Hecke Symmetries and Characteristic Relations on Reflection Equation Algebras

D.I. Gurevich

ISTV, Université de Valenciennes, 59304 Valenciennes, France

P.N. Pyatov

Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Moscow region, Russia

P.A. Saponov

Theory Department, IHEP, 142284 Protvino, Moscow region, Russia

Abstract

We discuss how properties of Hecke symmetry (i.e., Hecke type $\mathcal{R}$-matrix) influence the algebraic structure of the corresponding Reflection Equation (RE) algebra. Analogues of the Newton relations and Cayley-Hamilton theorem for the matrix of generators of the RE algebra related to a finite rank even Hecke symmetry are derived.

1 Introduction

There exist (at least) two matrix type algebras connected to the quantum groups. One of them is well known since the creation of the quantum groups theory. It is dual to a quantum linear group and is defined by the famous "RTT" relation (2.3). We restrict ourselves here to an $\mathcal{R}$-matrix of Hecke type (called the Hecke symmetry below). In this case the algebra under question is a flat deformation of the corresponding classical object.

Another "matrix algebra" is related to the so-called reflection equation (2.5) (RE algebra in what follows). It was introduced by I.Cherednik in the context of factorizable scattering on a half-line [1] and then found a number of different applications (see e.g. [2]). Both these algebras play an important role in the integrable systems theory and, moreover there exists a "transmutation procedure" [3] converting one of them to another.

For the latter algebra there exists an analogue of Newton relations between two natural families of invariant "functions". The principle aim of these notes is to present these relations and demonstrate a quantum analogue of Cayley-Hamilton theorem in the RE algebra.

In [4] and [5] the RE algebras related to the standard quantum linear group were considered from this point of view. Nevertheless, there exists a great deal of non-deformational Hecke symmetries constructed in [6]. If $R : V^{\otimes 2} :\to V^{\otimes 2}$ is such a symmetry acting in some linear space V it naturally generates "symmetric" and "skew-symmetric" algebras of the space V. Then the dimensions of homogeneous components of these algebras can be completely different from those of the classical (or super-) case. We show here that some versions of the Newton and Cayley-Hamilton relations are valid for any even (this means that the skew-symmetric algebra is finite dimensional) and closed (see below) Hecke symmetry.

Let us note that for the involutive $\mathcal{R}$-matrix ($R^2 = 1$) the RE algebras were considered under the name of monoidal groups in [6], where a version of the Newton relations was established for the simplest (i.e., of rank 2, see below) $\mathcal{R}$-matrix.

The paper is organized as follows. Section 2 contains the definitions and a survey of known facts about Hecke symmetries and RE algebras. In Section 3 we construct iterative relations connecting two different sets of the central elements of RE algebras. The characteristic identities for the matrix of generators of RE algebras are also derived here. These results may be viewed as a generalization of the classical Newton relations and Cayley-Hamilton theorem (see e.g. [7]) to the case of matrices with non-commuting entries.

2 Preliminaries

Hecke symmetries and related algebras.

First, let us recall several basic definitions. The matrix $R_{12} \equiv R_{i_1 i_2}^{j_1 j_2} \in Mat(N^2,\mathbb{C})$, $i_1, i_2, j_1, j_2 = 1, \ldots, N$, is called the *Hecke symmetry* if it satisfies the Yang-Baxter equation and Hecke condition, respectively,[1]

$$R_{12}R_{23}R_{12} \quad = \quad R_{23}R_{12}R_{23}\ , \tag{2.1}$$

$$R^2 = \mathbf{I} + \lambda R\ , \qquad \lambda = q - 1/q\ . \tag{2.2}$$

Here $q \in \mathbb{C}$ is an arbitrary parameter. We will assume later on that q is not a root of unity and, hence, for any natural p the corresponding q-number $p_q \equiv (q^p - q^{-p})/(q - q^{-1})$ is different from zero.

With any Hecke symmetry one associates in a canonical way the following two unital associative matrix type algebras.

