Beam–Wave Interaction in the Passbands and Stopbands of Periodic Slow-Wave Systems

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***Abstract*—A method of analysis of beam–wave interaction in passbands and stopbands of periodic slow-wave systems (SWSs), based on the use of the finite-difference equation of excitation of such systems by an electron beam, is presented. In contrast to equations of a well-known beam–wave interaction theory, it includes the local coupling impedance that characterizes the interaction of electrons with the total field of two waves (forward and counter propagating) of the SWS and does not tend to infinity at cutoff frequencies. It allowed to develop a general theory of interaction of electron beams and waves in passbands and stopbands of SWSs without using their equivalent circuits. A study of beam–wave interaction in the folded waveguide-type SWSs has been performed. An elementary analytical calculation of electrodynamic characteristics of such SWSs necessary to study the interaction is given. Specifics of interaction in passbands and near the cutoff frequencies of periodic SWSs are considered. Amplification conditions in stopbands of such SWSs are discovered. Properties of the beam–wave interaction at the cutoff frequency where the folded waveguide is an analog of multigap open resonators used in such electron devices, as an orotron, are examined.**

***Index Terms*—Backward wave, beam–wave interaction, folded waveguide, orotron, passband, periodic slow-wave system (SWS), stopband, traveling-wave tube (TWT).**

I. Introduction

IN POWERFUL traveling-wave tubes (TWTs), periodic slow-wave systems (SWSs) of the type of coupled cavity systems, interdigital systems, and other similar SWSs with complex configuration of individual elements are used [1]. Currently, for millimeter and submillimeter waveband TWTs, the possibilities of use of the simplest SWSs—folded or serpentine waveguides, for which a micrometer size element fabrication technology has been developed, are explored [2]–[4].

Beam–wave interaction theory in TWTs with periodic SWSs was developed in many works where mostly two methods were used. The first one is to use the equivalent circuits of periodic SWSs in the form of RLC circuits or chains of four-pole circuits, starting from [5]–[7]. Some beam–wave interaction properties in periodic SWSs were explored using equivalent circuits.

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However, the selection of adequate equivalent circuits is a difficult problem and requires justification for each particular SWS type.

The other method is based on the general theory of periodic SWS excitation developed in [8]. In this paper, Vainstein [8] analyzes the beam–wave interaction in periodic SWSs using the representation of all RF fields and currents by series of spatial harmonics. A characteristic equation for electron waves in a form similar to Pierce's equation [9] for smooth SWSs is obtained. This approach faces difficulties related to cutoff of periodic SWSs at the passband edges, when the energy flow of the electromagnetic wave tends to zero and the coupling impedance, characterizing the efficiency of interaction between one wave and electrons, tends to infinity.

In order to overcome the arising difficulties, Arkadakskiy and Chykin [10], Kuznetsov and Kuznetsov [11], and Kuznetsov *et al.* [12] proposed various modifications of the equation of excitation of periodic SWSs near cutoff frequencies by the RF current of the beam. Several related works are given in [13] and [14].

In this paper, we use the finite-difference excitation equation, obtained in [15] from the general waveguide excitation theory, and the local coupling impedance entering into this equation. In contrast to the theory developed in [9], where the coupling impedance describes the interaction between electrons and one SWS mode through one spatial harmonic of this wave, synchronous with the beam, the local coupling impedance describes the interaction between electrons and the total field of two (forward and counter propagating) SWS waves in interaction gaps and does not tend to infinity at cutoff frequencies [16]–[18]. It allowed to develop in [19], a generalized small-signal theory of interaction of electron beams and waves in passbands and stopbands of SWSs without using SWS equivalent circuits. A universal characteristic equation of electron waves in passbands and stopbands of periodic SWSs was obtained, including the known equations for smooth SWSs and periodic SWSs near cutoff frequencies as special cases [10]–[14].

Herein, a study of beam–wave interaction in a folded-type waveguide SWS is performed on the basis of the method developed in [15]–[20]. An elementary analytical calculation of electrodynamic characteristics of such SWSs necessary to explore the interaction is given in Section III. Specifics of interaction in passbands and near the cutoff frequencies of periodic SWSs are considered in Section IV. Analysis of amplification in stopbands of such SWSs is performed in Section V. Properties of the beam–wave interaction at the cutoff frequency where the folded

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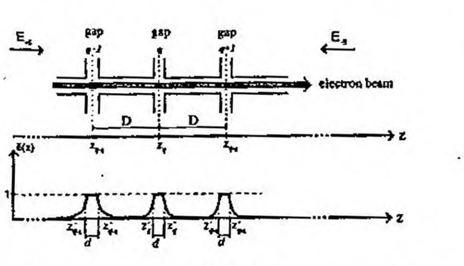

Fig. 1. Schematic of a periodic SWS with discrete beam–wave interaction and distribution of the field in gaps $\bar{e}(z)$, averaged over the beam cross section.

waveguide is an analog of multigap open resonators used in such electron devices, as an orotron, are explored in Section VI.

II. Method of Analysis of Beam-Wave Interaction in Periodic SWSs Based on Finite-Difference Excitation Theory

A. Finite-Difference Equation of Excitation of Periodic SWSs by an Electron Beam

The initial point is the theory of excitation of periodic SWSs (Fig. 1), which is based on the expansion of the total field $\vec{E}(x,y,z,t) = \mathrm{Re}\vec{E}(x,y,z)e^{-i\omega t}$, excited at frequency ω in series in terms of forward $(+s)$ and counter-propagating $(-s)$ modes of the system $\vec{E}_{\pm s}(x,y,z)$ [1], [8], [21]

$$\vec{E} = \sum_s (C_s\vec{E}_s + C_{-s}\vec{E}_{-s}) + \frac{1}{i\omega\varepsilon} j_z\vec{z}_0 \qquad (1)$$

where $C_{\pm s}(z)$ are the excitation coefficients determined by the density of the RF current $j(x,y,z)$ of the beam and the characteristics of modes

$$\frac{dC_{\pm s}}{dz} = \pm\frac{1}{N_s}\int_{S(z)} \vec{j}(x,y,z)\vec{E}_{\mp s}(x,y,z)dS$$

$$N_s = \int_S \{[\vec{E}_s \ \vec{H}_{-s}] - [\vec{E}_{-s} \ \vec{H}_s]\}\vec{z}_0 dS \qquad (2)$$

where N_s is the wave norm, and $S(z)$ is the cross section of the system.

