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INTERTEMPORAL THREE-PRODUCT GENERAL EQUILIBRIUM MODEL OF RUSSIAN ECONOMY[1]

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This paper presents a three-product dynamic model of Russian economy. The calculation of the model comes to a boundary problem on a long interval of time for a rigid dynamic system. Thus development of a supercomputer algorithm method is required. The model describes complete system of balances of uniform labor, three products (export, import and interior product) and six financial instruments: cash, operating accounts, loro accounts with Central Bank, bank loans, bank deposits, net deposits with Central Bank and foreign currency. The model describes dynamics of Russian economy as a result of interaction of the following agents: Producer, Bank, Household, Owner, Trader, Government, Central Bank, Exporter and Importer. The quarter model from 2004 to 2011 adequately reproduces such macroeconomic indicators as GDP (Consumption, Investments, Import, Export and their deflators), bank loans, bank liquidity, bank currency accounts, monetary reserve and many other. The main result obtained was the discovery of a strong turnpike property: although in the model we allow agents to know the future, this knowledge is unnecessary for them to find optimal behavior when the parameters of the model are properly identified.

Keywords: Macroeconomic model, GDP, General equilibrium, National accounts.

Introduction

Nowadays mathematical modeling is the main instrument for analyzing and modeling of national economy though there is still no versatile model of economic systems as reliable as those of technical

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systems. The reason is that the economy is capable for substantial inner development and constantly keeps generating phenomenon out of current theory. However our forty-years' experience of creation and application of economic models in the Department of Mathematical Modeling of Economic Systems in Computational Center of Academy of Sciences (CCAS RAS) shows that economic models serve a secure instrument both for economic mechanisms analysis and for forecasting consequences of macroeconomic decisions *provided constant economic relations* [4, 18, 19, 20, 24, 26].

The global economic crisis was exogenous for Russian economy; hence interior economic relations have not changed much. However exterior impulse was intense enough to turn previously appropriate one-product models of Russian economy inappropriate. This paper presents a three-product dynamic model of Russian economy capable of displaying global crisis consequences.

Russian national economy of the second half of first decade XXI century is examined. The object of research is dynamics of the most important macroeconomic indicators, particularly within global financial crisis. The goal is to develop a medium-term intertemporal general equilibrium model of Russian economy based on non-linear desagregation of national accounts by expenditures. The model has to be able to replicate statistical indicators and to perform experiments and forecasts.

The model describes complete system of balances of uniform labor, three products (export, import and interior product) and 6 financial instruments: cash, operating accounts, loro accounts with Central Bank, bank loans, bank deposits, net deposits with Central Bank and foreign currency. The model describes dynamics of Russian economy as a result of interaction of the following nine agents: Producer, Bank, Household, Owner, Trader, Government, Central Bank, Exporter and Importer.

The model has 30 adjustment parameters. Values of these parameters cannot be found in official statistics. These adjustment parameters are used for model optimization. This is done via supercomputer (mainframe) MVS100K (140.16 TFlops) in Joint Supercomputer Center (JSCC) of Russian Academy of Sciences (RAS).

The quarter model from 2004 to 2011 adequately reproduces a list of macroeconomic indicators. Moreover, one can change exogenous series to perform so called "analytical experiment". The model constantly demonstrates *turn-pike property*, i.e. any changes in exogenous series result in no significant changes in model solution before the changes in exogenous series take place. This fact strongly supports rational expectations preposition. Although in the model we allow agents to know the future, this knowledge is unnecessary for them to find optimal behavior when the parameters of the model are properly identified. Also by prolonging exogenous series one can calculate forecasts and analyze impacts of various policies on national economy.

Literature Review

In the modern economic science it is widely acknowledged that the most fundamental approach for economic modeling is to describe an economic system dynamics as a result of multiple rational agent interaction. Computable General Equilibrium (CGE) models have been the main practical tool for macroeconomic modeling and forecast since 1990s. It happened so because it was found out that accounting for only technological constraints (balance models), extrapolation of previous tendencies in data (econometric models) and application of exterior constraints (system dynamics models) are not enough for complete description of modern economy [31, 32]. Most CGE models represent economy of a state (might be divided by subregions of a country) or a group of states [6, 7, 14]. As for Russian economy such models are developed by Makarov and his colleagues [16]. There also global CGE models [9, 13].

System analysis of evolving economy (SAEE) is the name proposed in 1975 at Dorodnicyn computing centre of Russian academy of sciences by academician AA Petrov for a new direction of research in mathematical modelling of economy [17]. The initial purpose was to synthesize the methodology for mathematical modelling of complex systems, which was developed in natural sciences, and the achievements of modern economic theory [11]. In some sense, especially in what concerns formalization, SAEE models are close to computable general equilibrium models (CGE). However, our models devote more attention to specifics of the prevailing economic relations. Moreover, it is worth noting that these studies have a relatively longer history and started 15 years before the onset of CGE models.

In 1988, a model that reproduces the main qualitative features of the evolution of the planned economy was constructed. Therefore, by the beginning of economic reforms in the Soviet Union and then in Russia the SAEE approach had already been developed for the analysis of the changes in the economy. In particular, two years before the dramatic economic reforms in 1992, its short-term consequences had been correctly predicted.

Later we came up with:

- the model of “high inflation” of 1992-1995;
- the model of "financial stabilization" of 1995-1998, in which the 1998 crisis in Russia was predicted (the reaction to Asian 1997 crisis);
- the model for assessment of economic prospects after the 1998 crisis, based on hypotheses about the nature of the economic relations that formed in the corresponding period in Russia.

These models helped to understand the inner logic of economic processes, hiding behind the visible (often seemingly paradoxical) pattern of economic phenomena that do not fit into known theoretical schemes. Experience in application of the models showed that they serve as a reliable tool in analysing macroeconomic patterns, as well as forecasting consequences of macroeconomic decisions in the context of existing relationships. One could entitle it as a "chronicle" of Russian economic reforms, written in terms of mathematical models [18, 19, 20 and 4].

The main difficulty in modelling the Soviet and Russian economy during 1986-2004 was that creation of every subsequent model started from the very beginning, that is from system analysis of changed economic relations. Creating a new model takes lots of time and expertize. In fact, in case of the aforementioned models it took up to one year of work by the team of qualified professionals. However, time was not as critical as compatibility issue connected with these models. New types of economic relations were described by new variables and new expressions, often requiring new mathematical methods. As a result, after almost fourty years of research of the evolution of Soviet and Russian economy we still can’t say that our models form what is called a system. The reason is that above-mentioned models are difficult to compare with each other, just as it is difficult to compare the models created by different research groups. Unfortunately, this situation is typical in modelling complex systems [23].

In 2004, we partly changed our modelling approach. First, we abandoned simplified dynamic relationships characteristic of CGE models and earlier SAEE models. Second, we turned to a theoretically more coherent, but technically and conceptually more complex framework of general intertemporal equilibrium with control of capital. The first model made in equilibrium with control of capital conception was issued in 2004. It was a one-product intertemporal general equilibrium model made for the Russian Tax authorities to assess the magnitude of shadow economy based on non-tax data [4]. The model described in this paper is a next evolutionary step of 2004 model. It is worthwhile to discuss the position of this approach in the framework of economic modelling, so the basic principles of our approach are discussed in the following subsections of the paper.

The System of Material Balances

The majority of models is based on the following pattern of movement of material goods: there is a set of agents (individuals and legal entities), a set of material goods (resources, products and services), all material goods are distributed among agents at every moment of time, and change of every agent’s stock of each variety of material goods is described by the balance

Change of agent's stock of good	=	Agent's production of good	–	Agent's consumption of good	–	Agent's current and capital expenditure	–	Agent's transfer of good to other agents	+	Receipt of good from other agents

Of course, this description is quite relative. Particular lists of agents and material goods are immense and constantly changing their composition. Nevertheless, accounting and economic statistics traditionally are based on this pattern. Its power consists in that it expresses a fundamental property of *additivity* of goods: one agent loses exactly what another gets. It allows to aggregate balances. The total stock of any subset of agents is described by the balance of the same form as of a single agent [27].

