the probabilities of an audit that leave honest and dishonest defaulters indifferent in relation to whether they conceal their income or not. From proposition 1, it follows that

$$\beta^H=(T_H-T_L)/\Delta; \;\; \beta^D(K_T)=(T_H-T_L)/\{\Delta-G_A^*[\Delta(1-p)(1-\gamma)-pK_T]\}. \qquad (11)$$

Proposition 2. $\beta^D(K_T) > \beta^H$ *for all* $K_T < K_T^*$,

$\beta^D(K_T^*) = \beta^H$ and $d\beta^D(K_T)/dK_T < 0$. (12)

Say that the number of audits does not depend on the specific auditors, but is determined at a higher administrative level. In this model, there are two types of tax inspectors: naive and experienced. We will represent the probability of an audit conducted by these types of inspectors as β^N and β^S, respectively.

Definition 3. *Naive tax inspectors believe that all taxpayers are honest. Experienced tax inspectors accept the possibility of corruption.*

Assumption 7. Tax inspectors are neutral to risk. They maximize tax collections, excluding the cost of an audit and disregarding punishment for bribes, but taking into account penalties for concealing income.

Since naive tax inspectors deny the possibility of bribes, then according to assumption 7 they maximize

$$\pi^N(\alpha^H,\beta^N) = \beta^N[\mu^N(T_H + F) + (1-\mu^N)T_L - c] + (1-\beta^N)T_L =$$

$$= \beta^N(\mu^N\Delta - c) + T_L, \tag{13}$$

where μ^N is the probability of concealing a high income, if the possibility of bribes is eliminated. In other words, the Bayes probability is equal to

$$\mu^N = \alpha^H q/\alpha^H q + 1 - q). \tag{14}$$

Experienced tax inspectors run into a more complicated problem, because they understand that each dishonest defaulter acts according to the strategy that is best for him when declaring his income. Suppose that $\alpha^D(K_T)$ is the probability that a dishonest defaulter, for whom the punishment for a bribe is K_T, conceals his high income.

Let α^H be the probability that an honest defaulter conceals his income. In that case, an experienced tax inspector maximizes the following function:

$$\pi^S(\alpha^H,\alpha^D,\beta^S) = \beta^S\{\mu^{SD}[G_A^*(T_L + p\Delta) + (1-G_A^*)(T_H + F)] + \mu^{SH}(T_H + F) +$$

$$+ (1-\mu^{SD}-\mu^{SH})T_L - c\} + (1-\beta^S)T_L =$$

$$= \beta^S[(\mu^{SD}\Delta(1-G_A^* - pG_A^*) + \mu^{SH}\Delta - c] + T_L, \tag{15}$$

where μ^{SD} and μ^{SH} are the probabilities that a report of a low income will come from a dishonest and an honest defaulter, respectively. These probabilities can be found using the Bayes approach.

Definition 4. *For a naive tax inspector, the best answer, which we will represent as* $\psi^N(\alpha^H)$, *is the value of* β^N *that maximizes* $\pi^N(\alpha^H, \beta^N)$. *The best answer for an experienced tax inspector, which we will represent as* $\psi^S(\alpha^H, \alpha^D)$, *is the value of* β^S *that maximizes* $\pi^S(\alpha^H, \alpha^D, \beta^S)$.

The following proposition follows from the linearity of φ^N with respect to α^H and φ^S with respect to μ^{SH} and μ^D.

Proposition 3. *The best answer for a naive tax inspector is*

$$\psi^N(\alpha^H)=\begin{cases}0, & if\ \ \alpha^H<(1-q)c/q(\Delta-c),\\ \in[0,1], & if\ \ \alpha^H=(1-q)c/q(\Delta-c),\\ 1, & if\ \ \alpha^H>(1-q)c/q(\Delta-c).\end{cases} \tag{16}$$

The best answer for an experienced tax inspector is

$$\psi^D(\alpha^H,\alpha^D)=\begin{cases}0, & \text{if}\ \ \mu^{SD}\Delta(1-G_A^*+pG_A^*)+\mu^{SH}\Delta-c<0,\\ \in[0,1], & \text{if}\ \ \mu^{SD}\Delta(1-G_A^*+pG_A^*)+\mu^{SH}\Delta-c=0,\\ 1, & \text{if}\ \ \mu^{SD}\Delta(1-G_A^*+pG_A^*)+\mu^{SH}\Delta-c>0,\end{cases} \tag{17}$$

where μ^{SD} *and* μ^{SH} *are defined above.*

Equilibrium in the model of corruption in tax agencies. Two types of tax inspectors were considered above: naive and experienced. We will introduce the concept of equilibrium for each of these types.

Definition 5. *The pair* $(\overline{\alpha}^H,\overline{\beta}^N)$ *such that* $\overline{\alpha}^H=\varphi^H(\overline{\beta}^N)$ *and* $\overline{\beta}^N=\psi^N(\overline{\alpha}^H)$ *is called the "naive" equilibrium. The triad* $(\hat{\alpha}^H,\hat{\alpha}^D(K_T),\hat{\beta}^S)$ *such that* $\hat{\alpha}^H=\varphi^H(\hat{\beta}^S)$ $\hat{\alpha}^D(K_T)=\varphi^D(\hat{\beta}^S,K_T)$ *and* $\hat{\beta}^S=\psi^S(\hat{\alpha}^H,\hat{\alpha}^D)$ *is called the "experienced" equilibrium.*

The existence of a unique naive equilibrium follows from the same reasoning as the existence of an equilibrium in the GRW model.

Proposition 4. *It follows that there is a unique "naive" equilibrium. There can be two types of this equilibrium:*

(1) *If* $c>q\Delta$ *then* $\overline{\alpha}^H=1$ *and* $\overline{\beta}^N=0$. *The value of* α^D *corresponding to this equilibrium is equal to one.*

(2) *If* $c\geq q\Delta$ *then* $\overline{\alpha}^H=(1-q)c/q(\Delta-c)$ *and* $\overline{\beta}^N=(T_H-T_L)/\Delta\equiv\beta^H$. *The value of* α^D *corresponding to this equilibrium is equal to one.*

The first type of "naive" equilibrium is realized when the cost of an audit is high. In this case, audits are not conducted at all, and both categories of taxpayers, honest and dishonest, conceal their high income. The second type of "naive" equilibrium is realized when the cost of an audit is low. In this case, the decision to conduct an audit is random, as is the decision of honest defaulters to conceal their income. In this equilibrium, dishonest defaulters always conceal their high income.

