Differential Migration Prospects, Skill Formation, and Welfare

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Abstract

This paper develops a one sector, two-input model with endogenous human capital formation. The two inputs are two types of skilled labor: "engineering," which exerts a positive externality on total factor productivity, and "law," which does not. The paper shows that a marginal prospect of migration by engineers increases human capital accumulation of both types of workers (engineers and lawyers), and also the number of engineers who remain in the country. These two effects are socially desirable, since they move the economy from the (inefficient) free-market equilibrium towards the social optimum. The paper also shows that if the externality effect of engineering is sufficiently powerful, everyone will be better off as a consequence of the said prospect of migration, including the engineers who lose the migration "lottery," and even the individuals who practice law.

1   Introduction

Substantial research has led to a consensus that human capital is a key determinant of both economic efficiency and social welfare.[1] Ever since the influential contribution of Lucas (1988), much of the literature has underscored the role of the externality effect of human capital in accounting for its crucial importance as a factor of production.[2] Since human capital is inherently heterogeneous, it stands to reason that different types of human capital confer different human capital externalities which, in turn, bear upon economic performance. Indeed, with both micro data and macro data, the empirical literature highlights the importance of the heterogeneity of human capital. For example, Willis (1986) and Grogger and Eide (1995) underscore the importance of the heterogeneity of human capital in determining labor earnings. Krueger and Lindahl (2001) survey evidence showing that the heterogeneity of human capital helps explain variation in cross-country economic growth. Nonetheless, theoretical analyses of heterogeneous human capital are relatively rare. Notable exceptions include Iyigun and Owen (1998, 1999), who emphasize the importance of both "professional human capital" and "entrepreneurial human capital" in economic development. They show how economies that have too little of either type of human capital might be hindered in their pursuit of economic growth.

In this paper, we seek to complement the received literature by developing a model of heterogeneous human capital with a particular emphasis on the impact of international migration on individuals' incentive to acquire different types of human capital. We contribute to the received literature in two specific respects. First, we allow various types of human capital to differ significantly in terms of their externality effect. Second, we consider the differential international "portability" of different types of human capital in an open economy setting, where the migration of one type of human capital is possible whereas that of another is not.

Our presumption is that individuals who possess the types of human capital that have high social returns (strong externality effects) in a developing country, are more likely to land a rewarding job offer in a developed country than individuals who possess the types of human capital that have low social returns (weak externality effects) in the developing country. The intuition underlying this thinking is quite simple: while the types of human capital that confer high social returns and associated high externality effects, such as engineering, are fairly universal, the types of human capital that confer lower social returns in a developing country, such as law, are not. The individuals with the former types of human capital in a developing country have a much better chance of

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[1]For example, see Becker (1964), Barro (1991), Mankiw, Romer and Weil (1992), Weil (2005), and the literature reviewed therein.

[2]Among others, recent important contributions on the externality effect of human capital include Acemoglu (1996), Black and Henderson (1999), Glaeser and Saiz (2004), Moretti (2004), and Ciccone and Peri (2006).

working in a developed country.

The "architecture" of our paper is as follows. To begin with, we study the formation and allocation of heterogeneous human capital in a closed developing economy. Efficient resource allocation would assign skilled workers in optimal proportions to occupations requiring different types of human capital. However, without government intervention or any prospect of migration, the different degrees of positive externality of different types of human capital entail a market failure in terms of achieving efficient allocation of productive human capital. This failure arises from too few individuals choosing to invest in the types of human capital that generate high externality effects and low private returns (for example, pure science). In addition, from the perspective of social welfare, all the individuals choose to acquire too little human capital.

We then examine how the prospect of migration may correct this allocation inefficiency. When the economy is open, selective migration can substantially enhance social welfare: inefficient resource allocation can be mitigated when the expected private returns to individuals who accumulate human capital with high social returns are raised by conferring upon them a better chance of migrating and working in a richer, technologically advanced country. We show that the prospect of migration for these individuals increases human capital accumulation, redistributes talent in a socially desirable way, increases the ex-ante (before migration occurs) payoffs of all groups of workers, and, under certain sufficient conditions, increases welfare – even that of the workers who responded to the opportunity to migrate but ended up not migrating.