Nevertheless, these systems of material balances may be abandoned in the future. The reason lies in ever growing importance of two classes of *non-additive* goods: public and information goods. *Public goods* are not distributed among agents. *Information goods* are freely replicable. When an agent transmits information she is not deprived of it. Also, an agent does not increase her information when receives it again.

Accounting of information goods remains an unsolved problem. At the same time, its importance is rising, since the properties of information goods are becoming more and more characteristic for material goods. For instance, today one rarely buys the same model of a phone, computer, or shoes for the second time. This means that one pays not only for the material object, but also for its novelty. In an attempt to take information into account, at the present time, accounting "squeezes" the information goods as a part of the service item and then struggles with the valuation of intangible assets. Meanwhile, theory comes to a conclusion that for adequate handling with information goods it becomes necessary to change no less than arithmetic itself.

The Financial Balance System

The complex system of material balances requires aggregation of an enormous amount of information about differentiated material goods. It is necessary not only for a researcher of the economy, but primarily for an economic agent who lives in it. Money serves as the main tool that aggregates and transfers information about flows of material goods in the economy. In any developed economy each systematically repeated transfer of goods meets an opposite directed cash flow of payments:

Flow of payments between agents for the transfer of goods	=	Price	×	Volume of transfer of goods between agents

In this flow of payments, as might be generally assumed, the price does not depend on the pair of agents that exchange the goods. This enables to obtain a description of aggregate flows of goods between macroagents2 by multiplying the material balances by the prices and summing up within the groups of agents and products. In particular, summation of goods and services, but without resources, over the agents located on the territory of a country, gives the main macroeconomic balance of the country

GDP	=	Consumption	+	Accumulation (Gross Investments)	+	Exports	–	Imports

[2] In this paper the term 'macroagent' refers to a set of small individual agents with similar possibilities and incentives in the economic system (such as householders, entrepreneurs, banks, etc.) and significant individual agents (such as Government and Central Bank).

Agents' stocks (funds) of money are additive quantities. In present conditions, when natural emission[3] is absent, these stocks satisfy the following balance [27].

Change of agent's stock of money	=	Balance of payments for goods from each of other agents[4]	+	Balance of transfers from each of other agents	+	Total increase in obligations to other agents	–	Total increase in obligations of other agents

Today, all new money in the world emerges in the process of credit emission - a simultaneous increase in assets and liabilities. As a result, the possibility of formation of financial bubbles is embedded in the very nature of modern money. Nevertheless, since material balances lose their consistency, cash flows remain the only reliably measurable quantities.

Behavior of Agents: The Canonical Form of the Model as a Generalization of the Idea of Economic Equilibrium

The next more specific principle is to consider agents not just as a subset of material and financial flows, but also as decision-makers who choose the volume of flows within their "scope of choice". Material and financial balances are internal constraints in the problem of choice of agents. Other specific constraints are technological limitations on the ability to transform some goods into others.

The main task of economic system as a managing system is to determine flows of transfer (exchange), the magnitude of which the transactors decide simultaneously. The idea of economic equilibrium is that each agent offers her own *plan* of the volume of her flow (demand or supply in economic language). This plan is conditional - it depends on the values of specific *information* variables (prices, interest rates). The title '*information*' is given to these variables since they carry information about the system to the agent. Plans that are consistent with existing economic relations are described by the institutional (external) constraints that may also depend on information variables. The simplest example of such a restriction is aforementioned relation between the flow of money and good at a specified price. Plans of agents become consistent with each other through their interaction so that balances included in the model hold true throughout the entire system.

As a result, we arrive at a model in the canonical form [4], schematically depicted in Fig. 1,

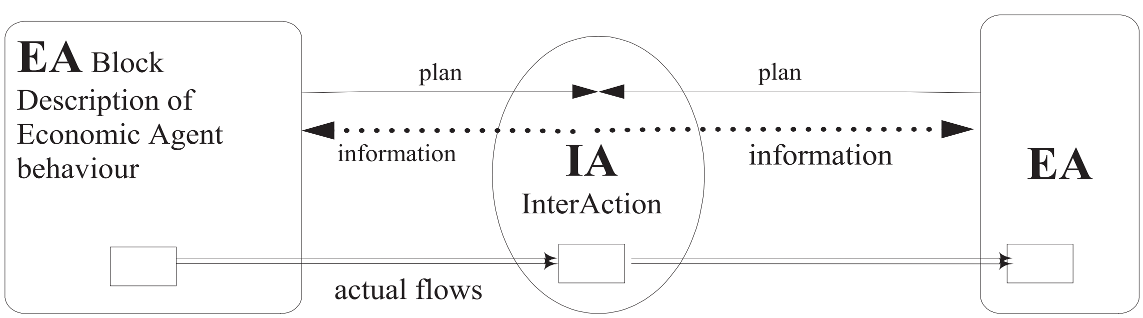


Figure 1.The scheme of the model in the canonical form.

[3] The idea to return to the gold (or energy) standard in order to overcome the crisis seems to be groundless. In the conditions of growing importance of innovation and dominance of the "virtual economy" it is unlikely that one could pick a single specific product, which could appropriately measure the level of production and consumption.

[4] Including financial services – balance of received and paid interest.

Relations of model in canonical form are split into blocks of two types: blocks that describe behaviour of agents (EA) and blocks that describe interactions between agents (IA). In general, presence of interaction between some agents does not necessary mean that the plans of these contractors are realized, but sometimes reduces simply to exchange of information between them. One may write in the canonical form not only any of the SAEE models, including the model of the planned economy [18] but also any of the CGE models, but not all econometric models. The classification of the relations in canonical form facilitates effective verification of correctness of the model at the level of formulas by means of developed computer modelling support system – ECOMOD [4].

The main problem in creating a model in canonical form is to choose a set of aggregate additive quantities. For these variables one writes a complete system of material and financial balances. This set of variables should be rich enough to express institutional constraints that reflect the economic relations. It is this form and constraints that reflect the difference between competitive and monopolistic markets [4], the difference between stock and share ownership [21], possibility to use monetary surrogates [27] and channels of shadow turnover [4]. The system of institutional constraints also shows the difference between CGE and SAEE models. In the former, institutional constraints are taken mainly from textbooks, whereas in the latter, we try to reflect the real state of affairs in the economy under consideration.

Rationality of Behavior: Representative Individuals or Macroagents?

The framework of the model usually leaves to an agent substantial freedom in choosing her plans. In imitation models the choice of an agent is described via direct prescription of decision rules. Probably, such approach could be justified in describing organizational routines within a corporation. At the same time, within the whole, self-organizing, economic system, such a direct solution "for every single economic unit" is too bold. In addition, modification of such solution without mistakes is difficult. Therefore, in economic theory, CGE models and SAEE models postulate rational behaviour for most of the agents, that is, a choice within the constraints of a strategy that maximizes some criterion: utility, profit, capitalization, etc.

This situation constantly attracts doubt from the experts and the sharp rejection of non-specialists. They say that economic theory is weak because it ignores the "human factor" and studies the behaviour of completely fictional «homo economicus». However, people are not atoms after all. One would think, why make up for their motives when you can simply ask why they act in a certain way? Such surveys are conducted rather often, but they do not add up to a coherent picture. Instead, the view of economists "from outside" can observe certain regularities.

Individual behaviour is determined by many factors indeed not taken into account by the economic theory (one can get married, sick, promoted at work etc.). It appears that any of us is more complicated than the economy as a whole. Therefore, all known attempts to establish economic theory on the laws of psychology failed completely, and it seems, we can understand something in economy just because it shows only a small portion of the richness of human inner world. In economics, people act under conditions of impersonal threats (legal or administrative sanctions, dismissal, bankruptcy, moral condemnation, etc.) [25]. As a result, their behaviour is standardized, and individual differences are levelled.

Still there are at least two different possible definitions of the subject of rational behaviour. The existing theory appeals to the "representative agents", each of which is characterized by a given constant interest. In microeconomic studies we sometimes try to identify difference of interests, but in macroeconomic models almost always only one representative agent of each type is considered (consumer, producer, marketer, etc.).