As shown in [20], convergence of the series may be significantly improved by separation of quasi-static field of the space charge $\hat{\vec{E}}(x,y,z)$.

To do this, right-hand side of (1) must be added the quasi-static field $\hat{\vec{E}}(x,y,z)$ and must be subtracted the field represented by a series (1) of the modes of the periodic system at a frequency tending to zero, $\bar{\omega} \to 0$.

For periodic SWSs, the following is exactly obtained:

$$\vec{E}(x,y,z) = \sum_s [C_s(z)\vec{E}_s + C_{-s}(z)\vec{E}_{-s} - \lim_{\bar{\omega}\to 0}\frac{\bar{\omega}}{\omega}(\bar{C}_s\bar{\vec{E}}_s + \bar{C}_{-s}\bar{\vec{E}}_{-s}) + \hat{\vec{E}}$$

$\bar{C}_{\pm s}$ and $\bar{E}_{\pm s}$ are taken at $\bar{\omega} \to 0$.

In the 1-D model of interaction, the density of the RF current is expressed in terms of the distribution function in the beam cross section $\psi(x,y)$

$$\vec{j}(x,y,z) = J(z)\psi(x,y)\vec{z}_0 \qquad (3)$$

normalized by the relationship

$$\int_{S_e} \psi(x,y)dS = 1$$

so that $J(z)$ is the RF current of the beam, and

$$S = \frac{1}{\int_{S_e}\psi^2(x,y)dS}$$

is the effective area of the beam cross section, $dS = dxdy$.

In accordance with the Floquet theorem, the mode field in a periodic waveguide is expressed as

$$\vec{E}_{\pm s}(x,y,z) = E^0_{\pm s}\vec{e}_{\pm s}(x,y,z)e^{\pm ih_s z}$$

where $\vec{e}_{\pm s}(x,y,z)$ are the distribution functions, which are periodic along the z-axis, h_s are the wavenumbers, and $E^0_{\pm s}$ is the amplitude of the chosen field component in point (x^0, y^0, z^0), where the distribution function of this component is equal to unity.

Allowing for (3), and introducing the longitudinal electric field averaged over the beam cross section

$$\bar{E}_z(z) = \int_{S_e} \psi(x,y)E_z(x,y,z)dS \qquad (4)$$

we can transform the equations for dimensionless excitation coefficients $C_{\pm s}(z)$ (2) into the equations for dimensional values (amplitudes) $C^0_{\pm s}(z) = E^0_{\pm s}C_{\pm s}(z)$

$$\frac{dC^0_{\pm s}}{dz} = \mp\frac{R^0_s}{2}J(z)\bar{e}_{\mp s}(z)e^{\mp ih_s z} \qquad (5)$$

where

$$\bar{e}_{\pm s}(z) = \int_{S_e} \psi(x,y)e_{\pm s,z}(x,y,z)dS$$

are the distribution functions of the longitudinal electric field of the forward and counter-propagating waves averaged over the beam cross section, and

$$R^0_s = -\frac{2E^0_s E^0_{-s}}{N_s} \qquad (6)$$

is the specific coupling impedance in point (x^0, y^0, z^0).

General expression (1) for the excited field includes one or several terms of series with spatial harmonics that are synchronous with the electron beam. The remaining terms of the series determine dynamic corrections to quasi-static field of the spatial charge $\hat{\vec{E}}(x,y,z)$. As a rule, these corrections can be disregarded due to rapid convergence of the series [21]. In considering beam–wave interaction in periodic SWSs at the frequencies close to the edges of the passband, the spatial harmonics of the forward and counter-propagating waves turn out to be synchronous with the beam here.

Therefore, let us consider the quasi-static field of the space charge and two terms of series (1) that correspond to these waves. Averaging in accordance with (4), we obtain the longitudinal field $\bar{E}_z(z)$, and after the separation of the sum of fields of forward and counter-propagating waves $\bar{E}(z)$, we obtain

$$\bar{E}_z(z) = \bar{E}(z) + \hat{\bar{E}}(z)$$
$$\bar{E}(z) = C_s^0(z)\bar{e}_s(z)e^{ih_s z} + C_{-s}^0(z)\bar{e}_{-s}(z)e^{-ih_s z}. \quad (7)$$

This expression takes all spatial harmonics of the excited field into account because distribution functions of the field of modes $\bar{e}_{\pm s}(z)$ are not expanded into series in terms of spatial harmonics. At the same time, due to separation of the forward ($+s$) and counter-propagating ($-s$) waves, the usual difficulty remains: their coupling impedance becomes infinitely large at the edges of the passbands. This difficulty can be eliminated, if the excited field of forward and counter-propagating waves is considered as a whole. The approach based on the use of the finite-difference equation of excitation of electrodynamic structures [15] is the most general.

The finite-difference equation of excitation for the averaged total field of forward and counter-propagating waves $\bar{E}_q(z) \equiv \bar{E}(z)$ can be obtained from (5) and (7) in the same way as was done in [15], i.e., by setting down the expressions for the finite differences of field $E_q(z)$ and excitation coefficients $C_{\pm s}^q(z)$. Derivation of the difference equation for the considered 1-D interaction model is given in the Appendix.

We will obtain the second-order finite-difference equation for the total excited field

$$\Delta^2 E_q + 2E_q(1-\cos\varphi_s) = -iR_s^0 \sin\varphi_s J_q d \quad (8)$$

where $\Delta^2 E_q = E_{q+1} + E_{q-1} - 2E_q$, q-gap's number.

The current induced in the qth gap is given by the following expression:

$$J_q = \frac{1}{d_q}\int_{z_q^-}^{z_q^+} J(z)\bar{e}_q(z)dz. \quad (9)$$

For flat interaction gaps of width d, we have $\bar{e}_q(z) \equiv 1$ inside the gap and $\bar{e}_q(z) \equiv 0$ outside the gap. For continuous functions $\bar{e}_q(z)$, an equivalent flat gap of width d_q can be introduced. Assuming that the field intensity in the middle of the equivalent flat gap is E_q and introducing the gap voltage U_q, the width can be determined from the following:

$$d_q = -\frac{U_q}{E_q} = -\frac{E_q\int_{z_q^-}^{z_q^+}\bar{e}_q(z)dz}{E_q} = \int_{z_q^-}^{z_q^+}\bar{e}_q(z)dz. \quad (10)$$

Differential equation of excitation (8) may also be rearranged into the equation for the gap voltages $U_q = -E_q d$

$$\Delta^2 U_q + 2U_q(1-\cos\varphi_s) = iR_s^0 J_q d^2 \sin\varphi_s. \quad (11)$$

It is important that the impedance $R_s^0 d^2 \sin\varphi_s$ on the right-hand side of this equation does not tend toward infinity at the edges of the SWS passband [16], [18].

