Guaranteed Control of Feedback Linearizable Nonlinear Object

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Abstract. An optimal control problem is formulated for a class of nonlinear systems for which there exists a coordinate representation transforming the original system into a system with a linear main part and a nonlinear feedback. In this case the coordinate transformation significantly changes the form of original quadratic functional. The penalty matrices become dependent on the system state. The linearity of the transformed system structure and the quadratic functional make it possible to pass over from the Hamilton–Jacoby–Bellman equation (HJB) to the state dependent Riccati equation (SDRE) upon the control synthesis. Note that it is rather difficult to solve the obtained form of SDRE analytically in the general case. In this study, we construct the guaranteed control method from the point of view of the system quality based on feedback linearization of the nonlinear system; the transformation of the cost function upon linearization is examined, as well as the system behavior in the presence of disturbance and the control synthesis for this case. The presented example illustrates the application of the proposed control method for the feedback linearizable nonlinear system.

Keywords: Optimal control, Differential game, State Dependent Riccati Equation, Feed Back Linearization

INTRODUCTION

This work focuses on the problem of output regulator for nonlinear systems, namely the problem of designing an output feedback controller.
The central idea feedback linearization is to algebraically transform a nonlinear system dynamic into (fully or partly) linear on so that linear control techniques can be applied. As opposite to conventional linearization feedback linearization is achieved by exact state transformation and feedback, rather that by linear approximation. Note that transform is not unique (details about this can be found in [4,6,11]. Several control design methods for that systems, based on a standard Lyapunov analysis [5,7,9,8].

In this paper an optimal control problem is formulated for a class of nonlinear systems being under the influence of uncontrollable disturbance for which there exists a coordinate representation transforming the original system into a system with a linear main part and a nonlinear feedback. Considering disturbance as actions of some player counteracting successful performance of a control problem, we will consider a task in a key of differential game with two players. Examining the problem of synthesis of the control law as the differential game of two players we introduce the quadratic functional. The coordinate transformation significantly changes the form of original quadratic functional. The penalty matrices become dependent on the system state. The linearity of the transformed system structure and the quadratic functional make it possible to pass over from the Hamilton–Jacoby–Bellman equation (HJB) to the state dependent Riccati equation (SDRE) upon the control synthesis.

Since the mid-90's, SDRE strategies have emerged as design method that provide a systematic and effective means of design nonlinear controllers. This method, first proposed by Pearson [11] and later expended by Wernli & Cook [13], was independently studied by Mracek & Clouter [10]. The method entails factorization of the nonlinear dynamics into the state vector and the product of a matrix-valued function that depends on the state itself (SDC). The theoretical contribution in Mrasek, Clouter has initiated an increasing use of SDRE techniques in a wide variety of nonlinear control applications [3].

Note that it is rather difficult to solve the obtained form of SDRE analytically in the general case. It is necessary to approximate the solution; this approximation is realized by numerical methods using symbolic computer packages or interpolation methods. In this study, we construct the guaranteed control method from the point of view of the system quality based on feedback linearization of the nonlinear system; the transformation of the cost function upon linearization is examined, as well as the system behavior in the presence of disturbance and the control synthesis for this case. The presented example illustrates the application of the proposed control method for the feedback linearizable nonlinear system.

STATEMENT OF THE PROBLEM

Consider the following continuous nonlinear system is described by the vector differential equation

$$\dot{x}(t) = f(x) + g_1(x)w(t) + g_2(x)u(t),$$
$$x(t_0) = x_0, \qquad (1)$$
$$y(t) = h(x).$$

Here $x(t) \in R^n$ state of systems; $x \in \Omega_x$, $X_0 \in \Omega_x -$ domain of possible initial conditions of system; $y \in R^m$, $m \le n -$ system exit; $u \in R^r$ control; $w \in R^k -$ disturbance variable;

$f(x),\ g_1(x),\ g_2(x),\ h(x) -$ matrixes are real and continuous, $f(0) = 0$. It is supposed that at all (x) system (1) is controllable and observable [1], $t \in R^+$. Besides, functions $f(x),\ g_1(x),\ g_2(x)$ we will assume to be rather smooth (C_∞) that through any $(0, x_0) \in t \times \Omega_x$ passed one and only one decision (1) $x(t, 0, x_0)$ and there would be the unique corresponding exit of system $y(t) = h(x(t, 0, x_0))$. The disturbance variable $w(t)$ are assumed to be bounded as follows:

$|w_i(t)| \le \sigma_i(x(t)), i = 1,...,k,\ t \ge 0$, where

$\sigma_i(x(t)) \ge 0$ for all $x(t) \in \Omega_x$. This condition we will write down in a look

$$|w(t)| \prec \sigma(x(t)),\ \forall\, t \ge 0. \qquad (2)$$

Considering disturbance $w(t)$ as actions of some player counteracting successful performance of a control problem, we will consider a task in a key of differential game with two players *U* and *W*.

