THE LOGIC OF FORBIDDEN COLOURS[1]

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The purpose of this paper is twofold: (1) to clarify Ludwig Wittgenstein's thesis that colours possess logical structures, focusing on his 'puzzle proposition' that "there can be a bluish green but not a reddish green", (2) to compare model-theoretical and game-theoretical approaches to the colour exclusion problem. What is gained, then, is a new game-theoretical framework for the logic of 'forbidden' (e.g., reddish green and bluish yellow) colours. My larger aim is to discuss phenomenological principles of the demarcation of the bounds of logic as formal ontology of abstract objects.

***Key words*:** abstract logic, formal ontology, invariance criterion, meaning postulates, opponent-processing model, 'stabilized-image' experiments, over-defined games, payoff independence, imaginary logic.

Logic *has no* ontology, but logic *is* formal ontology. Logical knowledge of reality is possible since logic deals with formal, metaphysically unchanging features of reality. But what does it mean exactly? How does our formal model of reality depend on more or less sophisticated understanding of logicality?

In this paper I discuss a classical problem of the relations between logic and ontology, more precisely, between abstract logics and formal ontologies. I argue that abstract logics may be considered as formal ontologies in the sense of Edmund Husserl's phenomenology. My proposal is based on the interpretation of the classes of isomorphism as model-theoretic analogues of phenomenological abstract categorical objects. What is gained, then, is a connection between model-theoretical and ontological approaches to different types of formal relations (e.g. psychological relations by Edmund Husserl, ideal relations by Alexius Meinong, internal relations by Ludwig Wittgenstein, logical relations by Alfred Tarski, and metalogical relations by Nikolay Vasiliev). I discuss some principles of the demarcation of the bounds of logic as formal ontology, focusing on the question: "Are the criteria proposed by Husserl, Meinong, Wittgenstein, Tarski, and Vasiliev necessary and sufficient for the demarcation of the bounds of formal relations?" The case-study is the oppositional relations of colours.

Certain hues (for example, green and blue) can combine in experience into a phenomenally composed colour. For a long time it has been accepted that no human observer can have an experience of a colour that is for him phenomenally composed of red and green (or yellow and blue) under normal circumstances. According to the opponent-processing model of colours, not only we never see a reddish green or a yellowish blue but rather it is in principle impossible to have an experience of these

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colours. Ludwig Wittgenstein claims that colours possess *logical* structures because of their internal relations. As he says in ‘*Remarks on Colour’*, “Among the colours: Kinship and Contrast. (And that is logic.)”[2]. Furthermore, Wittgenstein includes into the scope of logic the proposition “there can be a bluish green but not a reddish green”. However the necessity of this proposition has been recently challenged by reports that ‘forbidden’ reddish green and yellowish blue colours *can* be perceived under special artificial laboratory conditions[3].

My main concern is to discuss whether these surprising results cast doubt on the Wittgenstein’s thesis about the logical structure of colours. My aim is to interpret these empirical results as evidence for game-theoretical semantics. To argue for this advantage of the game-theoretical approach to the logic of colours I propose the uniform game-theoretical model both for standard opponent perception of colours and for its violations in neuropsychological experiments.

Abstract logics as formal ontologies

One of the attempts to demarcate the bounds of logic is a definition of abstract logic in generalized model theory. An abstract logic consists of a collection of structures closed under isomorphism, a collection of formal expressions, and a relation of satisfaction between the two[4]. This definition does not include any conditions concerning rules of inference. If we accept the principle “No logic without inference” the term ‘model-theoretic language’ seems to be more appropriate than the term ‘abstract logic’. My proposal is to interpret abstract logics as formal ontologies, i.e. as genuine logics at least in phenomenological sense.

The interpretation of logic as formal ontology, i.e. an a priori science of objects in general, goes back to Edmund Husserl. The project of formal ontology has been planned by Husserl already in his ‘*Logical Investigations’* (1901), but it has been completely developed only in his later works, especially in ‘*Ideas Pertaining to a Pure Phenomenology and to a Phenomenological Philosophy*’ (1913) and in ‘*Formal and Transcendental Logic*’ (1929).