Thus, for periodic waveguides, an everywhere-limited local coupling impedance that accounts for both the forward and counter-propagating waves simultaneously can be introduced

$$Z_s = R_s^0 d^2|\sin\phi_s|. \quad (12)$$

Its relationship with the coupling impedance $K_{s,m}$ of the mth spatial harmonic is determined from the following:

$$K_{s,m} = \frac{|\bar{E}_{s,m}|^2}{2h_{s,m}^2 P_s} = |R_s^0|\frac{|e_{s,m}|^2}{h_{s,m}^2}$$
$$e_{s,m} = \frac{1}{D}\int_{-\frac{D}{2}}^{+\frac{D}{2}}\bar{e}(z)e^{-ih_{s,m}}dz, \quad h_{s,m} = h_s + \frac{2\pi m}{D}$$

where $e_{s,m}$ and $h_{s,m}$ are the dimensionless amplitudes and wavenumbers of spatial harmonics. As a result, we have

$$K_{s,m} = Z_s\frac{|e_{s,m}|^2}{\phi_{s,m}^2|\sin\phi_s|}, \quad \phi_{s,m} = h_{s,m}D. \quad (13)$$

Within the passband, where only one synchronous wave is excited and R_s^0 is limited, either difference equation (8) or source expression (7), allowing for only one forward wave in it, which matches an equation in finite differences of the first order, may be used for the SWS field.

At the edges of passbands $\varphi_s = 0, \pi, 2\pi, \ldots$, the local impedance Z_s value remains limited, while $K_{s,m} \to \infty$. Outside the passbands, a reactive attenuation depending on the frequency appears, while the real parts of the wavenumber and the phase shift per period remain constant. Therefore

$$h_s = p\frac{\pi}{D} + ih_s''(\omega), \quad p = 0, 1, 2, \ldots$$

where the imaginary part of the wavenumber determines the attenuation of the waves in the SWS stopband.

Phase shift φ_s (in general, it is complex) and local coupling impedance Z_s, entering the finite-difference equation (8), are sufficient for a description of excitation of a periodic SWS inside, outside, and at the edges of the passband. They can be calculated either when processing the results of computer simulation of SWS fields. Thus, the finite-difference equation of excitation (8) makes it possible to describe the beam–wave interaction without using any equivalent circuits of periodic SWSs, as it was done in [5]–[7] and some other works. We also note that the finite-difference equation of excitation can be used in the theory of beam–wave interaction both the small-signal (linear) theory and a large signal for calculating the efficiency and output power electron devices. In this paper, this equation is applied to develop a general linear theory of beam–wave interaction in 0-type devices.

B. Equations of the Small-Signal (Linear) Theory of Electron Beam Bunching in Periodic SWSs

Now let us write the equations of the linear theory of electron bunching in the field $\bar{E}(z)$ of a periodic SWS.

Electron motion equations, continuity equation, and expressions for electron beam current harmonics [1], [21] are the source expressions. Let us write as usual

$$\omega t(z, t_0) = \omega t_0 + h_e z + \vartheta(z, t_0) \quad (14)$$

where ϑ is the electron phase perturbation induced by the RF field and t_0 is the electron transit moment of the SWS beginning $z = 0$. In the linear theory, it is assumed that $|\vartheta| \ll 1$ for the small signal and the field only at the frequency ω, that is also considered as small $\sim|\vartheta|$, is taken into account. The electron motion equation and the expression for the first harmonic of the electron beam current will look as follows:

$$\frac{\partial}{\partial z}\left(\frac{mv_e\tilde{v}}{e}\right) = \mathrm{Re}\left[\bar{E}(z) + \frac{\Gamma}{i\omega\varepsilon_0}\frac{J(z)}{S}\right]e^{-i(\omega t_0 + h_e z)} \quad (15)$$

$$\frac{\partial\vartheta}{\partial z} = -\frac{\omega}{v_e^2}\tilde{v} \quad (16)$$

$$J(z) = \frac{J_0}{\pi}\int_0^{2\pi} e^{i\omega t_0}\cdot i\vartheta d(\omega t_0)\cdot e^{ih_e z} \quad (17)$$

where v_e and $\tilde{v}$ are the constant and variable electron velocities, $h_e = (\omega/v_e)$—an electron wavenumber, and Γ—longitudinal quasi-static field depression coefficient.

Multiplying (15) and (16) by $e^{i\omega t_0}$, integrating the result over ωt_0, and taking (17) into account, we obtain

$$\frac{dI}{d\zeta} = -iV \quad (18)$$

$$\frac{dV}{d\zeta} = -i\sigma^2 I + F \quad (19)$$

where

$$I = \frac{1}{\pi}\int_0^{2\pi} i\vartheta e^{i\omega t_0}d(\omega t_0) \quad (20)$$

$$V = \frac{1}{\pi}\int_0^{2\pi}\frac{\tilde{v}}{\varepsilon v_e}d(\omega t_0) \quad (21)$$

$$F = \frac{De}{dm\omega v_e\varepsilon^2}E_z e^{-ih_e z} \quad (22)$$

are the dimensionless amplitudes of the RF current, the electron velocities, and the field in the gap; $\zeta = \varepsilon h_e z$ is a dimensionless coordinate; ε is a small parameter that in general case may be chosen arbitrarily, for example, as the gain parameter C of a TWT or the ratio of the plasma frequency to the operating frequency; $\sigma^2 = \Gamma(\omega_p/\varepsilon\omega)^2$ is the space charge parameter; and $\omega_p = (e/\varepsilon_0 m)(J_0/v_e S)$ is the plasma frequency without considering the depression.

On the basis of (18) and (19), it is not difficult to obtain the following linear equations for the dimensionless RF current:

$$\frac{d^2 I}{d\zeta^2} + \sigma^2 I = -iF \quad (23)$$

as well as the equation for the dimensional RF current J

$$\frac{d^2 J}{dz^2} - 2ih_e\frac{dJ}{dz} + \left(\Gamma h_p^2 - h_e^2\right)J = -ih_p^2 S\omega\varepsilon_0\bar{E}. \quad (24)$$

These equations have been repeatedly derived and used in the theory of TWTs and klystrons. Here, they are used in order to study the development of a self-consistent theory of beam-wave interaction in the periodic SWS with interaction gaps, spaced at step D.

