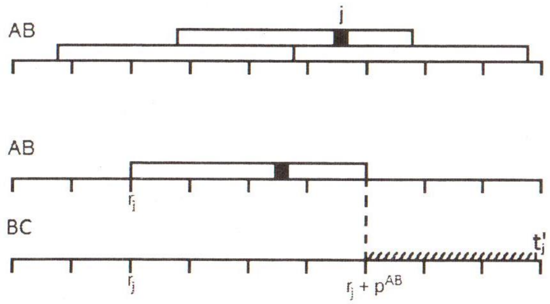
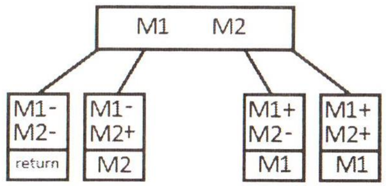
**Method 2.** We change the moments of departure of trains on the track $AB$.




We use the following scheme to choose the right method.




If there are no "bad" orders in our pair of schedules $\Theta(N^{AB\cup AC}, y)$ and $\Theta(N^{BC\cup AC}, y)$, then we depart trains on the track $AB$ according to $\Theta(N^{AB\cup AC}, y)$ and on the track $BC$ according to $\Theta(N^{BC\cup AC}, y)$. As a result we obtain the schedule $\Theta^3(N, y)$.

**Theorem 3.** *Algorithm 3 constructs the schedule $\Theta^3(N, y)$. If algorithm 3 terminates then there is no schedule which holds (4).*

To construct the optimal schedule we use algorithm 2. The only difference is that we should use the schedule $\Theta^3(N, y)$ instead of $\Theta(N, y)$.

**M station with tree topology.** The formulation of this problem and the problem for 3 stations is the same. The only difference is that we deals with $M$ stations with tree topology. Due to the tree topology there is only one way between each pair of stations. We also can enumerate stations from left to right (or from right to left).

**Algorithm 4.** We use algorithm 3 to get out of "bad" orders for each



station, from the left to the right follows the numeration (according to chosen direction of the train moving). When there are no "bad" orders on each station we obtain the schedule $\Theta^M(N, y)$. After that we use algorithm 2 to construct the optimal schedule for $M$ stations.

**Theorem 4.** *Algorithm 4 constructs the optimal schedule according to criterion $L_{\max}$ in $O(M^2 \frac{n^4}{k})$ operations.*

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## Existence theorems for elliptic equations in unbounded domains

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We consider the first boundary value problem for elliptic systems defined in unbounded domains, which solutions satisfy the condition of finiteness of the Dirichlet integral also called the energy integral

$$\int_{\Omega} |\nabla u|^2 dx < \infty.$$

### Basic concepts

Let $\Omega$ is an arbitrary open set in $\mathbb{R}^n$. As is usual, by $W^1_{2,\,loc}(\Omega)$ we denote the space of functions which are locally Sobolev, i.e.

$$W^1_{2,\,loc}(\Omega) = \{f : f \in W^1_2(\Omega \cap B^x_\rho), \forall \rho > 0\,, \forall x \in \mathbb{R}^n\}\,,$$

where $B^x_\rho$ – open ball with center at point $x$ and with radius $\rho$. If $x = 0$ then we will write $B_\rho$. We will denote by $\mathring{W}^1_{2,\,loc}(\Omega)$ set of functions from $W^1_{2,\,loc}(\mathbb{R}^n)$, which is the closure of $C_0^\infty(\Omega)$ in the system of seminorms

$\|u\|_{W_2^1(\mathcal{K})}$, where $\mathcal{K} \subset \mathbb{R}^n$ are various compacts. Let denote by $L_2^1(\Omega)$ a space of generalized functions in $\Omega$, which first derivatives belong to $L_2(\Omega)$ [4], in other words

$$L_2^1(\Omega) = \{f \in \mathcal{D}'(\Omega) : \int_\Omega |\nabla f|^2 dx < \infty\}.$$

Let $\omega \subseteq \mathbb{R}^n$ is an open set, $\mathcal{K} \subset \omega$ is a compact. We will denote by $\Phi_\varphi(\mathcal{K}, \omega)$ the set of functions $\psi \in C_0^\infty(\omega)$ such that $\psi = \varphi$ in the neighborhood of $\mathcal{K}$, or in other words $\psi - \varphi \in \overset{\circ}{W}{}^1_{2,\,loc}(\mathbb{R}^n \setminus \mathcal{K})$.

Let's define a capacitance of a compact $\mathcal{K}$ relative to the set $\omega$ [4]:

$$\operatorname{cap}_\varphi(\mathcal{K}, \omega) = \inf_{\psi \in \Phi_\varphi(\mathcal{K}, \omega)} \int_\omega |\nabla \psi|^2 dx\,.$$

The capacitance of arbitrary closed set $E \subset \omega$ in $\mathbb{R}^n$ is defined by the formula $\operatorname{cap}_\varphi(E, \omega) = \sup_{\mathcal{K} \subset E} \operatorname{cap}_\varphi(\mathcal{K}, \omega)$. If $\omega = \mathbb{R}^n$, then instead of $\operatorname{cap}_\varphi(E, \mathbb{R}^n)$ we will write $\operatorname{cap}_\varphi(E)$.