We believe that the idea of absolutely independent "representative individuals" contradicts the fact that people and organizations interact with each other. In large groups of actors who perform similar roles in the economy competition and imitation take place [26]. As a result, the collective behaviour of such a group is simpler and more consistent than the behaviour of any of its members [10], and can be described as a simple tendency to maximize consumption, income, wealth, etc. This can be confirmed by direct measurements [1, 3 and 18].

These surprising results can be explained (including at the model level) that the utility function is not "placed" in the minds of consumers, but comes from the interaction of non-rational consumers and traders manipulating prices to their advantage. Based on these arguments and observations, we attribute interests to macroagents, that is, consider these interests simply as variational principles that distinguish the observed behaviour of macroagents from all options allowed by limitations of the model [17].

This statement works backwards too. When we are dealing with powerful actors with a single will, for example with the state, we do not try to describe its operation principle of optimality. It is better simply to ask what it is going to do that is to describe its behaviour scenarios and possible actions.

The Principle of Rational Expectations and a Typical Problem of an Agent

The above principles are sufficient to build a static but not a dynamic model. In dynamic models an agent plans her actions for the future, and therefore should predict future changes in market conditions (information variables). There is a paradox: we construct a model to give a forecast of market conditions, and to build the model we need to know how agents forecast these conditions.

A radical solution to this paradox is the principle of rational expectations [2, 8]. Most simply, it is stated as follows: model agents use for their forecasts the very same model that we build. Although the principle of rational expectations gives rise to reasonable doubt, because it implies that the model agents "know everything in advance", we ventured to apply it to modelling the real Russian economy. As a result, we achieved success to a larger extent than it could be possible with simplification of the phenomenological description of agent behaviour, characteristic of CGE models and earlier SAEE models.

In deterministic case, which is in question here, the principle of rational expectations leads to a model of intertemporal general equilibrium [8]. In this model, each agent according to her goals, opportunities and forecasts, determines her supply and demand for products, resources and financial instruments in the current and all future times. Then these forecasts (same for all agents) are determined from the condition of matching of demand and supply. Intertemporal equilibrium model are widely known but so far they have been used exclusively to study some theoretical issues on the steady-state conditions of abstract models of economy [30]. We ventured to apply this approach to the observed dynamics of the Russian economy and, as will be shown below, it is only after some of the theoretical findings and overcoming serious difficulties in implementation that it led to success.

General Inter Temporal Equilibrium Model with Control of Capital

This subsection starts with description of the firm and owner interaction. Intertemporal equilibrium model can be obtained from the classical static general equilibrium model of Arrow-Debreu under assumption that goods that are produced and consumed at different moments of the time are different goods, and that storage is a technological process that transforms “goods today” into “goods tomorrow”.

Difficulties arise concerning the question who determines investment of capital and thus the capital stock. In the static model the value of capital stock is fixed. It is possible to assume that consumer owns the capital and the firm pays a rent to owner. Also it is possible to consider that the firm owns the capital stock and pays the same fee in the form of dividends. In the dynamic model the capital stock is changed by investments, and the question who invests and why is crucial.

The notion of equilibrium efficiency could be used as a criterion for determination of the solution correctness. From our point of view, the most important and original achievement of mathematical economics is welfare theory. According to the 1st welfare theorem [8] a perfectly competitive equilibrium (at least in the static) implements the same consumption and production volumes as the ideal planning authority acting in the interests of consumers. Moreover, the set of typical economic agents emerges from the decomposition of the ideal planning problem into a Nash equilibrium problem equivalent to market equilibrium. Such agents are consumers-owners (maximize consumption given prices and budget constraints), firms (maximize profit given prices and technological constraints) and traders (maximize profit over prices).

In the case of a dynamic model the decomposition is ambiguous in the sense of the question who determines the investment policy. As far as we know, intertemporal equilibrium models are usually based on the approach proposed by M. Sidrauski [29] nearly a half a century ago. This approach assumes direct identification of the consumers' savings with the firms' capital stock. The corresponding intertemporal equilibrium is effective5. However, we consider this approach to be limited. Firstly, at present time most of the owners and shareholders do not manage the capital stock, moreover, they even do not know the structure of the capital stocks they possess. Nowadays property is managed by financial instruments only. Secondly, Sidrauski's approach excludes banking system, the main function of which is to transform savings into investments.

In 2003 we succeeded in finding another way to construct efficient intertemporal equilibrium, which we call *general intertemporal equilibrium with control of capital* (GEC). According to this approach the firm manages fixed capital and determines capital investments, while the owner manages net assets (own capital) relying only on the price and return on the own capital. The price and return on own capital are determined by the firm according to capital maximization problem [4, 24].

Scaling Invariance and Agent's Capital

A GEC model becomes particularly simple and natural if production and consumption are homogeneous (scale-invariant). The model is homogeneous if the absolute size of the simulated system is not significant: doubling of the exogenous additive quantities (flows and stocks of products, resources and financial instruments, including their initial values) results in doubling of the endogenous variables. Homogeneity assumption (constant returns to scale) is present in all popular economical models in a varying degree. It is worth to note that most of the known economic relationships (for example, famous Solow's Golden Rule) were obtained from the balanced growth analysis, and the balanced growth is possible only under the condition of homogeneity.

The agents' goals are unified in the homogeneous GEC model. We assume that both firms and consumers maximize their properly defined capitalization6. The capitalization in the model should be defined as valuation of net wealth. Prices used for valuation are dual variables to balance constraints in optimality conditions. Valuating price is equal to market price for liquid asset or liability, and is less than market price for non-liquid asset, and are greater for non-liquid liability.

Capitalization satisfies the following simple equation:

Capitalization increment	=	Balance sheet profit	-	Distributed profit	×	(1+effective tax rate)

Producer's distributed profit is dividends. Consumer's "distributed profit" is consumer expenditures. Effective tax rate is determined inside the model. It includes both the actual taxes and the rates of indirect losses associated with, for example, the need to keep cash reserves to implement cash flow.

Balance sheet profit can be expressed in two forms:

1	Balance sheet profit	=	value added by production	+	value added by trade	+	balance of payments for resources	+	revaluation of liquid assets	–	depreciation of non-liquid assets

5 many examples studied in [30].

6 Recently, the same principles of unification were found worthy alternatives, see [21].

Value added consists of interest on savings in the case of consumer. We should emphasize, that this expression is not postulated, but it is derived from the model. The depreciation rates are determined from the model so that to reduce valuation in market prices to valuation in dual variables.

2	Balance sheet profit	=	agent's capitalization	×	internal rate of return

The variable "internal rate of return" does not depend on the absolute values of flows and stocks. It is a nontrivial convolution (different for different agents) of current and future real rates of return on financial instruments and technological processes. In the framework of GEC model this is exactly the rate of return that firm reports to the owner. The owner determines the rate of profit growth in accordance with the equation for the firm's capital.

In the homogeneous GEC model agent's capitalization coincides with expected profit, discounted by internal rate of return. Thus, if asset prices and discount factors are derived from optimality conditions, then, in the framework of GEC model, all conventional values of the firm (by value of net assets, by net present value, by market capitalization) *coincide*.

Finally, we have used agent's capital for the solution of well-known problem of imposing terminal conditions. We assume only one terminal condition: the capitalization should grow by a certain factor. Under this condition agent's capitalization is positive on the optimal trajectory [22]. So this condition substitutes "no Ponzi game condition", which is usually imposed in a very intricate form [30].

It is amazing that all these properties of the agent's capital do not require any additional assumptions. They are derived from the optimality and homogeneity in the same way as the energy conservation is derived from the principle of least action and time independency of physical system[7].

Turnpike Property

In intertemporal equilibrium models agents plan their actions for all future periods of time. One could think that optimal agents' actions should strongly depend on their knowledge about the future. For example, the optimal trajectory of a missile crucially depends on the starting and destination point. However, this is not the case for economic processes. Turnpike property is typical for economic models: *the impact of the future goals and exogenous variables exponentially decreases with the increase of the difference between present and future periods of the time* [8]. This property gives hope that model calculations may be valid without detailed knowledge of the future.

