

Coalition Homomorphisms of Games with Preference Relations

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Abstract: Let G be a game with preference relations of players I . Then we can construct a game of coalition if strategies and preference relations of coalitions are given. Let G, H be two games with preference relations. A homomorphism from game G into game H is said to be a coalition homomorphism if it preserves preference relations for coalitions. We study the following problems. We find conditions under which a homomorphism of games is a homomorphism for games of coalitions. We stand a correspondence between acceptable outcomes of games which are in homomorphic relations of various types. We find connections between acceptable outcomes of game with preference relations and a-core for games with payoff functions.

1. The cooperative aspect of a game is connected with its coalitions. Consider a game of players $N = \{1, \dots, n\}$ with preference relations

$$G = ((X_i)_{i \in N}, A_i, (\rho_i)_{i \in N}, F).$$

For any coalition $T \subseteq N$ we need define its set of strategies X_T and its preference relation ρ_T . We consider X_T in the form

$$X_T = \prod_{i \in T} X_i$$

and a preference relation for coalition T in one of the following ways:

A) $a_1 \stackrel{\rho_T}{\leq} a_2 \Rightarrow (\forall i \in T) a_1 \stackrel{\rho_i}{\leq} a_2$ – Pareto concordance of preferences of players,

OR

B) $\begin{cases} a_1 \stackrel{\rho_T}{<} a_2 \Rightarrow (\forall i \in T) a_1 \stackrel{\rho_i}{<} a_2, \\ a_1 \stackrel{\rho_T}{\sim} a_2 \Rightarrow (\forall i \in T) a_1 \stackrel{\rho_i}{\sim} a_2. \end{cases}$ – modified Pareto concordance of preferences of

players.

2. In this paper we consider the following optimal solutions:

K -equilibrium points and K -acceptable outcomes.

Let K be an arbitrary family of coalitions in game G .

Definition 1. A strategy $x_T^0 \in X_T$ is called a refutation of coalition T against outcome a if for any strategy $x_{N \setminus T} \in X_{N \setminus T}$ the condition $a \stackrel{\sigma_T}{\prec} F(x_T^0, x_{N \setminus T})$ holds. An outcome a is called K -acceptable if any coalition $T \in K$ does not have a refutation against this outcome.

Definition 2. A strategy $x_T^0 \in X_T$ is called a refutation of coalition T against situation $x \in X$ if the condition $F(x) \stackrel{\sigma_T}{\prec} F(x_T^0, x_{N \setminus T})$ holds. A situation $x^0 \in X$ is called K -equilibrium if any coalition $T \in K$ does not have a refutation against this situation.

3. In this section we find some connections between optimal cooperative solutions of games which are in homomorphic relations.

Let $\Gamma = \langle (U_i)_{i \in N}, B_i(\sigma_i)_{i \in N}, \Phi \rangle$ be one more game of the same players and $f = (\phi_1, \dots, \phi_n, \psi)$ be a homomorphism from game G into game Γ .

Definition 3. A homomorphism f is said to be:

- a *coalition homomorphism* if it preserves preference relations for coalitions, i.e. for any coalition $T \subseteq N$ the condition

$$a_1 \stackrel{\sigma_T}{\prec} a_2 \Rightarrow \psi(a_1) \stackrel{\sigma_T}{\prec} \psi(a_2)$$

holds;

- a *strict coalition homomorphism* if for any coalition $T \subseteq N$ the system of the conditions

$$\begin{cases} a_1 \stackrel{\sigma_T}{\prec} a_2 & \Rightarrow & \psi(a_1) \stackrel{\sigma_T}{\prec} \psi(a_2), \\ a_1 \stackrel{\sigma_T}{\sim} a_2 & \Rightarrow & \psi(a_1) \stackrel{\sigma_T}{\sim} \psi(a_2) \end{cases}$$

is satisfied;

- a *regular coalition homomorphism* if for any coalition $T \subseteq N$ the system of the conditions

$$\begin{cases} \psi(a_1) \stackrel{\sigma_T}{\prec} \psi(a_2) & \Rightarrow & a_1 \stackrel{\sigma_T}{\prec} a_2, \\ \psi(a_1) \stackrel{\sigma_T}{\sim} \psi(a_2) & \Rightarrow & \psi(a_1) = \psi(a_2) \end{cases}$$

is satisfied.

The main result of this paper is finding of a correspondence between sets of K -acceptable outcomes and K -equilibrium situations of games which are in homomorphic relations of indicated types.

References

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