The organization of controls $u(t)$ and $w(t)$ with use of a principle of feedback is supposed. Examining the problem of synthesis of the control law as the differential game of two players *U* and *W* we introduce the functional

$$J(x,u,w) == \min_u \max_w \frac{1}{2}\int_0^\infty \Big\{ y^T(t)Qy(t) + $$
$$+ u^T(t)Ru(t) - w^T(t)P\big(\sigma(x(t))\big)w(t)\Big\}\, dt. \qquad (3)$$

Here, the matrix Q can be positive semi-definite; the matrices R, P are positive definite. Positively definite matrix $P(\sigma(x(t)))$ is created so that to consider the greatest possible disturbance of a look (2) operating on system. Additional requirements to values of parameters of these matrixes will be defined further.

FEEDBACK LINEARIZATIION AND UPDATING OF STATEMENT OF A TASK

Let's assume that there is a function $x(t) = \Phi^{-1}(z(t))$ and this function is smooth. Thus, the smooth function $z(t) = \Phi(x(t))$ defined in domain Ω_z is a diffeomorfizm in domain Ω_x.

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$$\dot{z}(t) = A_0 z(t) + B_0 v(t) + G(z)w(t),$$
$$z(0) = \Phi(x_0), \qquad (4)$$
$$s(t) = H(z)z(t).$$

where

$$A_0 z = \left[\frac{\partial\Phi}{\partial x}(f(x) + g_2(x)\alpha(x))\right]_{x=\Phi^{-1}(z)}, \qquad (5)$$

$$B_0 = \left[\frac{\partial\Phi}{\partial x}(g_2(x)\beta(x))\right]_{x=\Phi^{-1}(z)},$$

$$G(z) = \left[\frac{\partial\Phi}{\partial x}(g_1(x)\right]_{x=\Phi^{-1}(z)} \qquad (6)$$

$$H(z) = \left[\frac{\partial\Phi}{\partial x}(h(x)\right]_{x=\Phi^{-1}(z)},$$

$$A_0 = diag\ (A_1,...,A_m),\ A_i \in R^{d_i \times d_i}, i = 1,...,m,$$

$$B_0 = diag\ (b_1,...,b_m),\ b_i \in R^{d_i \times 1},\ \sum_{i=1}^m d_i = n.$$

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 1 \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix},\ b_i = \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \qquad (7)$$

$v(t)$ is the new input vector.

The coordinate transformation $z(t) = \Phi(x(t))$ and feedback law

$$u(t) = \alpha(x) + \beta(x)v(t) \qquad (8)$$

transform original nonlinear system (1) into system (9) with the linear structure and state dependent coefficients (SDC). For linear object (4) with the control $v(t)$ we will enter, having substituted in (3) control (8), representation of an initial functional in a look

$$J(z,v,w) =$$

$$= \min_v \max_w \frac{1}{2}\int_0^\infty \Big\{ z^T(t)Q(z)z(t) + v^T(t)R(z)v(t) + \qquad (9)$$

$$+2z^T(t)N(z)v(t) - z^T(t)Q(z)z(t) + v^T(t)R(z)v(t) +$$

$$+2z^T(t)N(z)v(t) - w^T(t)P\left(\sigma(\Phi^{-1}(z))\right)w(t)\Big\}\,dt,$$

where

$$Q(z) = H^T(z)QH(z) + \theta^T(z)R\theta(z),$$

$$R(z) = \left(\beta(\Phi^{-1}(z))\right)^T R\beta(\Phi^{-1}(z)), \qquad (10)$$

$$P(z) = P\left(\sigma(\Phi^{-1}(z))\right),\ N(z) = \theta^T(z)R\beta(\Phi^{-1}(z)),$$

$$\theta(z) = \begin{bmatrix} \dfrac{\alpha_1(\Phi^{-1}(z))}{n\cdot z_1} & \cdots & \dfrac{\alpha_1(\Phi^{-1}(z))}{n\cdot z_n} \\ \vdots & \ddots & \vdots \\ \dfrac{\alpha_m(\Phi^{-1}(z))}{n\cdot z_1} & \cdots & \dfrac{\alpha_m(\Phi^{-1}(z))}{n\cdot z_n} \end{bmatrix}.$$