According to Husserl, logic is two-sided. On the one hand, logic is formal apophantic, the domain of judgment. On the other hand, it is formal ontology, the domain of formal objects. Husserl believed that the transcendental justification of logic is possible only if we postulate a special region of formal categorical objects. This region has to save logic from the ‘specific relativism’ of Immanuel Kant who gave his interpretation of logical structures in terms of universal human abilities. Husserl considered them as structures of some objective area of abstract higher-level objects. What is the nature of these objects? My suggestion is to consider classes (types) of isomorphism as model-theoretic analogues of categorical objects of Husserl’s formal region.

[2] Wittgenstein 1977. P. 23.

[3] See: Crane, Piantanida 1983; Billock, Gleason, Tsou 2001; Billock, Tsou 2010.

[4] See: Barwise 1983. P. 3.

Any two isomorphic structures represent the same abstract system. We do not know anything about an abstract system except the relations existing between its objects in the system. At the same time, classes of isomorphism are abstract individuals of higher order, i.e. hypostases of structurally invariant properties of models. Thus, formal ontologies do not distinguish between specific individuals in the domain, but they are not Kant's 'empty functions of unity' since they deal with individuals of higher order, i.e. classes of isomorphic structures.

Furthermore, classes of structures closed under isomorphism are generalized quantifiers. A predicate represents a property. So the semantic value of a predicate is a subset of the domain. A quantified expression has as semantic value a set of subsets of the domain. So a quantifier can be considered as second – level property, property of properties[5]. For example, Mostowski's generalized quantifiers are interpreted by classes of subsets of the universe and attribute cardinality properties to the extensions of one-place first-level predicates. Mostowski's infinite quantifier Q^M says that the extension of a suitable predicate has infinite cardinality $Q^M = \{X\text{: } X \text{ is } \textit{infinite}\}$. Mostowski's generalized quantifiers attribute second-order cardinality properties. More precisely, a Mostowski's quantifier is a function associating with every structure a family of subsets of its universe closed under permutations of the universe. Thus, Mostowski's quantifiers perfectly satisfy the permutation invariance criterion by Alfred Tarski[6].

Invariance criterion for logical notions

In his famous lecture '*What are Logical Notions?*' (1966) Tarski proposed to call a notion logical if and only if "it is invariant under all possible one-one transformations of the world onto itself"[7]. According to Tarski-Sher's criterion, it is better to discuss 'isomorphisms' (or 'bijections') and "structures" instead of 'permutations' (or 'transformations') and the 'world'. This criterion is historically traced to Lindström's generalization of Mostowski's approach[8].

[5] From phenomenological point of view, generalized quantifiers express so cold psychological properties. For Husserl, psychological properties, unlike physical (as, for example, to be philosopher or to play chess), do not influence on other properties, but exist because of them. Meinong preferred to speak about ideal and real properties. His terminology seems to be more successful because it doesn't presuppose irrelevant association with psychology. Phenomenology doesn't consider psychological properties as psychological phenomena. The point is that psychological (or ideal) properties are second-order properties of ideas or concepts.

[6] In fact, there is no conceptual necessity to consider quantifiers as second-order *properties*. The obvious challenge here is to generalize this understanding on second-order *relations*. This generalization of quantifiers was proposed by Per Lindström (see: Lindström 1966). His quantifiers are interpreted as second-order relations between first-order relations on the universe. Binary examples of Lindström's quantifiers are: Resher's quantifiers $Q^R = \{<X, Y>\text{: } X, Y \subseteq U \text{ and } card(X) < card(Y)\}$, Hartig's quantifiers $Q^H = \{<X,Y>\text{: } X,Y \subseteq U \text{ and } card(X) = card(Y)\}$, syllogistics quantifiers "All … are…" $= \{<X,Y>\text{: } X,Y \subseteq U \text{ and } X \subseteq Y\}$, "Some … are …" $= \{<X,Y>\text{: } X,Y \subseteq U \text{ и } X \cap Y \neq \varnothing\}$. Tarski's thesis of 'our logic' as 'logic of cardinality' may be fair for the theory of monadic quantification (logic of properties of classes of individuals), but not for the theory of binary quantification (logic of properties of classes of pairs of individuals) (see: Dragalina-Chernaya 2013).