A). The algebra $\mathcal{T}(R)$ [8] is generated by the set of N^2 generators $T_i^{\,j}$ subject to the relation

$$R_{12}T_1T_2 = T_1T_2R_{12}\ . \tag{2.3}$$

It is the Hopf algebra with the comultiplication

$$\triangle T_i^{\,j} = T_i^{\,k} \otimes T_k^{\,j}\ . \tag{2.4}$$

With some additional assumptions on the matrix R (see [6] and below) one can define also the antipodal mapping $\mathcal{S}(\cdot)$ such that

$$\mathcal{S}(T_i^{\,k})T_k^{\,j} = T_i^{\,k}\mathcal{S}(T_k^{\,j}) = \delta_i^{\,j}\ .$$

In case of R being the $\mathcal{R}$-matrix corresponding to linear quantum group, the algebra $\mathcal{T}(R)$ becomes the quantization of the algebra of functions on the linear group [8].

B). The *Reflection Equation* algebra $\mathcal{L}(R)$ is generated by the set of N^2 generators $L_i^{\,j}$ satisfying the condition

$$R_{12}L_1R_{12}L_1 = L_1R_{12}L_1R_{12}\ . \tag{2.5}$$

This algebra is naturally endowed with the structure of adjoint comodule under the left coaction of quantum linear group, and with the structure of trivial (invariant) comodule under its right coaction:

$$\delta_\ell(L_i^{\,j}) = T_i^{\,k}\mathcal{S}(T_p^{\,j}) \otimes L_k^{\,p} \quad , \quad \delta_r(L_i^{\,j}) = L_i^{\,j} \otimes 1\ . \tag{2.6}$$

[1] Here and in what follows the standard conventions of [8] are used in denoting matrix indices.

It is worth noticing that in case of R being the $\mathcal{R}$-matrix corresponding to a linear quantum group, the matrix elements L_i^j have a nice geometrical interpretation as quantum analogues of the basic right-invariant vector fields over the linear group.

Quantum trace.

Let us derive several consequences of the relation (2.3). Denote $\mathcal{R}_{12} \equiv P_{12}R_{12}$, where P_{12} is the permutation matrix, and rewrite (2.3) in a more conventional form

$$\mathcal{R}_{12}T_1T_2 = T_2T_1\mathcal{R}_{12}\ . \tag{2.7}$$

We say that the Hecke symmetry is *closed* if the matrix $\mathcal{R}_{12}^{t_1}$ is invertible. With this additional assumption one can turn (2.7) to the form

$$(\mathcal{R}^{t_1})^{-1}{}_{i_1i_2}^{k_1k_2}\mathcal{S}(T)_{k_2}^{j_2}T^t{}_{k_1}^{j_1} = T^t{}_{i_1}^{k_1}\mathcal{S}(T)_{i_2}^{k_2}(\mathcal{R}^{t_1})^{-1}{}_{k_1k_2}^{j_1j_2}\ . \tag{2.8}$$

Now, by performing the antipodal map of the above relation and summing up the indices j_1, j_2 we eventually obtain

$$\mathcal{S}^2(T)\mathcal{C} = \mathcal{C}T\ , \quad \text{where} \quad \mathcal{C}_i^j \equiv \sum_k (\mathcal{R}^{t_1})^{-1}{}_{ji}^{kk} = Tr_{(1)}\left[\left((\mathcal{R}^{t_1})^{-1}\right)_{12}^{t_1} P_{12}\right] \tag{2.9}$$

An analogous procedure with summing up indices i_1, i_2 gives

$$\mathcal{B}\mathcal{S}^2(T) = T\mathcal{B}\ , \quad \text{where} \quad \mathcal{B}_i^j \equiv \sum_k (\mathcal{R}^{t_1})^{-1}{}_{kk}^{ij} = Tr_{(2)}\left[\left((\mathcal{R}^{t_1})^{-1}\right)_{12}^{t_1} P_{12}\right] \tag{2.10}$$

By definition, all the entries of the matrix T are linearly independent and, hence, the matrix $\mathcal{B}\mathcal{C} = \mathcal{C}\mathcal{B}$ is scalar.