It can be seen that the matrices $Q(z)$, $R(z)$, $N(z)$ and $P\left(\sigma(\Phi^{-1}(z))\right)$ depend on the state of system (4). Initial control problem for object (1) with cost function (3) is transformed into the problem of the control actions synthesis for object (4) with functional (9).

The work has following goals:

G1) to find the solution of differential game in the form of a synthesis problem of optimum controls;

G 2) to find nonlinear control of a look (8);

G 3) to carry out analysis of a control system stability.

Directed by (4) and (9) the problem is referred to the class of problems whose parameters of system and functional of quality depend on system state (State-Dependent Coefficients).

CONTROL SYNTHESIS

Let us write the Hamilton–Jacoby–Bellman equation using functional (9)

$$\min_v \max_w \Bigg[\frac{\partial V(z)}{\partial z(t)}(A_0 z + B_0 v + G(z)w) +$$

$$+\frac{1}{2}\Big\{ z^T(t)Q(z)z(t) + v^T(t)R(z)v(t) +$$

$$+2z^T(t)N(z)v(t) - w^T(t)P(z)w(t)\Big\} \Bigg] = 0,$$

where $V(z)$ is the Bellman function. There is no boundary condition in the Hamilton–Jacoby–Bellman. Optimal control are determined by the following expressions [1]

$$v(t) = -R^{-1}(z)\left\{ B_0^T \left[\frac{\partial V(z)}{\partial z(t)}\right]^T + N^T(z)z(t) \right\} =$$

$$= \left\{\beta(\Phi^{-1}(z))\right\}^{-1} \times \qquad (11)$$

$$\times \left[R^{-1}\left\{\beta^T(\Phi^{-1}(z))\right\}^{-1} B_0^T S(z)z(t) - a(\Phi^{-1}(z)) \right],$$

$$w(t) = P^{-1}\left(\sigma(\Phi^{-1}(z))\right) G^T(z) \left[\frac{\partial V(z)}{\partial z(t)}\right]^T,$$

(11)

where vector $\{\partial V(z)/\partial z(t)\}^T$ is defined by the solution of the equation

$$\left[\frac{\partial V(z)}{\partial z(t)}\right] A_0 z(t) - \frac{1}{2}\left\{ B_0^T \left[\frac{\partial V(z)}{\partial z(t)}\right]^T + N^T(z)z(t) \right\}^T \times$$

$$- \times R^{-1}(z)\left\{ B_0^T \left[\frac{\partial V(z)}{\partial z(t)}\right]^T + N^T(z)z(t) \right\} + \qquad (12)$$

$$+\frac{1}{2} z^T(t)Q(z)z(t) +$$

$$+\frac{1}{2}\left[\frac{\partial V(z)}{\partial z(t)}\right] G(z)P^{-1}(z)G^T(z)\left[\frac{\partial V(z)}{\partial z(t)}\right]^T = 0.$$

Thus, controls (11) are organized according to the feedback principle, where $\partial V(z)/\partial z$ is the solution to Eq. (12). Let us define $\partial V(z)/\partial z$ as [6]

$$\left[\frac{\partial V(z)}{\partial z}\right]^T = S(z)z(t). \qquad (13)$$

We rewrite (11) with account of (13),

$$v(t) = -R^{-1}(z)\left\{B_0^T S(z)+N^T(z)\right\}z(t),$$
$$w(t) = P^{-1}G^T(z)S(z)z(t). \tag{14}$$

Expression (12) becomes the state dependent Riccati equation

$$S(z)(A_0 - B_0R^{-1}(z)N^T(z)) +$$
$$+(A_0-B_0R^{-1}(z)N^T(z))^T S(z)-$$
$$-S(z)\left[B_0R^{-1}(z)B_0^T-G(z)P^{-1}(z)G^T(z)\right]S(z)+ \tag{15}$$
$$+Q(z)-N(z)R^{-1}(z)N^T(z)=0.$$

