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ION-ACOUSTIC INSTABILITY AND ANOMALOUS THERMAL CONDUCTIVITY IN THE TRANSITION REGION

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Abstract. Within the framework of model calculations the possibility of occurrence of the ion-acoustic oscillation instability in a plasma without current and particle fluxes, but with an anisotropic distribution function, which corresponds to heat flux is shown. The model distribution function was selected taking into account the medium conditions. The increment of ion-acoustic oscillation is investigated as functional of the distribution function parameters. The threshold condition for the anisotropic part of the distribution function, under which the build-up of ion-acoustic oscillation with the wave vector opposite to the heat flux begins is studied. The critical heat flux, which corresponds to the threshold of ion-acoustic instability, is determined. For the solar conditions, the critical heat flux proved to be close to the heat flux from the corona into the chromosphere on the boundary of the transition region. The estimations show that outside of active regions and even in active regions with weaker magnetic fields ion-acoustic turbulence can be responsible for the formation of the sharp temperature jump. The generalized Wiedemann-Franz law for a non-isothermic quasi-neutral plasma with developed ion-acoustic turbulence is discussed. This law determines the relationship between electrical and thermal conductivities in a plasma with well-developed ion-acoustic turbulence. The anomalously low thermal conductivity responsible to the formation of high temperature gradients in the zone of the temperature jump is explained. The results are used to explain some properties of stellar atmosphere transition regions.

Key words: star - Sun - chromosphere - corona - transition region - turbulence - thermal conductivity

1. Introduction

A fundamental problem is the explanation of the temperature jump between the chromosphere and the corona in late-type stars (Jordan, 1996). Actually the temperature jump formation depends on many factors. For example, we must take into account the medium cooling during the ultraviolet radia-

tion, the medium composition changing along altitude, and the unsteady magnetic structures. In our opinion, primary importance has the low thermal conductivity of turbulent plasma. As the natural reason for turbulence serves the heat flux from the corona into the chromosphere.

At the present time space plasma physics accumulated many results on wave instabilities (Zhelezniakov, 1996). We will show additionally that the instability of ion-acoustic oscillations can be realized in the absence of ordinary particle fluxes for a supercritical heat flux through the medium. Naturally, when heat flux is present, there is the specific heterogeneity of the medium. We will consider that the wavelength of the ion-acoustic oscillations (it is compared with the Debye's radius) is small in comparison with the scale of the medium heterogeneity. Therefore during the instability increment calculation we will use the local homogeneous medium approximation.

We will pay special attention to the study of the conditions, which correspond to the threshold of the ion-acoustic instability and to the estimation of the maximum heat flux, which can pass through the medium in the weak turbulence regime. Let us recall that the plasma nonisothermicity is the necessary condition for the existence of ion-acoustic oscillations. We do not have data about the degree of the nonisothermicity in the transition region plasma, but observations showed that in the solar atmosphere the hydrogen temperature even at somewhat high altitudes grows relatively slowly in comparison with the electron temperature (Marsch *et al.*, 2000). This gives reasons to assume that the ion temperature in the transition region is substantially lower than the electron temperature.

In the space plasma with essential heat flux a significant level of ion-acoustic turbulence can be established. This turbulence provides conditions for formation of an anomalous collision frequency and comparatively low the plasma thermal conductivity. The obtained expressions for the plasma thermal conductivity give a possibility to explain processes governing the formation of the temperature jump between the chromosphere and the corona in late-type stars.

2. Heat Flux as a Reason for Ion-Acoustic Instability in Space Plasmas

Let the model distribution function $f\,(x, v)$ be the sum of the isotropic $F(v)$ and the anisotropic $\Phi(x, v)$ part:

$$f = F + \Phi,$$
$$F = f_\circ(v) + |f_1(v) + \frac{v}{4}\frac{\partial f_1}{\partial v}|, \tag{1}$$
$$\Phi = [f_1(v) + \frac{v}{4}\frac{\partial f_1}{\partial v}]x.$$

Here $F(v) = (1/2)\int_{-1}^{1} f dx$, $x = v_z/v$, the z axis is directed along the heat flux,$f_\circ(v)$ is a non-negative function, $f_1(v)$ is a differentiated function. In the paper by Bespalov and Savina (2009) we showed that possibly the distribution function in form (1) corresponds to the situation, when in the plasma there is no electron beam but there is heat flux.

