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Weakly Nonlinear Periodic Stokes Edge Waves

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Abstract—The structure and nonlinear dispersion equation of periodic Stokes edge waves of higher modes are found. A nonlinear Schrödinger equation for the wave amplitude is derived in a dimensionless form. The coefficients of this equation depend on the mode number alone. It is noted that periodic waves of any mode are modulationally unstable. It is shown that the nonlinearity of a higher-order wave (at the same steepness) decreases as the mode number increases, and, consequently, higher-mode Stokes edge waves are more stable.

1. INTRODUCTION

Edge waves relate to trapped waves. These are long waves that propagate along shore, reach a maximum amplitude at the boundary with land, and decay rapidly as the distance from shore increases. In recent years, such waves have been the subject of numerous studies in hydrodynamics, hydraulic engineering, and offshore engineering.

Edge waves were intensively studied within the framework of a linear theory of long waves [1, 2]. Weakly nonlinear Stokes edge waves of invariant form were examined in [3]. In [4], it was shown that the results obtained in the shallow-water approximation are in good agreement with those in the context of the complete theory. The properties of nonlinear edge waves were studied theoretically and experimentally in [5] within the framework of a nonlinear Schrödinger equation.

Since the coefficients of intermode three-wave interactions for edge waves propagating in one direction are equal to zero [6], a multimode field of edge waves can be represented as a superposition of the wave fields of individual modes. Such an approach was employed in [7, 8] to study the mechanisms of formation of anomalous edge waves.

In this work, the asymptotic procedure described in [3] is used to find the structure of weakly nonlinear periodic Stokes edge waves of any mode and to derive a nonlinear Schrödinger equation. It is shown that periodic waves of any mode are modulationally unstable.

2. STRUCTURE AND DISPERSION RELATION OF NONLINEAR PERIODIC STOKES EDGE WAVES

Nonlinear equations of shallow water for the case of an inclined shore can be written as

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}\left((\beta y + \zeta)\frac{\partial \Phi}{\partial x}\right) + \frac{\partial}{\partial y}\left((\beta y + \zeta)\frac{\partial \Phi}{\partial y}\right) = 0, \quad (1)$$

$$\frac{\partial \Phi}{\partial t} + \frac{1}{2}\left(\frac{\partial \Phi}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial \Phi}{\partial y}\right)^2 + g\zeta = 0, \quad (2)$$

where $\zeta(x, y, t)$ is the displacement of the water surface, $\Phi(x, y, t)$ is the velocity potential for the components of the horizontal velocity, t is time, x is the alongshore coordinate, y is the coordinate transverse to the shore, β is the bottom slope, and g is the acceleration of gravity. System (1), (2) is subject to the following boundary conditions: ζ is limited on the shore and at infinity.

Following the procedure described in [3], we will seek the solutions in the form of travelling waves of a constant form by representing ζ and Φ as functions of $\theta = kx - \Omega t$ and y. We take into account that the amplitude is small and expand ζ and Φ as power series in the amplitude a:

$$\zeta = a\beta\{\zeta_1(\theta, y) + ka\zeta_2(\theta, y) + k^2a^2\zeta_3(\theta, y) + \dots\}, \quad (3)$$

$$\Phi = ag\beta\Omega^{-1}\{\Phi_1(\theta, y) + ka\Phi_2(\theta, y) + k^2a^2\Phi_3(\theta, y) + \dots\}. \quad (4)$$

Secular terms are known to appear in such problems; therefore, for the problem's solvability, the expansion of the frequency Ω in powers of a is also used:

$$\Omega^2 = \omega_n^2\{1+\gamma k^2a^2+\ldots\}. \quad (5)$$

Taking into consideration the change of variables and expansions (3)–(5), we rewrite system (1), (2) to obtain the following equation for ζ_1 in the first order in a:

$$L\zeta_1 = 0, \quad (6)$$

where

$$L = y\partial_{yy}+\partial_y-(\omega_n^2/g\beta-k^2y)\partial_{\theta\theta}. \quad (7)$$

