Reflectionless Propagation of Acoustic Waves in the Solar Atmosphere

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Abstract—The possibility that vertical acoustic waves with frequencies lower than the cutoff frequency corresponding to the temperature minimum pass this minimum is investigated. It is shown that the averaged temperature profile in the solar atmosphere can be approximated by several so-called reflectionless profiles on which the acoustic waves propagate without internal reflection. The possibility of the penetration of vertical acoustic waves, including low-frequency ones, into the solar corona is explained in this way.

INTRODUCTION

Interest in studying acoustic-gravity waves continues to remain high among astrophysicists. This is primarily because the question about the heating of the chromosphere and corona of the Sun and other stars remains open (Stix 2002; Ulmschneider 2003).

Historically, the acoustic-gravity waves (to be precise, acoustic noise) generated by subphotospheric convection were defined as being among the prime candidates for the carriers of energy into the solar atmosphere (see, e.g., the review by Ulmschneider 2003). High-frequency waves after their transformation into shocks were believed to be mainly responsible for the chromospheric heating (Ulmschneider 1971; Kaplan et al. 1972), because long waves must experience strong reflection from the temperature profile inhomogeneity (Stix 2002; Ulmschneider 2003) and, consequently, their role in the energy balance of the solar atmosphere must be minor. However, a recent analysis of observational data calls this proposition into question (Pontieu et al. 2005; Fossum and Carlson 2005, 2006; Jefferies et al. 2006; Marsh and Walsh 2006).

In this paper, we suggest a new approach to explaining the penetration of waves to large distances without any energy loss that is based on analytical, so-called reflectionless solutions of the wave equation for acoustic waves in an atmosphere with steep temperature gradients. Solutions of this kind

were found for acoustic waves in the Earth's atmosphere (Petrukhin et al. 2011) and gravity waves in a stratified ocean (Talipova et al. 2009; Grimshaw et al. 2010), showing that the propagation of waves in a highly inhomogeneous medium is possible in principle under certain conditions imposed on the inhomogeneity profile.

VERTICAL DISTRIBUTIONS OF THE SOUND SPEED ADMITTING REFLECTIONLESS WAVE PROPAGATION IN THE SOLAR ATMOSPHERE

In the linear approximation, the one-dimensional system of Euler equations for an ideal compressible gas in a gravity field can be easily reduced to the wave equation for the vertical velocity of gas particles $V(x,t)$:

$$\frac{\partial^2 V}{\partial t^2} = c^2(z)\frac{\partial^2 V}{\partial z^2} - \gamma g \frac{\partial V}{\partial z}. \qquad (1)$$

The coefficients of this equation are determined by only one parameter: the vertical distribution of the sound speed $c(z) = (\gamma p_0/\rho_0)^{1/2}$. Here,

$$p_0(z) = p(0) \exp\left[-\int_0^z \frac{dz'}{H(z')} \right], \qquad (2)$$

$$\rho_0(z) = \rho(0)\frac{T(0)}{T(z)} \exp\left[-\int_0^z \frac{dz'}{H(z')} \right],$$

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where $p(0)$, $\rho(0)$, and $T(0)$ are, respectively, the pressure, density, and temperature at some fixed level ($z = 0$); $H(z) = c^2(z)/\gamma g$ is the pressure scale height on horizon z; g is the acceleration due to gravity, which is directed oppositely to the oz axis; and γ is the adiabatic index.

As will be shown below, there exist certain sound speed profiles for which the solutions of Eq. (1) describe traveling waves with variable amplitude and phase that are not reflected inside the atmosphere.

Generally, the solution of the linear equation (1) describes the transformation of the incident wave into the wave reflected from inhomogeneities of the medium and does not break up into two independent solutions corresponding to traveling waves in opposite directions. The existence of traveling waves is trivial in the case of partial differential equations with constant coefficients. Therefore, we will try to find the transformations that reduce Eq. (1) to an equation with constant coefficients; the basic idea of such a transformation was discussed previously (Petrukhin et al. 2011; Talipova et al. 2009) and is reproduced here briefly.