Among our most interesting and intriguing recent results is discovery of *strong turnpike property*. In all applied models built on the principles described above the turnpike property is so strong that the influence of the future becomes almost negligible in one-two periods of time (if parameters of the model are such that the model approximates the statistics well) [1]. In other words, we allow agents to foresee the future, but it appears to be enough for them to know the current state to determine optimal behaviour. We suggest this effect emerges because institutional constraints (under correct parameters) narrow the choice. However, it is still necessary to formulate intertemporal decision problem in order to derive optimality conditions.

Model of the Russian Economy in Crisis Period

The applied GEC model can be obtained from the model of perfect competition by introducing additional constraints and opportunities into the description of agents that reflect the existing economic relations, as well as the introduction of additional agents, for example, banking system. Description of additional agents is facilitated by the fact that goals of agents in the GEC model are unified. In addition, it is

[7] Mathematically speaking, the capital of an agent is the first integral of the field of extremals corresponding to the scaling symmetry of the agent.

important to take into account actions of individual agents (Ministry of Finance, Central Bank, the external world), whose behaviour is described by the scenario.

If one combines all optimality conditions of macroagents (EA blocks in Fig. 1) with the scenario of individual agents' actions and add descriptions of their interactions (IA blocks in Fig. 1), which contain balances of financial and material flows between the agents, as well as some information flows, one will have a model of intertemporal equilibrium with control of capital (GEC), which we now investigate and use. Specific character of this model is expressed, firstly, by a set of aggregate additive values (material and financial assets), which describe the behaviour and interactions of agents, and secondly by a set and a form of institutional constraints.

Equilibrium in this model, of course, is not necessary effective, but other properties are generally preserved. For some agents, however, homogeneity may be violated due to monopoly of the agent in the market or economies of scale or restrictions such as quotas. Then on the right-hand side of the equation for capitalization, one will have additional components with the meaning of indirect revenues or losses of profit. Integrals of these components over future time discounted at the rate of return can be interpreted as objectively determined monetary value of nonmaterial assets of the agent.

Realistic macro model of this type reduces to a system that initially contains more than hundreds of nonlinear finite and differential equations. Moreover, because the model requires agents to decide for the optimal control problem, it is not available to calculate step by step, starting from the initial condition. For GEC model calculations have to use very sophisticated algorithms. At the same time, the number of tuning (adjustment) parameters in the model occurs to be small: 20-30 against hundreds in econometric or CGE models. Therefore, when a GEC model poorly reproduces the statistics, we have to change the shape of the original relations and redo all the calculations.

It is almost impossible to write a model on a paper and to translate it into a programming language in a reasonable time without errors and distortions of the underlying assumptions. We see the only method to work effectively with the GEC model – to use the above-mentioned support system for mathematical modelling "ECOMOD" implemented in the computer algebra system Maple. ECOMOD

- checks correctness of balances of the model;
- automatically formulates optimality conditions for given constraints;
- allows automatic repeating calculations after changes in the original equations;
- applies numerical algorithm to a model expressed in standard mathematical notation;
- records the model in a form that could be used for publication.

Three Products and Desagregation

At this point we have to elaborate on the "three-product" basis. In fact, some general equilibrium models are based on one-product approach. The single product can be produced as GDP; consumed as consumption, gross investments, exports; and imported as imports. However, the market equilibrium requires for the single product to have one price. Provided that in Russian national GDP; consumption, gross investments, exports and imports have different growth rates of price indexes, we cannot use the one-product approach.

Our approach is not based on aggregation of various products into large cumulative group as it is commonly done by government and non-government agencies. We desagregate the high-level data, i.e. the national account balance by consumption (NBC), which is gathered in current prices by the Russian Federal Statistical Agency. In constant prices we can write:

$$Y(t) = J(t) + C(t) + E(t) - I(t) \tag{1}$$

where $Y(t)$ – GDP, $J(t)$ – Gross Investments, $C(t)$ – Consumption, $E(t)$ – Exports, $I(t)$ – Imports. Hence, the balance in current prices can be written as:

$$p_Y(t)Y(t) = p_J(t)J(t) + p_C(t)C(t) + p_E(t)E(t) - p_I(t)I(t), \tag{2}$$

where price indexes $p_Y(t)$, $p_J(t)$, $p_C(t)$, $p_E(t)$ and $p_I(t)$ are determined as ratio of nominal (in current prices) component of NBC to real (in constant prices) components of NBC.

Obviously, the one-product model cannot replicate both (1) and (2) at the same time if price indexes are not equal because the one-product model implies single price for single product. That is the reason we had to give up one-product model and develop a method for desagregation of NBC based on differences in price indexes.

Assume that the components $Y(t)$, $J(t)$ and $C(t)$ of NBC in constant prices (1) are non-linear homogenous aggregates of three low-level products: observed in statistics export product E and import product I and of unobserved in statistics interior product X. At the same time it is supposed that import product usage (construction materials, details, etc.) is determined not by technological constraints but by consumers demands. So we incorporate intermediate components into final products and assume model balances:

$$X(t) = J_X(t) + C_X(t) \quad I(t) = J_I(t) + C_I(t), \tag{3}$$

where

- J_I – imports for investments, e.g., import equipment for factories and plants (final product) and import material for its construction (intermediate);
- C_I – imports for consumption, e.g., cars (final product) and details (intermediate) for a Russian TV-set;
- J_X – interior production for investments, e.g., construction company services (final services) and electro energy wasted on construction (intermediate);
- C_X – interior production for consumption, e.g., trade services at car sale (final services) and electro energy wasted at sales (intermediate).

Then it is assumed that exports $E(t)$ and imports $I(t)$ in (1) reflect complete production and inflow/outflow of model products E and I. The observed consumption and gross investments in (1) are described as:

$$C(t) = g(C_I(t), C_X(t)), \tag{4}$$

$$J(t) = h(J_I(t), J_X(t)) \tag{5}$$

Finally, GDP can be produced by various combinations of interior and export products:

$$Y(t) = f(X(t), E(t)) \tag{6}$$

The functions $g(\cdot,\cdot)$, $h(\cdot,\cdot)$, $f(\cdot,\cdot)$ are, at least, monotonous and linear homogeneous. The latter assumption is natural for aggregate function since it has to be independent of measurement units (such as meter, dollar, year). Linear homogeneity provides additivity of financial flows while undertaking non-linear functions in real flows.

Agent Rationality

Since the series of $X(t)$, $J_X(t)$, $C_X(t)$, $C_I(t)$, $J_I(t)$ cannot be found in official data the equations (3) -(6) are not sufficient to identify the functions $g(\cdot,\cdot)$, $h(\cdot,\cdot)$, $f(\cdot,\cdot)$ in (4)-(6), However we have yet unused information on price indexes (deflators). This information can be used if one contemplates aggregation functions $g(\cdot,\cdot)$, $h(\cdot,\cdot)$, $f(\cdot,\cdot)$ as those of **utility** or **costs**. To do so we have to introduce to the model a new deflator $p_X(t)$ – the price index for interior product.