Matrix $S(z)$ in (14) is the unique, symmetric, positive-definite solution of (15).
From this equation it is visible that matrixes

$$Q^*(z) = \tag{16}$$
$$= H^T(z)QH(z) + \theta^T(z)R\theta(z) - N(z)R^{-1}(z)N^T(z),$$

$$\varphi(z) = B_0R^{-1}(z)B_0^T - G(z)P^{-1}(z)G^T(z) =$$
$$=B_0\left\{\left[\beta(\Phi^{-1}(z))\right]^T R\left[\beta(\Phi^{-1}(z))\right]\right\}^{-1} \times \tag{17}$$
$$\times B_0^T - G(z)P^{-1}(z)G^T(z).$$

should be positive, at least, semidefinite at $\forall z \in \Omega_z = \Phi(\Omega_x)$ that will be shown at carrying out the analysis of stability of system with controls (15).

The control $v(t)$ is possible to present as

$$v(t) = \left\{\beta(\Phi^{-1}(z))\right\}^{-1} u(t) - \alpha(\Phi^{-1}(z)) =$$
$$= -\left\{\beta(\Phi^{-1}(z))\right\}^{-1} \times \tag{18}$$
$$\times\left[R^{-1}\left\{\beta^T(\Phi^{-1}(z))\right\}^{-1} B_0^T S(z)z(t)+a(\Phi^{-1}(z))\right].$$

Coming back to system (1), we will make return substitution in the law of feedback (8)

$$u(t) = \alpha(x) + \beta(x)v(t) =$$
$$= \alpha(x) - R^{-1}\left\{\beta^T(x)\right\}^{-1} B_0^T S(\Phi(x))\Phi(x) - \alpha(x) =$$
$$= -R^{-1}\left\{\beta^T(x)\right\}^{-1} B_0^T S(\Phi(x))\Phi(x). \tag{19}$$

Thus, optimum controls for system (1) is defined by expressions

$$u(t) = -R^{-1}\left\{\beta^T(x)\right\}^{-1} B_0^T S(\Phi(x))\Phi(x),$$
$$w(t) = P^{-1}G^T(\Phi(x))S(\Phi(x))\Phi(x). \tag{20}$$

where, $z(t) = \Phi(x)$ — the diffeomorfizm, and a matrix $S(\Phi(x)) = S(z)$ is the solution of the equation of Riccati (15).
The equations (20) describe optimum strategy with feedback for players of U and W as functions of current state and this strategy corresponds to a saddle point of a task on a minimax. In that case the control system is described by expression

$$\dot{x}(t) = f(x) +$$
$$+\left[g_1(x)P^{-1}G^T(\Phi(x))-g_2(x)R^{-1}\left\{\beta^T(x)\right\}^{-1} B_0^T\right]\times$$
$$\times S(\Phi(x))\Phi(x),\ x(t_0) = x_0, \tag{21}$$
$$y(t) = h(x).$$

Theorem 1. Let's consider system (1), being under the influence of the uncontrollable disturbance satisfying to bound (2). Let's assume existence of a diffeomorfizm $z(t) = \Phi(x))$ and transformation by feedback (8) such that the equation (1) is input-state linearized and is representable in the form of (4). Then controls at performance of the conditions (16), (17) and delivering a minimum to a functional (3) are defined by expressions (20) where the matrix $S(z) = S(\Phi(x))$ is the solution of the state dependent Riccati equation (15).

ANALYSIS OF STABILITY

Let us use the second Lyapunov method for investigation of the system stability [1]. The function $\tilde{V}(z) = 2V(z)$ where $V(x)$ is the Bellman function for system (1) and cost function (3), is the Lyapunov function. Let $\omega_i\{|x|\}, i = 1,2,3$, be the scalar nondecreasing functions, such that $\omega_i(0) = 0, \omega_i\{|x|\} > 0$ for $x \neq 0.$. The function $\tilde{V}(x)$ satisfies the condition $\omega_1\{|x|\} \le \tilde{V}(x) \le \omega_2\{|x|\}, \forall x$.
It follows from the second Lyapunov theorem that if the following condition is proved:

$$\frac{\partial\tilde{V}(x)}{\partial x}\frac{dx(t)}{dt} \le -\omega_3\{|x|\}, \tag{22}$$

then system is stable.