Our analysis complements recent research on the "beneficial brain drain," which demonstrates that a policy of controlled migration from a developing country encourages individuals there to accumulate more human capital than they would have chosen to do in the absence of such a policy, and consequently, that welfare increases for both the migrants and for those who stay behind in the developing country.[3] We identify an additional channel - other than the incentive effect on the acquired quantity of human capital – through which controlled migration can increase social welfare: the chance of migrating influences individuals' decisions regarding the type of human capital that they form. It makes it more attractive for individuals to acquire human capital with a high externality effect but low private returns. The prospect of migration attached to one type of human capital can revise the composition of the human capital acquired in an economy in a socially desirable manner. A policy that enables workers of a specific type to migrate can benefit workers of all types.

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[3]See, for example, Stark, Helmenstein, and Prskawetz (1997, 1998), Stark and Wang (2002), Fan and Stark (2007), and Stark, Casarico, Devillanova, and Uebelmesser (2011).

2 A closed-economy model

2.1 Setup

2.1.1 Workers

Consider a model with one consumption good, the price of which is normalized to unity, and two production inputs: engineering, which we denote by M (think of mechanical engineering), and law, L. The economy is populated by a continuous set $\mathcal{N}$ of individuals with linear preferences over the consumption good. Prior to employment, each individual chooses which type of human capital – engineering or law – to acquire; the set of all the individuals is thus partitioned into $\mathcal{N}_M$ individuals who study engineering, and $\mathcal{N}_L$ individuals who study law.

After the occupational choices are made, individuals of both specializations choose *how much* human capital to acquire. The cost of acquiring θ_x units of human capital of either type by individual $x \in \mathcal{N}$ is $\frac{K}{2}\theta_x^2$, where $K > 0$ measures the difficulty of human capital acquisition. For the sake of simplicity, we assume that the said cost is the same for both types of human capital; relaxing this assumption will change the results that follow quantitatively, but not qualitatively.

2.1.2 Firms

Competitive firms produce the consumption good by means of a constant returns to scale Cobb-Douglas technology, and use labor of both types as inputs of production. Denoting the discrete set of firms by $\mathcal{I}$, the production function of a firm $i \in \mathcal{I}$ is

$$Y_i = A \left(\int_{x \in \mathcal{N}_{iM}} \theta_x dx \right)^{\alpha} \left(\int_{x \in \mathcal{N}_{iL}} \theta_x dx \right)^{1-\alpha} \tag{1}$$

where $\mathcal{N}_{ij}$ is the set of workers of type $j \in \{M, L\}$ hired by firm i, θ_x is the human capital of worker x, and $\alpha \in [0, 1]$ is the output elasticity of engineering. The parameter A is total factor productivity (henceforth TFP) which, we assume, depends on the average knowledge of engineering (but not law) in the entire population:

$$A = \left(\frac{\int_{x \in \mathcal{N}_M} \theta_x dx}{N} \right)^{\eta} \tag{2}$$

where $\eta \in [0, 1)$ is the elasticity of TFP with respect to the average knowledge of engineering[4] and $N = |\mathcal{N}|$ is the measure of the set $\mathcal{N}$, that is, the population size.[5] We

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[4]We assume that η does not exceed unity because otherwise the maximization problems solved below would become convex with no finite solutions. A "modest" value of $\eta < 1$ is empirically quite plausible.

