Saratov State University

Presenting
Academic Achievements to the World.
Natural Science

Papers from the conference for young scientists
«Presenting Academic Achievements to the World»

March 3–4, 2011
Saratov

Issue 2

Saratov
Saratov University Press
2011

Саратовский государственный университет им. Н. Г. Чернышевского

Представляем научные достижения миру. Естественные науки

Материалы научной конференции молодых ученых
«Presenting Academic Achievements to the World»

Март 3–4, 2011
Саратов

Выпуск 2

Саратов
Издательство Саратовского университета
2011

УДК 5(082)
ББК 20я43
П71

**Представляем научные достижения миру. Естествен-
П71 ные науки** : материалы научной конференции молодых ученых «Presenting Academic Achievements to the World». – Саратов : Изд-во Сарат. ун-та, 2011. – Вып. 2 – 116 с. : ил.
ISBN 978-5-292-04093-4

В данном сборнике опубликованы материалы участников секции естественных наук научной конференции молодых ученых «Presenting Academic Achievements to the World», которая состоялась в Саратовском государственном университете 3–4 марта 2011 года. В сборник включены статьи с результатами исследований в области физики, химии, географии, геологии и информационных технологий.

This publication assembles papers given at the conference for young scientists «Presenting Academic Achievements to the World» which was held in March 3-4, 2011 at Saratov State University. The articles present the results in such fields of natural science as Physics, Chemistry, Geograghy, Geology and Information Technology.

УДК 5(082)
ББК 20я43

Работа издана в авторской редакции

ISBN 978-5-292-04093-4

EQUILIBRIUM POINTS IN ANTAGONISTIC GAMES WITH PREFERENCE RELATIONS

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We consider some optimality solutions of antagonistic games with preference relations. In such games, instead of payoff functions, reflexive binary preference relations are given. A game with preference relations *in the normal* form is a system

$$G = \langle X, Y, A, \rho, F \rangle \qquad (1)$$

where X is the set of *strategies* of player 1, Y is the set of *strategies* of player 2, A is the set of *outcomes*, $r \subseteq A^2$ is a preference relation of the player 1, F is the realization function, that is, a map from the set of all *situations* $X \times Y$ into the set of outcomes A. Assertion $a_1 \overset{\rho}{\leq} a_2$ means that outcome a_1 is less preference than a_2 for player 1.

Remark. For antagonistic game a preference relation of the player 2 is ρ^{-1}.

Given a preference relation $r \subseteq A^2$, we denote $\rho^s = \rho \cap \rho^{-1}$ its symmetric part and $\rho^* = \rho \setminus \rho^s$ its strict part. We write $a_1 \overset{\rho}{\leq} a_2$ istead of $(a_1, a_2) \in \rho$, $a_1 \overset{\rho}{\sim} a_2$ instead of $(a_1, a_2) \in \rho^s$, $a_1 \overset{\rho}{<} a_2$ instead of $(a_1, a_2) \in \rho^*$.

We introduce three types of equilibrium concepts: equilibrium (Rozen, 2010), Pr–equilibrium and saddle point. We consider antagonistic games with various preference structure: linear, acyclic, transitive and antisymmetric (Savina, 2010).

Definition 1. Situation $(x_0, y_0) \in X \times Y$ is called

- *an equilibrium point* if for any strategies $x \in X, y \in Y$ the condition

$$F(x, y_0) \overset{\rho}{\not>} F(x_0, y_0) \overset{\rho}{\not>} F(x_0, y) \qquad (2)$$

holds;

- *a Pr–equilibrium point* if for any strategies $x \in X, y \in Y$ the condition

$$F(x, y_0) \overset{\rho}{\not>} F(x_0, y) \qquad (3)$$

is satisfied;

- *a saddle point* (or *Nash equilibrium*) if the condition

$$F(x, y_0) \overset{\rho}{\leq} F(x_0, y_0) \overset{\rho}{\leq} F(x_0, y) \qquad (4)$$

holds.

Example. Consider antagonistic game G of the form (1) in which the set of strategies of player 1 is $X = \{x_1, x_2\}$, the set of strategies of player 2 is $Y = \{y_1, y_2\}$, the set of outcomes is $A = \{a, b, c, d\}$, the realization function is given by the Table 1 and the preference relation is given by diagram (Fig. 1).

Table 1

The realization function *F*

F	y_1	y_2
x_1	a	b
x_2	c	d

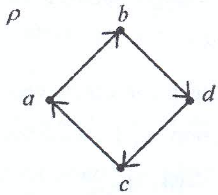

Fig.1

Situations (x_1, y_1) and (x_2, y_2) are saddle points but outcomes in theirs are different.

From this example it is easy to show that the following assertion is true.

Proposition. In antagonistic games with acyclic preference structure of the form (1) the outcomes in saddle points are the same.

Definition 2. In antagonistic game G of the form (1) outcome a is called

- *acceptable for player 1* if

$$\neg(\exists x \in X)(\forall y \in Y)\, F(x,y) \overset{\rho}{>} a;$$

- *quite acceptable for player 1* if

$$(\exists y \in Y)(\forall x \in X)\, F(x,y) \overset{\rho}{\ngtr} a;$$

- *acceptable for player 2* if

$$\neg(\exists y \in Y)(\forall x \in X)\, F(x,y) \overset{\rho}{<} a;$$

- *quite acceptable for player 2* if

$$(\exists x \in X)(\forall y \in Y)\, F(x,y) \overset{\rho}{\nless} a.$$

Definition 3. Outcome a is said to be *acceptable (quite acceptable) in game* G if it is acceptable (quite acceptable) for both players.