[7] Tarski 1986. P.149.

[8] See: Sher 1991.

Felix Klein's famous *Erlangen Program* (1872) proposed the classification of various geometries according to invariants under suitable groups of transformations. Klein suggested that each geometric field can be characterized by the invariance condition satisfied by its notions. We can restrict or increase the transformations taken into account, getting more specific or more general geometrical notions. For example, affine geometry is more general than Euclidean geometry in the sense that it can distinguish fewer objects (for example, all triangles are the same in affine geometry), since its notions are invariant under more general group of transformations. Permutation invariance takes all one-one transformations into account and, as a result, according to Tarski, characterizes the most general notions. For Tarski, the science which studies these notions is logic.

The idea that logic is characterized by an invariance condition, i.e. by the things it does not distinguish between, has a long history. For Kant, for example, general logic "treats of understanding without any regard to difference in the objects to which the understanding may be directed"[9]. For Willard Quine, logic cannot assume any special entities as existing ones. Thus if logic is supposed to be independent of ontology, not only set theory but also second-order logic as 'set theory in sheep's clothing' go beyond the bounds of logic[10].

If we interpret formality of a theory as its invariance under permutations of the universe it means that the theory does notdistinguish between individual objects and characterizes only those properties of model which do not depend on its nonstructural transformations. Formal property should be preserved under the arbitrary switching of individual objects. For instance, 'red' and 'green' are non-formal properties, since they distinguish between things which are red and green.

However the standard argument in favor of invariance under permutation, which relies on the generality of logic, may be challenged. Ludwig Wittgenstein, for example, does not consider generality as a defining attribute of logicality: "The mark of a logical proposition *is not* general validity…[11] The general validity of logic might be called essential, in contrast with the accidental general validity of such propositions as 'All men are mortal'"[12]. Yet, what kind of general validity is essential and, as a result, logical for Wittgenstein?

The colour exclusion problem

According to '*Tractatus Logico-Philosophicus*' (1922), it is logically impossible for two colours to be at one place at the same time. This is because of the 'logical structure of colour'. As Wittgenstein pointed out, "Just as the only necessity that exists is logical necessity, so too the only impossibility that exists is logical

[9] Kant 1929. A52.

[10] See: Quine 1970.

[11] Wittgenstein 1922, 6.1231.

[12] Ibid. 6.1232.

impossibility[13]. <…> For example, the simultaneous presence of two colours at the same place in the visual field is impossible, in fact logically impossible, since it is ruled out by the logical structure of colour (It is clear that the logical product of two elementary propositions can neither be a tautology nor a contradiction. The statement that a point in the visual field has two different colours at the same time is a contradiction.)”[14].

Wittgenstein suggests that colour-ascriptions should be elementary. But, as the concluding remark implies, they cannot be elementary. The point is that the colour ascriptions are logically interdependent, and Wittgenstein tells us that elementary propositions are independent. This is a well-known problem of *colour exclusion*.