Let us take known expressions for the dispersion equation and the increment of ion-acoustic oscillations (Kadomtsev, 1975). For relatively small phase speeds, when $v_{Ti}^2 \ll (\omega/k)^2 \ll \langle v^2 \rangle$, the increment for an isotropic ion distribution function and the selected electron distribution function (1) is written down in the form:

$$\gamma(y,k) = \omega(\frac{\omega}{k})^3 \frac{m_i}{m}\frac{\pi^2}{n}\{-F(\frac{\omega}{k}) - \frac{m}{m_i}F_i(\frac{\omega}{k}) +$$
$$+\frac{ky}{\pi\omega}\int_0^\infty\int_{-1}^{1}\left[f_1(v) + \frac{v}{4}\frac{\partial f_1}{\partial v}\right] \times \mathrm{Re}(1 - x^2 - y^2)^{-1/2} dx dv\}, \tag{2}$$

where $y = k_z/k$, m and m_i are the electron and ion mass, n is the electron plasma density.

We assume that the ion distribution function is Maxwellian with the temperature T_i. The isotropic part of the electron distribution function in the region of comparatively low velocities is also assumed to be Maxwellian but with a considerably higher temperature T. In the expression for the increment we will carry out integration for x and for the velocity value v. Then we will consider that the maximum on the angular coordinate y is equal to $\gamma(k) = \max|_y \gamma(y,k)$ and corresponds to $y = 1$, if $\int_0^\infty f_1 dv > 0$, and $y = -1$, if $\int_0^\infty f_1 dv < 0$. Then the dimensionless increment $\gamma(k)/\omega_i$, as a function from the dimensionless phase velocity $\xi = \omega/kv_{Ti}$ has the following form:

$$\frac{\gamma(k)}{\omega_{pi}} = \xi^2\left(1 - \pi^{-1/2}B^{2/3}C^{1/3}\xi^2\right)^{1/2} \times \left[A - B\xi - C\xi\exp\left(-\xi^2\right)\right]. \tag{3}$$

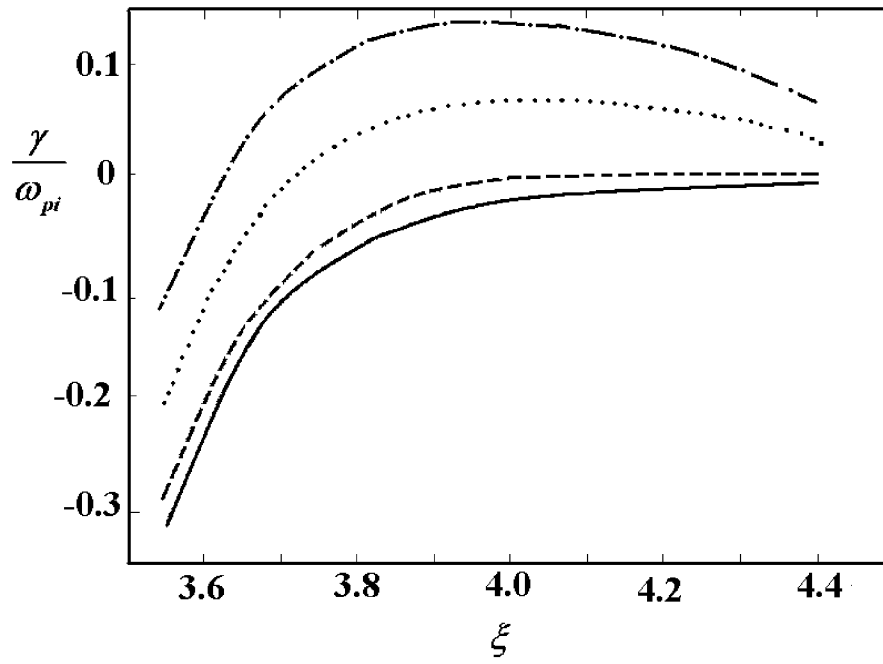

Figure 1: The dependence of the ion-acoustic oscillations increment on the phase velocity for several values of heat flux: A = 0 is the solid line (there is no heat flux), A = 0.005 is the broken line, A = 0.025 is the dotted line, A = 0.045 is the prime dotted line.