In this case, Φ_1 is determined from linearized equation (2):

$$\frac{\partial\Phi_1}{\partial\theta} = \zeta_1. \quad (8)$$

In accordance with a linear theory of shallow water, we choose

$$\zeta_1 = f(y)\cos\theta, \quad \Phi_1 = f(y)\sin\theta, \quad (9)$$

where $f(y)$ are the eigenfunctions of the homogeneous boundary-value problem

$$\begin{gathered} yf''+f'+(\omega_n^2/g\beta-k^2y)f = 0, \\ f(y=0)\le M = \text{const}, \quad f(y\longrightarrow\infty)\longrightarrow 0. \end{gathered} \quad (10)$$

It is well known [2] that boundary-value problem (10) has an infinite set of eigenfunctions

$$f_n(y) = e^{-ky}L_n(2ky) \quad (11)$$

and eigenvalues

$$\omega_n^2 = (2n+1)g\beta k, \quad (12)$$

where n is the number of the corresponding mode of edge waves and L_n is the Laguerre polynomial.

If we choose Stokes wave (9) with an arbitrary but fixed mode number as the solution of the first approximation, we obtain inhomogeneous equations for ζ_m in each subsequent order in the amplitude a. For example, the second-order equation has the form

$$L\zeta_2 = P(y)+Q(y)\cos 2\theta, \quad (13)$$

where

$$\begin{aligned} P(y) = \frac{\beta g}{2\omega_n^2k}(&-k^2yff_{yy}-yf_{yy}^2-f_yf_{yy} \\ &-k^2yf_y^2-yf_yf_{yyy}-k^2ff_y), \end{aligned} \quad (14)$$

$$\begin{aligned} Q(y) = \frac{1}{2\omega_n^2k}(&-2f_y^2\omega_n^2+\beta gf_yf_{yy}-2ff_{yy}\omega_n^2 \\ &+\beta gyf_{yy}^2+2\beta k^4gyf^2+4k^2f^2\omega_n^2 \\ &+\beta gyf_yf_{yyy}-3\beta k^2gyf_y^2-k^2g\beta ff_y-\beta gyk^2ff_{yy}), \end{aligned} \quad (15)$$

and Φ_2 is determined from the relation

$$\frac{\partial\Phi_2}{\partial\theta} = \frac{1}{2k\omega_n^2}(g\beta\Phi_{1y}^2+2k\omega_n^2\zeta_2+k^2g\beta\Phi_{1\theta}^2). \quad (16)$$

In view of the fact that the right-hand side in (13) is completely determined and the boundary-value problem is solvable (there are no internal resonances at the zeroth and second harmonics), its solution is found in the form

$$\zeta_2 = f_2^{(1)}(y)+f_2^{(2)}(y)\cos 2\theta, \quad (17)$$

where the functions $f_2^{(1)}(y)$ and $f_2^{(2)}(y)$ are found as the solutions of the boundary-value problems

$$yf_{2yy}^{(1)}+f_{2y}^{(1)} = P(y), \quad (18)$$

$$yf_{2yy}^{(2)}+f_{2y}^{(2)}+4(\omega_n^2/g\beta-k^2y)f_2^{(2)} = Q(y). \quad (19)$$

In the third order in the amplitude, we obtain the equation

$$L\zeta_3 = R(y)\cos\theta+S(y)\cos 3\theta. \quad (20)$$

The expressions for $R(y)$ and $S(y)$ are not presented here because of their awkwardness.

The quantity Φ_3 is determined from the relation

$$\frac{\partial\Phi_3}{\partial\theta} = \frac{1}{\omega_n^2k}(g\beta k^2\Phi_{1\theta}\Phi_{2\theta}+\omega_n^2k\zeta_3+g\beta\Phi_{1y}\Phi_{2y}). \quad (21)$$