Definition 4. Situation $(x_0, y_0) \in X \times Y$ is said to be *acceptable (quite acceptable) in game* G if outcome $F(x_0, y_0)$ is acceptable (quite acceptable) in game G.

The basic properties of equilibrium concepts are the following result.

Theorem

1. In antagonistic game with preference relations of the form (1) the following conditions hold:

a) saddle points are equilibrium points;

b) Pr–equilibrium points are equilibrium points also.

2. Saddle points are Pr–equilibrium points in antagonistic game with transitive preference structure.

3. In antagonistic game with linear transitive preference structure three types of equilibrium concur.

4. Any quite acceptable outcome is acceptable in antagonistic game with preference relations of the form (1).

5. Outcome in equilibrium point is quite acceptable (and acceptable also) in antagonistic game with preference relations of the form (1).

6. Let G be an antagonistic game with linear and antisymmetric preference structure. If set of quite acceptable outcomes is nonempty, then set of saddle points is nonempty also.

Proof.

1. a) Let (x_0, y_0) be a saddle point. Suppose that situation (x_0, y_0) is not an equilibrium point. Then there exist $x' \in X, y' \in Y$ such that the condition $F(x', y_0) \overset{\rho}{>} F(x_0, y_0)$ or $F(x_0, y_0) \overset{\rho}{>} F(x_0, y')$ holds. Let the first condition is satisfied. Putting in condition (4) $x = x'$ we get $F(x', y_0) \overset{\rho}{\leq} F(x_0, y_0)$ which is contradictory with the condition $F(x', y_0) \overset{\rho}{>} F(x_0, y_0)$. Thus, (x_0, y_0) is an equilibrium point.

b) Let (x_0, y_0) be a Pr–equilibrium point. Putting in condition (3) $x = x_0$ we get $F(x_0, y_0) \overset{\rho}{\ngtr} F(x_0, y)$. Now putting in condition (3) $y = y_0$ we obtain $F(x, y_0) \overset{\rho}{\ngtr} F(x_0, y_0)$. Hence, $F(x, y_0) \overset{\rho}{\ngtr} F(x_0, y_0) \overset{\rho}{\ngtr} F(x_0, y)$. Thus, (x_0, y_0) is an equilibrium point.

2. Let G be antagonistic game with transitive preference structure. Let (x_0, y_0) be a saddle point in game G. Suppose that situation (x_0, y_0) is not a Pr–equilibrium point. Then there exist $x' \in X, y' \in Y$ such that the condition $F(x', y_0) \overset{\rho}{>} F(x_0, y')$ holds. Putting in (4) $x = x', y = y'$ we get $F(x', y_0) \overset{\rho}{\leq} F(x_0, y_0) \overset{\rho}{\leq} F(x_0, y')$. Since

relation ρ is transitive then $F(x',y_0)\overset{\rho}{\leq}F(x_0,y')$ which is contradictory with the condition $F(x',y_0)\overset{\rho}{>}F(x_0,y')$. Thus, (x_0, y_0) is a Pr–equilibrium point.

3. Let G be an antagonistic game with linear transitive preference structure. It is sufficiently to proof that equilibrium points are saddle ones. Let (x_0, y_0) be an equilibrium point. Since relation ρ is linear then condition (2) will be written in the form $F(x,y_0)\overset{\rho}{\leq}F(x_0,y_0)\overset{\rho}{\leq}F(x_0,y)$. Thus, (x_0, y_0) is a saddle point.

4. The assertion follows from logical rule of changing quantifiers $\exists\forall\Rightarrow\forall\exists$.

5. Let (x_0, y_0) be an equilibrium point. For $x=x_0$ we get outcome $F(x_0, y_0)$ is quite acceptable for player 2 and for $y=y_0$ we obtain outcome $F(x_0, y_0)$ is quite acceptable for player 1. Thus, $F(x_0, y_0)$ is a quite acceptable in game G.

6. Let G be an antagonistic game with linear and antisymmetric preference structure. Let a be a quite acceptable outcome then the system of the conditions

$$\begin{cases}(\exists x_0\in X)(\forall y\in Y)\,F(x_0,y)\overset{\rho}{\nless}a,\\(\exists y_0\in Y)(\forall x\in X)\,F(x,y_0)\overset{\rho}{\ngtr}a.\end{cases}$$

is satisfied. Since relation ρ is linear then we get

$$F(x,y_0)\overset{\rho}{\leq}a\overset{\rho}{\leq}F(x_0,y). \tag{5}$$

Putting in the last condition $x=x_0, y=y_0$ we obtain $F(x_0,y_0)\overset{\rho}{\leq}a\overset{\rho}{\leq}F(x_0,y_0)$. Since relation ρ is antisymmetric then $F(x_0, y_0) = a$. Put in condition (5) $a = F(x_0, y_0)$. Thus, (x_0, y_0) is a saddle point.

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CONTENT

Научное издание

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Материалы научной конференции молодых ученых
«Presenting Academic Achievements to the World»

Выпуск 2

Ответственный за выпуск *Н. И. Иголкина*
Корректор *И. Ю. Астахова*
Оригинал-макет подготовила *Н. И. Степанова*

Подписано в печать 19.12.2011. Формат 60x84 $^1/_{16}$.
Усл. печ. л. 6,74 (7,25). Тираж 90 экз. Заказ 107.

Издательство Саратовского университета.
410012, Саратов, Астраханская, 83.
Типография Издательства Саратовского университета.
410012, Саратов, Астраханская, 83.