The derivative of the Bellman function with respect to the coordinate can be rewritten in the following form:

$$\frac{\partial\tilde{V}(x)}{\partial x} = 2\frac{\partial V(x)}{\partial z}\frac{\partial z}{\partial x} = 2\frac{\partial V(z)}{\partial z}\frac{\partial\Phi(x)}{\partial x}.$$

Considering (5), (6) and (11), from the last expression in view of (13) we will have

$$\frac{\partial \tilde V(x)}{\partial x} = 2z^T(t)S(z)\times$$
$$\times\left\{A_0 - B_0R^{-1}(z)\left[B_0^T S(z)+N^T(z)\right]+ \right. \quad (23)$$
$$\left. +G(z)P^{-1}D^T(z)S(z)\right\}z(t).$$

Let's appoint $\omega_3\{|x|\}$. Let

$$\omega_3\{|x|\} = \omega_3\{|\Phi^{-1}(z)|\} = z^T(t)Q^*(z)z(t), \quad (24)$$

where matrix

$$Q^*(z) = H^T(z)QH(z) +$$
$$+\theta^T(z)R\theta(z) - N(z)R^{-1}(z)N^T(z)$$

at least, the positively semi-certain. Let's note that $z^T(t)Q^*(z)z(t) > 0$ at $z(t) \neq 0$ that is provided with the corresponding purpose of a matrix Q in a functional of quality (3).

After all transformations (22) takes the following form:

$$2z^T(t)S(z)\left\{A_0 - B_0R^{-1}(z)\left[B_0^T S(z)+N^T(z)\right]+\right.$$
$$\left.+2G(z)P^{-1}G^T(z)S(z)\right\}z(t) \le \quad (25)$$
$$\le -z^T(t)Q^*(z)z(t).$$

If the right hand side is moved to the left, the Riccati equation similar to (16) can be found in the square brackets; then (25) is simplified,

$$z^T(t)S(z)\times$$
$$\times\left[B_0R^{-1}(z)B_0^T - G(z)P^{-1}G^T(z)\right]S(z)z(t) > 0$$

at all $z(t) \neq 0$. Therefore the matrix

$$\varphi(z) = B_0\left\{\left[\beta(\Phi^{-1}(z))\right]^T R\left[\beta(\Phi^{-1}(z))\right]\right\}^{-1}\times$$
$$\times B_0^T - G(z)P^{-1}(z)G^T(z)$$

should be positive semi definite.

Thus, performance of a condition of positive definiteness of matrixes

$$\begin{aligned} Q^*(z) &= H^T(z)QH(z) + \\ &+\theta^T(z)R\theta(z) - N(z)R^{-1}(z)N^T(z), \end{aligned} \quad (26)$$

$$\varphi(z) = B_0R^{-1}(z)B_0^T - G(z)P^{-1}(z)G^T(z), \quad (27)$$

that was supposed at synthesis of optimum control (14), provides stability to nonlinear system.

Theorem 2. Let's consider the system (1) being under the influence of uncontrollable disturbance satisfying to bound (2). Let's assume existence of a diffeomorfizm $z(t) = \Phi(x))$ and transformation by feedback (8) such that the equation (1) is input-state linearized and is representable in the form of (4). Controls (14) where the matrix $S(z)$ is defined by the solution of the equation (15), provide to system stability if matrixes of a penalty Q and R in a functional (3) are appointed so that matrixes $Q^*(z)$ in (26) and $\varphi(z)$ in (27) would be, at least, positively semidefinite.