It is logical to seek the solution to problem (20) in the form

$$\zeta_3 = f_3^{(1)}(y)\cos\theta+f_3^{(2)}(y)\cos 3\theta. \quad (22)$$

The boundary-value problem

$$yf_{3yy}^{(2)}+f_{3y}^{(2)}+9(\omega_n^2/g\beta-k^2y)f_3^{(2)} = S(y) \quad (23)$$

for the function $f_3^{(2)}(y)$ is completely solvable. At the same time, the boundary-value problem for the function $f_3^{(1)}(y)$ contains an indefinite correction to the dispersion relation. In view of internal resonance, the inhomogeneous boundary-value problem

$$yf_{3yy}^{(1)}+f_{3y}^{(1)}+(\omega_n^2/g\beta-k^2y)f_3^{(1)} = R(y) \quad (24)$$

is solvable only if the condition that the right-hand side $R(y)$ in (24) is orthogonal to the eigenfunction of the linear operator determining boundary-value problem (10) is fulfilled. In view of the fact that this operator is self-conjugate, this condition has the form

$$\int_0^\infty f(y)R(y)dy = 0 \tag{25}$$

and makes it possible to unambiguously determine the nonlinear correction to the dispersion relation, i.e., the coefficient γ.

Similarly, the higher-order corrections can be obtained; however, technical difficulties in solving the boundary-value problems increase dramatically. We present several solutions describing the structure of nonlinear periodic Stokes waves and dispersion relations for different modes.

First, we will briefly discuss the lowest mode of edge waves, which was already analyzed in [5]. In particular, the modal structure and the linear dispersion relation have the form

$$f_0(y) = e^{-ky}, \quad \omega_0^2 = gk\beta. \tag{26}$$

It is important to emphasize that, in the following approximations, the terms with $\cos 2\theta$ and $\cos 3\theta$ do not appear in the expressions for ζ_2 and ζ_3. This implies that nonlinear Stokes waves of zeroth mode remain virtually sinusoidal. However, such a wave is accompanied by an average lowering of the level, which is obtained from (13):

$$\zeta_2 = -\frac{1}{2}e^{-2ky}, \quad \Phi_2 = 0. \tag{27}$$

The third-order solution of Eq. (20) is expressed in the form

$$\zeta_3 = f_3^{(1)}(y)\cos\theta, \quad \Phi_3 = f_3^{(2)}(y)\sin\theta, \tag{28}$$

where Eq. (24) for $f_3^{(1)}(y)$ is determined by the expression

$$yf_{3yy}^{(1)} + f_{3y}^{(1)} + (k-k^2y)f_3^{(1)} = -\gamma k e^{-ky} + ke^{-3ky}. \tag{29}$$

For solvability, the right-hand side must be orthogonal to $f_0(y) = e^{-ky}$. Hence, it follows from (25) that $\gamma = \frac{1}{2}$, so that the dispersion equation has the form

$$\Omega^2 = g\beta k\left(1 + \frac{1}{2}a^2k^2\right). \tag{30}$$

In this case, series (3) and (4) for ζ and Φ are written as

$$\zeta = a\beta\Big\{e^{-ky}\cos\theta - \frac{1}{2}kae^{-2ky} + k^2a^2\Big(e^{-ky}\int -\frac{1}{2}E_1(-2ky)k(e^{2ky}-2)e^{-4ky}dy - \frac{1}{4}E_1(-2ky)e^{-3ky} + \frac{1}{4}E_1(-2ky)e^{-5ky}\Big)\cos\theta + \dots\Big\}, \tag{31}$$

$$\Phi = ag\beta\Omega^{-1}\Big\{e^{-ky}\sin\theta + k^2a^2 \times\Big(e^{-ky}\int -\frac{1}{2}E_1(-2ky)k(e^{2ky}-2)e^{-4ky}dy - \frac{1}{4}E_1(-2ky)e^{-3ky} + \frac{1}{4}E_1(-2ky)e^{-5ky}\Big)\sin\theta + \dots\Big\}, \tag{32}$$

where $E_1(z) = -\ln z + \psi(1) - \sum_{l=1}^{\infty}\frac{(-z)^l}{l\cdot l!}$ is the integral exponential function with the exponent 1 and $\psi(1) = -0.5772156649\ldots$.