In the "experienced" equilibrium, there are the two cases described above, plus two more, in which $\hat{\beta}^S>\beta^H$. These additional equilibria are characterized by the fact that some dishonest taxpayers always conceal their income, and some always report their high income. There is only one type of indifferent dishonest defaulter, and for this type, we will define the punishment for a bribe as $H(\hat{\beta}^S)$ from proposition 1, we get

$$H(\hat{\beta}^S) = K_T^* - [\hat{\beta}^S \Delta - (T_H - T_L)] / \hat{\beta}^S p G_A^* \tag{18}$$

Proposition 5. *There is a unique "experienced" equilibrium. There can be four types of this equilibrium:*

(a) if $c > q\Delta[1 - G_T^* G_A^*(1-p)]$ *then* $\hat{\alpha}^H = \hat{\alpha}^D(K_T)$ *for all* $K_T < K_T^*$ *and* $\hat{\beta}^S = 0$;

(b) if $q\Delta[1 - G_T^* G_A^*(1-p)] \geq c \geq q G_T^* \Delta(1 - G_A^* + pG_A^*)/(1 - q + qG_T^*)$ then $\hat{\alpha}^H = [c(1 - q + qG_T^*) - qG_T^*\Delta(1 - G_A^* + pG_A^*)] / q(1 - G_T^*)(\Delta - c)$ and $\hat{\alpha}^D(K_T) = 1$ *for all* $K_T < K_T^*$ *and* $\hat{\beta}^S = (T_H - T_L)/\Delta$;

(c) if $qG_T^*\Delta(1 - G_A^* + pG_A^*)/(1 - q + qG_T^*) > c > qG_T^*[K_T^* - (F/pG_A^*)]\Delta(1 - G_A^* + pG_A^*)/$ $/\{1 - q + qG_T^*[K_T^*(F/pG_A^*)]\}$ *then* $\hat{\alpha}^H = 0$,

$$\alpha^D(K_T) = \begin{cases} 0, & \text{if } K_T > H(\hat{\beta}^S), \\ \in [0,1], & \text{if } K_T = H(\hat{\beta}^S), \\ 1, & \text{if } K_T < H(\hat{\beta}^S), \end{cases} \tag{19}$$

and $\hat{\beta}^S$ *is the solution of* $G_T[H(\hat{\beta}^S)] = c(1-q) / q[\Delta(1 - G_A^* + pG_A^*) - c]$

(d) if $gG_T^*[K_T^* - (F/pG_A^*)]\Delta(1 - G_A^* + pG_A^*)/\{1 - q + qG_T^*[K_T^* - (F/pG_A^*)]\} \geq c$ ***then*** $\hat{\alpha}^H = 0$,

$$\alpha^D(K_T) = \begin{cases} 0, & \text{if } K_T > K_T^* - (F/pG_A^*), \\ \in [0,1], & \text{if } K_T = K_T^* - (F/pG_A^*), \\ 1, & \text{if } K_T < K_T^* - (F/pG_A^*), \end{cases} \tag{20}$$

and $\hat{\beta}^S = 1$.

The model makes it possible to investigate the comparative statics of these equilibria, assuming that the probability of punishment for a bribe is the same for all dishonest defaulters. For the sake of simplification, premises are adopted in relation to the distribution of the amount of punishment for a bribe among taxpayers. In this case, it is pointed out that there is a unique "experienced" equilibrium, which is one of the four specific types. The value of each taxpayer's average income can be specified.

Basic conclusions. The equilibria that exist in the model have the following properties.

1. More often than "naive" tax inspectors, "experienced" ones do not conduct any audits at all.

2. On the other hand, if some number of audits does take place, the cost of an audit in an "experienced" tax agency will be higher than the cost of an audit in a "naive" one.

3. When none of the taxpayers wants to give a bribe, the "naive" and "experienced" equilibria coincide with the equilibrium in the GRW model. Nev-

ertheless, there is no such effect in the absence of punishment for bribery.

The essence of this phenomenon is that there is a fundamental asymmetry between taxpayers who give bribes and auditors who take them. As the percentage of taxpayers who are willing to give a bribe tends to zero, the operation of the entire tax system straightens out, and we approach a state of the complete absence of corruption. But as the number of bureaucrats who take bribes decreases, the system's operation also improves, but the improvement is limited and never reaches the level where corruption is completely absent.

In another study concerning the problem of corruption in tax agencies, T. Besley and J. McLaren [2] present a model for calculating alternative ways of paying tax inspectors in the presence of corruption. Their article investigates the role of wages in combating corruption, and three levels of wages are defined. First, a tax inspector can be paid a wage that he can earn in any other job (a reservation wage). Second, he can be paid a wage that solves the problem of moral hazard, that is, one that prevents bribery (an efficiency wage). Third, the government can pay a wage below the reservation wage. In that case, only dishonest people become tax inspectors. This is a capitulation wage. The conditions in which each of the versions brings in the greatest income from tax collection minus administrative expenses are precisely determined in the article.

In [3], A.A. Vasin and O. Agapova suggest a model of the optimum organization of a tax inspectorate within the framework of interaction among the Center (principal), Inspectors (agents), and Businessmen (clients).

The article by J. Hendriks, M. Keen, and A. Muthoo [4] considers questions of extortion and evasion in tax collection. It is assumed that taxpayers can be the victims of tax collectors, who extort bribes by threatening to assign a higher income than they actually received. An interesting result is the conclusion that the poorest citizens always become victims of the tax inspectors, and rich ones never do.

2. Models with a hierarchical structure

2.1. Social costs of bribes in a bureaucracy with a hierarchical structure

The model suggested by L. Hillman and F. Katz [5] is one of the first models of corruption in a bureaucracy with a hierarchical structure. In this study, the chief object of investigation is corruption within an organization that has some hierarchical structure, and the main goal of the analysis is to evaluate the social costs of corruption in such a system. As is noted in [5], it is not the bribes themselves that are the source of social costs, but the fact that bribes are an object of competition on the part of the bureaucrats (agents). The bureaucracy's organizational structure is supposed to be fixed; however, competition arises for specific spots in it. For a bribe, a lower-level bureaucrat "sells" a client the competitive product proper (rent), and this bribe is divided with a bureaucrat at a higher level, who, in turn, shares part of it with his boss, and so on. Thus, the bribe is distributed among bureaucrats at various levels of the hierarchy. At each level, the bureaucrats compete for the best spot, where they can join in on dividing up the bribe, and there is only one such spot at a given level of the hierarchy.

The model is based on the idea that, first of all, bribes redistribute profit, "shifting" it from the person who acquires rent (the client) to the bureaucrat who receives the bribes, and second, activity connected with the bureaucrat's struggle to hold a "good spot" involves real resources, which has social costs.

Basic Premises and Brief Description of the Model

Suppose that, according to government regulations, there is a certain rent. We will call it the initial rent (for example, a government order). This rent creates rivalry and leads to the appearance of two types of competition: one for the rent itself, and the other to determine which lower-level bureaucrat will receive a bribe in the course of competition for the initial rent. The next competition occurs if lower-level bureaucrats use bribes in the course of the second competition. These bribes, which are handed over to a bureaucrat at a higher level of the hierarchy, also become the object of rivalry, and so on. The sequence of competitions comes to an end when it reaches a noncompetitive position, or in the stage in which the participants do not pay bribes and only use real resources to receive a position. The real resources spent in all of these competitions have to be taken into account cumulatively in the total value of the social costs occurring as a result of government activity attempting to regulate the market. The sequence of competitions is a vertical hierarchical structure that arises when lower-level bureaucrats have to relinquish part of their bribes to their superiors.

So, in a bureaucratic hierarchy, there are several levels: the number of them is taken as equal to $(n-1)$, so that the total number of competitions is equal to n. Each one consists of one bureaucrat and each position in the hierarchy is competitive. As a result, the hierarchy consists of $(n-1)$ winners, although there may be many more contenders for the corresponding spots.