We will seek the solution of Eq. (1) in a form similar to the expression for a wave field in the WKB approximation but without any constraints on the wavelength

$$V(z,t) = A(z)\Phi(\tau,t), \quad \tau = \tau(z), \quad (3)$$

where all functions are to be determined. After substituting (3) into (1), we obtain the Klein–Gordon equation with variable coefficients

$$A(z)\left[\frac{\partial^2\Phi}{\partial t^2} - c^2(z)\left(\frac{d\tau}{dz}\right)^2\frac{\partial^2\Phi}{\partial\tau^2}\right] \quad (4)$$

$$-\left[2c^2\frac{dA}{dz}\frac{d\tau}{dz} + c^2A\frac{d^2\tau}{dz^2} - \gamma gA\frac{d\tau}{dz}\right]\frac{\partial\Phi}{\partial\tau}$$

$$-\left(c^2\frac{d^2A}{dz^2} - \gamma gA\right)\Phi = 0.$$

This equation is transformed into the Klein–Gordon equation with constant coefficients ($P = \text{const}$)

$$\frac{\partial^2\Phi}{\partial t^2} - \frac{\partial^2\Phi}{\partial\tau^2} = P\Phi \quad (5)$$

if we impose the following conditions:

$$\tau(z) = \int\frac{dz}{c(z)}, \quad (6)$$

$$A(z) \sim \sqrt{c(z)}\exp\left[\int\frac{dz}{2H(z)}\right],$$

$$P = \frac{1}{A}\left[c^2(z)\frac{d^2A}{dz^2} - \gamma gA(z)\right]. \quad (7)$$

The physical meaning of the function $\tau(z)$ is obvious; this is the wave travel time. Note that the wave amplitude is specified by the same expression as that in the WKB approach for a smoothly changing medium, although, in this case, the inhomogeneity is arbitrary, including a strong one. This gives additional arguments for justifying the solutions obtained in the form of reflectionless waves. Thus, as a result of the transformations, the original equation (1) with variable coefficients was reduced to Eq. (5) with a constant coefficient P. Its solutions in the form of traveling waves will be considered in the next section. Here, we will study Eq. (7), which, in view of (6), is the sought-for second-order ordinary differential equation for finding the "reflectionless" sound speed profiles. Let us rewrite it in a dimensionless form containing only one parameter β:

$$\frac{d^2u}{dh^2} - \frac{1}{2u}\left(\frac{du}{dh}\right)^2 - \frac{2}{u^2}\frac{du}{dh} - \frac{1}{2u^3} = -\frac{\beta}{2u}, \quad (8)$$

where

$$u = c(z)/c_0, \quad h = z/H_0, \quad H_0 = c_0^2/\gamma g,$$

$$\beta = -4P/\omega_0^2, \quad \omega_0 = \gamma g/2c_0.$$

Here, c_0 is the sound speed at some height $z = 0$ and ω_0 is the cutoff frequency of acoustic waves corresponding to an isothermal atmosphere in which the sound speed is c_0.

Equation (8) cannot be solved analytically in general form. Only at $\beta = 0$ can we obtain two profiles that in dimensional variables are

$$c^2(z) = c^2(0) - \frac{2}{3}\gamma gz, \quad (9)$$

$$c^2(z) = c^2(0) - 2\gamma gz.$$

The sound speed distributions (9) correspond to the well-known model of a polytropic atmosphere, i.e., an atmosphere with constant temperature gradients that are, respectively,

$$\frac{dT}{dz} = -2\mu g, \quad \frac{dT}{dz} = -2\frac{\mu g}{R}, \quad (10)$$

where μ is the molecular weight and R is the gas constant. Since the adiabatic index can take on values within the range $1 \leq \gamma \leq 5/3$, the well-known Schwarzschild convective instability criterion (Sobolev 1985) holds for such gradients:

$$-\frac{dT}{dz} > -\frac{dT}{dz}\bigg|_{ad} = \frac{\gamma - 1}{\gamma}\frac{\mu g}{R}. \quad (11)$$

Profiles (9) can be used in modeling the convection zones of the Sun and other stars.

Other reflectionless sound speed (temperature) profiles can be determined by numerically integrating Eq. (8).