The isomorphic level lines of the function $g(\cdot,\cdot)$ can be interpreted as indifference curves which show the possibilities of substitution between interior and import product for the aggregate consumer. And the function $g(\cdot,\cdot)$ can be interpreted as the **utility function** of purchases. The consumer purchases the products $C_I(t)$, $C_X(t)$ at the moment of time t at the cost of $\Phi(t) = p_I(t)C_I(t) + p_X(t)C_X(t)$. This quantity we know from official statistics: $\Phi(t) = p_C(t)C(t)$. Provided (4) we come up with

$$p_C(t)\,g(C_I(t), C_X(t)) = p_I(t)\,C_I(t) + p_X(t)\,C_X(t) \tag{7}$$

Why has the consumer purchased the combination $C_I(t)$, $C_X(t)$ but not any other? The trivial answer is that with any other set of products at the same prices the equation (4) is inconsistent with the **budget constraint**

$$p_C(t)\,g(C_I, C_X) \le p_I(t)\,C_I + p_X(t)\,C_X \text{ for all } C_I, C_X \ge 0. \tag{8}$$

From (7) and (8) it comes that the values of $C_I(t)$, $C_X(t)$ bring maximum to the equation

$$p_C(t)\,g(C_I, C_X) - p_I(t)\,C_I - p_X(t)\,C_X \to \max_{C_I, C_X \ge 0} \tag{9}$$

To make the task (9) mathematically correct we make common for utility functions assumptions: it has to be smooth and concave. Since we are interested in interior solution only (both import and interior products are purchased) we get necessary and sufficient conditions for (9)

$$\partial_1 g\left(C_I(t), C_X(t)\right) = \frac{p_I(t)}{p_C(t)}, \quad \partial_2 g\left(C_I(t), C_X(t)\right) = \frac{p_X(t)}{p_C(t)}. \tag{10}$$

Since the function $g(\cdot,\cdot)$ is linear homogeneous, the maximization task (9) is degenerate. It has non-trivial solution not for all prices ratio. And when there is a solution, it is not single. At the solution the equation (7) has to be active (sufficed, i.e. the equality in (7) should be strict with no discrepancy). From algebraic standpoint, the partial derivatives of $g(\cdot,\cdot)$ are functions of arguments ratio. Which is why, the equations (10) determine not the values of $C_I(t)$, $C_X(t)$ but the ratio $C_I(t)/C_X(t)$ and the price $p_X(t)$. Since we can use the symmetric in (10), together with (7) we come up with

$$\frac{\partial_1 g\left(C_I(t), C_X(t)\right)}{\partial_2 g\left(C_I(t), C_X(t)\right)} = \frac{p_I(t)}{p_X(t)}, \quad p_C(t)\,g(C_I(t), C_X(t)) = p_I(t)\,C_I(t) + p_X(t)\,C_X(t). \tag{11}$$

Contemplating the concave function $h(\cdot,\cdot)$ in (5) as a utility function of purchases of aggregate investor and applying the same logic as for the consumer we arrive to

$$\frac{\partial_1 h\left(J_I(t), J_X(t)\right)}{\partial_2 h\left(J_I(t), J_X(t)\right)} = \frac{p_I(t)}{p_X(t)}, \quad p_J(t)h\left(J_I(t), J_X(t)\right) = p_I(t)\,J_I(t) + p_X(t)\,J_X(t). \tag{12}$$

For the function $f(\cdot,\cdot)$ the logic has to be changed. The value $V(t) = p_X(t)X(t) + p_E(t)E(t)$ represents the revenue of a producer. In official statistics we can find it as $p_Y(t)Y(t) = p_Y(t)\,f(X(t), E(t))$ and it can be interpreted as costs for factor of production $Y(t)$ required to produce $X(t)$ and $E(t)$. Then the rational aggregate producer has to choose such combination of $X(t)$ and $E(t)$ that maximizes current profit

$$p_X(t)X + p_E(t)E - p_Y(t)f\left(X,E\right) \to \max_{X,E} . \tag{13}$$

Provided homogeneity of $f(\cdot,\cdot)$ the maximal profit is zero. And the equations similar to (11), (12) arise

$$\frac{\partial_1 f\left(X(t),E(t)\right)}{\partial_2 f\left(X(t),E(t)\right)} = \frac{p_X(t)}{p_E(t)}, \quad p_Y(t)f\left(X(t),E(t)\right) = p_X(t)X(t) + p_E(t)E(t) . \tag{14}$$

We receive **11** equations (3), (4), (5), (6), (11), (12) and (14) for **6** unobserved in statistics series $X(t)$, $C_X(t)$, $J_X(t)$, $C_I(t)$, $J_I(t)$ and $p_X(t)$. If we manage to find such constant functions $g(\cdot,\cdot)$, $h(\cdot,\cdot)$ and $f(\cdot,\cdot)$ that this system of equations suffices with good proximity for every period of time, then it will be proved that NBC can be desagregated into balances of three low-level model products. It is worth noting, that at the same time we get a strong argument towards common preposition that macroagents' behaviour can be described as rational.

Assume that $f(\cdot,\cdot)$, $g(\cdot,\cdot)$, $h(\cdot,\cdot)$ are as follows

$$\begin{aligned} f(X,E) &= A_f\left(\sigma_f X^{e_f} + (1-\sigma_f)E^{e_f}\right)^{1/e_f}, \\ h(J_I,J_C) &= A_h\left(\sigma_h J_I^{e_h} + (1-\sigma_h)J_C^{e_h}\right)^{1/e_h}, \\ g(C_I,C_X) &= A_g\left(\sigma_g C_I^{e_g} + (1-\sigma_g)C_X^{e_g}\right)^{1/e_g}. \end{aligned} \tag{15}$$

Such functions (Constant elasticity of substitution – CES) are widely applied in mathematical economics. These function are obviously linear homogeneous. Whenever the exponential parameter (like e_f, e_g, e_h in (15)) is greater than 1 (less than 1) the function is convex (concave).

As a result the identification task arises. The task can be informally expressed as follows: one wants to find the coefficients A_f, σ_f, e_f, A_h, σ_h, e_h, A_g, σ_g, e_g in (15) such that, provided known statistical series $Y(t), J(t)$, $C(t)$, $E(t)$, $I(t)$ $p_Y(t)$, $p_J(t)$, $p_C(t)$, $p_E(t)$, $p_I(t)$, the system of equations (3)-(6) and (11)-(14) suffices as precise as possible. To correctly state the problem we have to determine the measure of "precision".

Assume the financial balances (second equation in (11)-(14)) and material balances (3) have to suffice with zero discrepancy, and the aggregation conditions (4)-(6) and the rationality conditions (first equations in (11)-(14)) may have little discrepancies (residuals). Since absolute values in left hand sides of (4)-(6) grow exponentially because of economic growth, and the ratios of prices in the right hand sides of (11)-(14) remain more or less constant over time, the discrepancies in aggregation conditions (4)-(6) will be measured in logarithms, and discrepancies in rational conditions – in natural units. Thus, the equations (4)-(6) are transformed into

$$\begin{aligned} \ln Y(t) &= \ln A_f + \frac{1}{e_f}\ln\left(\sigma_f X^{e_f} + (1-\sigma_f)E^{e_f}\right) + \varepsilon_1(t), \\ \ln J(t) &= \ln A_h + \frac{1}{e_h}\ln\left(\sigma_h J_I^{e_h} + (1-\sigma_h)J_C^{e_h}\right) + \varepsilon_2(t), \end{aligned} \tag{16}$$

$$\ln C(t) = \ln A_g + \frac{1}{e_g}\ln\left(\sigma_g C_I^{e_g} + (1-\sigma_g)C_X^{e_g}\right) + \varepsilon_3(t), \tag{17}$$

And the first equations in (11)-(14) are transformed into

$$\begin{aligned}
&\frac{\sigma_f}{(1-\sigma)_f}\left(\frac{X(t)}{E(t)}\right)^{e_f-1}-\frac{p_X(t)}{p_E(t)}=\varepsilon_4(t),\\
&\frac{\sigma_h}{(1-\sigma)_h}\left(\frac{J_I(t)}{J_X(t)}\right)^{e_h-1}-\frac{p_I(t)}{p_X(t)}=\varepsilon_5(t),\\
&\frac{\sigma_g}{(1-\sigma)_g}\left(\frac{C_I(t)}{C_X(t)}\right)^{e_g-1}-\frac{p_I(t)}{p_X(t)}=\varepsilon_6(t).
\end{aligned}\tag{18}$$

and minimize the total sum of square residuals (TSS)

$$\sum_{t=1}^{T}\sum_{i=1}^{6}\varepsilon_i(t)^2\to\min\tag{19}$$

by $X(t),\ C_X(t),\ J_X(t),\ C_I(t),\ J_I(t),\ p_X(t),\ a_f,\ b_f,\ a_h,\ b_h,\ a_g,\ b_g>0,\ e_f,\ e_h,\ e_g$ and $\varepsilon_i(t)$ for $i=1...6,\ t=1...T$, where T – observation period length; under the following constraints

$$\begin{aligned}
&X(t)=J_X(t)+C_X(t),\ I(t)=J_I(t)+C_I(t),\\
&p_C(t)\,g(C_I(t),C_X(t))=p_I(t)\,C_I(t)+p_X(t)\,C_X(t),\\
&p_J(t)h\big(J_I(t),J_X(t)\big)=p_I(t)\,J_I(t)+p_X(t)\,J_X(t),
\end{aligned}\tag{20}$$

$$p_Y(t)f\big(X(t),E(t)\big)=p_X(t)X(t)+p_E(t)E(t).\tag{21}$$

Equations-equalities in (20)-(21) allow to exclude some variables and to simplify the task (19) with $(9+6T)$ unknowns to a task with only 9 unknown parameters in production function (15). Still the task remains quite complex so we had to apply some non-standard algorithms calculated on the supercomputer (mainframe) MVS-100K of the Joint Supercomputer Centre of Russian Academy of Sciences (JSCC RAS).