With the use of the matrix $\mathcal{C}$ one introduces an analogue of the trace operation in the algebra $\mathcal{L}(R)$ [9]. Namely, the *quantum trace* of the matrix L is defined as

$$Tr_qL \equiv Tr(\mathcal{C}L)\ , \tag{2.11}$$

and it extracts the invariant part of the left-adjoint comodule L_i^j. Indeed, using formula (2.9) one easily checks

$$\begin{aligned}
\delta_\ell(Tr_qL) &= Tr\left\{\mathcal{C}TL\mathcal{S}(T)\right\} = Tr\left\{\mathcal{S}^2(T)\mathcal{C}L\mathcal{S}(T)\right\} \\
&= \mathcal{S}\left(Tr\left\{T\mathcal{S}(T)\mathcal{C}L\right\}\right) = \mathbf{I}\otimes Tr_qL\ .
\end{aligned} \tag{2.12}$$

Here we omit symbol $\otimes$ in calculations. The above reasonings also work for any left-adjoint comodule, for example, the quantum trace of any power of the matrix L is invariant.

When realizing (2.9) and (2.12) in specific T-representations, one gets several useful relations for the quantum trace and the matrix $\mathcal{C}$. Consider the representations $\rho(T_a) = \mathcal{R}_{ab}$, and $\rho'(T_b) = \mathcal{R}_{ab}^{-1}$. Then

$$\begin{array}{ll} \rho(\mathcal{S}(T_a)) = \left(\left((\mathcal{R}_{ab}^{-1})^{t_a} \right)^{-1} \right)^{t_a} & , \quad \rho(\mathcal{S}^2(T_a)) = \mathcal{R}_{ab}^{-1} \; , \\ \rho'(\mathcal{S}(T_b)) = \mathcal{R}_{ab} & , \quad \rho'(\mathcal{S}^2(T_b)) = \left((\mathcal{R}_{ab}^{t_b})^{-1} \right)^{t_b} = \left((\mathcal{R}_{ab}^{t_a})^{-1} \right)^{t_a} \; . \end{array}$$

The relation (2.9) in the representation ρ looks like

$$\mathcal{C}_a \mathcal{R}_{ab} = \left(\left((\mathcal{R}_{ab}^{-1})^{t_a} \right)^{-1} \right)^{t_a} \mathcal{C}_a \; ,$$

whereas for the representation ρ' it can be cast into the form

$$\mathcal{R}_{ab} \mathcal{C}_b = \mathcal{C}_b \left(\left((\mathcal{R}_{ab}^{-1})^{t_a} \right)^{-1} \right)^{t_a} \; .$$

Combining these two relations we obtain

$$R_{12} \mathcal{C}_1 \mathcal{C}_2 = \mathcal{C}_1 \mathcal{C}_2 R_{12} \; . \tag{2.13}$$

In a similar way one deduces from (2.12)

$$Tr_{q(2)}(R_{12} X_1 R_{12}^{-1}) = Tr_{q(2)}(R_{12}^{-1} X_1 R_{12}) = (Tr_q X)\mathbf{I} \; , \tag{2.14}$$

$$Tr_{q(1,2)}(R_{12} X_{12} R_{12}^{-1}) = Tr_{q(12)} X_{12} \; . \tag{2.15}$$

Here X_1 and X_{12} are arbitrary operator-valued matrices, and the last relation is a consequence of (2.13). Now, applying (2.14) to the relations (2.5) one easily checks that all the elements $Tr_q L^k$, $k \geq 1$ are central in the algebra $\mathcal{L}(R)$.

Quantum antisymmetrizers and quantum Levi-Civita tensors.

Now let us remind some of the results of the paper [6] which will be relevant to considerations below. Note that the notation adopted here is slightly different from that of Ref.[6]. The correspondence is the following: our parameter q corresponds to $q^{1/2}$ of Ref.[6]; R corresponds to $\frac{1}{q^{1/2}} S$ of Ref.[6].