Certain real resources are spent in the competition for a position and bribes are redistributed to the next level. The winner who holds a position in the hierarchy receives profit in the form of a share of the bribes passing from lower to higher levels. The point of the analysis is to determine the ratio between the level of the initial rent and the amount of real resources used in the competitions (and, consequently, wasted from the point of view of society), that is, how much of the initial rent is wasted in the hierarchy of competitions. There are n competitions, one for the rent and $n-1$ for the bureaucrats' positions. In each competition, the contenders can choose whether to influence the outcome by using real resources or paying bribes. The resultant parameter of choice is the portion α_i $(0 \leq \alpha_i \leq 1)$ of the participants' expenditures, which represents expenditures of real resources in competition i, and the portion $(1-\alpha_i)$ of expenditures on bribery.

Say that the value of the initial rent is equal to 1, and p is the portion of it spent on acquiring the rent.

The total spending by participants at the level i is equal to

$$r_i = p^i(1-\alpha_1)(1-\alpha_2)...(1-\alpha_{i-1}),\ i = 2, \dots, n. \tag{21}$$

These expenditures are divided into those that are turned over to the next level:

$$T_i = p^i(1-\alpha_1)...(1-\alpha_i), \tag{22}$$

and those that are spent on real resources:

$$w_i = p^i(1-\alpha_1)...(1-\alpha_{i-1})\alpha_i = r_i - t_i. \tag{23}$$

Then the social costs of the competition for the initial rent can be calculated as

$$W_n = \Sigma_{i=1,n} W_i = p^1\alpha_1 + p^2(1-\alpha_{i-1})\alpha_2 + p^n(1-\alpha_1)(1-\alpha_2)(1-\alpha_{n-1})\alpha_n. \tag{24}$$

With perfect competition, that is, when $\pi = 1$, and setting $\alpha_i = a$, we get:

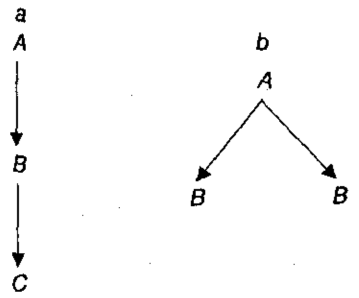
$$W_n = \alpha + (1-\alpha)\alpha + \dots + (1-\alpha)^{n-1}\alpha = 1-(1-\alpha)^n, \tag{25}$$

which means that the rent is completely wasted when $n \to \infty$ With finite n, complete waste occurs in the last level of the hierarchy.

If there is a barrier preventing a new participant from entering the competition for a position, then $p < 1$. Competition with a small number of participants m occurs, and each participant chooses his expenditures in accordance with the Nash equilibrium. In contrast to the case of perfect competition, complete waste of the rent does not happen here.

From the model, it follows that if real resources and bribes are used in the competition for rent, and the positions where bribes are received are competitive, then the structure of the bureaucracy that emerges will most likely be multilevel, rather than single-level. From calculations according to the model, it also follows that in the case of perfect competition, as well as competition with a small number of participants, the social costs rise with an increase in the number of levels of the hierarchy. This result supports the proposals of a number of economists that the number and structure of government organizations be constitutionally limited.

Figure 1. Types of Hierarchies



(a) "vertical" h_1; (b) "plane" h_2.

2.2. Combating corruption in a bureaucracy with a hierarchical structure

A number of articles have studied models of the mechanism of internal corruption occurring in a hierarchically organized bureaucracy that receives bribes from outside. In such a bureaucracy, higher-level bureaucrats have to keep an eye on lower-level ones in order to prevent corruption. Within the framework of a "principal–agent" scheme, this means that the agent is a hierarchically organized bureaucracy. Hereinafter, we will call its members simply agents. The principal sets the "rules of the game," and the agents play it, pursuing (as far as possible) their own goals. F. Kofman and J. Lawarree [6] investigated the optimum motivational scheme in a two-level organization with the possibility of a deal between the boss (supervisor) and his subordinate (the agent), and without such a possibility (albeit at the price of additional expenditures). The result of the study is the conclusion that corruption may be present in an organization even in the optimum state.

This research was further developed by M. Bec [7]. He introduces a third agent, who can be either a middle link in a "vertical" hierarchy, or one more subordinate agent in a "plane" one (see Figure 1).

In [7], it is assumed that each agent (bureaucrat) at any level of the hierarchy is offered a bribe from outside. The bureaucrat may take the bribe or reject it. The number of bureaucrats in the hierarchy who take bribes from outside is considered the level of "external" corruption. In addition, lower-level bureaucrats can buy off higher-level ones (in this case, there is internal corruption in the organization). The principal's goal is to minimize the funds spent on attaining some assigned level of the organization's "external" corruption. This system is used to investigate what the optimum form of hierarchical structure and motivational scheme for bureaucrats are in the presence and absence of internal corruption in the organization.

Basic premises and brief description of the model

Four participants figure in the model: the principal and three agents ($i = 1, 2, 3$). These participants are characterized by the following parameters: w_i = the wages of agent i; w—the agents' guaranteed wages; z—the bribe by which an agent can be bought off (an external constant); $b_i \in [0, 1]$—agent i's decision about accepting a bribe ($b_i = 1$ if he accepts the bribe, otherwise $b_i = 0$). The overall level of "external" corruption in the organization is determined as $t = \Sigma b_i / 3$. The principal has to develop a hierarchical management structure in order to attain a certain level of "external" corruption. Say that $H = \{h_1, h_2\}$ is the set of possible structures of the organization, where h_1 is a "vertical" hierarchy and h_2 is a "plane" one (see Figure 1); $c(nm)$ is management costs, where m is the level of effort to supervise one subordinate, and $n = 1, 2$ is the number of subordinates. The probability $\mu(nm, n)$ of discovering the fact of bribery increases with a rise in the level of supervisory efforts. (It is assumed that $c(nm)$ increases and is strictly convex, while $\mu(nm, n)$ increases and is strictly concave with m; and $\mu(nm, n) \to 1$ when $m \to \infty$. Moreover, $c(0) = \mu(0, n) = 0$, and $\mu(nm, n) \to 0$ when $m \to 0$.) The supervisor i is paid a monetary reward p_i for exposing a deal. If the agent i is a higher-level boss and supervises his subordinates with the same level of effort m, then his expected profit is equal to $R_i(n, p_i t) = p_i \mu(nm, n) \Sigma_j b_j \Sigma$, where $\Sigma_j b_j$ is the expected number of subordinates who have been bribed, and t is the target level of corruption. The principal's goal is to minimize costs, which are equal to the sum of wages and rewards paid out, while attaining the level of corruption t. To achieve this goal, he chooses rewards $\{p_i\}$, wages $\{W_j\}$, and the hierarchy h. Each agent i is supposed to have his own utility function $U_i(m_i, b_i)$, with all of the agents being neutral to risk. Therefore, each agent chooses the level of effort that is optimum for himself and decides whether or not he will take an "external" bribe based on the desire to maximize $EU_i(m_i, b_i)$ (E is the symbol of mathematical expectation). An agent's utility function is his profit, which is made up of his wages, rewards, bribes from outside, and possible bribes from subordinates, minus the costs of supervising his subordinates.

So, the principal's problem looks like this:

$$\min_{h \in H, p_i, w_j} \sum_j R_i(n, p_i, t) + \sum_j w_j, \tag{26}$$

with the following constraints:

$$b_j^* \in \arg\max_{b_j} [EU_j(m_i^*, b_j)] \tag{27}$$

$$m_i^* \in \arg\max_{m_i} [EU_i(m_i, b_j^*)] \tag{28}$$

$$\left(\sum_n b_n^*\right)/3 = t < 1, \tag{29}$$

$$EU_i \geq w, i = 1, 2, 3, \tag{30}$$

$$NC_i(h, p_i, w_j, t) \geq 0, \tag{31}$$

where (31) is the condition that there is no internal corruption between the supervisor i and his subordinates, and it may or may not be included as one of the constraints.