For identification non-smoothed quarter data on national accounts by consumption in current prices and in constant prices (for the year 2008) were used. The time period starts from 1st quarter of 2004 and ends with 4th quarter of 2010 (44 points altogether). The identified parameters of production functions (15) are as follows:

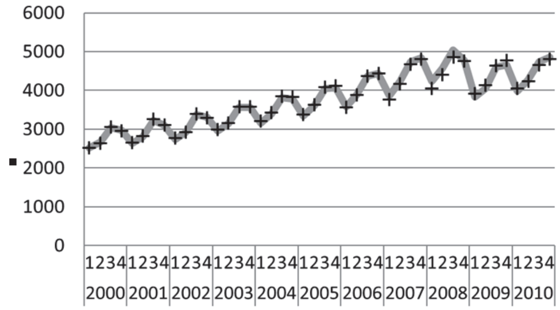
$$f=1.88\left(.68X^{1.05}+.32E^{1.05}\right)^{\frac{1}{1.05}},\ h=1.75\left(.22J_I^{\ .005}+.78J_C^{\ .005}\right)^{\frac{1}{.005}},\ g=1.85\left(.28C_I^{\ .003}+.72C_X^{\ .003}\right)^{\frac{1}{.003}}.$$

The function f appeared to be convex, and the functions g and h concave, as it should be though we placed no ad-hoc boundaries. The achieved precision in aggregation production functions and rational conditions are reflected on the pictures -.

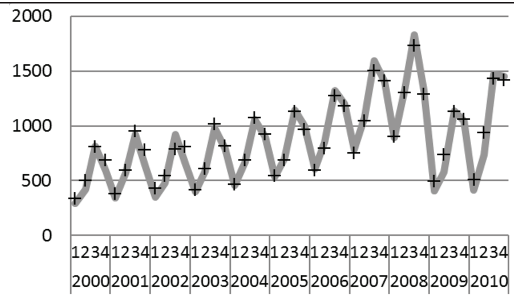
One can observe that relations (16)-(17) are sufficed almost exactly including the crisis period of 2008-2009. This also implies that the way we chose (19) was not crucial. Rationality conditions (18) are sufficed less precisely. However the both sides of equations have common trend and seasonal vibration.

From algebraic standpoint, the identified relations inexplicitly provide four *non-linear* connections between 10 observed series: real GDP $Y(t)$, real gross investments $J(t)$, consumption $C(t)$, Exports $E(t)$, Imports $I(t)$ and their price indexes $p_Y(t)$, $p_J(t)$, $p_C(t)$, $p_E(t)$ and $p_I(t)$. And these connections are fulfilled on non-smoothed data for 44 quarters including crisis period.

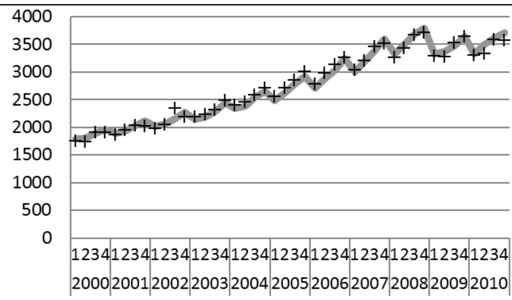
Table 1. Non-linear functions (15) (gray line – statistics, black X's – calculations) and rationality conditions (18) (gray line – price ratio, black X's – first partial derivatives ratio).



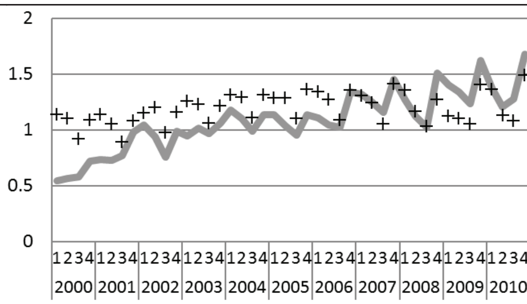
Picture 1. GDP in (15)



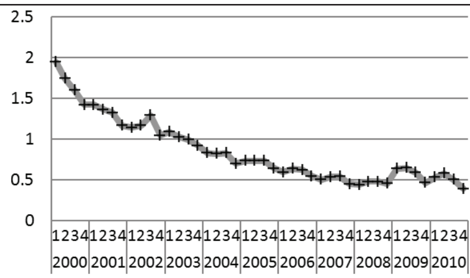
Picture 2. Gross investments in (15)



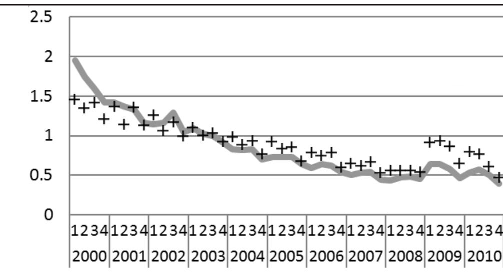
Picture 3. Consumption in (15)



Picture 4. Producers' rationality in (18)



Picture 5. Investors' rationality in (18)



Picture 6. Consumers' rationality in (18)

In order to ease the description of the macroagent Producer in the model described in this paper the above mentioned relations of three-product desagregation were put into a bit unnatural macroagent called Trader.

The Agents of the Model and its Results

In this subsection of the paper 9 agents of the three-product intertemporal general equilibrium model of Russian economy are described. In fact, total description of all agents (all the equations, optimization problems and solutions, commentaries), interactions of all agents; and the final set of equations for calculation would multiply the number of pages in this paper by almost three times. So we provide only informal description.

The latest version of the model of the Russian economy contains description of the real sector, generating domestic and export products and consuming domestic and imported products; and financial sector. Financial flows that accompany production, distribution and consumption are described as turnover of 6 financial instruments: cash balances, current accounts, balances of correspondent accounts in the Central Bank, bank loans, bank deposits, deposits / loans of banks in the Central Bank, foreign currencies. Products, labour and financial instruments form a set of additive values for which in the model is written a complete system of balances.

Producer represents an aggregate of commercial non-financial organizations. Hires labour, invests into own fixed capital by purchases of investment product[8], produces all GDP by means of labour and capital, sells GDP, draws loans from the Bank, keeps current accounts within Bank, pays taxes to Government and dividends to Owner.

The plans for all of the mentioned in the previous paragraph variables are determined by the Producers' maximization of capitalization problem under certain technological constraints (given the production function for GDP from labour and capital), under constraints of loan balances and current accounts, liquidity constraints based on forecast for prices, wages and loan interest rates. The description of Producer contains sufficient conditions for optimal plans.

Bank represents the totality of financial commercial organizations, and the material costs are not taken into account. The applied model for the Bank was developed beforehand by MY Andreev, NP Pilnik and IG Pospelov [3]. The Bank issues the loans to the Producer, draws on deposits of Household, non-residents and of Central Bank, maintains non-cash payments of Producer via its current account within Central Bank and uses the residual on the current account of Producer as a credit source with zero interest rate. Also Bank satisfies Central Bank reserve requirements, pays dividends to the Owner.