By its definition any Hecke symmetry determines a series of local representations of Hecke algebras $H_q(\mathbb{C})$ (see e.g. [10]). We will consider some central projectors of these

Hecke algebras, namely, antisymmetrizers ${P_-}^k$. They are iteratively defined by the relations

$$ {P_-}^1 = \mathbf{I} \; , \quad {P_-}^k = \frac{1}{k_q} \left(q^{k-1}\mathbf{I} - q^{k-2}R_{k-1} + \ldots + (-1)^{k-1}R_1 \cdot \ldots \cdot R_{k-1} \right) {P_-}^{k-1} \; . $$

Here we use the notation $R_i \equiv R_{i(i+1)}$ and ${P_-}^i \equiv ({P_-}^i)_{12\ldots i}$. One can give several alternative definitions for the antisymmetrizers:

$$ {P_-}^k = \frac{1}{k_q} {P_-}^{k-1} \left(q^{k-1}\mathbf{I} - q^{k-2}R_{k-1} + \ldots + (-1)^{k-1}R_{k-1} \cdot \ldots \cdot R_1 \right) \; , $$

or

$$ {P_-}^k = \frac{1}{k_q} \left(q^{k-1}\mathbf{I} - q^{k-2}R_1 + \ldots + (-1)^{k-1}R_{k-1} \cdot \ldots \cdot R_1 \right) {P_-}_2^{k-1} \; , $$

where ${P_-}_j^i \equiv ({P_-}^i)_{j(j+1)\ldots(j+i-1)}$.

The following properties of the projectors ${P_-}^k$ are proved in [6]:

$$ \begin{aligned} {P_-}^k R_i &= R_i {P_-}^k = -\frac{1}{q} {P_-}^k \; , \quad \text{for} \quad i \leq k-1 \; ; && (2.16) \\ {P_-}^k {P_-}_j^i &= {P_-}_j^i {P_-}^k = {P_-}^k \; , \quad \text{for} \quad i+j-1 \leq k \; ; && (2.17) \\ {P_-}^k R_k {P_-}^k &= -\frac{(k+1)_q}{k_q} {P_-}^{k+1} + \frac{q^k}{k_q} {P_-}^k \; ; && (2.18) \\ {P_-}_2^k R_1 {P_-}_2^k &= -\frac{(k+1)_q}{k_q} {P_-}^{k+1} + \frac{q^k}{k_q} {P_-}_2^k \; . && (2.19) \end{aligned} $$

The last pair of relations may be equally used for an iterative definition of ${P_-}^k$.

The closed Hecke symmetry R is called the *even Hecke symmetry of rank* p if the following condition on antisymmetrizer is satisfied

$$ dim {P_-}^{p+1} = 0 \; . \tag{2.20} $$

As a consequence of (2.20) one gets for the closed symmetry that the image of ${P_-}^p$ is one-dimentional [6]: $dim {P_-}^p = 1$. Therefore ${P_-}^p$ may be presented in the form

$$ ({P_-}^p)_{i_1 i_2 \ldots i_p}^{j_1 j_2 \ldots j_p} = u_{i_1 i_2 \ldots i_p} v^{j_1 j_2 \ldots j_p} \equiv u_{|12\ldots p\rangle} v^{\langle 12\ldots p|} \; , \tag{2.21} $$

where

$$ v^{i_1 i_2 \ldots i_p} u_{i_1 i_2 \ldots i_p} = 1 = v^{\langle 12\ldots p|} u_{|12\ldots p\rangle} \; , $$

and in the r.h.s. of the formulas we introduce the symbolic notation to be employed in what follows. The quantities $u_{|\ldots\rangle}$ and $v^{\langle\ldots|}$ are the analogues of left and right Levi-Civita tensors. They are defined (up to a numerical factor) by the conditions

$$R_i u_{|12\ldots p\rangle} = -\frac{1}{q} u_{|12\ldots p\rangle} , \qquad v^{\langle 12\ldots p|} R_i = -\frac{1}{q} v^{\langle 12\ldots p|} , \qquad i = 1, \ldots, p-1 . \tag{2.22}$$

One can easily check that the tensors $u_{|\ldots\rangle}$, $v^{\langle\ldots|}$ defined by (2.22) really enter into formula (2.21) for $P_-{}^p$ (simply apply $P_-{}^p$ to these $u_{|\ldots\rangle}$ and $v^{\langle\ldots|}$).