## Problem statement

Let $L$ is a divergent operator

$$L = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial}{\partial x_j}\right),$$

where $a_{ij}$ are bounded measurable functions in $\mathbb{R}^n$ satisfying condition

$$\gamma|\xi|^2 \le \sum_{i,j=1}^{n} a_{ij}(x)\,\xi_i\,\xi_j\,, \quad \xi \in \mathbb{R}^n, \gamma > 0\,.$$

The solution of the Dirichlet problem

$$\begin{cases} Lu &= 0 \text{ in } \Omega \\ u|_{\partial\Omega} &= \varphi, \end{cases} \tag{1}$$

where $\varphi \in W^1_{2,\,loc}(\mathbb{R}^n)$, is a function $u \in W^1_{2,\,loc}(\Omega)$ such that:



1) $u - \varphi \in \overset{\circ}{W}{}^1_{2,\,loc}(\Omega)$, i.e. $(u - \varphi)\mu \in \overset{\circ}{W}{}^1_2(\Omega)$ for any function $\mu \in C_0^\infty(\mathbb{R}^n)$;

2) function $u$ has bounded Dirichlet integral

$$\int_\Omega |\nabla u|^2 dx < \infty\,;$$

3)

$$\int_\Omega \sum_{i,j=1}^{n} a_{ij}(x)\frac{\partial u}{\partial x_j}\frac{\partial \psi}{\partial x_i}\, dx = 0$$

for any function $\psi \in C_0^\infty(\Omega)$.

## Basic results

**Theorem 1.** *Let's $cap_{\varphi - c}(\mathbb{R}^n \setminus \Omega) < \infty$ for some constant $c \in \mathbb{R}^n$. Then the problem (1) has a solution.*

**Theorem 2.** *Let the problem (1) has a solution and it is true that*

$$\int_{\mathbb{R}^n \setminus \Omega} |\nabla \varphi|^2 dx < \infty\,.$$

*Then there is such constant $c \in \mathbb{R}^n$, that $cap_{\varphi - c}(\mathbb{R}^n \setminus \Omega) < \infty$.*

**Theorem 3.** *Let $n \ge 3$. Then $cap_{\varphi - c}(\mathbb{R}^n \setminus \Omega) < \infty$ if and only if*

$$\sum_{k=N}^{\infty} cap_{\varphi - c}((\overline{B}_{2^{k+1}} \setminus B_{2^{k-1}}) \cap (\mathbb{R}^n \setminus \Omega), B_{2^{k+2}} \setminus \overline{B}_{2^{k-2}}) < \infty$$

*for some $N \in \mathbb{N}$.*

## Particular cases

Let consider the space $\mathbb{R}^n$ with a set of coordinates $(x_1, x_2, \ldots, x_n)$ and let $\varphi_\alpha = (1 + |\,x_1|)^\alpha$. Domain $\Omega_{1,i}$ is upper half-plane relative to $x_i$, where $i \ne 1$, in other words $\Omega_{1,i} = \{(x_1, x_2, \ldots, x_n) | x_i \ge 0, i \ne 1\}$. Domain $\Omega_2$ is the outer part of the space formed by surface of revolution relative to $x_1$ of the curve from Fig.1.

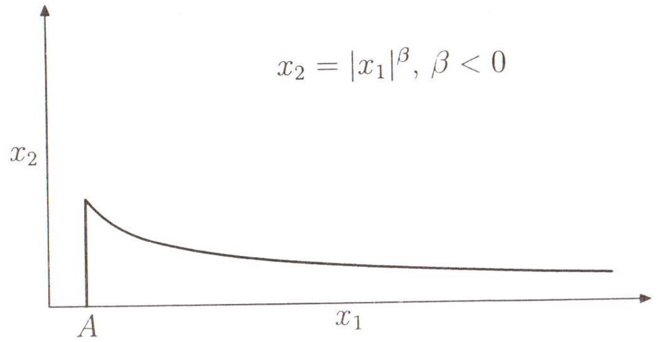


Fig. 1: Domain $\Omega_2$

**Corollary 1.** *Let $n \geq 2$. Then for the domain $\Omega_{1,i}$ and for bounded function $\varphi_\alpha$ the existence of solutions of the problem (1) is equivalent to either an inequality $\alpha < -\frac{1}{2}$ or $\alpha = 0$.*

**Corollary 2.** *Let $n \geq 3$. Then for the domain $\Omega_2$ and for bounded function $\varphi_\alpha$ the existence of solutions of the problem (1) is equivalent to either an inequality $\alpha < -\frac{1+\beta(n-3)}{2}$ or $\alpha = 0$.*


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# On the determination of the earthquake slip distribution via linear programming techniques


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The description that one can have of the seismic source is the manifestation of an imagined model, obviously outlined from Physic Theories and supported by mathematical methods. In that context, the modelling of earthquake rupture consists in finding values of the parameters of the selected physics-mathematical model, through which it becomes possible to reproduce numerically the records of earthquake effects on the Earths surface. Actually, these effects are the elastic records at near field source and at far field source, and inelastic deformations recorded by geodetic techniques. The detail and accuracy level, with which the characteristic parameters for large earthquakes are computed, depends on the combination of two factors - the applied methods and the used data.

Under the hypothesis of constant slip direction and constant rise time of individual source time function, the problem of complete seismic slip time history and distribution reconstruction reduces to the solution of a system of linear equations. It is well-known that this inverse problem is ill-posed [6]. The usual regularization techniques [8] can hardly be applied in this case because of a very high dimension of this problem (see, e.g., [3]). The problem can be overcome by introducing some additional regularizing constraints. Some additional physical hypotheses, like no-backslip constraint, result in condition of non-negativeness of solutions to the system of linear equations.

The positivity that prohibits negative seismic moment values, is a constraint naturally assumed when used the Non Negative Least Squares algorithm (NNLS) [5] to inverts seismic waveforms to slip distribution (e.g., [7]).

We present and test a Linear Programming (LP) inversion in dual form, for reconstructing the kinematics of the rupture of large earthquakes through space-time seismic slip distribution on finite faults planes. The proposed method can be considered as a continuation of the work started