In ‘*Some Remarks on Logical Form*’ (1929) Wittgenstein offered a solution to this problem. Here he is interested in examining what he calls the ‘logical structure’ or the ‘logical form’ of the ‘phenomena’. He writes, “we can only arrive at a correct analysis by, what might be called, the logical investigation of the phenomena themselves, i.e., in a certain sense a posteriori, and not by conjecturing about a priori possibilities”[15]. A color-incompatibility claim is a tautology and “does not express experience”, however, being result of “logic investigation of the phenomena themselves”, it is “in a certain sense a posteriori”. Wittgenstein said that a proposition “reaches up to reality”, and by this he meant that “the forms of the entities are contained in the form of the proposition which is about these entities. For the sentence, together with the mode of projection which projects reality into the sentence, determines the logical form of the entities”[16]. Finally, Wittgenstein came to the conclusion that propositions such as ‘A is red’ should be seen as atomic, but with numbers entering into their logical forms to reflect the degrees of quality involved. If so, atomic propositions which attribute degrees to qualities should be seen in the framework of systems of co-ordinates. He considered the *geometry* of logical space of colour representation as an objective basis for the necessity of the colour-incompatibility claims.

In a conversation recorded by Friedrich Waismann in 1929, Wittgenstein remarks that statements about colour can be represented in geometrical terms by assigning them a position along certain colour axes. He writes, “Every statement about colours can be represented by means of such symbols. If we say that four elementary colours would suffice, I call such symbols of equal status *elements of representation*. These elements of representation are the ‘objects’”[17]. In ‘*Philosophical Remarks*’ (1930) Wittgenstein adapts Alois Höfler’s colour-octahedron (1897) based on Ewald Hering’s opponent-processing model of colours[18]. The basic colour pairs of this model, i.e. its elements of representation (red – green, blue – yellow, white – black) are situated at opposite points of colour-octa-

[13] Wittgenstein 1922, 6.375.

[14] Ibid. 6.3751.

[15] Wittgenstein 1929. P. 163.

[16] Ibid. P. 169.

[17] Wittgenstein 1993. P. 43.

[18] See: Wittgenstein 1975. P. 278.

hedron axes. Thus, we can define 'orange', for instance, as what lies between red and yellow. To say that something is orange, then, is to say that it has a colour between red and yellow (possibly with a number reflecting the degree of the colour involved). The degree of a colour is not its quantity. According to Wittgenstein, "If I say in the ordinary sense that red and yellow make orange, I am not talking here about a quantity of the components. And so, given an orange, I can't say that yet more red would have made it a redder orange"[19]. Wittgenstein proposed to represent the colours by means of a double-cone. As he pointed out, "If we represent the colours by means of a double-cone, instead of an octahedron, there is only one *between* on the colour circle, and red appears on it between blue-red and orange in the same sense as that in which bluered lies between blue and red. And if in fact that is all there is to be said, then a representation by means of a double-cone is adequate, or at least one using a double eight-sided pyramid is"[20]. Wittgenstein's double-cone represents the logical structure of colour. This is a grammatical representation, not a psychological or physical one.

If our logic takes into account a spectrum of invariance which preserves several additional structures, for example, a *logical* structure of colour space, we may get various types of logical invariance. Johan van Benthem suggests that the permutation invariance criterion may be viewed as "only one extreme in a spectrum of invariance, involving various kinds of automorphisms on the individual domain"[21]. Therefore, following Wittgenstein we turn back from Tarski's permutation invariance criterion to Klein's original program. From the point of view of Klein's ideology, the logic of colours may be considered as a member of a family of various logics of abstract objects whose notions are invariant for one-one transformations which respect additional formal structures, in particular, the formal relations of colours. The invariance criterion which is generalized in this way is wide enough to include not only one extreme type of invariance (i.e. permutation invariance), but a variety of invariances which respect different types of ordering of the universe.

Yet, what kinds of abstract objects are formal? What does it mean to be a formal abstract object? Gila Sher states that "Speaking in terms of objects we can say that formal objects are not just elements of formal structures, they are themselves formal structures"[22]. Logic takes certain general laws of formal structures and turns them into general laws of reasoning.

Now the key question is the following: Why did Wittgenstein consider relations between colours to be logical? My main concern is to clarify so cold Wittgenstein's 'puzzle proposition' from '*Remarks on Colour*' that "there can be a bluish green but not a reddish green".