The vector (h, p_i, w_j), which is the solution of this extremum problem, gives rise to a game between the supervisors and subordinates. In this game, the Nash equilibrium strategies are the vectors (m_j^*, b_j^*) (see constraints (27) and (28)). Constraint (29) is the condition of attaining the average expected level of corruption; and constraint (30) is the requirement that the solution (h^*, p_i^*, w_i^*) satisfy the condition of the agents' participation in the game.

This model is analyzed within the framework of the classical "principal–agent" model with hierarchies H_1 and H_2. The following basic questions are investigated in the study:

(a) can the principal achieve the dual goal of reducing external corruption to the assigned level without the risk of simultaneously increasing internal corruption?

(b) how does the form of hierarchy minimize its members' profit from internal corruption?

(c) what form of hierarchy is preferable from the principal's point of view?

Basic conclusions

1. A reduction in the level of external corruption may lead to increased internal corruption in the organization, that is, the answer to the first question, generally speaking, is no.

2. Corruption gives more to someone who stands higher on the hierarchical ladder: the profit from internal corruption is greater in the upper part of a "vertical" hierarchy than in the lower part.

3. The answers to the second and third questions depend on the management parameters: if the probability of discovering a deal when supervising two agents is significantly greater than when supervising one, then the "plane" hierarchy is more efficient than the "vertical" one for reducing corruption. But if internal corruption is possible, the "plane" hierarchy may not have this advantage.

As is pointed out in [7], it would be interesting to include in the model factors such as more complex supervisory systems, for example, direct supervision of agents at all lower levels by a higher-level supervisor. Competition between lower-level agents for bribes and for spots in the hierarchy itself can also be included in the model.

At present, intensive work is under way in connection with modeling corruption in a hierarchical structure. In particular, such research constitutes one direction of contract theory, because corruption can appear as an illegal agreement between a supervised agent and his supervisor. Traditionally, it was thought that the possibility of a supervisor and an agent entering into an illegal agreement has a negative effect on the principal, increasing his expenses either for preventing corruption or combating it. In a number of models, such negative consequences of corruption were due to the fact that the original contracts were executed once and for all and were not subsequently revised. A. Lambert-Mogiliansky's study [8, pp. 52–101] examined the possibility of renegotiating the contracts in a three-level principal–supervisor–agent model, and the conclusion drawn from accepting this premise is that, at the optimum, the existence of corruption may be more beneficial to the principal than its absence, if the costs of a deal between the supervisor and the agent are insignificant. The study by T. Olsen and G. Torsvik [9], which allows revision of contracts and considers what is essentially a dynamic model of corruption, shows that the positive dynamic effects of corruption may exceed the negative static effects. A dynamic (two-period) version of the three-level principal–supervisor–agent model is examined. In it, the principal "speculates" that the supervisor enters into agreements with the agent illegally and with limited commitments, and in this case they cannot last very long. As a result, the principal profits in the long run from that which he loses in the short run.

The article by K. Basu, S. Bhattacharya, and A. Mishra [10] is also devoted to a hierarchical structure. It studies the problem of recursion, which is that, when the agent (an auditor or policeman) strikes a deal with someone whom he should arrest, he has to take into account the fact that he, in turn, could be caught taking a bribe and become part of a similar deal, but this time as the one giving the bribe. The article analyzes the situation of endless and finite chains of catching people involved in bribery. In analyzing certain conditions of controlling corruption in such chains, the authors of [10] showed that, when attempting to reduce corruption, increasing the probability of punishment is more effective than raising the amount of the fines. (While the standard approach asserted that the roles of the probability of punishment and the amount of the fines are symmetrical, which led to the claim that by adding zeros to the amount of the fines a stroke of the pen is equivalent to expenditures on additional supervision of the agents.)

A.P. Mikhailov [11] also considers a hierarchical structure in the form of a chain of institutions, each of which is subordinate to a superior one. Within the framework of the proposed macroeconomic dynamic model, the "authority–society" system is investigated, where authority belongs to the aforementioned chain of institutions and society is capable of influencing the redistribution of power in the chain of institutions. There can be internal corruption in the hierarchy of institutions, which is reflected in the model as the possibility of a lower-level institution buying "a share of power" from a superior institution for a bribe. When the effectiveness of measures included in the model for suppressing corruption was investigated, the conclusion obtained was the opposite of traditional ideas: the main efforts to reduce corruption should be aimed at the lower links in the chain of institutions.

3. Multiple corruption equilibria

One direction in the use of models to study corruption is to study different levels of corruption in the same socioeconomic system. Multiple equilibria are a natural phenomenon in equilibrium economic models and models of economic games. Nevertheless, in view of the autonomy of models of corruption as a direction of socioeconomic modeling, the study of such phenomena observed in practice requires a special explanation. The importance of this avenue is determined by the fact that these models examine the effectiveness of measures to combat corruption. One of the results of a number of studies is the conclusion that the same anti-corruption measures may lead to significantly different levels of corruption, which is observed in practice. Therefore, the prevention of corruption apparently requires the construction of special anti-corruption schemes, which have to take into account these unexpected effects.

We will briefly describe several models of this type. In [12], O. Cadot considers the supply and demand for corrupt deals within the framework of a situation where government bureaucrats control the issuing of permits, for example, hiring for some position. (The situation is similar to a dual monopoly from the point of view that each bureaucrat meets once with one candidate.) A corrupt bureaucrat may ask the candidate for a bribe and the candidate may agree to give it or refuse to do so. Moreover, he can also inform a superior agency of this extortion. The bureaucrats differ in the degree of their corruption, and the candidates differ in their suitability for the position. A game takes place between the bureaucrat and the candidate, which, as is shown in the article, has several equilibrium points if different assumptions are made about the information structure. In their article [13], J. Andvig and K. Moene analyze a more general situation, in which the portions of bureaucrats "infected" by corruption and bribe-givers "infected" by corruption are taken into account. The model is a dynamic one: when making decisions, the agents consider their consequences (specifically, the agents' utility function includes their expected profits in the next period). This model also has several equilibria, which are characterized by different portions of corrupt bureaucrats with the same parameters of the economy (a bureaucrat's wages, the size of the bribes, the level of the discount rate, and the probability that a deal will be discovered). Among the equilibria that appear, there are stable ones, with corruption at high or low levels, and also an intermediate, unstable equilibrium. The system can slip from an "average" level of corruption to one of the stable states even with small variations. Nevertheless, changes in the model's parameters, for example, a bureaucrat's wages, do make it possible to move from one stable level of corruption to the other. The next question is: At what price? In other words, the problem arises of balancing expenditures on anti-corruption measures against the profit from reducing corruption. This aspect must be taken into account, for example, in setting wages in the public sector, when the bureaucrats themselves compare them with incomes in the economy's private sector.

This field of research also includes the work of C.M. Asilis and V.H. Juan-Ramon [14], who suggest a dynamic model that investigates the interrelation of corruption and the accumulation of capital. They also studied the effect of government anti-corruption measures on the state of equilibrium and social welfare.