Here,

$$A = \frac{9\pi^2 m^{1/2} T_i |q_z|}{2^{5/2} n \kappa^{3/2} T^{5/2}}, \quad B = (\pi \frac{m}{m_i})^{1/2} (\frac{T_i}{T})^{3/2}, \quad C = \pi^{1/2} \frac{m_i}{m},$$

q_z is the heat flux density which is transferred by electrons with this distribution function (1). We considered during the increment (3) evolution that $m\omega^2/2\kappa T k^2 < m/m_i$ and therefore $\exp(-m\omega^2/2\kappa T k^2) \simeq 1$. The dependence of the ion-acoustic oscillations increment on the phase velocity for several values of heat flux is shown in Figure 1: A = 0 is the solid line (there is no heat flux), A = 0.005 is the broken line, A = 0.025 is the dotted line, A = 0.045 is the prime dotted line. The function $\gamma(k)/\omega_{pi}$ has the maximum value for $\sqrt{\ln(C/\sqrt{2}A)} \leq \xi$. The increment maximum value grows with an increase in the parameter$A$. The threshold of instability is achieved satisfying the conditions $\gamma/\omega_{pi} = 0, \partial/\partial\xi\,(\gamma/\omega_{pi}) = 0$ and the critical heat flux is determined:

$$|q_z| = \delta \frac{n\,(\kappa T)^{3/2}}{m_i^{1/2}}, \tag{4}$$

where $\delta = (2^{5/2} T_i^{1/2} \xi_{th})/(9\pi^{3/2} T^{1/2}) \approx 1$. This result practically coincides with the expression, obtained by us in the paper (Bespalov and Savina, 2008) by another method. If in the heterogeneous plasma the heat flux is constant, then the maximum increment is realized in a certain intermediate region, where the product $nT^{3/2}$ is minimal.

The experimental data about the altitude distributions of the plasma density and electron temperature in the solar atmosphere (Aschwanden, 2004) are well known. As a result of the analysis of these data it is clear, that the increment can become positive first of all in the transition region, where for $q_z \approx 5 \cdot 10^5\ erg \cdot cm^{-2} \cdot s^{-1}$ the electron plasma density $n \approx 10^{10} cm^{-3}$ is much less than in the chromosphere, and the temperature $T \approx 10^5 K$ is much less than in the corona.

3. Self-Consistent Model for the Temperature Jump in the Transition Region of Stellar Atmospheres

We will consider that on the boundaries of the transition region there are no regular velocities of electrons and ions $\vec{u} = \vec{u}_i = 0$, electric currents and macroscopic electric fields are absent. Naturally, in this case the density of electrons and ions at each point must be equal ($n = n_i$). We will limit ourselves to the examination of the one-dimensional temperature jump in the co-ordinate space. Let us select the z axis in the direction of the temperature jump from the corona to the chromosphere, and the reference point is in the centre of the temperature jump.

Outside the heat source the structure of the stationary temperature jump in a nonisothermic plasma is described by a sufficiently complex system of quasilinear equations (see [3,8-9]). For determining the general properties of processes in the temperature jump region it is convenient to introduce for each thin layer of the medium into consideration more well studied experimentally and theoretically the task about the anomalous electrical conductivity in the plasma with the same base parameters. In the paper by Bespalov and Savina (2007) a conversion is introduced, which to every solution of the anomalous electrical conductivity problem places in the correspondence solution of the anomalous thermal conductivity problem. Practical use of this conversion made it possible to formulate the generalized Wiedemann-Franz law, which connects the thermal conductivity of the turbulent plasma χ_Σ with the anomalous electrical conductivity $\sigma_\Sigma = \sigma + \sigma_{eff}$

by means of the following relationship:

$$\chi_\Sigma = \frac{\kappa^2}{e^2} T \delta \sigma_\Sigma, \tag{5}$$

where κ is Boltzmann constant, e is the electron charge, T is the temperature, σ is the electrical conductivity due to Coulomb collisions, $\sigma_{eff}(E_z)$ is the anomalous electrical conductivity which depends on the electric field E_z. The calculations showed that for the correct determination of the thermal conductivity χ_Σ, including the coefficient of usual and anomalous thermal conductivity, it is necessary to replace in formula (5) the electric field E_z by the expression: $-(3\kappa/2e)\partial T/\partial z$. Let us note that, if the distribution function is close to Maxwell's function, then calculations give the possibility to define concretely the numerical coefficient $\delta = 5/3$.