As a consequence of (2.18), (2.19) and the definitions of $\mathcal{C}$ and $\mathcal{B}$ (2.9), (2.10) we have

$$\mathcal{C}_{|a\rangle}^{\langle b|} = \frac{p_q}{q^p} v^{\langle b1\ldots p-1|} u_{|a1\ldots p-1\rangle} , \qquad \mathcal{B}_{|a\rangle}^{\langle b|} = \frac{p_q}{q^p} v^{\langle 1\ldots(p-1)b|} u_{|1\ldots(p-1)a\rangle} , \tag{2.23}$$

and also

$$Tr_q \mathbf{I} = Tr\mathcal{C} = \frac{p_q}{q^p} = Tr\mathcal{B} , \qquad Tr_{q(2)} R_{12} = Tr_{(1)} \mathcal{B}_1 R_{12} = \mathbf{I} . \tag{2.24}$$

Let us derive a few more useful relations between the quantum trace and the quantum Levi-Civita tensors. For any (possibly operator-valued) $N \times N$ matrix X_i^j define the *symmetrizing map* $S_+{}^k$: $Mat(N) \to Mat(N)^{\otimes k}$

$$S_+{}^k(X) \equiv X_1 + R_1 X_1 R_1 + \ldots + R_{k-1} R_{k-2} \cdot \ldots \cdot R_1 X_1 R_1 R_2 \cdot \ldots \cdot R_{k-1} . \tag{2.25}$$

Obviously, for any X,

$$S_+{}^k(X) R_i = R_i S_+{}^k(X) , \quad i = 1, \ldots, k-1 .$$

Now performing simple calculations one obtains the formula

$$P_-{}^p S_+{}^p(X) = S_+{}^p(X) P_-{}^p = \frac{p_q}{q^{p-1}} P_-{}^p X_1 P_-{}^p ,$$

whereof the desired relations follows:

$$v^{\langle 12\ldots p|} S_+{}^p(X) = q Tr_q X \, v^{\langle 12\ldots p|} , \qquad S_+{}^p(X) u_{|12\ldots p\rangle} = u_{|12\ldots p\rangle} \, q Tr_q X . \tag{2.26}$$

3 Newton relations and Cayley-Hamilton theorem

For any RE algebra associated with rank p even Hecke symmetry there are two canonical possibilities for introducing a set of p central elements being at the same time invariants of the adjoint coaction δ_ℓ (2.6). The first set contains the q-traces of powers of the matrix L

$$s_q(i) \equiv q\,Tr_q L^i\,, \quad i = 1, \ldots, p. \tag{3.1}$$

The centrality and invariance of these elements have been already demonstrated in the previous Section.

The second set is not so obvious as the first one. It is formed by combinations of quantum minors of the matrix L:

$$\sigma_q(i) \equiv \alpha_i v^{\langle 12\ldots p|}\,(L_1 R_1 \ldots R_{i-1})^i\,u_{|12\ldots p\rangle}\,. \tag{3.2}$$

Here α_i are some normalizing constants to be fixed below. As a partial explanation to this definition one may note that the combinations of $\mathcal{R}$-matrices enter the r.h.s. of (3.2) namely in such a way that the whole expression would be adjoint invariant. One can also directly check the centrality of the elements $\sigma_q(i)$, but this will follow from the connection between the first and the second sets established below.

The elements $s_q(i)$ and $\sigma_q(i)$ play, respectively, the role of the basic power sums and the basic symmetric polynomials for the quantum matrix L. In case of the quantum linear groups and for q being not the root of unity both sets are known to be complete, i.e., they generate the center of RE algebra. As for other Hecke symmetries, it seems that the problem of completeness of these sets should be treated in each case separately. The relation between these two sets is established by the following q-analogue of Newton's formulas:

Proposition 1. *The sets $\{\sigma_q(i)\}$ and $\{s_q(i)\}$, $i = 1, \ldots, p$ are connected by the relations*

$$\frac{i_q}{q^{i-1}}\sigma_q(i) - s_q(1)\,\sigma_q(i-1) + \ldots + (-1)^{i-1}s_q(i-1)\,\sigma_q(1) + (-1)^i s_q(i) = 0\,, \tag{3.3}$$

provided that the numerical factors α_i are fixed as

$$\alpha_i = q^{-i(p-i)}\binom{p}{i}_q\,. \tag{3.4}$$

Here $\binom{p}{i}_q = i_q!(p-i)_q!/p_q!$ *— are the q-binomial coefficients, and* $i_q! = i_q\,(i-1)_q!$, $1_q! = 1$.