[19] Ibid. P. 275.

[20] Ibid. P. 278.

[21] Van Benthem 1989. P. 320.

[22] Sher 1996. P. 678.

Wittgenstein's 'puzzle proposition': meaning postulates vs mapping functions

In his famous paper '*Reds, Greens, and Logical Analysis*' Hilary Putnam suggests that Wittgenstein's 'puzzle proposition' is analytic, in the sense in which 'analytic' means 'true on the basis of definitions plus logic'. He proposed to define the second-level predicates "Red (F)" (for "F is a shade of red") and "Grn (F)" (for "F is a shade of green"). In defining these predicates we must be restricted, in particular, by the postulate: "Nothing can be classified as both a shade of red and a shade of green (i.e., "that shade of red" and "that shade of green" must never be used as synonyms)"[23]. Putnam's approach to color-incompatibility has gained widespread acceptance among recent eminent writers on perception. As Larry Hardin says in '*Color for Philosophers*', "Perhaps not being red is part of the concept of being green. Yet it seems that all a normal human being has to do to have the concept of green is to experience green in an appropriately reflective manner"[24].

Nevertheless, the introduction of certain meaning postulates seems to be irrelevant to the exegesis of Wittgenstein's ideas. The meaning postulates expand a family of analytic truths by means of dictionary conventions. On the contrary, for Wittgenstein, any attempt to explain truth of the colour incompatibility claims is misguided, since the question of truth doesn't make sense for *rules* of logical syntax. As he pointed out in the so cold '*Big Typescript*', "The proposition «at one place at one time there is only room for one colour» is of course a masked proposition of grammar. Its negation is not a contradiction; rather it speaks against a rule of our accepted grammar. "Red and green don't go together at the same place" does not mean, they are never actually together, rather it means that it is nonsense to say that they are at the same place at the same time and therefore also nonsense to say they are never at the same place at the same time"[25]. Wittgenstein writes further in '*Philosophical Remarks*', "Grammatical conventions cannot be justified by describing what is represented. Any such description already presupposes the grammatical rules. That is to say, if anything is to count as nonsense in the grammar which is to be justified, then it cannot at the same time pass for sense in the grammar of the propositions that justify it"[26]. To sum up, the meaning postulates deal with lexicon, but internal relations of colours concern grammar.

Contrary to the meaning postulates approach, Jaakko Hintikka and Merrill Hintikka proposed to represent the concept of colour "by a function c which maps points in visual space into a color space. Then the respective logical forms of 'this patch is red' and 'this patch is green' would be c(a) = r and c(a) = g, where r and g are the two separate objects red and green, respectively. The logical incompatibility of the two color ascriptions is then reflected according to Wittgensteinian principles by

[23] Putnam 1956. P. 216.

[24] Hardin 1988. P. 122. See also: Westphal 2005.

[25] See: Noë 1994. P. 25.

[26] Wittgenstein 1975. P. 55.

the fact that the colors red and green are represented by different names. And if so, the two propositions are logically incompatible in the usual logical notation. Their incompatibility is shown by their logical representation: a function cannot have two different values for the same argument because of its 'logical form', i.e., because of its logical type"[27]. For Wittgenstein, as Jaakko Hintikka tells us, "the conceptual incompatibility of color terms can be turned into a logical truth simply by conceptualizing the concept of color as a function mapping points in a visual space into color space"[28]. Thus, "nonlogical analytical truths sometimes turn out to be logical ones when their structure is analyzed properly"[29].

My proposal is to generalize Hintikka's approach on binary colours, e.g., on the phenomenal structure of reddish green or bluish yellow experiences.