A. Antoci and P.L. Sacco's study [15] also considers a game of entering into contracts as it changes in time. According to the model suggested in this article, corruption is extremely sensitive to the "culture," that is, the transmission of experience by the imitation of behavior, and to historical conditions (the initial distribution of types of behavior that the population inherits from the past).

In conclusion, part 3 of this survey cites studies showing that systematically recurring violations of the law can turn from artifacts into tradition [16], and it is considerably easier to counteract arguments than society's traditional behavior [16, 17].

3.1. Model of corruption deterrence

We will consider F.T. Lui's dynamic model [18] in greater detail. This is a simple model with overlapping generations, which can explain why, for example, the level of corruption in a country may rise drastically in comparison with some periods in the past, while the parameters of the punishments do not change very much. On the other hand, it explains why in a heavily corrupted society the usual measures to combat corruption, for example, intensified surveillance of bureaucrats, are an expensive "gratification" for society that is not comparable to their effect.

Brief description of the model and conclusions

In the economy, there are two overlapping generations of bureaucrats in each period: young and old. The number of bureaucrats is the same in the two generations. In each period, each bureaucrat is offered a unit of income in the form of a bribe and he decides whether or not he will accept it. If a young bureaucrat accepts a bribe and is later checked, then with unitary probability he must pay a monetary fine of C units. He can continue his work in the next period. However, if he takes a bribe once again and is caught, then the new fine will now be equal to C' units. In this case, C' is so large that a bureaucrat who is punished while still young will not take another bribe if there is any positive probability that he will be checked. The probability $p(t)$ of being checked during the time t is the same for each bureaucrat.

The bureaucrats in one generation differ only in how honest they are. If a bureaucrat with honesty h takes a bribe, then he simply values it as 1 (unity). It is assumed that h is a random quantity with a uniform distribution function $F(h)$, $h \in [0; 1/f]$. The distribution function $F(h)$ is the same for each generation. It is also assumed that all bureaucrats are neutral to risk.

During the time t, an old bureaucrat who has not been punished before will take a bribe if and only if his expected profit will be

$$1 - h - p(t)C \geq 0. \quad (32)$$

Suppose that $W_0(t) = 1 - p(t)C$. An old bureaucrat with honesty h belongs to the group that will be corrupted if and only if

$$W_0(t) \geq h. \quad (33)$$

During the time t, a young bureaucrat must take into account the expected profit when he becomes old during the period $t + 1$. Say that the probability of being checked at the point in time $t + 1$ that is expected at the time t is $p^e(t + 1)$. Then, since a young bureaucrat who has been punished actually loses the opportunity to take a bribe in the future, a young bureaucrat with honesty h will take a bribe at the time t if and only if

$$1 - h - p(t)[c + \max[1 - h - p^e(t + 1)C, 0]] \geq 0. \quad (34)$$

Because $\max[1 - h - p^e(t + 1)C, 0] \geq 0$, the possible value of a bribe for a young bureaucrat is no greater than for an old one. This indicates that an old bureaucrat is more sensitive to corruption than a young one, since the old bureaucrat has not been previously punished.

Say that $W_0(t + 1) = 1 - p^e(t + 1)C$. At the time t, a young bureaucrat with honesty h expects to take a bribe at the time $t + 1$ if and only if

$$W_0(t + 1) \geq h. \quad (35)$$

If (35) is satisfied, then (34) is equivalent to

$$1 - \frac{p(t)C[1 - p^e(t+1)]}{1 - p(t)} \geq h. \tag{36}$$

We will introduce the notation:

$$\overline{W}_Y(t) = 1 - \frac{p(t)C[1 - p^e(t+1)]}{1 - p(t)}. \tag{37}$$

A young bureaucrat with honesty h will take a bribe if and only if

$$\overline{W}_Y(t) \geq h. \tag{38}$$

If (35) is not satisfied, then at the time t a young bureaucrat does not expect to take a bribe during the period $t+1$. Then (34) is equivalent to $1 - h - p(t)C \geq 0$.

Suppose that $W_Y(t) = 1 - p(t)C$. A young bureaucrat with honesty h will take a bribe if and only if

$$W_Y(t) \geq h. \tag{39}$$

In this case, $\overline{W}_Y(t) \leq W_Y(t) \leq W_0(t+1)$ if and only if $p(t) > p^e(t+1)$. It is proved that, when $p(t) > p^e(t+1)$, the portion of young corrupt bureaucrats at the time t is given by the function $F(\overline{W}_Y(t))$, and when $p(t) \leq p^e(t+1)$ the portion of young corrupt bureaucrats at the time t is given by $F(W_Y(t))$. It is assumed that $p^e(t) \geq p(t-1)$ if and only if $p(t) \geq p(t-1)$. In other words, the hypothesis in relation to the expected change in probability of being checked turns out to be true. It is proved that when $p^e(t) \geq p(t-1)$, the proportion of old corrupt bureaucrats at the time t is given by $(1-p)(t-1)F(W_0)(t)$, and when $p^e(t) < p(t-1)$, the proportion of old corrupt bureaucrats at the time t is given by $F(W_0)(t) - p(t-1)F(\overline{W}_Y(t-1))$.

Let $B(t)$ be the portion of corrupt bureaucrats among all the bureaucrats at the point in time t. $B(t)$ is the arithmetic mean of the portions of old and young corrupt bureaucrats who take bribes at the time t. This quantity is used to measure the level of corruption in the economy at the time t. The preceding results can be represented by the following four cases:

$$B(t) = \begin{cases} (1/2)[F(W_Y(t)) + (1 - p(t-1))(F(W_0(t)))], \\ \text{in version 1}: p(t) \leq p^e(t+1), p(t) \leq p^e(t+1); \\ (1/2)[F(\overline{W}_Y(t)) + F(W_0(t)) - p(t-1)F(\overline{W}_Y(t-1))], \\ \text{in version 2}: p(t-1) > p^e(t), p(t) > p^e(t+1); \\ (1/2)[F(W_Y(t)) + F(W_0(t)) - p(t-1)F(\overline{W}_Y(t-1))], \\ \text{in version 3}: p(t-1) > p^e(t), p(t) \leq p^e(t+1); \\ (1/2)[F(\overline{W}_Y(t)) + (1 - p(t-1))(F(W_0(t)))], \\ \text{in version 4}: p(t-1) \leq p^e(t), p(t) > p^e(t+1). \end{cases} \tag{40}$$

If all of the proportions $F(\cdot)$ are less than one, then the corresponding expressions for the values of W can be substituted into expressions (40). Then we get

$$B(t) - (f/2)[(2-p(t-1))(1-p(t)C) - J_1 + J_2], \tag{41}$$

where the functions J_1 and J_2 depend on $p(t-1)$, $p(t)$, C, $p^e(t)$, and $p^e(t+1)$. From (41), it follows that $B(t)$ depends on the probabilities of being checked, which are determined below.

With higher $B(t)$, the cost of effective checking is higher. To include this circumstance in the model, the following assumptions are made.

In each period, the government spends R units of resources on checking. The resources needed for effective checking of one person at the time t are $r(t)$. It is assumed that

$$r(t) = 1/(m - nB(t)), \text{ where } m > n > 0. \tag{42}$$

Let N be the total number of bureaucrats. Then

$$p(t) = A - kB(t), \text{ where } A = Rm/N \text{ and } k = Rn/N. \tag{43}$$

Substituting (43) into (41), we can obtain the pattern of change in $B(t)$. The assumptions that were made enable us to show that by assigning R we can get several stable equilibrium levels of corruption. Say that the initial level of corruption in the economy is low. Due to the small costs of checking each person, R can be spent on a larger number of people. Consequently, fewer people will choose to become corrupt. The situation is analogous in the opposite case, with a high level of corruption.