[5]In the analysis that follows we will denote by N_{ij} the measure of a set $\mathcal{N}_{ij}$ of workers of type $j \in \{M, L\}$ hired by firm i. By N_j we will denote the measure of a set $\mathcal{N}_j$ of workers of type j.
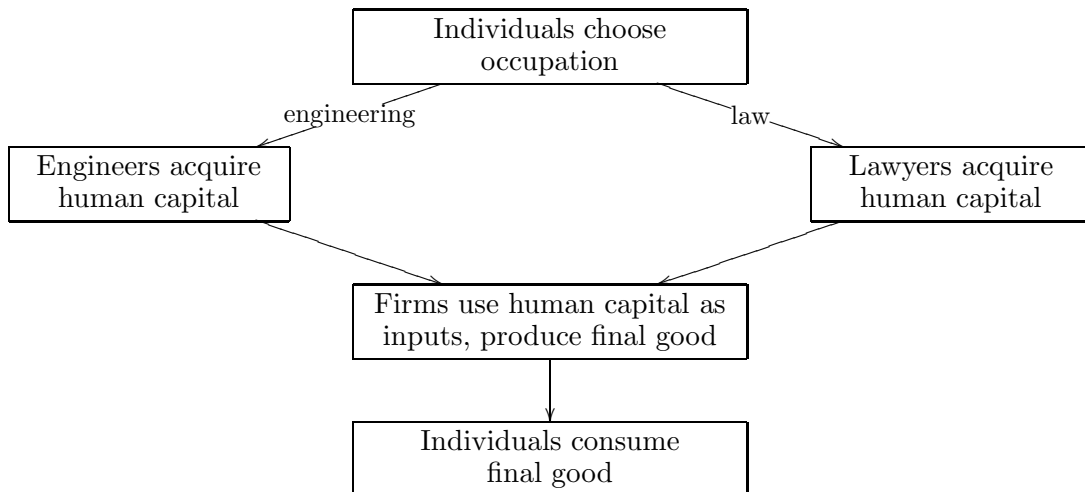
Figure 1: The timing of events in the model of the closed economy

assume that each firm is small enough, and that it treats the total factor productivity as given.

The timing of events is illustrated in Figure 1.

As shown in the subsequent analysis, all the individuals of a given occupation acquire the same amount of human capital; denote this amount by θ_M for engineers, and by θ_L for lawyers. This allows us to rewrite (1) and (2), respectively, as

$$
Y_i = A \left(\theta_M N_{iM}\right)^\alpha \left(\theta_L N_{iL}\right)^{1-\alpha} \tag{3}
$$

$$
A = \left(\theta_M \frac{N_M}{N}\right)^\eta \tag{4}
$$

As follows from a well-known property of the constant-returns-to-scale Cobb-Douglas production function, the number of firms in the market, as well as their size distribution, is immaterial for computing the aggregate output and aggregate demand for the two types of labor. A corollary of this property is that we can compute the aggregate output by assuming that there is only one firm that hires all the workers, and that this firm has the following production function:

$$
Y(\theta_M, \theta_L, N_M, N_L) = \left(\theta_M \frac{N_M}{N}\right)^\eta \left(\theta_M N_M\right)^\alpha \left(\theta_L N_L\right)^{1-\alpha} \tag{5}
$$

2.2 The social planner's problem

The social planner seeks to bring to a maximum the aggregate output net of the aggregate cost of human capital acquisition, hence to solve the following problem:

$$
\max_{\{\theta_M, \theta_L, N_M, N_L\}} \left[Y(\theta_M, \theta_L, N_M, N_L) - N_M \frac{K}{2} \theta_M^2 - N_L \frac{K}{2} \theta_L^2 \right]
$$

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subject to the size-of-population constraint: $N_M + N_L \leq N$.

The first order conditions are

$$(\alpha + \eta)\frac{Y(\theta_M, \theta_L, N_M, N_L)}{\theta_M} - N_M K \theta_M = 0 \tag{6}$$

$$(1 - \alpha)\frac{Y(\theta_M, \theta_L, N_M, N_L)}{\theta_L} - N_L K \theta_L = 0 \tag{7}$$

$$(\alpha + \eta)\frac{Y(\theta_M, \theta_L, N_M, N_L)}{N_M} - \frac{K}{2}\theta_M^2 - \lambda = 0 \tag{8}$$

$$(1 - \alpha)\frac{Y(\theta_M, \theta_L, N_M, N_L)}{N_L} - \frac{K}{2}\theta_L^2 - \lambda = 0 \tag{9}$$

where λ is the Lagrange multiplier for the size-of-population constraint. Upon multiplying (6) by $\frac{\theta_M}{N_M}$ and subtracting the resulting expression from (8), we obtain:

$$\frac{K}{2}\theta_M^2 - \lambda = 0 \tag{10}$$

By conducting similar operations with (7) and (9), we obtain an expression for θ_L:

$$\frac{K}{2}\theta_L^2 - \lambda = 0 \tag{11}$$

From comparing (10) to (11) it follows that the levels of human capital (say years of university education) acquired in both occupations are equal to each other: $\theta_M = \theta_L \equiv \theta$; this feature helps to simplify considerably the subsequent analysis. Given this result, (6) and (7) imply that $\frac{N_M}{N_L} = \frac{\alpha + \eta}{1 - \alpha}$. Combined with the size-of-population constraint, we get that the numbers of engineers and lawyers in the economy are, respectively,

$$N_M = \frac{\alpha + \eta}{1 + \eta}N \tag{12}$$

and

$$N_L = \frac{1 - \alpha}{1 + \eta}N$$

The aggregate output Y can now be expressed as follows:

$$
\begin{aligned}
Y &= \left[\frac{\alpha + \eta}{1 + \eta}\theta\right]^\eta \left[\frac{\alpha + \eta}{1 + \eta}\theta N\right]^\alpha \left[\frac{1 - \alpha}{1 + \eta}\theta N\right]^{1-\alpha} \\
&= \frac{(\alpha + \eta)^{\alpha + \eta}(1 - \alpha)^{1-\alpha}}{(1 + \eta)^{1+\eta}}\theta^{1+\eta}N \\
&= C_0 \theta^{1+\eta}N
\end{aligned}
\tag{13}
$$

where

$$C_0 \equiv \frac{(\alpha + \eta)^{\alpha + \eta}(1 - \alpha)^{1-\alpha}}{(1 + \eta)^{1+\eta}}$$

Insertion of (13) and (12) into (6) yields

$$(\alpha + \eta)C_0\theta^\eta N - \frac{\alpha + \eta}{1 + \eta}NK\theta = 0$$

which generates the following expression for the optimal level of human capital, θ_0:

$$\theta_0 = \left((1 + \eta)\frac{C_0}{K}\right)^{\frac{1}{1-\eta}} \qquad (14)$$

From (13) and (14) it follows that per capita welfare (that is, per capita output less the per capita cost of human capital acquisition) is

$$
\begin{aligned}
W_0 &= \frac{Y}{N} - \frac{K}{2}\theta_0^2 \\
&= \left[(1 + \eta)^{\frac{1+\eta}{1-\eta}} - \frac{1}{2}(1 + \eta)^{\frac{2}{1-\eta}}\right]C_0^{\frac{2}{1-\eta}}K^{-\frac{1+\eta}{1-\eta}} \qquad (15)
\end{aligned}
$$

2.3 The market equilibrium

2.3.1 The labor market: supply

The labor market is characterized by the equilibrium wages w_j for an efficiency unit of human capital of type $j \in \{M, L\}$. Given the wages, each worker of each type j decides how much human capital to acquire by solving

$$\max_{\theta_j}\left[w_j\theta_j - \frac{K}{2}\theta_j^2\right] \qquad (16)$$

which yields a unique solution of $\theta_j^* = \frac{w_j}{K}$. Therefore, the welfare of a worker of type j is

$$W_j = w_j\theta_j^* - \frac{K}{2}(\theta_j^*)^2 = \frac{1}{2}\frac{w_j^2}{K}$$

Since workers are free to choose their occupation, in equilibrium they enjoy the same welfare in both occupations. This means that equilibrium wages must be equal across the two occupations

$$w_M = w_L \equiv w \qquad (17)$$

which also implies that workers acquire the same level of human capital in both occupations

$$\theta_1 \equiv \theta_j^* = \frac{w}{K}, \forall j \qquad (18)$$

and therefore, that welfare is equal to

$$W_1 = \frac{1}{2}\frac{w^2}{K}$$

2.3.2 The labor market: demand

We assume that there is a discrete set $\mathcal{I}$ of price-taking firms. The firms treat the total factor productivity (4) as given. Because of the constant-returns-to-scale production technology and perfect competition, a firm of any size will make zero profit in equilibrium, hence, analytically speaking, firm size does not matter.