Proof: Consider the quantities $s_q(i-j)\sigma_q(j)$ for $1 \le j < i \le p$. With the help of (2.26) and taking into account the commutativity relations

$$R_k(L_1R_1\dots R_{j-1})^j = (L_1R_1\dots R_{j-1})^j R_k\ , \quad k = 1,\dots,j-1\ ,$$

following from (2.1), (2.5), one can perform transformations:

$$\begin{aligned} s_q(i-j)\sigma_q(j) &= \alpha_j v^{\langle 12\dots p|} S_+{}^p(L^{i-j})(L_1R_1\dots R_{j-1})^j u_{|12\dots p\rangle} = \\ &\alpha_j \frac{j_q}{q^{j-1}}\, v^{\langle 12\dots p|}(L_1^{i-j+1}R_1\dots R_{j-1})(L_1R_1\dots R_{j-1})^{j-1}u_{|12\dots p\rangle} + \\ &\alpha_j \frac{(p-j)_q}{q^{p-j-1}}\, v^{\langle 12\dots p|}(R_j\dots R_1L_1^{i-j}R_1\dots R_j)(L_1R_1\dots R_{j-1})^j u_{|12\dots p\rangle}\ . \end{aligned}$$

Now using relations

$$(L_1R_1\dots R_{j-1})^j R_j\dots R_1 = (L_1R_1\dots R_j)^j$$

we complete the transformation of the second term of the sum, and finally have

$$\begin{aligned} s_q(i-j)\sigma_q(j) &= \alpha_j \frac{j_q}{q^{j-1}}\, v^{\langle 12\dots p|}(L_1^{i-j+1}R_1\dots R_{j-1})(L_1R_1\dots R_{j-1})^{j-1}u_{|12\dots p\rangle} + \\ &\alpha_j \frac{(p-j)_q}{q^{p-j-1}}\, v^{\langle 12\dots p|}(L_1^{i-j}R_1\dots R_j)(L_1R_1\dots R_j)^j u_{|12\dots p\rangle}\ . \end{aligned} \tag{3.5}$$

In the boundary cases $j=1$, $j=i-1$ this equation reads

$$\begin{aligned} s_q(i-1)\sigma_q(1) &= \alpha_1 \frac{q^{p-1}}{p_q} s_q(i) + \alpha_1 \frac{(p-1)_q}{q^{p-2}}\, v^{\langle 12\dots p|}(L_1^{i-1}R_1)(L_1R_1)u_{|12\dots p\rangle}\ ; \\ s_q(1)\sigma_q(i-1) &= \alpha_{i-1} \frac{(i-1)_q}{q^{i-2}}\, v^{\langle 12\dots p|}(L_1^2R_1\dots R_{i-2})(L_1R_1\dots R_{i-2})^{i-2}u_{|12\dots p\rangle} + \\ &\frac{\alpha_{i-1}}{\alpha_i}\frac{(p-i+1)_q}{q^{p-i}}\sigma_q(i)\ . \end{aligned}$$

Let us choose $\alpha_1 = p_q/q^{p-1}$ in order that $\sigma_q(1) = s_q(1)$ Now putting

$$\alpha_j = q^{2j-1-p}\frac{(p-j+1)_q}{j_q}\alpha_{j-1}\ ,$$

and, then, making an alternating sum of (3.5) for different values of $1 \le j \le i-1$ one obtains the relation (3.3). ■

The elements $s_q(k)$ for $k > p$ can also be expressed in terms of $\{\sigma_q(i)\}$, or $\{s_q(i)\}$, $i \le p$. The relation is provided by an analogue of the Cayley-Hamilton Theorem:

Proposition 2. *The L-matrix satisfies the following characteristic identity*

$$\sum_{i=0}^{p}(-L)^i\sigma_q(p-i) \equiv 0\ . \tag{3.6}$$

Here we imply $\sigma_q(0) = 1$.