C. Universal Characteristic Equation of Electron Waves and Its Special Cases

Equations (23) and (24) together with the use of finite-difference equation of excitation (8), taking (9) and (22) into account, allow to find the matrix of coefficients a_{ij}, relating the dimensionless values of RF current of electron beam I, velocities of electrons (kinetic potential) V, and field F in the $(q+1)$th SWS interaction gap with their values in one, and for the field in two previous gaps

$$\begin{aligned} I_{q+1} &= a_{11}I_q + a_{12}V_q + a_{13}F_q \\ V_{q+1} &= a_{21}I_q + a_{22}V_q + a_{23}F_q \\ F_{q+1} &= a_{31}I_q + a_{32}V_q + a_{33}F_q + a_{34}F_{q-1}. \end{aligned} \quad (25)$$

In the small-signal theory, one can look for the solution in the form of electron waves, for which

$$I_{q+1} = \lambda I_q,\quad V_{q+1} = \lambda V_q,\quad F_{q+1} = \lambda F_q.$$

At that, from (25), we come to the following system of third-order homogeneous linear equations:

$$\begin{aligned} (a_{11} - \lambda)I_q + a_{12}V_q + a_{13}F_q &= 0 \\ a_{21}I_q + (a_{22} - \lambda)V_q + a_{23}F_q &= 0 \\ a_{31}I_q + a_{32}V_q + \left(a_{33} - \lambda + \frac{a_{34}}{\lambda}\right)F_q &= 0. \end{aligned} \quad (26)$$

Equating the determinant of this system to zero, we obtain, however, a quadratic equation for eigenvalues $\lambda = e^{i\psi}$, what is explained by the fact that excitation equation (8), relating the dimensionless field values $F_q \sim U_q$ at three steps of the SWS, is a difference equation. As a result, expanding system (26) determinant, we come to the universal characteristic equation of electron waves in periodic SWSs, obtained in [19]. The equation determines the complex perturbation ψ of the electron wave phase change per an SWS step with respect to an unperturbed phase change in the electron beam $\varphi_e = (\omega/v_e)D$, because $U_{q+1} = U_q\exp(i(\varphi_e + \psi))$. In the general case, the obtained equation coefficients depend on many parameters—electron beam current and voltage, electron conductivity of interaction gaps, and other parameters of the beam and SWS. In particular, ε has a meaning of amplification parameter C in a TWT with the distinction that it is expressed via the local coupling impedance and, therefore, has no particularities at cutoff frequencies and is defined in SWS stopbands. Therefore, the characteristic equation of electron waves, obtained in [19], is true in SWS passbands, stopbands, and at cutoff frequencies. Here, it is applied in order to study the electron beam interaction in a folded waveguide when the space charge is small and the interaction gaps are thin, and when the analytical equation solutions are possible. In this case, as shown in [19], coefficients a_{ij} take the following form:

$$\begin{aligned} &a_{11} = 1,\quad a_{21} = 0,\quad a_{31} = \mp 2i\varepsilon\varphi_e^2\cdot e^{-i\varphi_e} \\ &a_{12} = -i\varepsilon\varphi_e,\quad a_{22} = 1,\quad a_{32} = 0 \\ &a_{13} = -i(\varepsilon\varphi_e)^2,\quad a_{23} = \varepsilon\varphi_e \\ &a_{33} = 2\cdot\cos\varphi_s\cdot e^{-i\varphi_e},\quad a_{34} = -e^{-2i\varphi_e} \end{aligned} \quad (27)$$

where the small parameter ε has been chosen by analogy with the TWT gain parameter

$$\varepsilon^3 = \frac{Z_s(-J_0)}{4U_e\varphi_e^3},\quad -J_0 > 0. \quad (28)$$

The upper character corresponds to the SWS with normal dispersion of the fundamental spatial harmonic $n = 0$, which is the operating one at $0 \le \varphi_s \le \pi$, the lower character—to the folded waveguide-type SWS, where the first spatial harmonic is used at $\pi < \varphi_s \le 2\pi$. It is also easy to obtain (27) directly from (11), (18), and (19), assuming that the interaction gaps are flat and thin and ignoring the space charge.

As usually, the gain parameter is small, $\varepsilon \sim C \ll 1$ and $\psi \ll 1$. Ignoring the terms of higher than the third order of smallness with respect to ε in (16), we obtain the following characteristic equation:

$$2(1-\cos\psi)[\cos\varphi_s - \cos(\varphi_e+\psi)] \pm \varepsilon^3\varphi_e^4 = 0. \quad (29)$$

Assuming also that $\cos\psi \cong 1-(\psi^2/2), \sin\psi \approx \psi$, we obtain from (29), the following quadratic algebraic characteristic equation for the complex electron waves phase perturbation:

$$\frac{\psi^4}{2}\cos\varphi_e + \psi^3\sin\varphi_e + \psi^2(\cos\varphi_s - \cos\varphi_e) \pm \varepsilon^3\varphi_e^4 = 0. \quad (30)$$

In the theory of electron devices, this small perturbation is usually normalized using the gain parameter ε [4] by introducing the value η (according to Vainstein [8]) or δ (according to Pierce [9])

$$\eta = \frac{\psi}{\varepsilon\varphi_e} = \frac{\psi' + i\psi''}{\varepsilon\varphi_e} = \eta' + i\eta'' = i\delta^* = -y + ix. \quad (31)$$

Then, the characteristic equation for electron waves in a periodic structure takes the following form:

$$\frac{\varepsilon}{2}\cos\varphi_e\eta^4 + \frac{\sin\varphi_e}{\varphi_e}\eta^3 + \frac{\cos\varphi_s - \cos\varphi_e}{\varepsilon\varphi_e^2}\eta^2 \pm 1 = 0. \quad (32)$$

III. Dispersion Characteristics and Local Coupling Impedance of the Folded Waveguide

Let us apply the derived equations to study the electron wave interaction in the uniform folded waveguide-type SWS without taking reflections from its bends into account, for example, in the serpentine waveguide with smooth bends.