Fourier's formula determines the heat flow in the medium

$$q_z = -\chi_\Sigma \frac{\partial T}{\partial z}. \tag{6}$$

We will use the generalised Wiedemann-Franz law (5) and well-known expressions for the anomalous electrical conductivity for the record of explicit expression for the heat flux in the plasma with developed ion-acoustic turbulence [5,11]. Thus, we obtain the dependence of the heat flux on the temperature and its gradient:

$$\frac{q_z}{T} = \frac{8.4 \cdot 10^{-2} \delta (\kappa \mathrm{T})^{3/2}}{\mathrm{e}^4 \mathrm{m}^{1/2}} \left| \frac{\partial T}{\partial z} \right|, \quad if \quad q < q_{cr}; \tag{7a}$$

$$\frac{q_z}{T} = 1.7 \delta \kappa n \left(\frac{\kappa T}{m_\mathrm{i}} \right)^{1/2}, \quad if \quad q \simeq q_{cr}; \tag{7b}$$

$$\frac{q_z}{T} = 1.7 \delta \kappa \left(\frac{\kappa T}{\mathrm{m}} \right)^{1/2} \frac{\kappa^{1/2}}{e^{1/2} \left(8\pi n \kappa T\right)^{1/4}} \left| \frac{\partial T}{\partial z} \right|^{1/2}, \quad if \quad q > q_{cr}. \tag{7c}$$

Here $\partial T/\partial z|_{cr} = -(2em/3\kappa m_i)(8\pi n \kappa T)^{1/2}$, $q_{cr} = 1.7\delta n (\kappa T)^{3/2} m_i^{-1/2}$ is the critical heat flux, which can pass through the plasma in the regime of small temperature gradients. The critical heat flux corresponds to the transition of the medium to the state with developed ion-acoustic turbulence.

For study effects, connected exclusively with the anomalous thermal conductivity, let us make the simplifying assumption about the absence

of a significant energy input and radiation losses in the lower corona and the transition region. Then the heat flux from the solar corona into the chromosphere is constant and equal to (Gibson, 1973)

$$q = 5 \cdot 10^5 erg \cdot cm^{-2} \cdot s^{-1}. \tag{8}$$

We take for the temperature $z = z_1$ and the electron density $n_1 = 10^8 cm^{-3}$ as the boundary conditions for the equation of thermal conductivity in the solar corona. These boundary conditions correspond to a lower heat flux than the critical one. Therefore for the study of the coordinate temperature dependence it is necessary to use the Equation (7a), which corresponds to the heat flux transferred by electrons during Coulomb collisions

$$\frac{\partial T}{\partial z} = -\frac{1.2 \cdot e^4 m^{1/2} q}{\delta \kappa^{7/2} T^{5/2}}. \tag{9}$$

Integrating Equation (9), we obtain the following space dependence of the temperature

$$\frac{T(z)}{T_1} = \left[1 - \frac{(z - z_1)}{(\Delta z)_1}\right]^{2/7}, \tag{10}$$

where $\Delta(z) = 5\delta\kappa^{7/2} T_1^{7/2} / 21 e^4 m^{1/2} q$ does not depend on plasma density. A temperature decrease according to the formula (10) will be continue to the locally critical heat flux, that corresponds to the developed ion-acoustic turbulence. According to the formulae (7a) and (10) when in the lower corona the condition $nT = const$ is satisfied we obtain, that the heat flux (8) will become critical for $z_2 - z_1 = 4.95 \cdot 10^{10} cm$ and $T = T_2 = 5.5 \cdot 10^5 K$. Note that according to experimental data, the temperature on the boundary of the temperature jump actually is equal to $T_2 = 5 \cdot 10^5 K$. During calculations the mass of ions was set equal to the proton mass and the plasma density on the boundary of the temperature jump was $n_2 = (2.3 \div 6.3) \cdot 10^8 cm^{-3}$ (Gallagher *et al.*, 1999).