'Forbidden' binary colours: the opponent-processing model vs. 'stabilized-image' experiments

We perceive many colours to be binary. Purple, for example, as a mixture of blue and red. We may see bluish red, but it seems impossible to see a colour that would be described as a 'reddish green' or a 'bluish yellow'. Thus, certain antagonistic pairs of colours seem not to be combined to form a binary colour. According to the opponent-processing model of colours which goes back to Ewald Hering (1892), there are different types of retinal photoreceptors with optimal spectral sensitivity to specific wavelengths (e.g., short, middle or long wavelength receptors). Signals from the cones are assumed to be combined in an opposing fashion to produce opposing signals in retinal ganglion cells. This means that the cells are excited by the presentation of a given colour and inhibited by presence of its antagonist. Red-green and blue-yellow are supposed to be spectrally opposing channels. Thus, it would be impossible for a human observer to perceive both red and green (blue and yellow). The point is that it would presuppose the simultaneous transmission of positive and negative signals in the same channel. As red cancels green and blue cancels yellow, reddish green and bluish yellow are considered to be 'forbidden' binary colours by the opponent-processing model.

Perhaps one of the most surprising results in modern neuropsychological literature on colour vision is the report that reddish green and yellowish blue colors *can* be perceived in so cold 'stabilized-image' experiments. In order to see, the eye needs contrast, which is provided by its very fast movements. If the eye totally lacks contrast for a few seconds then the image will fade out. A stabilized image is an image that is projected on a part of the visual field and which follows the movements of the eye, so that the fading out of the image is restricted only to the stabilized portion of the visual field. This can be done, for example, with special eyetracker. If

[27] Hintikka and Hintikka 1983. P. 161.

[28] Hintikka 2009. P. 52.

[29] Ibid.

an image is stabilized on a part of the retina for a certain time, thus producing a sort of ‘informational hole’, then the brain tends to complete the image by so cold *filling in* process using the information of the surround. In ‘stabilized-image’ experiments, the subjects were presented with red and green (or blue and yellow) stripes on a black field, such that the red and green stripes had a common border. The red-green field was stabilized using an eyetracker. This was done in order to provoke a filling-in process in which the information from the non-stabilized parts of the image should be used.

In violation of the classical opponent-processing model, ‘stabilized-image’ experiments have shown that by stabilizing the retinal image between an antagonistic pair of red/green or blue/yellow equiluminant fields the entire region can be perceived simultaneously as both red and green (blue and yellow) or, to be more precise, as a ‘forbidden’ mixture colour whose red and green (blue and yellow) components were as clear as, for example, the green and blue components of aqua.

The first attempt at modeling these opponency violations by Hewitt Crane and Thomas Piantanida was based on the hypothesis that there is an extra stage of cortico-cortical rather then retinocortical visual processing, i.e. a non-opponent filling-in mechanism[30]. I suggest that the game-theoretical approach allows us to offer the uniform explanation both to standard opponent perception and to its violations in ‘stabilized-image’ experiments.

The logic of colours in game-theoretical perspective

From the very beginning, the opponent-processing model of colours developed in the game-theoretical framework. It suggested that the basis for colour sensations lies in a process of *winner-take-all competition* between red and green (blue and yellow). Now it is clear that this model must take into account the interactions between *teams* of color-labeled cells. As Vincent Billock, Gerald Gleason and Brian Tsou write, “Recent models of cortical color processing suggest that cortical color opponency may not be based on hard-wired wavelength opponency within a single cell but rather on (potentially fragile) interactions between cortical color-sensitive cells”[31]. They assumed that the struggle between red- and green- (or blue- and yellow-) teams is simply blocked by the border synergy of equilumininance and stabilization.

I suppose that there is no need to block the game processing since a variety of game-theoretical independences provides important insights into the theory of opponent-processing. In particular, the border synergy effect may be captured by the game-theoretical notion of *payoff independence.*

Payoff independence logic (PI logic) has developed by Ahti-Veikko Pietarinen and Gabriel Sandu[32]. They distinguish two types of independences in semantical

[30] Crane and Piantanida 1983. P. 1079.

[31] Billock, Gleason and Tsou 2001. P. 2399.