The field phase change along the waveguide over a loop of length l corresponding to step $D = L/2$

$$\phi_s = hl, \quad h = \sqrt{k^2 - k_s^2}$$

where $k = 2\pi/\lambda$ is the wavenumber in vacuum and $k_s = 2\pi/\lambda_s$ is the critical wavenumber. Phase advance of the nth spatial harmonic along the z-axis of the SWS on period L will make up: $\varphi_{s,n} = 2\varphi_s + 2\pi n, n = 0, \pm1, \pm2, \ldots$, and on step $D = L/2$, allowing for the geometric rotation of the waveguide of 180°

$$\varphi_n = \varphi_{s,n}/2 - \pi = \varphi_s + \pi(2n-1). \quad (33)$$

The fundamental spatial harmonic is a backward wave, because $\varphi_0 = \varphi_s - \pi < 0$ at $0 \le \varphi_s < \pi$, for the first operating spatial harmonic $\varphi_1 = \varphi_s + \pi$. The operating harmonic slowing factor is determined from the relationship

$$\frac{c}{v_1} = \frac{\phi_l}{kD} = \frac{l}{D}\sqrt{1-\left(\frac{\lambda}{\lambda_\pi}\right)^2} + \frac{\lambda}{2D}$$
$$= \frac{c}{v_{1,\pi}}\left(\frac{2l}{\lambda_\pi}\sqrt{1-\left(\frac{\lambda}{\lambda_\pi}\right)^2} + \frac{\lambda}{\lambda_\pi}\right). \quad (34)$$

Here, λ_π is the low-frequency edge of the main passband associated with $\varphi_1 = \pi$; it is determined by the critical frequency of the folded waveguide

$$\lambda_\pi = \lambda_{\kappa p}, \quad h = 0, \quad \frac{c}{v_{1,\pi}} = \frac{\lambda_\pi}{2D}.$$

It is seen from (34) that the form of the dispersion characteristic is governed by only one parameter, (l/λ_π). The RF boundary $\lambda_{2\pi}$ of the main passband, associated with $\varphi_{s,1} = 2\pi$ (the Bragg resonance), is absent in an ideal waveguide without reflections; however, it appears in a real system because of reflections from waveguide bends.

It is easy to calculate the local coupling impedance of a uniform folded waveguide according to (12), using (2) for the wave norm N_s, (7) for the specific coupling impedance, and expressions for the R_s^0-type operating wave field components for a rectangular waveguide

$$H_z = C_s\cos(k_s x)e^{ihz}$$
$$H_x = -i\frac{h}{k_s}C_s\sin(k_s x)e^{ihz}$$
$$E_y = i\frac{k}{k_s}C_s Z_0\sin(k_s x)e^{ihz}$$
$$E_x = E_z = 0, \quad H_y = 0$$

where $k = \omega/c$, $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ is the wavenumber and impedance in free space and $k = \pi/a$ is the critical wavenumber of wave H_{10} of the waveguide. x-, y-, and z-axis are directed along the a size wide side, b size narrow side and longitudinal axis of the considered waveguide section, respectively. Given that for the counter-propagating wave $(-s)$, the wavenumber h changes its sign, we obtain

$$N_s = C_s C_{-s}\frac{kh}{(\frac{\pi}{a})^2}\cdot ab\cdot Z_0, \quad R_s^0 = 2\frac{k}{h}\frac{1}{ab}Z_0$$
$$Z_s = 2\frac{k}{h}\frac{b}{a}Z_0\cdot|\sin\varphi_s|. \quad (35)$$

The dispersion curves of the uniform folded waveguide in the passband and in the stopband are analytically calculated from the expression for the wavenumber of the waveguide h by (33), (34), and (36).

Fig. 3(a) shows the results of calculation of dispersion of the +1st spatial harmonic of the forward and backward waves (the −1st spatial harmonic of the counter-propagating wave), which have the same phase velocity v_π at the π-form cutoff frequency. The calculation is performed for three typical SWS variants, when the slowing factor $c/v_\pi = 4$. The wave characteristics are given not only in the SWS passband but also in the low-frequency stopband $\lambda > \lambda_\pi$, what is necessary

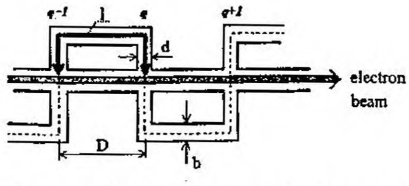

Fig. 2. Rectangular folded waveguide. Cross section of the waveguide $a * b$, for a uniform waveguide $d = b$, D-step, and $2D = L$—period of folded waveguide.

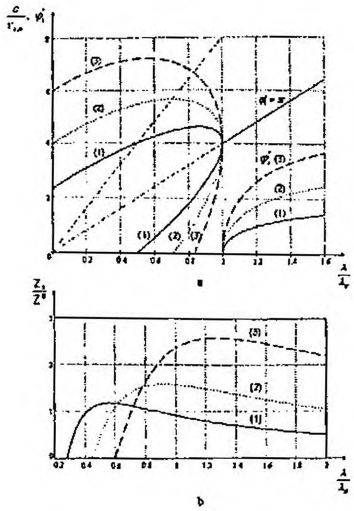

Fig. 3. Electrodynamic characteristics of a uniform folded waveguide as a function of wavelength. (a) Slowing factor of forward and backward waves in the passband $\lambda/\lambda_\pi < 1$ and decay constant φ_s'' in the stopband at $\lambda/\lambda_\pi > 1$. (b) Normalized local coupling impedance Z_s/Z^0 and λ_π—cutoff wave length. 1—$l/\lambda_\pi = 0.29$, 2—$l/\lambda_\pi = 0.5$, and 3—$l/\lambda_\pi = 0.75$.

for the analysis of the interaction in this band, where the phase shift is complex

$$\phi_s = \phi_s' + i\phi_s'', \quad \phi_s' = 0, \quad \phi_1' = \pi$$
$$\phi''_{\pm s} = \pm h'' \cdot l = \pm\frac{2\pi l}{\lambda_\pi}\sqrt{1-\left(\frac{\lambda_\pi}{\lambda}\right)^2}. \tag{36}$$

Local coupling impedance dependencies on the wavelength, given in Fig. 3(b), show its continuous variation when passing from the passband to the stopband, moreover the maximum of Z_s may be in one or another band when the waveguide dimensions change. The absolute value of Z_s depends on the shape of the folded waveguide. Z_s may increase significantly for a folded H-waveguide or a folded slot line.

Three areas of electron wave interaction in a periodic waveguide may be singled out. They are shown in Brillouin's diagram (Fig. 4).

1) TWT-type interaction in the passband at $\omega_\pi < \omega < \omega_{2\pi}$, with the area center at $\varphi_s = (3\pi/2)$.