Proof: Consider the quantity

$$w(x)^{\langle 12\ldots p|} \equiv \prod_{i=0}^{p-1}\Big[(L_1 - q^{2i}x\mathbf{I})R_1\ldots R_{p-1}\Big]\, u_{|12\ldots p\rangle}\ .$$

Generalizing in an obvious way an observation that the commutator $\left[R_1\, ,\, (L_1 - x\mathbf{I})R_1(L_1 - q^2x\mathbf{I})\right]$ is proportional to the q-symmetric term $(R_1 + 1/q)$, one can convince himself that $R_i w(x)^{\langle 12\ldots p|} = 0$ for $i = 1, \ldots, p-1$. Hence

$$w(x)^{\langle 12\ldots p|} = \triangle(x)v^{\langle 12\ldots p|}\ . \tag{3.7}$$

Here the scalar coefficient $\triangle(x) \equiv w(x)^{\langle 12\ldots p|}u_{|12\ldots p\rangle}$ is an analogue of the characteristic polynomial for the matrix L. It can be calculated with the use of (2.1), (2.5), (2.22), and the q-combinatorial relations. The result is

$$\triangle(x) = \sum_{i=0}^{p}(-x)^i\sigma_q(p-i)\ . \tag{3.8}$$

Now, simply repeating the classical arguments of [7] one can prove that $\triangle(L) \equiv 0$. The starting point for this is provided, e.g., by the relation

$$w(x)^{\langle 2\ldots p+1|}u_{|12\ldots p\rangle} = \triangle(x)v^{\langle 2\ldots p+1|}u_{|1\ldots p\rangle}\ .$$

$\blacksquare$

To conclude it is worth making one note. In the case $q \to 1$ (that is $R^2 = 1$) formulae (3.3) and (3.6) tend to their classical limits. We would like to emphasize that the key role is played by the value of *rank* p of Hecke symmetry. The specific value N of the dimension of the space V where this symmetry is realized does not matter here.

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References

[1] Cherednik I.V., Theor. Math. Phys. **61** (1984) 35 (in Russian).

[2] – Quantum current algebras: Reshetikhin N.Yu. and Semenov-Tian-Shansky M.A., Lett. Math. Phys. **19** (1990) 133.

– Braided groups: Majid S., in: *Quantum Groups*, Lect. Notes in Math. **1510** (1992) 79; J.Math.Phys **32** (1991) 3246; ibid. **34** (1993) 2045.

– Quantum group differential geometry: Jurčo B., Lett. Math. Phys. **22** (1991) 177; Alekseev A.Yu., Faddeev L.D., Commun. Math. Phys. **141** (1991) 413; Schupp P., Watts P., Zumino B., Lett. Math. Phys. **25** (1992) 139-147; Commun. Math. Phys. **157** (1993) 305; Isaev A.P., Pyatov P.N., J.Phys.A: Math.Gen. **28** (1995) 2227-2246; Faddeev L.D., Pyatov P.N., hep-th/9402070, to appear in F.A.Berezin memorial volume.

– q-Minkowski space-time: Kulish P.P., Algebra & Analiz **6** (1994) 195.

[3] Drinfeld V.G., Algebra & Analiz, **1** No. 2 (1989) 30-46; Majid S., in: M.-L.Ge and H.J.de Vega (Eds), *Quantum Groups, Integrable Statistical Models and Knot Theory*, World Scientific, Singapore, 1993, p.231.

[4] Pyatov P.N., Saponov P.A., J.Phys.A: Math.Gen. **28** (1995) 4415-4421.

[5] Nazarov M. and Tarasov V., Publications RIMS **30** (1994) 459.

[6] Gurevich D.I., Algebra & Analiz **2** (1990) 119-148 (in Russian); English translation in: Leningrad Math. J. **2** (1991) 801-828.

[7] Lankaster P., *'Theory of Matrices'.* Academic Press, New York – London, 1969.

[8] Faddeev L.D., Reshetikhin N.Yu., Takhtajan L.A., Algebra & Analiz **1** (1989) 178-206 (in Russian); English translation in: Leningrad Math. J. **1** (1990) 193.

[9] Reshetikhin N.Yu., Algebra & Analiz **1** (1989) 169 (in Russian); English translation in: Leningrad Math. J. **1** (1990) 491.

[10] Wenzl H., Invent. Math. **92** Fasc.2. (1988) 349.