[32] See, for instance, Pietarinen 2006; Pietarinen and Sandu 2009.

games: informational independence, i.e. players' ignorance concerning the choices made in the game, and strategical independence that affect players' strategic decisions. Players may lack information concerning the structural meta-properties of the game, including the strategies used in the game, the values of the players' payoff functions, the number of agents in the opponent team or the size of one's own team, etc. PI logic is interested in the strategical independence. It goes back to John Harsanyi's pioneering work on games with incomplete information played by 'Bayesian' players[33].

The main idea of my proposal is the interpretation of opponency violations as payoff independence in 'stabilized-image' games between red/green or blue/yellow teams of cortical color-sensitive cells. In winner-take-all games, the following holds. If there is a winning strategy of the red team then there does not exist a winning strategy of the green team, and vice versa. In 'stabilized-image' games the information exchange between the opponent teams is blocked by the synergy of equilumininance and stabilization on the cortical strategic meta-level. Consequently, both red and green (blue and yellow) teams have winning strategies in these games. In other words, 'stabilized-image' games are *over-defined*. Thus, the law of non-contradiction fails in the generalized logic of colours allowing the simultaneous perception of antagonistic pairs of colours. In contrast to winner-take-all games, 'stabilized-image' games are non-strictly competitive.

To clarify the interpretation, let me borrow a fanny 'chair analogy' from '*Some Remarks on Logical Form*'. For Wittgenstein, the proposition "Red and green don't go together at the same place at one time" is similar to the propositions "Brown and Jones now sit in this chair". As he says, "For if the proposition contains the form of an entity which it is about, then it is possible that two propositions should collide in this very form. The propositions, "Brown now sits in this chair" and "Jones now sits in this chair" each, in a sense, try to set their subject term on the chair. But the logical product of these propositions will put them both there at once, and this leads to a collision, a mutual exclusion of these terms"[34].

Obviously, this 'chair analogy' gives rise to a worry. Wittgenstein seems to speak here about physical, not about visual space. In my point of view, there is no need to worry if we don't consider grammatical rules to be empirical statements. As Wittgenstein pointed out in '*The Blue book*', "We don't say that the man who tells us he feels the visual image two inches behind the bridge of his nose is telling a lie or talking nonsense. But we say that we don't understand the meaning of such a phrase. It combines well-known words, but combines them in a way we don't yet understand. The grammar of this phrase has yet to be explained to us"[35]. The proposition "A is red and green" has no sense, because internal relations are scaffolding for logical space. Internal relations exist in logical, that is, informational space. So, if red team does not know that green team already has winning strategies, it is logically possible that red and green go together at the same place at one time in

[33] See: Harsanyi 1967.

[34] Wittgenstein 1929. P. 169.

[35] Wittgenstein 1958b. P. 9.

the informational space. Brown and Jones can sit together in one chair if this chair is a part of the logical space furniture.