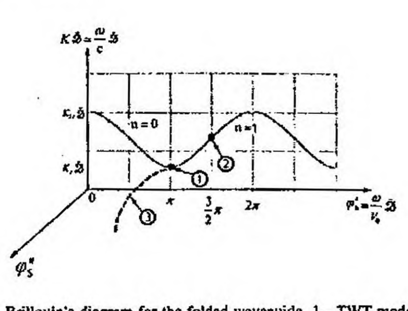

Fig. 4. Brillouin's diagram for the folded waveguide. 1—TWT mode on the first operating spatial harmonic in the passband at $\varphi_s' \approx (3\pi/2)$. 2—orotron mode at the cutoff frequency ω_π at $\varphi_s' = \pi$. 3—klystron mode in the stopband at $\omega < \omega_\pi$.

2) Orotron-type interaction at the cutoff frequency at $\omega = \omega_\pi$, when the waveguide is similar to open resonators, used in such devices, as an orotron.
3) Klystron-type interaction in the stopband at $\omega < \omega_\pi$ (Section V).

IV. Properties of Beam–Wave Interaction in the Passband of Periodic Folded Waveguide-Type SWSs

Let us consider the special features of electron beam interaction with folded waveguide-type SWS waves, when its first spatial SWS harmonics ($n = 1$) is operating in the TWT. Let us use (32) at $\pi \le \varphi_s = \varphi_{s,1} \le 2\pi$. In case of exact synchronism, when $\varphi_e = \varphi_{s,1}$, in some cases, it is possible to obtain its analytical solutions.

In the middle of the passband, when $\varphi_s = (3\pi/2)$, we obtain a cubic equation and the corresponding solution

$$\eta^3 = -\frac{3\pi}{2}, \quad \eta_{1,2} = \sqrt[3]{\frac{3\pi}{2}}\left(\frac{1}{2} \pm i\frac{\sqrt{3}}{2}\right)$$
$$\eta_3 = -\sqrt[3]{\frac{3\pi}{2}}, \quad \psi_j = \varepsilon\frac{3\pi}{2}\eta_j. \tag{37}$$

Here, the counter-propagating SWS wave has no effect on the solution form. In contrast, at the passband edge at $\varphi_s = \pi$, one of the spatial harmonics of both SWS forward and counter-propagating waves is synchronous with the electron beam, and we come to a quadratic equation, determining four electron waves

$$\eta^4 = -\frac{2}{\varepsilon}, \quad \psi^4 = -2\varepsilon^3\pi^4$$
$$\psi_{1,2} = \pm 2^{-\frac{1}{4}}\pi(1+i)\varepsilon^{\frac{3}{4}}$$
$$\psi_{3,4} = \pm 2^{-\frac{1}{4}}\pi(1-i)\varepsilon^{\frac{3}{4}}. \tag{38}$$

If the interaction decreases, then $\varepsilon \to 0$, we obtain $\eta \to \infty$, what points to incorrectness of normalization (31) at the edge of the SWS passband. However, we have a finite solution for ψ, which shows that constant increases in electron waves at the edge of the passband are $\sim\varepsilon^{3/4}$ (rather than ε, as in the middle of the passband or in smooth SWSs). In other words, $\varepsilon^{3/4}$ is the gain parameter at the edge of the passband.

V. Conditions of Wave Amplification in the Folded Waveguide-Type Periodic SWS Stopband

In periodic SWSs in stopbands, the field phase throughout the SWS is either the same or jumps to π in certain cross sections as in cavity resonators or standing waves if ohmic losses are ignored. In this case, synchronous interaction of the electron beam and the field is possible. This interaction conditions are slightly different in various SWS bands, but anyway the phase advance value in the electron beam φ_e in the case of exact synchronism is a multiple of π, because $\varphi_e = \varphi_s$. In (30) and (32), the cubic summand disappears and we arrive at a biquadratic characteristic equation, whose analytical solutions allow to explore amplification in SWS stopbands. Similarly, analytical solutions and more general characteristic equation (29) are derived in this case.

In the low-frequency stopband of the folded waveguide-type SWS, we have on the first spatial harmonic and in the case of exact synchronism of electrons with it, in accordance with (33) and (36)

$$\varphi_s = \pi + i\varphi_s^{//}, \quad \varphi_e = \pi. \tag{39}$$

We obtain the following characteristic quadratic equation with respect to $\cos\psi$:

$$2(1-\cos\psi)\left[ch\varphi_s^{//} - \cos\psi\right] - \varepsilon^3\pi^4 = 0. \tag{40}$$

Its solutions look as follows:

$$\cos\psi = 1 + \frac{1}{2}\left(ch\varphi_s^{//} - 1\right) \mp \frac{1}{2}\sqrt{\left(ch\varphi_s^{//} - 1\right)^2 - 2\varepsilon^3\pi^4}. \tag{41}$$

If there is no interaction of electrons and SWS field, then $\varepsilon = 0$ and we obtain two values

$$\cos\psi = \begin{cases} 1 & \text{in case of the upper character} \\ ch\varphi_s^{//} & \text{in case of the lower character.} \end{cases}$$

The first of them determines two coincident waves of the electron beam without considering the spatial charge, $\psi_{1,2} = 0$; if it is considered they are split into two space-charge waves—fast and slow. The second value of $\cos\psi$ corresponds to two reactively decaying SWS waves in the stopband, at $m = 1$, we have $\psi_{3,4} = \pi \pm i\varphi_s^{//}$.

As the amplification parameter ε grows, the $\cos\psi$ value changes, however, it remains real up to the critical point, determined by the relationship

$$\sqrt{2\varepsilon_g^3}\pi^2 = ch\varphi_s^{//} - 1. \tag{42}$$

At $\varepsilon > \varepsilon_g \cos\psi$, values are complex and electron wave amplification in the SWS stopband is possible. Thus, the electron waves may amplify near the cutoff frequency, but bunching of the electron beam occurs inside the stopband, as in a klystron.