DESIGN OF GUARANTEED CONTROL

The equation (4) with controls (14) looks like

$$\begin{aligned} &\dot z(t) = A_0 z(t) - \\ &-\left\{\left[B_0R^{-1}(z)B_0^T - G(z)P^{-1}G^T(z)\right]S(z) + \right. \\ &\left. +B_0R^{-1}(z)N^T(z)z(t),\right. \\ &z(0) = z_0,\ s(t) = H(z)z(t). \end{aligned} \quad (28)$$

We determine one of the possible trajectories $z_G(t)$ [2]:

$$\begin{aligned} &\dot z_G(t) = A_0 z_G(t) - B_0(R^*)^{-1}(N^*)^T z_G(t) - \\ &-\left[B_0(R^*)^{-1}B_0^T - G^*(P^*)^{-1}(G^*)^T\right]S^* z_G(t), \quad (29) \\ &z_G(0) = z_0,\ s_G(t) = H^* z_G(t). \end{aligned}$$

Here R^*, G^*, N^*, H^* = const such that, if

- $R(z), G(z), N(z), H(z)$ are measurable on the set Ω_z for any fixed R, G, N, H and z;
- $R(z), G(z), N(z), H(z)$ are continuous on z for any fixed R, G, N, H;
- and for a fixed t the functions $R(z), G(z), N(z), H(z)$ are continuous in aggregate of the variables z and R, G, N, H,

then there exist functions $m(t)$ and $n(t)$ that are Lebesgue-integrable and such that if

$$\left\|\left[A_0 z_G(t) - B_0(R^*)^{-1}B_0^T - G^*(P^*)^{-1}(G^*)^T\right]S^* z_G(t) - \right.$$
$$\left. -B_0(R^*)^{-1}(N^*)^T z_G\right| = m(t)$$

and $\left|H^* z_G(t)\right| = n(t)$, where S^* is a positive-definite solution of the equation of Riccati with constant matrixes

$$S^*(A_0 - B_0(R^*)^{-1}(N^*)^T)^* -$$
$$-S^*(B_0(R^*)^{-1}B_0^T - G^*(P^*)^{-1}(G^*)^T)S^* + Q^* - \quad (30)$$
$$-N^*(R^*)^{-1}(N^*)^T + S^*(A_0 - B_0(R^*)^{-1}(N^*)^T)^T S = 0,$$

then

$$\Big\|\Big[A_0 z_G(t) - B_0(R^{-1}(z)B_0^T -$$
$$-G(z)P(\sigma(z))^{-1}(G^*)^T\Big]S^* z_G(t) -$$
$$-B_0(R^*)^{-1}(N^*)^T z_G(t)\Big\| \geq m(t)$$

and $|H(z)z(t)| \leq n(t)$.

Therefore, the parameters of model (29) R^*, G^*, N^*, H^* may be called the "worst".

Definition. Under the condition

$$\Big\|\Big[A_0 z_G(t) - B_0(R^*)^{-1}B_0^T - G^*(P^*)^{-1}(G^*)^T\Big]S^* z_G(t) -$$
$$-B_0(R^*)^{-1}(N^*)^T z_G(t)\Big\| \leq$$
$$\leq \Big\|\Big[A_0 z_G(t) - B_0(R(z))^{-1}B_0^T -$$
$$-G(z)(P(\sigma(z))^{-1}(G^*)^T\Big]S^* z_G(t) -$$
$$-(R(z)) - B_0(R(z))^{-1}(N^*)^T z_G(t)\Big\|$$

the system (29) is majorant-model of a system (28).
By this definition, linear controls

$$\begin{aligned} v^*(t) &= -(R^*)^{-1}\left\{B_0^T S^* + (N^*)^T\right\}z(t), \\ w^*(t) &= (P^*)^{-1}(G^*)^T S^* z(t) \end{aligned} \qquad (31)$$

for the system (1) we will call guaranteeing.

Therefore the initial system (1) with controls (31) is described by expression

$$\dot{x}(t) = f(x) +$$
$$+\left[g_1(x)P^{-1}\left(G^*\right)^T - g_2(x)R^{-1}\left\{\beta^T(x)\right\}^{-1}B_0^T\right]\times$$
$$\times\left[S^*\right]\Phi(x),\ x(t_0) = x_0,$$
$$y(t) = h(x).$$