On the other hand, the appeal to neuroscience seems to be unsuitable for the interpretation of Wittgensteinian notion of 'logical space' since he clearly does not think that the science, and particularly neuroscience, is relevant to the resolution of philosophical problems. In a famous passage from '*Philosophical Investigations*' §109 Wittgenstein stresses the distinction between his methods and those of sciences: "It was true to say that our considerations could not be scientific ones <…> There must not be anything hypothetical in our considerations. We must do away with all explanation, and description alone must take its place. And this description gets its light, that is to say its purpose, from the philosophical problems. These are, of course, not empirical problems; they are solved, rather, by looking into the workings of our language, and that in such a way as to make us recognize those workings: in despite of an urge to misunderstand them. The problems are solved, not by giving new information, but by arranging what we have always known. Philosophy is a battle against the bewitchment of our intelligence by means of language"[36]. Moreover, Wittgenstein considered psychological concepts to be everyday concepts. As he says in '*Remarks on the Philosophy of Psychology*', "Psychological concepts are just everyday concepts. They are not concepts newly fashioned by science for its own purposes, as are the concepts of physics and chemistry. Psychological concepts are related to those of the exact sciences as the concepts of the science of medicine are to those of old women who spend their time nursing the sick"[37]. According to Wittgenstein, nothing that science discovers will affect the application of psychological concepts. Thus, neuropsychological data cannot influence the geometry of *our* colour space. Furthermore, he claimed in '*Remarks on Colour*', "But even if there were also people for whom it was natural to use the expressions 'reddish-green' or 'yellowish-blue' in a consistent manner and who perhaps also exhibit abilities which we lack, we would still not be forced to recognize that they see colours which we do not see. There is, after all, no commonly accepted criterion for what is a colour, unless it is one of our colours"[38]. Wittgenstein put the question: "But can I describe the practice of people who have a concept, e.g. 'reddish-green' that we don't possess?" In any case I certainly can't teach this practice to anyone"[39]. If we ask: "Now to what extent is it a matter of logic rather than psychology that someone can or cannot learn a game?" Wittgenstein thinks it sufficient to reply: "The person who cannot play *this* game does not have *this* concept"[40]. Speaking about 'logical space' we deal with logic rather than with psychology. As Wittgenstein says, "When dealing with logic, 'One cannot imagine that' means: one doesn't know what one should imagine here"[41].

[36] Wittgenstein 1958a. P. 47.

[37] Wittgenstein 1980. P. 12.

[38] Wittgenstein 1977. P. 4.

[39] Ibid. P. 32.

[40] Ibid. P. 31.

[41] Ibid. P. 6.

I suppose however that some new neuropsychological experiments may influence our language games, which, in turn, constitute what the colours are. Howard Lovecraft showed in his famous novel '*The Colour Out of Space*' how the experience of the extra-cosmic colour which is impossible in human viewing may destroy our 'form of life'. In fact, our language-games with colours are historically changeable; and neuropsychological experiments may contribute to our phenomenal history of colours. It is possible, for example, that tomorrow the invention of special glasses with a built-in eyetracker will make reddish green and bluish yellow new common colours of our everyday 'form of life'.

The imaginary logic of 'forbidden' colours

Furthermore, I suggest that PI logic of 'forbidden' colours may confirm Nikolay Vasiliev's project of imaginary logic. Vasiliev classified all judgments into judgments on facts and judgments on concepts. He called the logic of concepts 'imaginary', taking the term from Lobachevski's definition of his geometry. As Aristotelian logic (like Euclidean geometry) concerns the real world, so Vasiliev's imaginary logic (like Lobachevski's geometry) concerns imaginary worlds. Logical structures are divided into the two levels: of metalogic, the level of necessary laws which cannot be eliminated without distracting the logic itself, and of ontology, which includes laws depending on some specific properties of the object investigated. First level is connected with epistemological commitments and second level depends on ontological commitments. According to Vasiliev, the universally valid law of excluded self-contradiction, which tells us that 'no proposition can be simultaneously true and false', belongs to the level of metalogic, but the law of excluded contradiction, which Vasiliev formulated as 'no object can have a predicate which contradicts it', belongs to the level of ontology and therefore its validity depends on the characteristics of the objects being investigated. As Vasiliev says, the law of excluded contradiction is empirical and real, i.e. it is the reduced formula comprising the uncountable facts, like that red is incompatible with dark blue, white, black, etc.; the silence is incompatible with noise, rest with movement, etc.[42].

Thus, we can reject the 'empirical' law of excluded contradiction, and, as a result, the law of colour incompatibility, because the opposite is not unthinkable. For Vasiliev, contradictions do not occur in the world of facts but only in the world of concepts. However, if we would be able to perceive, for example, red and green together at the same place at one time, we can reject the law of excluded contradiction in our empirical world of facts. If we interpret the oppositional relation of red and green as a kind of independent negation, the perception of reddish green in 'stabilized-image' experiments gives us an empirical example of the violation of empirical law of excluded contradiction.

[42] См.: Васильев 1989. С. 67.

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