The conclusions regarding the possibility of electron waves amplification at $\varepsilon > \varepsilon_g$ may be substantiated by applying the third conservation law in electron beams (see [21, Appendix VI]), from which the following kinetic theorem Chu follows in the linear theory:

$$P_K(z) - P_K(0) = V_e J_0 \varepsilon \mathrm{Re} I V^* \tag{43}$$

where $P_K(z)$ is the kinetic energy flow of electrons at cross section z, V_e, and J_0 are the accelerating voltage and current of the beam. In case of discrete interaction, for each jth electron wave, we can obtain from it

$$P_{q+1,j} - P_{q,j} = V_e J_0 \exp\left(-2\psi_i^{//}\right)\varepsilon \mathrm{Re} I_{q,j} V_{q,j}^* \tag{44}$$

where $P_{q,j}$—this wave kinetic energy flow in the qth interaction gap. Using the first two relationships (25) and eliminating F_q from them, let us also find the complex impedance of the electron wave $Z_j = I_{q,j}/V_{q,j}$, and the value

$$\begin{aligned}\mathrm{Re}\left(I_{q,j}V_{q,j}^*\right) &= |V_{q,j}|^2 Z_{q,j} \\ &= -|V_{q,j}|^2 \mathrm{Re}\frac{a_{23}a_{12} - a_{13}(a_{22} - \lambda_j)}{a_{23}(a_{11} - \lambda_j) - a_{13}a_{21}}\end{aligned} \tag{45}$$

characterizing its active power change at the SWS step. Let us calculate this value in the considered case of a small space charge and thin interaction gaps, when matrix coefficients are determined by (27), and taking that $\psi \ll 1$ into account, so that $\lambda_j = \exp(i\psi_j) \approx 1 + i\psi_j$.

We will obtain

$$\mathrm{Re} I_{q,j} V_{q,j}^* = -|V_{q,j}|^2 \varepsilon\varphi_e \frac{\psi_j^/}{|\psi_j|^2} = -|V_{q,j}|^2 \frac{\eta_j^/}{|\eta_j|^2}. \tag{46}$$

From (44) and (46), it is seen that at $\psi_j^/ = \varepsilon\varphi_e\eta_j^/ > 0$, we have $P_{q+1,j} - P_{q,j} < 0$, i.e., the beam gives up energy to the electron wave, otherwise quite the opposite. At the same time, the wave amplitude grows in the positive direction at $\psi_j^{//} < 0$, because

$$\begin{aligned}U_{q+1,j} &= U_{q,j}\exp(i(h_e + \psi_j)) \\ &= U_{q,j}\exp\left(i\left(h_e + \psi_j^/\right)\right)\cdot\exp\left(-\psi_j^{//}\right).\end{aligned}$$

The above analytical solutions allow to analyze various cases.

VI. Beam–Wave Interaction at the Cutoff Frequency of the Folded Waveguide as an Analog of Orotron

At the cutoff frequency, the rectangular waveguide may be considered as an open resonator with mirrors, formed by narrow walls, spaced at intervals a. There is a standing electromagnetic wave with the resonating wavelength $\lambda_s = 2a$ between these walls. This wave determines the cutoff frequency ω_s of the lowest mode H_{10}. When passing to the folded rectangular waveguide (Fig. 2), this cutoff frequency is preserved, if the waveguide bends are assumed to be sufficiently smooth and reflections from them are ignored. As the wave phase velocity at the cutoff frequency is equal to infinity, the field phase along the straight waveguide is constant, and in the folded waveguide, it differs by π in adjacent interaction gaps due to geometric rotation of the waveguide. Synchronous beam–wave interaction is obtained when the transit time of one SWS step is

$$T_e = \frac{D}{v_e} = \frac{T_s}{2} = \frac{1}{2f_s} \quad \text{or} \quad \frac{c}{v_e} = \frac{\lambda_s}{2D}.$$

Thus, at the cutoff frequency, the folded waveguide is similar to a multigap open resonator, where the beam–wave interaction as in an orotron is possible. Phase constants of so-generated electron waves are determined by (38) that may be easily rearranged in the following form:

$$\psi_j^2 = \mp i\sqrt{2\varepsilon^3\pi^4}$$
$$\psi_j = e^{i\frac{\pi}{2}\left(j+\frac{1}{2}\right)}\pi\sqrt[4]{2\varepsilon^2}, \quad j = 0, 1, 2, 3.$$

Let us consider various waves.

At $j = 0$, the electron beam gives up energy to the wave ($\psi' > 0$), growing in the negative direction ($\psi'' > 0$). This case corresponds to the backward-wave oscillator (BWO).

At $j = 1$, the electron beam takes energy from the wave ($\psi' < 0$), decaying in the positive direction ($\psi'' > 0$). This case corresponds to the decaying forward wave in a TWT.

At $j = 2$, the electron beam takes energy from the wave ($\psi' < 0$), the beam decays in the negative direction ($\psi'' < 0$). This case corresponds to the decaying backward wave in a BWO.

At $j = 3$, the electron beam gives up energy to the wave ($\psi' > 0$), growing in the positive direction ($\psi'' < 0$). This case corresponds to the amplified forward wave in a TWT.

Combination of these waves leads to different beam–wave interaction modes—amplifying or generating. Their selection is determined by the value of the load at the folded waveguide ends, consideration of which is a singular boundary problem.

VII. Conclusion

The theory of beam–wave interaction in SWSs, developed on the basis of the finite-difference equation of SWS excitation, has a number of advantages compared with the traditional wave analysis. It allows to study the interaction not only in passbands but also in stopbands, regardless of the used SWS type. The obtained analytical solutions of the universal characteristic equation of electron waves allowed to explore the special features of interaction in passbands and near the cutoff frequencies of periodic folded waveguide-type SWSs, to find conditions of amplification in stopbands of such SWSs. Properties of the beam-wave interaction at the cutoff frequency, where the folded waveguide is an analog of multigap open resonators, used in such electron devices, as an orotron, are examined.

Appendix
Finite-Difference Equation of Excitation of Periodic SWSs

Allow finite differences of the first and second orders, relating the excited field in three SWS cross sections, z, $z \pm D$

$$\begin{aligned}
\pm\Delta_\pm \bar{E} &= \bar{E}(z \pm D) - \bar{E}(z)\\
\Delta^2 \bar{E} &= \Delta_+ \bar{E} - \Delta_- \bar{E}\\
&= \bar{E}(z + D) - 2\bar{E}(z) + \bar{E}(z - D)\\
\pm\Delta_\pm C_s^0 &= C_s^0(z \pm D) - C_s^0(z)\\
\pm\Delta_\pm C_{-s}^0 &= C_{-s}^0(z \pm D) - C_{-s}^0(z).
\end{aligned} \tag{1A}$$

Let us calculate these differences near the qth interaction gap, representing the field as a sum of forward and counter-propagating waves in the form of (7), and apply it to the field in gaps, assuming $z = z_q$, $z_q \pm D$. As usually, the gaps are loosely coupled along the beam channel, it is possible to ignore the field phase changes along the z-axis in each interaction gap and assume that it is constant. Therefore, in expressions for the field (7), we have within the limits of the qth gap

$$\bar{e}_{\pm s}(z)e^{\pm i h_s z} = \bar{e}(z)e^{\pm i\psi_q} \tag{2A}$$

where $\bar{e}(z)$ is a real field distribution function, that is the same for forward and counter-propagating waves in all gaps, and $\psi_q = h_s z_q$ is the constant field phase in a gap.