In the region of the temperature jump for $z_2 < z$ developed ion-acoustic turbulence is realized. Therefore as the dependence of the heat flux on the temperature and its gradient it is necessary to use formula (7c), according to which exists the almost exponential temperature jump

$$\frac{\partial T}{\partial z} = -q^2 \frac{(2\pi)^{1/2} me}{\delta^2 \kappa^{7/2} n_2^{3/2} T_2^{15/4}} T^{5/4}. \tag{11}$$

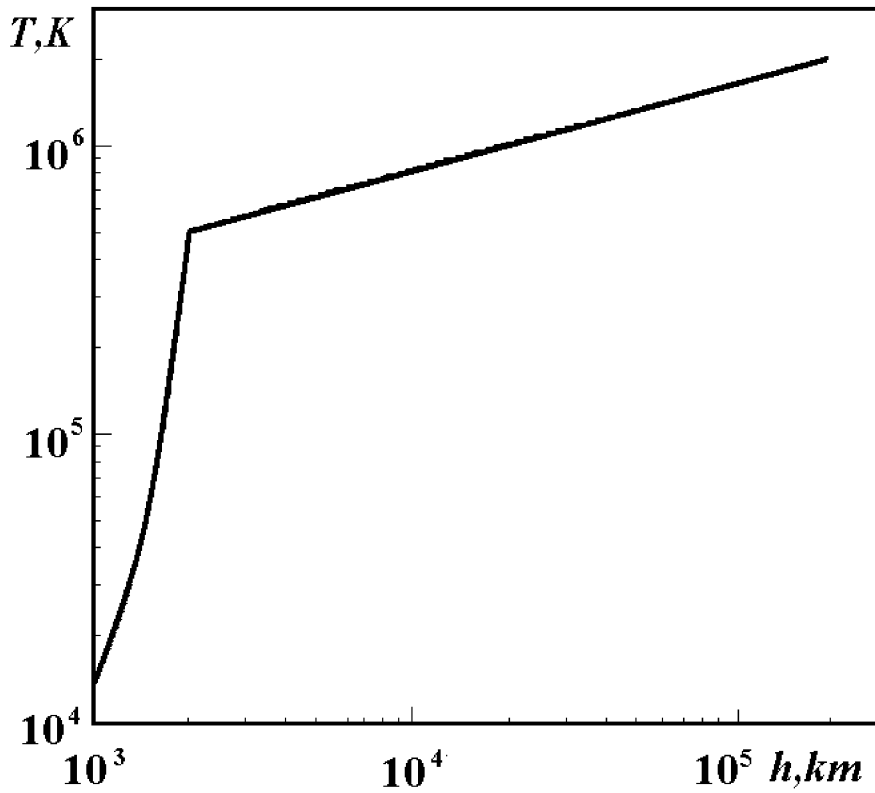

Figure 2: Height dependence of temperature

Solution of the equation (11) gives the temperature distribution in the transition region:

$$\frac{T(z)}{T_2} = \left[1 + \frac{(z - z_2)}{(\Delta z)_2}\right]^{-4}, \tag{12}$$

where $(\Delta z)_2 = 4.4(m^{1/2}/n_2^{1/2}e)(\kappa T_2/m)^{7/2}(n_2 m/q)^2 \geq 10^3 cm$. Lower than transition region determined by the formula (12) the temperature trend by the recombination processes is limited. As a result we obtain the height dependence of temperature shown in Figure 2. In this figure the height h is counted off upward from the convective zone of the solar photosphere.

The obtained results hold for a sufficiently weak magnetic field, when $B < (4\pi nmc^2)^{1/2} \min[(m_i/m)^{1/2}, 0.1(T/T_i)^{1/2}]$. These inequalities will be satisfied, if $B < (4\pi nmc^2)^{1/2} \approx 100G$ and $n \approx 10^9 cm^{-3}$ because the typical value of magnetic field in the transition region of the solar atmosphere (outside the active regions) is not more than several Gauss.

4. Conclusion

We examined a new model of the formation of a stationary turbulent temperature jump in the transition region in a late-type star atmosphere. The regime of developed ion-acoustic turbulence is realized for significant local heat flux from the corona into the chromosphere. As a result of this the thermal conductivity of the medium falls, and the temperature gradient grows. The results of our study shows that there is a critical heat flux, which can pass through the plasma of the lower corona in the regime of comparatively small temperature gradients. The temperature jump is formed near to the local critical heat flux.

In our opinion the experimental diagnostics of ion-acoustic oscillations in the transition region is an important independent task. Probably, for this purpose it is possible to use data of X-ray and millimetre observations. The direct measurements of the local electric field based on the Stark effect are especially interesting.

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