It is also assumed that $1 > A > k > 0$, $C > 1 > AC$, and $f > 1$.

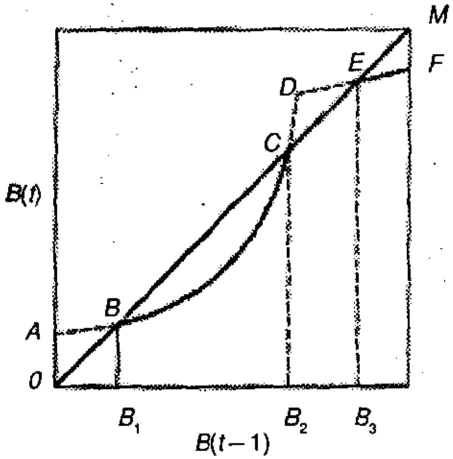
As is shown in [18], there are three stationary equilibrium levels of corruption $B^* = B(t)$ for all t. Because of (42), $p(t) = p^*$ for all t, where p^* is the probability of being checked that corresponds to B^*. The stationary levels are possible only if $p(t) \le p^e(t+1)$ and $p(t-1) \le p^e(t)$. This is the case that is considered below. Since $F(W_Y(t)) = F(W_0(t)) = f(1-p(t))C \le 1$, if $f(1-p(t))C \le 1$ and $F(W_Y(t)) = F(W_0(t)) = 1$, if $f(1-p(t)C > 1$, then

$$B(t) = \begin{cases} (f/2)(2-p(t-1))(1-p(t)C), \text{ if } f(1-p(t)C) \le 1, & (44) \\ 1-(1/2)p(t-1), \text{ if } f(1-p(t)C) > 1. & (45) \end{cases}$$

The solution can be represented on a phase diagram. Figure 2 shows a diagram of the change in $B(t)$ depending on $B(t-1)$. The curve $ABCD$ corresponds to (44). It intersects the line PM (this line has a slope of 45 degrees) at two points, B and C. The straight line DF, which represents equation (45), intersects the line OM at point E. Thus, there are three equilibrium points, and it can easily be shown that the equilibrium is stable only at points B and E.

If we change the assumption about the relationships of the probabilities of

Figure 2. **Phase Diagram**



being checked and their expected values, then, as is shown in [18] (with the help of numerical modeling), instead of Figure 2 we get Figure 3. From the figure, we can see that if the initial value of the variable $B(t-1)$ is greater than B_2, or if $B(t-1)$ so small that $B(t)$ is above point *C*, then $B(t)$ converges to point *E*. In other cases, $B(t)$ converges to point *B*.

How the equilibrium levels of corruption depend on the model's parameters is investigated in [18], and the difference between small and significant changes in the parameters is emphasized, because their consequences are different.

Algebraically, it is more convenient to deal with the stationary probability of being checked p^* than with B^*. According to equation (43), p^* is related to the stationary equilibrium level of corruption B^* by the following equation:

$$p^* = a - kB^*. \qquad (46)$$

From Figure 3, it follows that there are three possible states of equilibrium for $B(t)$. These are *B*, *C*, and *D*. The first two are determined from the equation

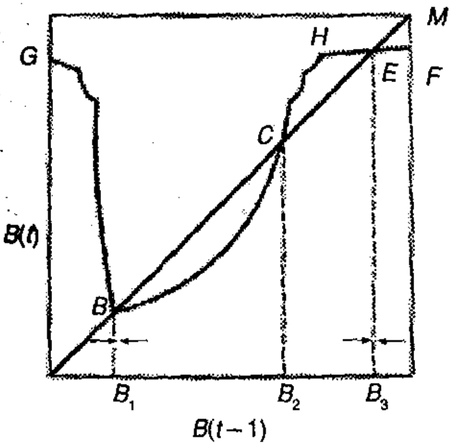
$$Cfkp^{*2} - [fk(2C+1)]p^* + (2fk - A) = 0. \qquad (47)$$

From (46), it is easy to see that B^* is negatively related to P^*. Consequently, the results obtained from the last equation can be interpreted in the following way. If the fine *C* or the resources for checking *R* rise slightly, then B^* falls. On the other hand, if the average level of honesty in the economy declines, then *f* and B^* rise, which seems natural.

Since, in addition to point *D*, point *E* is also a stationary solution, it is interesting to investigate its dependence on the parameters. From (45), it follows that at point *E*

$$B^* = (2 - A)/(2 - k). \qquad (48)$$

Figure 3. Phase Diagram



This level of B^* does not depend on the amount of the fine C. However, when the resources for checking R rise, then both A and k rise in the same proportions, therefore B^* falls. Consequently, for an economy with a high level of corruption, changes in R can reduce the level of corruption, while small changes in C cannot.

The question of an economy's transition from one level of corruption to another is of particular interest. Investigation of this question indicates that when the society becomes more lenient with corrupt bureaucrats, then a sharp rise in the level of corruption is possible. What is more, once it appears, a high level of corruption remains, even if the parameters of the measures to limit it are returned to their former level. This explains the existence of societies with sharply differing levels of corruption and the same anti-corruption measures. The intuitive explanation of this fact is that, once it appears, corruption requires higher expenditures on checking and restraint. The government's efforts become less effective.

Moreover, it follows from the model that, due to the possibility of transition from one state of equilibrium to another, sometimes severe limiting measures that seem not to be optimum in the short run do become optimum in the long run. At the same time, in a number of cases, rough measures (for example, the introduction of high fines C) may cause the opposite effect, taking an economy from a low level of corruption (at point B) to a high level (at point E). This happens if corruption "breaks out" of an unstable stationary state (point C) at some time, due to fluctuations in the transition process.

What has been said above is an example of the numerous conclusions that come from detailed analysis of this model.

3.2. Model of exchange of popularity for a bribe

Problems of stationary levels augmented by actually observed effects of fluctuations in the level of corruption are considered within the scope of a macroapproach in the study by G. Feichtinger and F. Wirl [19]. As the authors of this article themselves note, its purpose is to explain several facts observed in the case of "rational" political activity, in particular, to study the dynamics of corruption and the possibility of the occurrence of cycles and instability in the rational behavior of politicians. The article explains one frequently encountered fact: periods of campaigns against corruption are often followed by periods of tacit toleration of bribery. They suggest a dynamic model of the optimum behavior of a politician whose utility function depends on popularity, on the one hand, and the level of personal income (including bribes), on the other. The solution of the extremum problem is a trajectory in "corruption–popularity" space. The study analyzes properties of the stability of optimum strategies and shows that the equilibrium may be a saddle point (attained either monotonically or through damped oscillations). In addition, cyclic fluctuations and various types of instability may take place. The article also proves the existence of stable limiting cycles and studies the effect of the model's parameters (the importance of popularity, people's memory, and the discount rate) on the dynamics of corruption and its stability.