THE WAVE FIELD IN A REFLECTIONLESS ATMOSPHERE

The Klein−Gordon equation (5) in the case of $P = 0$ has the solutions that describe traveling waves

$$V(t,z) = GA(z)\Phi\left[t - \int \frac{dz}{c(z)}\right], \qquad (12)$$

where $\Phi(t)$ defines the wave field on the emitter and G is an arbitrary constant. At $P \neq 0$, the elementary solution of Eq. (5) is found for a monochromatic wave

$$V(t,z) = GA(z)\exp\left[i\left(\omega t - K\int \frac{dz}{c(z)}\right)\right] \quad (13)$$

with the dispersion relation

$$l = \pm\sqrt{\sigma^2 - \beta}. \qquad (14)$$

Here, $l = K/\omega_0$ and $\sigma = \omega/\omega_0$.

It is well known (Eckart 1960) that the dispersion relation for vertical acoustic waves in an isothermal atmosphere is

$$\omega^2 = c_0^2 k^2 + \omega_0^2, \qquad (15)$$

from which it follows that waves with a frequency lower than the cutoff frequency ω_0 cannot propagate in such an atmosphere. It is assumed that such a prohibition must also be extended to a nonisothermal atmosphere. In particular, it is assumed in astrophysical literature that acoustic waves with a frequency lower than the cutoff frequency corresponding to the sound speed at the temperature minimum cannot pass through an atmospheric layer with this minimum or, equivalently, the sound speed minimum. It can be clearly seen even from a simple analysis of Eq. (8) that this assertion is, in general, incorrect. It is well known that at the minimum of a differentiable function, its first derivative must be zero, while its second derivative must be positive. Taking this into account, from Eq. (8) we obtain

$$\left(\frac{d^2u}{dz^2}\right)_{\min} = \frac{1-\beta}{2}. \qquad (16)$$

Here, $(\frac{d^2u}{dz^2})_{\min}$ is the second derivative of the function at its minimum, and the value of the function u at this point is taken to be unity ($u = c(z)/c(0)$, where $c(0)$ is the sound speed at the level of the temperature minimum). In particular, it follows from (16) that for $\beta < 0$ waves with any frequencies and, consequently, with frequencies lower than the cutoff frequency in an equivalent isothermal atmosphere with a temperature equal to the temperature at the level of the minimum can pass through this layer. For positive β, the cutoff frequency ω_{cut} is determined from the dispersion relation (14) as

$$\omega_{cut} = \beta^{1/2}\omega_0, \qquad (17)$$

i.e., it differs from the traditional cutoff frequency ω_0 by the factor $\beta^{1/2}$ and $\omega_{cut} < \omega_0$ for $\beta < 1$. A fundamentally new result follows even from this simple example: vertical acoustic waves with a frequency lower than the cutoff frequency corresponding to the temperature minimum can pass through the region of this minimum without reflection.

Figure 1 presents the reflectionless profiles for various values of β in the range $(-1, +1)$ in the region of the temperature minimum. For convenience, the height of the minimum point is taken to be $h = 0$ and the dimensionless sound speed at this point is $u = 1$. The sound speed variation in the region of the temperature minimum of the solar atmosphere is also indicated here by the dots for the VAL3c model (Vernazza et al. 1981). Since good agreement with the model used is obtained at $\beta = 0.9$, the difference in cutoff frequencies compared to the isothermal case ($\beta = 1$) is small, see (17).

All components of the wave field can be found from the linear equations of gas dynamics. Thus, the wave part of the pressure is defined by the formula (Lamb 1947)

$$p' = \frac{i\rho_0}{\omega}\left[c^2(z)\frac{\partial V}{\partial z} - gV\right]. \qquad (18)$$

We can also easily calculate the vertical energy flux density (Eckart 1960)

$$\Pi = \frac{1}{2}\left[p'V^* + Vp'^*\right], \qquad (19)$$

where ($*$) denotes complex conjugation. Substituting here (13) and (19), in view of (2), we obtain

$$\Pi = \frac{\gamma|G|^2Kp(0)}{\omega}. \qquad (20)$$

Consequently, the energy flux does not depend on z and is conserved, although the atmosphere is highly inhomogeneous. As a result, a monochromatic wave can propagate to great heights without any energy loss. This conclusion is valid for waves on any reflectionless profile irrespective of the magnitude and sign of the parameter β.