EXAMPLE

The synchronous generator dynamics [8] in the established regime can be defined by the dynamic system of form (1),

$$\begin{cases} \dot{x} = f(x) + g_1(x)w(t) + g_2(x)u(t), \\ x(0) = x_0, \end{cases}$$

where

$$f(x) = \begin{bmatrix} x_2 \\ -p\left[(1+x_3)\sin(x_1+d) - \sin d\right] - qx_2 \\ -rx_3 + s\left[\cos(x_1+d) - \cos d\right] \end{bmatrix},$$

$$g_1(x) = g_2(x) = [0 \quad 0 \quad 1]^T,$$

p, q, r, s, d – are the real parameters. The coordinate transformation for this system,

$$\Phi(x) = \begin{bmatrix} x_1 \\ x_2 \\ -p\left[(1+x_3)\sin(x_1+d) - \sin d\right] - qx_2 \end{bmatrix}$$

together with the feedback functions

$$\alpha(x) = \frac{-px_2(1+x_3)\cos(x_1+d) - pq\sin d + q^2 x_2}{p\sin(x_1+d)} +$$
$$+ q(1+x_3) + rx_3 - s\left[\cos(x_1+d) - \cos d\right]$$

and $\beta(x) = -\dfrac{1}{p\sin(x_1+d)}$

form the system

$$\begin{cases} \dot{z} = A_0 z + B_0 v + D(z)w, \\ z(0) = z_0. \end{cases}$$

The matrices $A_0, B_0, D(z)$ have the form

$$A_0 = A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},\ B_0 = b_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$D(z) = \left[\frac{\partial\Phi}{\partial x}\cdot g_1(x)\right]_{x=\Phi^{-1}(z)} = \begin{bmatrix} 0 \\ 0 \\ -p\sin(z_1+d) \end{bmatrix}$$

for al $x \in \Omega_x = \left\{x : 0 < x_1 + d < \pi\right\}$. In this case $\Omega_z = \Phi(\Omega_x) = \left\{z : 0 < z_1 + d < \pi\right\}$.

The guaranteed control $u(t)$ is possible to present as

$$u(t) = -\left(R^*\right)^{-1}\left\{\beta^T(x)\right\}^{-1} B_0^T S^* \Phi(x).$$

The simulation was performed in Simulink of MATLAB software with the following plant parameters:

$p = 136.0544, \ q = 4, \ r = 0.4091,$

$s = 0.2576, \ d = \pi / 4,$

initial conditions $x(0) = (1 \quad 18 \quad -9)^T$ and weighting matrices

$Q = diag \ (|x_1(0)|, |x_2(0)|, |x_3(0)|), R = 0.0001, P = 1000.$

White noise with the intensity $W = 1$ was used as *w(t)*.

The plots of the states x_1, x_2, x_3 of the system shown in Figure.

FIGURE 1. This is the Style for Figure Captions. Center this if it doesn't run for more than one line.

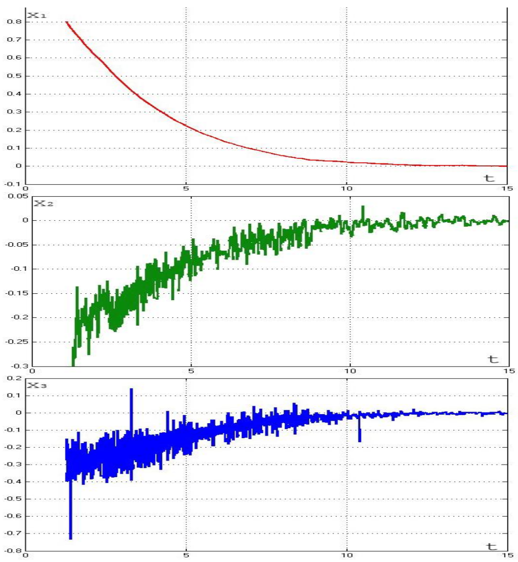

FIGURE. Plot of state x_1, x_2, x_3 in the presence of control $u(t)$ and disturbance variable $w(t)$ (White noise).

The plots in Fig. testify successful plant stabilization in the presence of the control action.

CONCLUSION

In this paper a problem of control for nonlinear systems is formulated for a class of a perturbed feedback-linearizable systems. Examining the problem of synthesis of the control law for those systems as the optimal differential game of two players. The linearity of the transformed system structure and the infinite-time performance quadratic criterion make it possible to pass over from the Hamilton–Jacoby–Bellman equation (HJB) to the state dependent Riccati equation (SDRE) upon the control synthesis. In general case an analytical solution of SDRE cannot be obtained. For the problem control low design of a perturbed feedback-linearizable systems we have proposed to search of realizing decision using of minimax principle witch based on application of majorant-model of system. One of the variants of further study can be formulated as the problem of non unique a feedback-linearizable, factorization (SDC parameterizations) of matrix systems and creation of a systems majorant-model.

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