Given that $\bar{e}(z)$ is a periodic function, i.e., $\bar{e}(z + D) = \bar{e}(z)$ and allowing for (1A) and (2A), we obtain from (7) in the qth gap

$$\bar{E}(z) = \bar{e}(z)\left[C_s^0(z)e^{i\psi_q} + C_{-s}^0(z)e^{-i\psi_q}\right] \tag{3A}$$

and in $q \pm 1$ gaps

$$\begin{aligned}
\bar{E}(z \pm D) = \bar{e}(z)\big\{&\left[C_s^0(z) \pm \Delta_\pm C_s^0\right]e^{i\psi_{q\pm1}}\\
&+ \left[C_{-s}^0(z) \pm \Delta_\pm C_{-s}^0\right]e^{-i\psi_{q\pm1}}\big\}.
\end{aligned} \tag{4A}$$

By substituting (3A) and (4A) into the expression for finite difference of the second order (1A), we obtain

$$\begin{aligned}
\Delta^2 \bar{E} &= \bar{E}(z + D) - 2\bar{E}(z) + \bar{E}(z - D)\\
&= \bar{e}(z)\big\{\left[e^{i\psi_{q+1}} - 2e^{i\psi_q} + e^{i\psi_{q-1}}\right]C_s^0(z)\\
&\quad + \left[e^{-i\psi_{q+1}} - 2e^{-i\psi_q} + e^{-i\psi_{q-1}}\right]C_{-s}^0(z)\\
&\quad + \Delta_+ C_s^0 e^{i\psi_{q-1}} + \Delta_+ C_{-s}^0 e^{-i\psi_{q+1}}\\
&\quad - \Delta_- C_s^0(z)e^{i\psi_{q-1}} - \Delta_- C_s^0(z)e^{-i\psi_{q-1}}\big\}.
\end{aligned} \tag{5A}$$

In the SWS with one interaction gap for the period (when $L = D$), we have in the qth gap $\psi_q = h_s z_q$ and in the $(q + 1)$th gap $\psi_{q\pm1} = \psi_q \pm \varphi_s$, where $\varphi_s = h_s D = \varphi_0$ is the phase shift on the step (Fig. 1) for the fundamental spatial harmonic of the field.

For the folded waveguide (when $L = 2D$, Fig. 2) allowing for geometric rotation of 180°, we have in adjacent gaps $\psi_{q\pm1} = \psi_q \pm \varphi_1$, $\varphi_1 = \varphi_s + \pi$ is the first spatial harmonic phase shift. In the SWS passband, we have $0 \leq \varphi_0 \leq \pi$, $\pi \leq \varphi_1 \leq 2\pi$.

Taking the last expressions into account, we can obtain from (5A), the finite-difference equation for the field E_q amplitude

$$\Delta^2 E_q + 2E_q(1 - \mathrm{Cos}\varphi_n) = G_q \tag{6A}$$

where $n = 0, 1$ and the field distribution function decreases, because $\bar{E}(z) = E_q\bar{e}(z)$. G_q is as follows:

$$\begin{aligned}
G_q = e^{i\psi_q}&\left[\Delta_+ C_s^0 e^{i\varphi_n} - \Delta_- C_s^0 e^{-i\varphi_n}\right]\\
+ e^{-i\psi_q}&\left[\Delta_+ C_{-s}^0 e^{-i\varphi_n} - \Delta_- C_{-s}^0 e^{i\varphi_n}\right]
\end{aligned} \tag{7A}$$

and it determines the field excitation by the electron beam current via finite differences $\Delta_\pm C_{\pm s}^0$ in accordance with (5A).

To calculate the finite differences $\Delta_\pm C_{\pm s}^0$, let us use (5), taking (2A) into account and field phase change in adjacent interaction gaps, $\psi_{q\pm1} = \psi_q \pm \varphi_n$. Introducing the following notation for multiplicity $f(z) = (R_s^0/2)J(z)\tilde{e}(z)$, we can write (5) in the following form:

$$\frac{dC_{\pm s}^0}{dz} = \begin{cases} \mp f(z)e^{\mp i\psi_q} & \text{for } z_q - \frac{D}{2} < z < z_q + \frac{D}{2} \\ \mp f(z)e^{\mp i\psi_{q+1}} & \text{for } z_q + \frac{D}{2} < z < z_q + D \\ \mp f(z)e^{\mp i\psi_{q-1}} & \text{for } z_q - D < z < z_q - \frac{D}{2}. \end{cases} \tag{8A}$$

Integrating this equation, we find

$$\begin{aligned} \Delta_+ C_{\pm s}^0 &= C_{\pm s}^0(z_q + D) - C_{\pm s}^0(z_q) \\ &= \mp e^{\mp i\psi_q} \int_{z_q}^{z_q+\frac{D}{2}} f(z)dz \mp e^{\mp i\psi_{q+1}} \int_{z_q+\frac{D}{2}}^{z_q+D} f(z)dz \\ \Delta_- C_{\pm s}^0 &= C_{\pm s}^0(z_q) - C_{\pm s}^0(z_q - D) \\ &= \mp e^{\mp i\psi_{q-1}} \int_{z_q-D}^{z_q-\frac{D}{2}} f(z)dz \mp e^{\mp i\psi_q} \int_{z_q-\frac{D}{2}}^{z_q} f(z)dz. \end{aligned} \tag{9A}$$

Substituting the obtained expression into (7A), we find simple expressions for the qth gap excitation function

$$G_q = -iR_s^0 \sin\varphi_m \int_{z_q-\frac{D}{2}}^{z_q+\frac{D}{2}} J(z)\tilde{e}(z)dz. \tag{10A}$$

It is important that the integrals of the current in adjacent gaps $q \pm 1$ annihilate each other, therefore the qth gap excitation is determined only by the electron beam current in this gap. If the current, induced in the qth gap, is now introduced by the following relationship:

$$J_q = \frac{1}{d} \int_{z_q-\frac{D}{2}}^{z_q+\frac{D}{2}} J(z)\tilde{e}(z)dz \tag{11A}$$

excitation equation (6A) will take on the form of (8).

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