At $n = 1$, the eigenfunctions of problem (10) have the form $f_1(y) = e^{-ky}(1-2ky)$ and its eigenvalues are $\omega_1^2 = 3gk\beta$. In the next order, the boundary-value problems for $f_2^{(1)}(y)$ and $f_2^{(2)}(y)$ are written as

$$yf_{2yy}^{(1)} + f_{2y}^{(1)} = -\frac{1}{3}ke^{-2ky}(42ky - 36k^2y^2 + 8k^3y^3 - 9) \tag{33}$$

and

$$yf_{2yy}^{(2)} + f_{2y}^{(2)} + 4(3k - k^2y)f_2^{(2)} = 2ke^{-2ky}(-7 + 10ky), \tag{34}$$

whence it is found that

$$f_2^{(1)} = \frac{1}{6}e^{-2ky}(-4k^2y^2 + 8ky - 5) \text{ and } f_2^{(2)} = \frac{1}{15}e^{-2ky}(50ky - 26). \tag{35}$$

Hence,

$$\zeta_2 = \Big(\frac{1}{6}(-4k^2y^2 + 8ky - 5) + \frac{1}{15}(50ky - 26)\cos 2\theta\Big)e^{-2ky}, \tag{36}$$

$$\Phi_2 = \frac{1}{15}(30ky - 18)e^{-2ky}\sin 2\theta.$$

Thus, the nonlinear Stokes edge wave of the first mode has a nonzero second harmonic and is more nonsinusoidal than the wave of the zeroth mode. We also emphasize that the propagation of the first mode is also accompanied by an average lowering of the level.

The expression for $R(y)$ at $n = 1$ is written as

$$R(y) = \frac{1}{15}ke^{-ky}(-60e^{-2ky} + 280k^3y^3e^{-2ky} - 1052k^2y^2e^{-2ky} + 900kye^{-2ky} + 90\gamma ky - 45\gamma). \quad (37)$$

Then, from orthogonality condition (25), we obtain the nonlinear correction to the dispersion equation $\gamma = \frac{19}{180}$, and the dispersion equation has the form

$$\Omega^2 = 3g\beta k\left(1 + \frac{19}{180}a^2k^2\right). \quad (38)$$

Series (3) and (4) for ζ and Φ are given by

$$\zeta = a\beta\Bigg\{e^{-ky}(1-2ky)\cos\theta + \frac{1}{15}ka\left(\frac{15}{6}(-4k^2y^2 + 8ky - 5) + (50ky - 26)\cos 2\theta\right) \times e^{-2ky} + k^2a^2\Bigg(\Bigg(e^{-ky}(-1+2ky) \times \int\frac{1}{60}(e^{2ky} - E_1(-2ky) + 2E_1(-2ky)ky)k(3600ky + 1120k^3y^3 - 4208k^2y^2 - 240 - 19e^{2ky} + 38kye^{2ky}) \times e^{-4ky}dy - \frac{139}{30}kye^{-3ky} + \frac{259}{60}E_1(-2ky)kye^{-5ky} - 34k^2y^2E_1(-2ky)e^{-5ky} - \frac{19}{120}e^{-3ky} + \frac{19}{120}E_1(-2ky)e^{-5ky} - \frac{152}{5}k^3y^3e^{-3ky} + \frac{19}{120}e^{-ky} \quad (39)$$
$$+ \frac{1198}{15}k^3y^3E_1(-2ky)e^{-5ky} - \frac{1052}{15}k^4y^4E_1(-2ky)e^{-5ky} + \frac{371}{15}k^2y^2e^{-3ky} + \frac{19}{30}k^2y^2e^{-ky} - \frac{19}{120}E_1(-2ky)e^{-3ky} + \frac{19}{60}E_1(-2ky)kye^{-3ky}$$
$$+ \frac{28}{3}k^4y^4e^{-3ky} + \frac{56}{3}k^5y^5E_1(-2ky)e^{-5ky} - \frac{19}{30}k^2y^2E_1(-2ky)e^{-3ky} + \frac{19}{15}k^3y^3E_1(-2ky)e^{-3ky}\Bigg) \times \cos\theta + \left(\frac{357}{5}ky - \frac{1744}{5}k^2y^2 + \frac{6976}{15}k^3y^3 - \frac{872}{5}k^4y^4\right)e^{-3ky}\cos 3\theta\Bigg) + \ldots\Bigg\},$$