Plans for all these variables are determined by the Banks' problem of maximization capitalization given certain balance constraints for loans, current accounts and deposits, reserve requirements of Central Bank based on forecast of interest rates (for loans and deposits) and Central Bank fund rate. The description of Bank contains sufficient conditions for optimal plans.

Household represents individuals - consumers and employees. The agent purchases and consumes final consumption good, sells labor and receives wages, receives transfers from the public budget, keeps accumulated money in Bank and receives interest payments. The labor supply is considered an exogenous[9] variable. The consumption, savings and liquid cash are planned by Household in order to maximize expected utility of consumption under balance constraints and liquidity constraints based on the forecast of future prices, wages, interest rates on deposits and employment rate. The description of Household contains sufficient conditions for optimal plans.

Owner Usually in general equilibrium models owners' responsibilities are attributed to households. However in Russian economy we have to separate these agents. The macroagent Owner represents individuals and entities that control the movement of capital between the sectors of national economy and outside the country. Owner owns Producer, Bank and Trader. According to GEC approach Owner receives forecast of information variables from Producer and Bank. These variables reflect the return on capital and hence inform the Owner about current and future state of Producer and Bank from financial standpoint. Provided this information Owner derives optimal dividends dynamics. The goal functional of Owner is expected value of foreign assets at a random moment of time. The description of Owner contains sufficient conditions for optimal plans.

Trader represents a pure intermediary between consumers, producers, exporters and importers. This agent holds all the non-linear technologies described above. Trader purchases real GDP $Y(t)$ from Producer at a price of $p_y(t)$ and 'produces' out of it export product $E(t)$ and interior product $X(t)$ according to technology (6). Trader sells the export product $E(t)$ to an agent Exporter at a price of $p_e(t)$. The interior product $X(t)$ and the import product $I(t)$ (bought from an agent Importer at a price of $p_i(t)$) are transformed into consumption product $C(t)$ (4) (sells to Household and Government) and into investment product $J(t)$ (5) (sells to Producer and Government). The goal of Trader is to maximize its profit. This problem is decomposed into (9) and (13) which come up with rationality conditions. These conditions result in zero profit. However in the model the profit is not equal to zero. It is equal to government debt to a Trader. The loss is covered by the Owner.

[8] Overall fixed capital in the economy is accumulated via public investment from the Russian budget.

[9] In this paper the term 'exogenous' refers to series that come to the model from outside and cannot be derived inside the model.

Government (State) whose work is represented in the model as an aggregated description of the activities of the Ministry of Finance and implicitly - the establishment of various parameters of economic policy (tax rates, government spending, regulations and provisions, etc.). Government is an individual agent, it has no utility function and optimization problem and its behaviour is described by scenarios.

Central Bank is presented in the model by its functions as the issuer of domestic currency, the holder of currency reserves, the settlement centre and the creditor of commercial banks. Central Bank is an individual agent, it has no utility function and optimization problem and its behaviour is described by scenarios.

Importer purchases foreign product at exterior currency prices, sells it to Trader at the interior rouble import product price, receiving revenue. The revenue is used to pay import duties, import taxes and purchases currency. Zero profit binds together exterior currency prices and interior rouble import product price. Importer is an individual agent, it has no utility function and optimization problem.

Exporter acts almost like Importer and also transfers foreign deposits into Russian Bank.

The **interactions** of the agents are described in 11 blocks: market for consumption product c, market for investment product j, real GDP market y, interior product market u, import product market i, export market e, loan market l, labor market r and deposit market s; interaction of Bank and Central Bank cb, interaction of Government and Central Bank g, taxes and duties x, Producers' capital up, Bank capital ub, Trader dividends uq, currency market w and current accounts maintenance n. These interactions describe material and financial balances between agents and transfers of information.

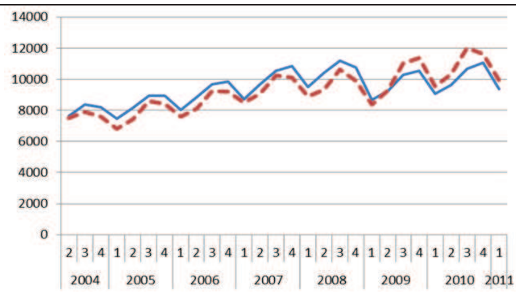
By combining all equations of all agents and interactions together the model is derived. It consists of 175 equations. By the means of ECOMOD the system was checked for correctness. Almost half of equations can be cut from the system as they determine some insignificant variables. After simplification the model core has 38 equations for 38 variables. Variables with "$\_s$" suffix are exogenous (see Table 2 below.)

Table 2.

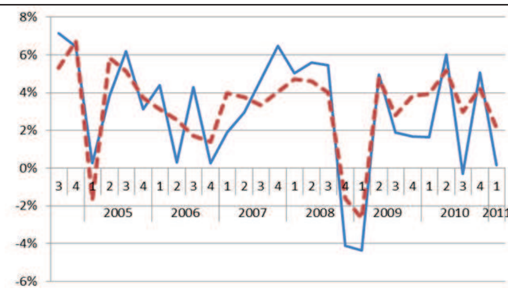
Name	Dimension	Data
Uncontrolled exterior factors		
$R\_s(t)$	labour/time	Number of active employees, including those working with the Government.
$bl\_s(t)$	1/ time	Loan duration
$bs\_s(t)$	1/ time	Deposit duration
$pim\_s(t)$	currency/product	Exterior (USD) import product price
$pex\_s(t)$	currency/product	Exterior (USD) export product price
$Df\_s(t)$	currency	Non-resident deposits
Public policies parameters		
$G\_s(t)$	product/time	Real Government consumption
$e\_s(t)$	1	Export duties rate
$i\_s(t)$	1	Import duties rate
$u\_s(t)$	1	Central Bank reserve rate
$w\_s(t)$	money/currency	USD exchange rate
$Gp\_s(t)$	product/time	Transfers for real Government consumption
$Dw\_s(t)$	1/time	USD exchange rate growth rate
$JG\_s(t)$	product/time	Real public investments
$SP\_s(t)$	money/ time	Subsidies to Producer
$Su\_s(t)$	money/time	Transfers to Household
$rlcb\_s(t)$	1/time	Central Bank fund rate (loans)
$rscb\_s(t)$	1/time	Central Bank fund rate (deposits)

The model has 70 constant parameters (40 are determined outside the model, 30 are used for adjustment). The basic algorithm for boundary problem calculation was taken from the 2004 model. However this algorithm does not allow for identification by adjustment parameters. The model was calculated from 1st quarter of 2004 till 1st quarter of 2011. To adjust the model on 30 adjustment parameters we developed parallel (MPI) algorithms and applied them on the supercomputer (mainframe) MVS-100K of JSCC RAS (140.16 TFlops).

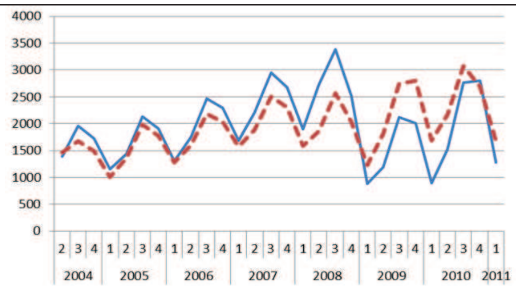
The Pictures - show the result of the model for basic observed series (slim line – official statistics, dotted line – calculation). All the real values are in bil. roubles of 2008, financial values are in bil. roubles.