REFLECTIONLESS WAVE PROPAGATION THROUGH THE SOLAR ATMOSPHERE

Observations show (Schrijver et al. 1997; Rutten 2007) that the solar atmosphere has a rather complex structure. Its parameters are nonstationary, change with height, and, in addition, are locally inhomogeneous in all directions. In particular, the chromosphere consists of a host of small-scale magnetic flux tubes even in quiet-Sun regions. Allowance for all these factors makes the problem of analyzing the propagation of wave motions in
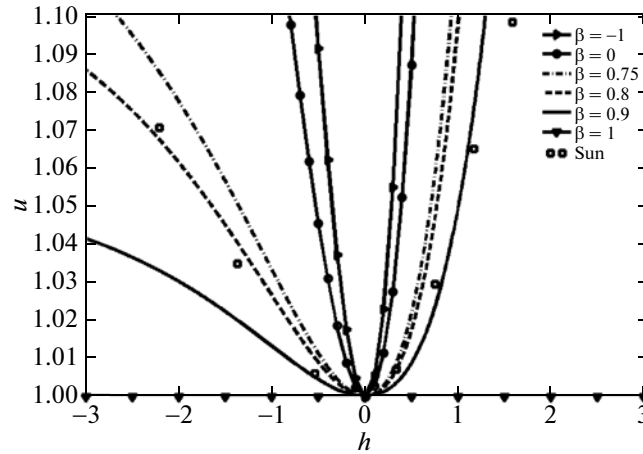
Fig. 1. Reflectionless sound speed profiles passing through the temperature minimum. The dots correspond to the sound speed distribution in the Sun's VAL3c model (Vernazza et al. 1981).

such a medium very complicated even numerically. Therefore, one-dimensional averaged atmospheric models, including the VAL3c model, are commonly used to qualitatively estimate the properties of wave motions with parameters much greater than the temporal and spatial local inhomogeneities. However, analytical solutions of the wave equation usually cannot be obtained even in such an approximation. The study of short acoustic waves for which the WKB approximation is valid (Gossard and Hook 1978) and the propagation of acoustic-gravity waves in an atmosphere with a constant temperature gradient (Lamb 1947; Petrukhin 1983a, 1983b) constitutes an exception. Therefore, in most papers, the propagation of wave disturbances in the solar atmosphere was studied numerically (Malins and Erdelyi 2007; Fedun et al. 2009). The results of all these works suggest that acoustic-gravity waves pass through the atmosphere relatively freely. At the same time, these works do not answer the question of why weak wave reflection is possible in a medium whose parameters are essentially inhomogeneous, while the temperature gradients are significant.

In our view, the answer is related to the possibility of approximating the distribution of solar-atmosphere parameters by reflectionless profiles. One of such approximations is shown in Fig. 2. We use the VAL3c model as a model of the solar atmosphere. Here, we normalized the height to the pressure scale height $H_0 = 120\,\text{km}$ and the sound speed to $c(0) = 7\,\text{km s}^{-1}$ (both parameters correspond to the solar temperature minimum for the VAL3c model).

We see from Fig. 2 that the observed sound speed distribution in the solar atmosphere from the acoustic wave generation zone in the convection zone to the

lower corona can be approximated by seven reflectionless profiles obtained by solving Eq. (15) for various values of β and for initial conditions corresponding to the Sun's profile.

At the joining points, the jump in the sound speed gradient is small and, in fact, only the jump in the second derivative occurs. The smallness of the jumps in the sound speed gradient at the boundaries of reflectionless layers suggests the weakness of the wave energy reflection and efficient wave penetration into the upper atmosphere.

Let us consider in more detail the wave transformation at the joining of reflectionless profiles. The boundary conditions at the boundary express the continuity of the vertical gas velocity and the pressure

$$[V]\Big|_-^+ = 0, \quad [p]\Big|_-^+ = 0, \tag{21}$$

where [] denotes the difference of quantities on both sides of the jump. Using expression (18), conditions (21) can be represented as

$$[V]\Big|_-^+ = 0, \quad \left[\frac{\partial V}{\partial z}\right]\Big|_-^+ = 0. \tag{22}$$