The model considers an aggregate agent (politician). At each moment in time, his utility function depends on two "particular" utility functions $V(P)$ and $U(c)$. The utility function $V(P)$ takes into account all types of benefit from popularity P; V is such that it can become highly negative if the politician's public approval drops below a certain threshold. The utility function $U(c)$ depends on the amount of bribes c. Corruption is measured by the parameter K. Both functions are supposed to be decreasing and concave: $U' > 0$, $U'' < 0$, $V' > 0$, and $V'' \leq 0$. Bribes c can be negative, when the politician spends money trying to win popular support, speaking out against widespread corruption.

The model is represented in the form of an optimum-control problem of the following type:

$$\max_{\{c(t)\}} \int_0^\infty e^{-rt} [U(c(t)) + V(P(t))] dt, \tag{49}$$

$$\dot{P} = g(P) - f(K), P(0) = P_0, \tag{50}$$

$$\dot{K} = c - \delta K, K(0) = K_0. \tag{51}$$

In (49), utility is maximized with two dynamic constraints. First of all, popularity ($P(t)$) is a dynamic process (according to (50)). In this case, it becomes negative when reports of corruption appear. However, public opinion does not react to isolated manifestations of corruption, because some level of corruption is considered inevitable. But it does react to a large number of accumulated reports of corruption K. According to the differential equation (51), such an accumulation of news about corruption foresees that the people on whose support the politicians must count have an inclination to forget (exponentially diminishing memory ($\delta \geq 0$)).

The function $g(P)$ can be an arbitrary, but concave ($g'' < 0$) process of diffusion, for example, following a logarithmic law. The diffusion process presumes that speeches aimed at bolstering a positive reputation are a decisive factor. The function $f(K)$ measures the loss of popularity, depending on the memory (accumulation) of observed corruption K. It is assumed that $f' > 0$ and $f'' > 0$.

The action expressed by the function f depends on several parameters, for example, on the local culture, suppression of freedom, and how interested certain circles are in exposing corruption. The system (49)–(51) presumes that the voters or the population solve the problem of politicians' competency and honesty by a majority of votes, acting purely rationally, but looking back, rather than ahead. This constraint, in the form of the assumption that the voters are rational, is perfectly true because "rational" voters will always be minimally informed, due to their "laziness" and because collecting information is an expensive "gratification" for them.

Applying the standard approach (Potriagin's maximum principle), next we solve the extremum problem and find the optimum trajectories $K(t)$, $c(t)$, and $P(t)$. They are investigated by traditional methods of analyzing dynamic systems. Along with mathematical results confirming the existence of different types of trajectories, the authors of the study draw a number of institutional conclusions.

> The ruling class (dictators, politicians, bureaucrats) considers bribes as its consumer goods. Obviously, the public does not like this kind of "consumption." At present, any government, even a dictatorial one, is limited by popularity conditions, which often lie below the same conditions for democratic regimes. The main result of the study is that these institutional constraints, which are expressed in the requirement of high popularity, also provide for a stable level of corruption. The difference in the requirements of "high" and "low" levels of popularity (i.e., "democracy" and "dictatorship") affects how stable the system is, but does not affect the level of corruption itself, which can be high in either case. Even with a stable equilibrium, it may be rational (for politicians) to achieve this equilibrium not monotonically, but through damped cycles. Moreover, complex (cyclical and unstable) measures may be rational for governments that only encounter slight popularity constraints. This

can be explained, to a certain degree, by the fact that, in the final analysis, a democracy is accompanied by some level of corruption, even more than in a dictatorship. [19]

3.3. *Model of collective reputation*

Why is it so hard to fight corruption? This question is covered in a number of articles on the modeling of effects such as transformation of individual corrupt deals into a social "tradition" [17]. We will briefly consider one of these studies, J. Tirole's model of the formation of a group reputation [16]. He introduces the concept of a group's collective reputation, which plays an important role in the economy and in social sciences. Some companies receive considerable rents thanks to their reputation as producers of high-quality products. The study looks at collective reputation as a result that depends on a group's history. By definition, a group's collective reputation reflects the average behavior of the group's members in the past. This means that:

(a) a group's collective reputation will be good if the reputation of its members is positive;

(b) in contrast to the group's behavior, the behavior of an individual in the past is not fully traced;

(c) consequently, the group's past behavior is used to predict the individual behavior of its members, and the group's reputation also affects the welfare of each member of the group and the motives of his actions; and

(d) the behavior of new members of the group depends on the past behavior of their predecessors.

In the model, a principal (the purchaser of a service) enters into a contract with an agent (the seller of the service) only if he is sure that the agent is not involved in corruption. In the economy under consideration, at the point in time t agents are remembered until the date $t + 1$ with probability $\lambda \in (0; 1)$. The size of the population is considered constant. This is a model of competition. At each time t, any agent contends with a new principal. The principal decides whether to offer the agent job 1 or 2. Job 1 is productive. Job 2 is less productive, but, in the principal's opinion, more suitable for an agent who decides to enter into corrupt relations. (It is assumed that it is always better for the principal to offer an agent job 2 than to fire him completely.)

Having gone to work, the agent decides whether or not to "deceive" his principal. During the period, the principal's profit from the first job is H if the agent is honest and D if he is not. We will represent the profit from the second job as h or d, respectively. In order to force the principal to make a nontrivial choice, it is assumed that corruption in performing job 1 affects the profit more than corruption in performing job 2, that is, $H > h \geq d > 0$. In order to ensure optimum conditions for hiring the agent, it is assumed that $d \geq 0$.

Honest agents exist in the proportion α; dishonest ones, in the proportion β; and opportunists, in the proportion γ, where $\alpha + \beta + \gamma = 1$. These proportions are the same for the entire population. Honest and dishonest agents behave in a predetermined way; therefore, the focus of the analysis is on the behavior of opportunists. Their decision depends on the profit from corruption and the loss of reputation. Their profit in solving problems 1 and 2 (even if they do not deceive the principal) is B and b, respectively, with $B > b \geq 0$. In both problems, the additional profit from corruption is $G > 0$. The role of an anti-corruption campaign is not modeled in this case. Or, G could be the expected profit from corruption, including the probability of application of legal sanctions. In addition, the model takes into account the agent's discount rate.

It is assumed that the agents know their preferences. The principal knows their proportions α, β, and γ, but does not fully observe their past behavior. There are several ways of forming incomplete observability of past behavior, the simplest one of which is chosen to illustrate the main ideas. With probability x_k, the principal learns that an agent has been involved in corrupt activity in the past at least once, if the agent has actually been "deceptive" k times. In other words, the principal's information about the agent with whom he is dealing is binary: either the principal knows that the agent has been corrupt, or he does not have such knowledge.

It is assumed that information leaks about corruption appear with greater probability, the more the agent has been deceptive in the past.

There can be two equilibrium points in this model. The low-level one exists only if the principal is well informed, while the equilibrium with a high level of corruption is observed with a sufficient number of opportunists and dishonest agents, and when the principal's information is not entirely accurate.

The main conclusion from the model is that the society's history is important: if one society is more corrupt today than another similar society, then there is a greater probability that the former one will be corrupt tomorrow, rather than the latter. One interesting result is that the economy "remembers" corruption in both the short run and the long run. Thus, a society consisting of individuals who are "infected" with corruption turns into a society where corruption becomes a normal phenomenon (a tradition). As a result, a "vicious circle of corruption" emerges, when the new generation suffers from the original sin of its predecessors' bribery. In this model, a society's transition from a low level of corruption to a higher one is more likely than vice versa. As is noted in [16], this may be why it takes constant efforts to control corruption, rather than brief *anti-corruption campaigns sometimes just for publicity*.