$$\Phi = ag\beta\Omega^{-1}\Bigg\{e^{-ky}(1-2ky)\sin\theta + \frac{1}{15}ka(30ky - 18)e^{-2ky}\sin 2\theta + k^2a^2\Bigg(\Bigg(\frac{19}{120}e^{-ky} - \frac{4}{9}ke^{-3ky}y\cos^2\theta + \frac{331}{15}k^2e^{-3ky}y^2 - \frac{152}{5}k^3e^{-3ky}y^3 + \frac{37}{90}ke^{-3ky}y + \frac{14}{27}e^{-3ky}\cos^2\theta - \frac{3119}{1080}e^{-3ky}$$
$$+ \frac{259}{60}e^{-5ky}E_1(-2ky)ky - 34e^{-5ky}k^2y^2E_1(-2ky) + \frac{1198}{15}e^{-5ky}k^3y^3E_1(-2ky) - \frac{1052}{15}e^{-5ky}k^4y^4E_1(-2ky) + \frac{56}{3}e^{-5ky}k^5y^5E_1(-2ky) + \frac{19}{120}e^{-5ky}E_1(-2ky)$$
$$+ \frac{19}{15}e^{-3ky}k^3y^3E_1(-2ky) + \frac{19}{30}e^{-ky}k^2y^2 - \frac{19}{30}e^{-3ky}k^2y^2E_1(-2ky) + \frac{28}{3}e^{-3ky}k^4y^4 + \frac{1}{60}(-1+2ky)e^{-ky}k \quad (40)$$
$$\times\int(e^{2ky} - E_1(-2ky) + 2E_1(2ky)ky)(-4208k^2y^2 + 3600ky + 1120k^3y^3 - 240 + 38kye^{2ky} - 19e^{2ky})e^{-4ky}dy + \frac{19}{60}e^{-3ky}E_1(-2ky)ky - \frac{19}{120}e^{-3ky}E_1(-2ky)\Bigg)\sin\theta$$

Nonlinear corrections to the dispersion equation for edge waves of different modes

n	γ
0	$\frac{1}{2}$
1	$\frac{19}{180}$
2	$\frac{87}{1600}$
3	$\frac{14691}{407680}$
4	$\frac{7862567}{293289984}$
5	$\frac{18661637}{876249088}$
6	$\frac{664537459}{37656985600}$
7	$\frac{113832960079}{7560364032000}$
8	$\frac{38385791862043}{2924684823756800}$
9	$\frac{2137977196204769}{183835365604327424}$
10	$\frac{118300847013803761}{11332375011265609728}$
11	$\frac{6435795760259199221}{679685530721032601600}$
12	$\frac{35617503544441300439}{4111519278463713280000}$
13	$\frac{5967755227480020029641}{747561272291776423526400}$
14	$\frac{140874326546896802771}{19032561335713814020096}$
15	$\frac{530864273958449559523171}{76945364004198585632555008}$
16	$\frac{1499770068264849538565771792207}{232145492329743584150527200460800}$
17	$\frac{634466754224924150789468495669}{104454812894007673309236297728000}$

$$+\left(\frac{127}{270}+\frac{1046}{45}ky-\frac{1744}{15}k^2y^2+\frac{6976}{45}k^3y^3-\frac{872}{15}k^4y^4\right)e^{-3ky}\sin 3\theta\Big)+\ldots\Big\}.$$

We do not present very cumbersome expressions for higher-mode nonlinear waves and indicate only nonlinear corrections to the dispersion relation, which play an important role in understanding the processes of stability of Stokes waves.

The nonlinear corrections determined for the first 18 modes of edge waves are listed in the table. The dependence of γ on the mode number n, along with the regression curve, is shown in the figure. Thus, as the mode number increases, the coefficient of the nonlinear correction in dispersion equation (5) decreases approximately as $1/(2 + 8n)$.