Since at the joining points the sound speed $u(h)$ remains continuous and its derivative du/dh is discontinuous (as is the parameter β), we find the transmission coefficient T for a monochromatic wave (we consider the wave variable V) passing through the boundary that separates two layers j and s from (22):

$$T_{js} = \frac{4il_j}{(u'_j - u'_s) + 2i(l_j + l_s)}, \tag{23}$$
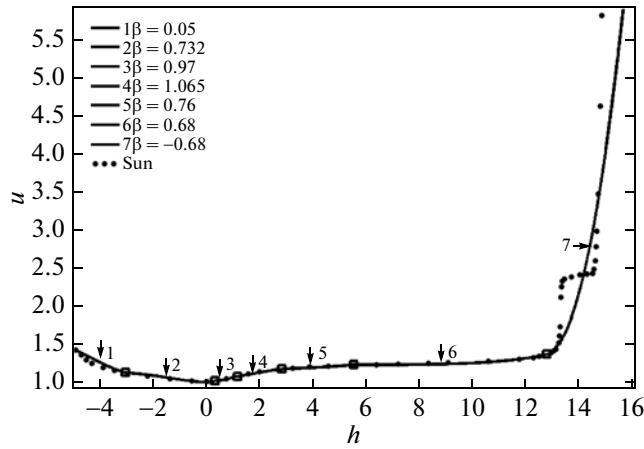
where $u' = du/dh$.

Fig. 2. Approximation of the model solar atmosphere by reflectionless profiles. The dots correspond to the sound speed distribution in the VAL3c model.
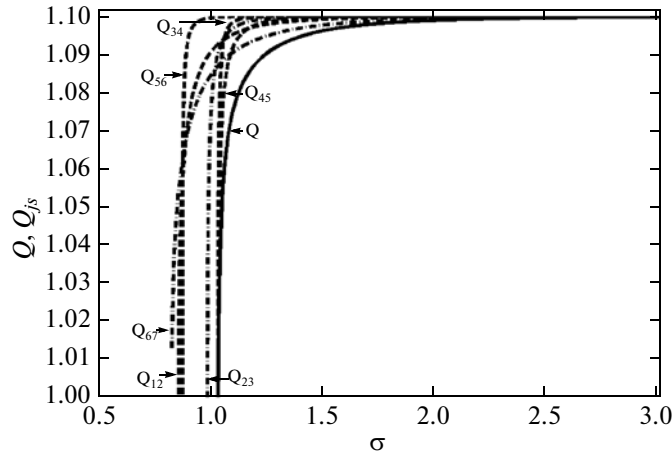

Fig. 3. Energy transmission coefficients of acoustic waves from the generation zone into the solar corona.

Using the model presented above, let us estimate the transmission coefficient for a wave through the solar atmosphere from the generation zone into the corona. As has already been noted, the energy losses can take place only at the joining points. When the wave propagates through boundary js, the energy transmission coefficient will be defined as

$$Q_{js} = \frac{\Pi_s}{\Pi_j} = \frac{l_s |T_{js}|^2}{l_j} \quad , \qquad (24)$$

where Π_j is the wave energy flux in layer j.

To estimate the energy flux Q passing through all boundaries of the layers of the reflectionless profiles, quantities (24) should be multiplied at all joining points.

In Fig. 3, the functions Q_{js} and Q are plotted against the dimensionless frequency σ. The values of Q_{js} and Q are given without allowance for the re-reflection of waves from each discontinuity. As follows from Fig. 3, allowance for the secondary reflections for all waves with $\sigma > 1.1$ or with periods less than 200−230 s is not necessary, because the atmosphere is virtually transparent for such waves. Thus, we basically show that the model of the solar atmosphere under consideration has parameters close to the reflectionless ones, which explains the good penetration of waves into the upper layers observed in numerical experiments.

CONCLUSIONS

Here, we obtained a new class of exact solutions to the linear hydrodynamic equations for a compressible gas in a gravity field that correspond to "reflectionless" vertical propagation of acoustic waves in a plane-stratified atmosphere. In this case, the waves can propagate to great distances without any energy loss. These solutions were obtained for peculiar laws of temperature variation in the atmosphere and the number of such profiles in the theory is fairly large. Comparison with the averaged solar atmosphere (the VAL3c model) showed that it could be approximated by several "reflectionless" profiles. At the boundaries of the layers that are joined fairly smoothly, the reflection is also fairly weak. As a result, we showed that the energy of acoustic waves could penetrate to great distances.