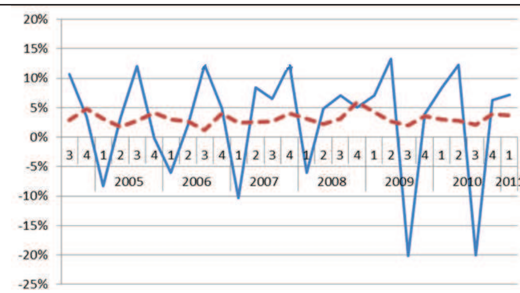
Picture 7. GDP



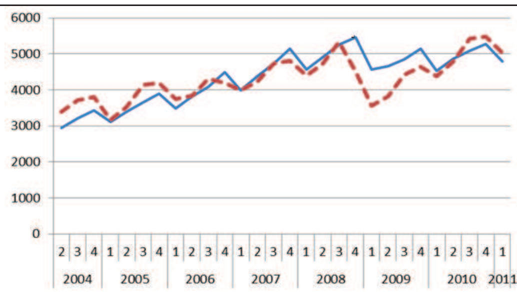
Picture 8. Inflation of GDP deflator



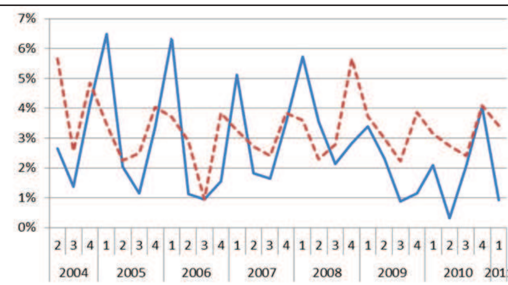
Picture 9. Gross Investments



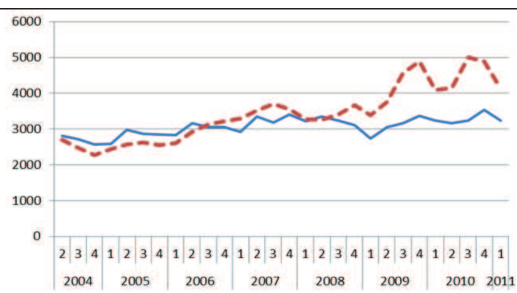
Picture 10. Inflation of Gross Investments deflator



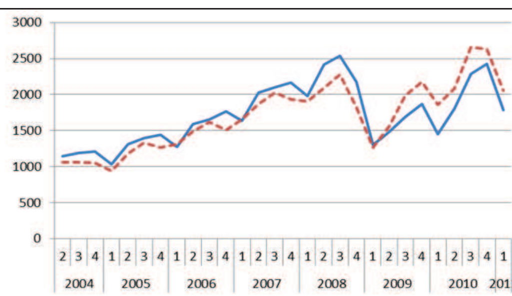
Picture 11. Consumption



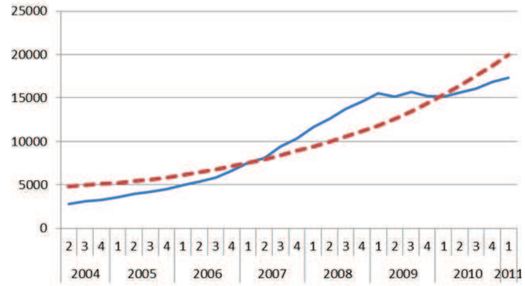
Picture 12. Consumer Price Index (CPI)



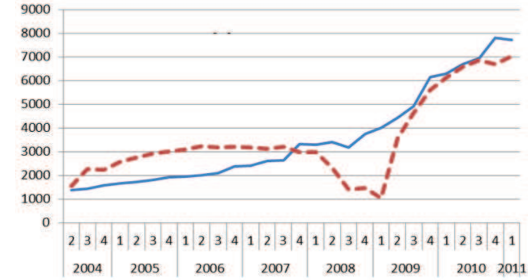
Picture 13. Exports



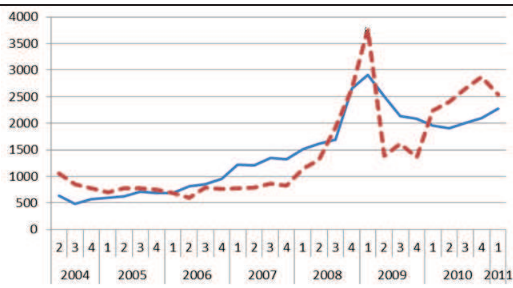
Picture 14. Imports



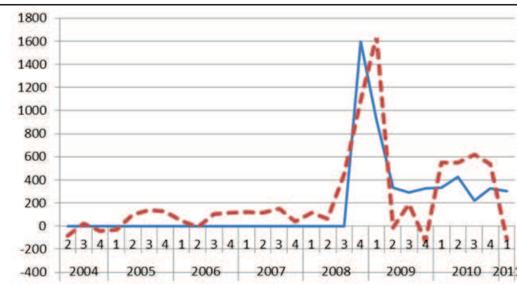
Picture 15. Loans



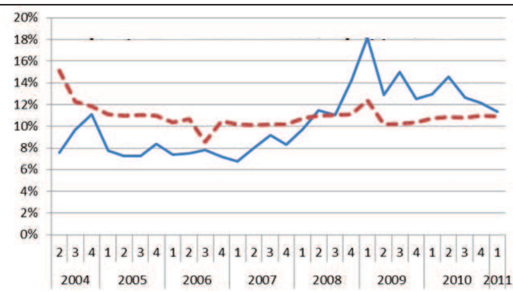
Picture 16. Deposits



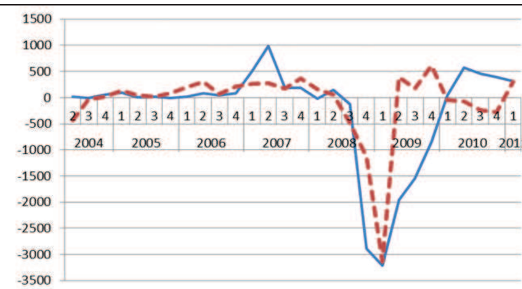
Picture 17. Bank Liquidity



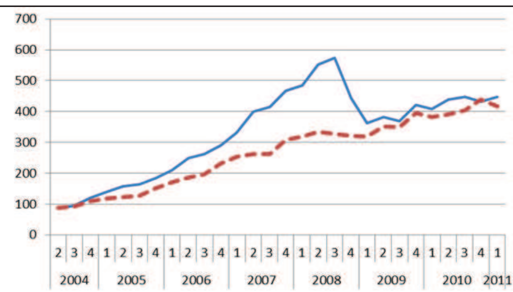
Picture 18. Bank Currency Accounts



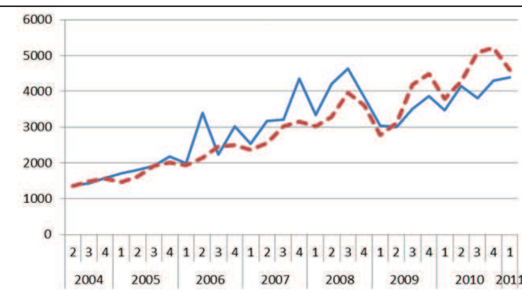
Picture 19. Loan Interest rates



Picture 20. Net loan of Central Bank to Bank



Picture 21. Foreign reserves



Picture 22. Budget Income

Moreover, by changing the exogenous series one can perform so called ‘scenario experiments’, that is to provide answers to question like “what would happen if…” The described model shows strong **turnpike effect**, i.e. any change in exogenous series results in no (or very small) changes in calculations before the changes in exogenous series take place. This is a strong argument towards application of **rational expectations** preposition.

Also by prolonging exogenous series one can calculate forecasts. Naturally, these forecasts would strongly depend on the way exogenous series are prolonged. The experiments we performed reveal that public investments increase result not only in GDP growth rate increase. It also provokes Owner to allocate more capital outside Russian economy (that is budget transfers flow to the foreign savings via investment product market). The same result is obtained for experiments with foreign deposits increase. The experiments reveal that no significant impact on economy will last within one-two years after such measures (public investments or foreign deposits increase) because of the Owner will to reallocate its capital in foreign currency assets. Such a result was not imbedded in the model. It came with identification, i.e. the parameters identified are such that we have such experiment results. The authors are currently developing a new much more detailed model for Russian economy. One of the tasks is to find ways to stimulate economic growth provided such Owner incentives as revealed by the mentioned above experiments.

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