3. NONLINEAR SCHRÖDINGER EQUATION FOR STOKES EDGE WAVES

In the preceding section, nonlinear corrections to the dispersion relations for Stokes edge waves of any mode were determined. Thus, the dispersion equation for weakly nonlinear edge waves has the form

$$\Omega_n^2 = g\beta k(2n+1)(1+\gamma_n a^2k^2). \quad (41)$$

As is well known, knowledge of the nonlinear dispersion relation makes it possible to write the nonlinear Schrödinger equation for the wave amplitude [9]. In the most general form, this equation is written as

$$i[A_t + c_{gr}A_x] + \frac{1}{2}\frac{\partial^2\omega_l}{\partial k^2}A_{xx} - \frac{\partial\Omega_n}{\partial a^2}|A|^2A = 0, \quad (42)$$

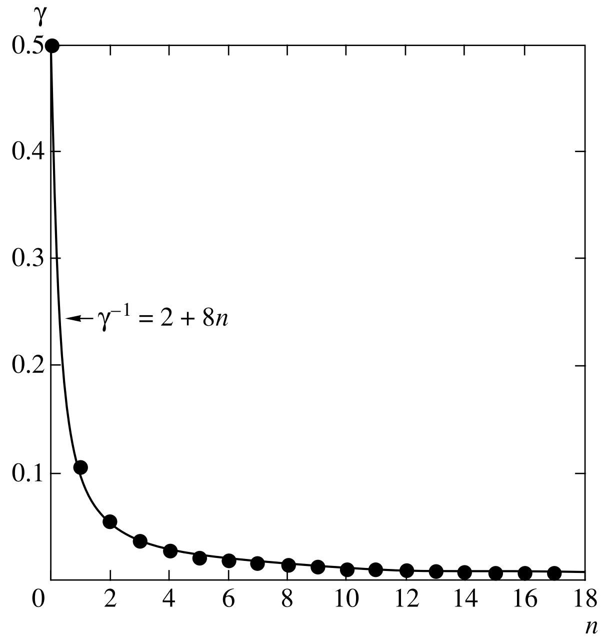
where $c_{gr} = \partial\omega_l/\partial k$ is the linear group velocity and ω_l is the wave frequency in a linear approximation, which is determined by formula (12) at different n. Based on nonlinear dispersion relation for a Stokes wave (41), all the coefficients are calculated in an explicit form, so that the nonlinear Schrödinger equation can be specified as

$$i\left[A_t + \frac{\omega_l(k_0)}{2k_0}A_x\right] - \frac{\omega_l(k_0)}{8k_0^2}A_{xx} - \frac{1}{2}\gamma_n\omega_l(k_0)k_0^2|A|^2A = 0, \quad (43)$$

where $|A| = a$.

In terms of the dimensionless variables

$$\xi = k_0x, \quad \tau = \omega_l(k_0)t, \quad A' = k_0A, \quad (44)$$



Dependence of the coefficient γ on the mode number and an approximating curve.

Eq. (43) has the form

$$i\left[A'_\tau + \frac{1}{2}A'_\xi\right] - \frac{1}{8}A'_{\xi\xi} - \frac{1}{2}\gamma_n|A'|^2A' = 0. \tag{45}$$

It is significant that such signs in the nonlinear Schrödinger equation correspond to the modulational instability of wave packets with any modal structure. Previously, this result was inferred for the lowest mode [3, 5], and it remains valid for a wave of any mode. Since the nonlinear coefficient decreases as the mode number increases, it follows that high-mode waves are more linear and more stable if their steepness and wave number are identical.

4. CONCLUSIONS

The structure and nonlinear dispersion equation of periodic Stokes edge waves of higher modes are found. Previously, this was done for the lowest mode alone. It is shown that, unlike the lowest mode, higher-mode waves have a more nonsinusoidal form. The nonlinear correction to the dispersion relation is positive for any mode number, and it decreases as the mode number increases. A dimensionless nonlinear Schrödinger equation is derived for the wave amplitude. The coefficients of this equation depend on the mode number alone. It is noted that edge waves of any modal structure are modulationally unstable. It is found that the nonlinearity of higher-mode waves (at the same steepness) decreases as the mode number increases, and, consequently, higher-mode Stokes edge waves are more stable.

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