3.4. "Evolution and revolution" model

One of the models that takes into account the dynamics of corruption was suggested by C. Bicchieri and C. Rovelli [20]. In this article, the evolution of a system of corruption is analyzed as an example of a more general study of the development, spread, and breakdown of social norms.

Corruption is considered as an illegal exchange of bribes between politicians and contractors for receiving contracts; but, as the authors note, this narrowing of the concept of corruption does not affect the study's conclusions. A similar exchange could be represented as an informal association, cooperation between politicians and those receiving the contract, but this study emphasizes the noncooperative aspect of the agreement. In other words, it models the fact that a politician will struggle with other politicians for a scarce resource (bribe), and the people competing for the contract will struggle to get it. Thus, both the politicians and the people competing for the contract are included in a sequence of games with the "prisoner dilemma": it is better for one player to be corrupt, but then the total profit will be less than if everyone decides to play honestly. The possible states of equilibrium in such a model were studied, and also a possible sudden switch from one equilibrium system to another.

The model's basic premises

Series of supergames are supposed to be played (modeling the interactive relations of the same group of players), with randomly selected adversaries. The players demonstrate rational behavior, that is, they try to maximize their profit. They adapt readily, so a strategy that has worked well in the past is maintained in the future, while one that worked poorly is changed. A player's strategy changes with time in response to the relative success of the strategies of the player's surroundings. In an equilibrium situation, there is one dominant strategy.

We will assume that there is a slow positive accumulation of social costs leading to catastrophe, that is, to an unexpected abrupt transition of the entire system to a new state.

We will consider the model in greater detail. Say that the number of players P associating within small groups is fixed. The players must select one of two strategies. For the sake of simplicity, it is also assumed that the number of players in each group is fixed: n. The supergame within such a group consists of repeating one game between n players. Each step of the game is a "prisoner dilemma" with the possibility of choosing to be honest (behavior h) or cor-

Figure 4. Payoffs for One Step of the Game

(I) all *h*; (II) at least one *c*

	I	II
h	*a*	*c*
c	*b*	*d*

rupt (behavior *c*). Each player faces a group of $(n - 1)$ identical players. The players complete a series of supergames (a series of N repetitions of one step of the game). After each round, everyone finds out which strategies were chosen and the profits. The payoff matrix for one step of the game is shown in Figure 4. The result of each round also becomes obvious to everyone. In each supergame, a player may choose between several strategies of behavior S: corrupt or honest. A corrupt strategy C calls for a choice of behavior c in each step of the game. An honest strategy H consists of choosing behavior h in the first step of a supergame and behavior c if at least one of the opponents selected behavior c in the previous step. There are two types of players: players of the opportunistic type, who can change their strategy (they are in the majority), and players who choose a strategy once and for all. Among such players, there is a small portion who choose to be honest at the beginning of a new supergame.

The strategy of an opportunistic player changes as $p_{st+1} = Zf(u_{st})p_{st}$, where p_{st} is the proportion of opportunistic players who chose strategy s in the supergame that began at the time t; u_{st} is the expected profit from choosing strategy s; and Z is a normalizing factor that does not depend on s.

It is also assumed that the profits from the game change slowly with time, namely, that there is a decrease or erosion of all profits. Profits a, b, c, and d satisfy conditions $b > a > d > c$. For the sake of simplicity, c is supposed to be equal to zero.

The proportions of players with different strategies is represented as $\pi_H = m_H/P$—always honest; $\pi_C = m_C/P$—always corrupt; $p_{Ht} = n_{Ht}/P$—honest opportunists; and $p_{Ct} = n_{Ct}/P$—corrupt opportunists, figured as the number of the corresponding players in relation to the total number of players. The number of opportunists is taken as N. Since the supergame is played with randomly selected opponents, the probability of playing against $n - 1$ relatively honest players is equal to $(p_{Ht} + \pi_H)^{n-1}$, and the probability of running up against at least one dishonest player is $(1-(p_{Ht}+\pi_H)^{n-1})$. In each supergame, the expected profit from the strategies is added up from the total profits of each step. The values of u_{Ht} and u_{Ct} are computed accordingly, and the value of the normalization factor Z is derived. For the sake of simplicity, we will represent p_{Ht} as p_t and, consequently, p_{Ct} as $(N/P)-p_t$, so we get the principal equation for the evolution of the portion of opportunistic honest players in time:

$$p_t = N/P \frac{f(u_{Ht}(p_t))p_t}{f(u_{Ht}(p_t))p_t + f(u_{Ct}(p_t))(N/P - p_t)} \tag{52}$$

where

$$u_{Ht}(p_t) = Na(p_t + \pi_H)^{n-1} + (N-1)d[1-(p_t+\pi_H)^{n-1}] \tag{53}$$

$$u_{Ct}(p_t) = [(N-1)d + b](p_t + \pi_H)^{n-1} + Nd[1-(p_t+\pi_H)^{n-1}] \tag{54}$$

that is, there are three equilibria:

i. $p_t = 0$

ii. $p_t = N/P$

iii. $f(u_{Ht}(p_t)) = f(u_{Ct}(p_t))$, which means that $u_{Ht}(p_t) = u_{Ct}(p_t)$, on the strength of the monotonically increasing function f.

(i) corresponds to the choice of corrupt behavior in each supergame; (ii)—to the choice of honest behavior; and (iii)—to the case when the relative advantages of honest and corrupt behavior are the same. This equilibrium exists when there is a very small number of players who are always honest (we will call this the system's first mode), and is absent when there is a considerable number of honest players (we will call this the system's second mode). These two situations are separated by the condition $\pi_H^{n-1} < d/[N(a-d)+2d-b]$.

When the stability of the states of equilibrium is investigated for the two modes, it is found that: in the first mode, equilibrium (i) is stable, but it ceases to be so in the second mode. Equilibrium (ii) is stable in both modes, and equilibrium (iii), which exists only in the first mode, is unstable.

When the erosion effect is included ($a_t = a - \varepsilon t$, $b_t = b - \varepsilon t$, $d_t = d - \varepsilon t$), it is assumed that ε is fairly small, so that the "prisoner dilemma" situation always remains in effect. The critical time of transition from the first to the second mode, with the assumption made above, will have the order of $t_{critical} \sim [d - N(a - d)\ \pi_H^{n-1}]/\varepsilon$. Thus, when it reaches a certain time, the system changes from one state of equilibrium to another, that is, an "honesty revolution" occurs. In order for such a transition to be possible, it is very important that there be at least a very small percentage of players in the system who always choose the honest strategy.

Basic conclusions

In [20], it is shown that the combination of the presence of a small group of "honest" players and aggregate social costs can be enough to bring the system to a critical (i.e., catastrophic) point at which the stable equilibrium level of corruption suddenly becomes unstable. When the system is at such a catastrophic point, the slightest push is sufficient to shift it to a different state of equilibrium. The "honesty revolution" occurs in a similar way. In the new cooperative state of equilibrium, all of the players choose to be relatively honest. This equilibrium will always be stable according to the conditions of the model. Such a catastrophic jump to a new equilibrium serves as an example of sudden and spontaneous grounds for cooperative patterns of behavior.

We hope that, in spite of the numerous publications, the study of corruption with the help of mathematical models just beginning.

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