Of course, the description of the actual solar atmosphere by the averaged model is fairly rough. However, the propagation of waves in a nonstationary and inhomogeneous atmosphere is very difficult to calculate even numerically, because adaptation to the rapidly changing conditions in the atmosphere is needed. That is why for a numerical analysis of the wave transformation in the solar atmosphere, the atmosphere is often considered to be stationary and averaged, as, for example, in Bryson et al. (2005), Fedun et al. (2009), and Fontenla et al. (2006). As regards the analytical methods, they were applied previously only for the simplest geometries: an isothermal (Gossard and Hooke 1975), polytropic atmosphere (Petrukhin 1983a, 1983b), a two-layer model (Taroyan and Erdelyi 2008). A new result of this work is the demonstration that the waves in a nonisothermal atmosphere can also propagate to great distances without reflection even at a very strong inhomogeneity, for example, when passing from the chromosphere to the corona.

It should be noted that the analytical approach considered here is universal in character. In our view, this technique can be useful in analyzing the magnetosonic and Alfvén waves in magnetic flux tubes of the solar atmosphere.

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REFERENCES

1. S. Bryson, A. Kosovichev, and D. Levy, Physica D **201**, 1 (2005).
2. C. Eckart, *Hydrodynamis of Oceans and Atmospheres* (Pergamon, Oxford, 1960; Regulyar. Khaotich. Dinamika, Izhevsk, 2004).
3. V. Fedun, R. Erdelyi, and S. Shelyag, Solar Phys. **258**, 219 (2009).
4. J. M. Fontenla, E. Avret, G. Thuiller, et al., Astrophys. J. **639**, 441 (2006).
5. A. Fossum and M. Carlson, Nature **435**, 919 (2005).
6. A. Fossum and M. Carlson, Astrophys. J. **646**, 579 (2006).
7. E. E. Gossard and W. H. Hooke, *Waves in the Atmosphere: Atmospheric Infrasound and Gravity Waves: Their Generation and Propagation* (Elsevier Sci., Amsterdam, New York, 1975; Mir, Moscow, 1978).
8. R. Grimshaw, E. Pelinovsky, and T. Talipova, J. Phys. Oceanogr. **40**, 802 (2010).
9. S. M. Jefferies, S. V. McIntosh, J. D. Armstrong, et al., Astrophys. Lett. **648**, 151 (2006).
10. S. A. Kaplan, L. A. Ostrovskii, N. S. Petrukhin, et al., Astron. Zh. **49**, 1267 (1972).
11. H. Lamb, *Hydrodynamics* (Cambridge Univ. Press, Cambridge, 1985; Gostekhizdat, Moscow, Leningrad, 1947).
12. C. Malins and R. Erdelyi, Solar Phys. **246**, 41 (2007).
13. M. S. Marsh and R. W. Walsh, Astrophys. J. **643**, 540 (2006).
14. N. S. Petrukhin, Astron. Rep. **27**, 70 (1983a).
15. N. S. Petrukhin, Astron. Rep. **27**, 408 (1983b).
16. N. S. Petrukhin, E. N. Pelinovskii, and E. K. Batsyna, JETP Lett. **93**, 564 (2011).
17. B. de Pontieu, R. Erdelyi, and I. de Moortel, Astrophys. Lett. **624**, 61 (2005).
18. R. Rutten, ASP Conf. Ser. **368**, 27 (2007).
19. C. Schrijver, A. Title, A. von Ballegoonen, et al., Astrophys. J. **487**, 424 (1997).
20. V. V. Sobolev, *Course in Theoretical Astrophysics* (Nauka, Moscow, 1967; NASA, Washington, 1969).
21. M. Stix, *The Sun. An introduction* (Springer, Berlin, 2002).
22. T. G. Talipova, E. N. Pelinovskii, and N. S. Petrukhin, Oceanology **49**, 622 (2009).
23. Y. Taroyan and R. Erdelyi, Solar Phys. **251**, 523 (2008).
24. P. Ulmschneider, Astron. Astrophys. **12**, 297 (1971).
25. P. Ulmschneider, *Lecture on Solar Physics* (Springer-Verlag, Antia, Bhatnagar, Ulmschneider, Berlin, 2003), p. 232.
26. J. E. Vernazza, E. H. Avrett, and R. Loeser, Astrophys. J. Suppl. Ser. **45**, 635 (1981).

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