Consider a firm $i \in \mathcal{I}$ that seeks to produce Y_i units of output at minimal cost; its optimization problem is

$$\min_{N_{iM}, N_{iL}} (w_M \theta_M N_{iM} + w_L \theta_L N_{iL}) \tag{19}$$

subject to (cf.(3))

$$A(\theta_M N_{iM})^\alpha (\theta_L N_{iL})^{1-\alpha} = Y_i$$

where $N_{ij}, j \in \{M, L\}$ is the number of workers of type j hired by firm i. The first-order conditions for this problem are

$$w_M \theta_M - \mu \alpha \frac{Y_i}{N_{iM}} = 0$$

$$w_L \theta_L - \mu(1-\alpha)\frac{Y_i}{N_{iL}} = 0$$

where μ is the Lagrange multiplier. From the first-order conditions we conclude that the ratio of engineers to lawyers demanded by any firm i is

$$\frac{N_{iM}}{N_{iL}} = \frac{\alpha}{1-\alpha} \frac{w_L}{w_M} \frac{\theta_L}{\theta_M} \tag{20}$$

Recalling the equilibrium wage equality (17) and human capital equality (18), we conclude that in equilibrium the ratio of the aggregate quantities demanded is equal to $\frac{N_M}{N_L} = \frac{\alpha}{1-\alpha}$. Given that the total supply of workers is N, the equilibrium division of labor is

$$N_M = \alpha N \tag{21}$$
$$N_L = (1-\alpha)N$$

2.3.3 Equilibrium analysis

We can now write the firms' aggregate profits $\sum_i \pi_i$, and due to the assumption of perfect competition, set them equal to zero. Recalling the expression for total factor productivity (4), as well as (21), we get that

$$\sum_i \pi_i = (\theta\alpha)^\eta (\theta\alpha N)^\alpha (\theta(1-\alpha)N)^{1-\alpha} - w\theta N = 0 \tag{22}$$

Upon dividing (22) throughout by θN, we obtain the following expression for the equilibrium wage:

$$w = \theta^{\eta} \alpha^{\alpha+\eta}(1-\alpha)^{1-\alpha} = \theta^{\eta} C_1 \qquad (23)$$

where $C_1 \equiv \alpha^{\alpha+\eta}(1-\alpha)^{1-\alpha}$. By solving the system of equations (18) and (23), we obtain unique solutions for the equilibrium level of human capital θ, the wage w, and the worker's welfare W_1:

$$\theta_1 = \left(\frac{C_1}{K}\right)^{\frac{1}{1-\eta}} \qquad (24)$$

$$w = C_1^{\frac{1}{1-\eta}} K^{-\frac{\eta}{1-\eta}} \qquad (25)$$

$$W_1 = \frac{1}{2} C_1^{\frac{2}{1-\eta}} K^{-\frac{1+\eta}{1-\eta}} \qquad (26)$$

2.3.4 Free equilibrium versus social optimum

We compare the social optimum with the free market equilibrium. As a preliminary, it is helpful to establish the following two technical results.

Lemma 1

$$\frac{(\alpha+\eta)^{\alpha+\eta}(1-\alpha)^{1-\alpha}}{(1+\eta)^{1+\eta}} \equiv C_0 \geq C_1 \equiv \alpha^{\alpha+\eta}(1-\alpha)^{1-\alpha}$$

with strict inequality if and only if $\eta \in (0,1)$ and $\alpha < 1$.

Lemma 2

$$(1+\eta)^{\frac{1+\eta}{1-\eta}} - \frac{1}{2}(1+\eta)^{\frac{2}{1-\eta}} \geq \frac{1}{2}$$

with strict inequality if and only if $\eta \in (0,1)$.

The proofs are in the Appendix.

We can now establish the following important results.

Proposition 1 *In the free-market equilibrium, the welfare per capita, the amount of accumulated human capital, and the share of engineers in the population are below the socially desirable level if $\eta \in (0,1)$ and $\alpha < 1$.*

Proof. The first part of the Proposition follows from a comparison of (15) and (26), using Lemmas 1 and 2. The second part follows from a comparison of (14) and (24), using Lemma 1. The third part follows directly from a comparison of (12) and (21). ∎

We next investigate how a selective migration prospect affects the free-market equilibrium level of welfare, the accumulated human capital, and the share of engineers in the population.
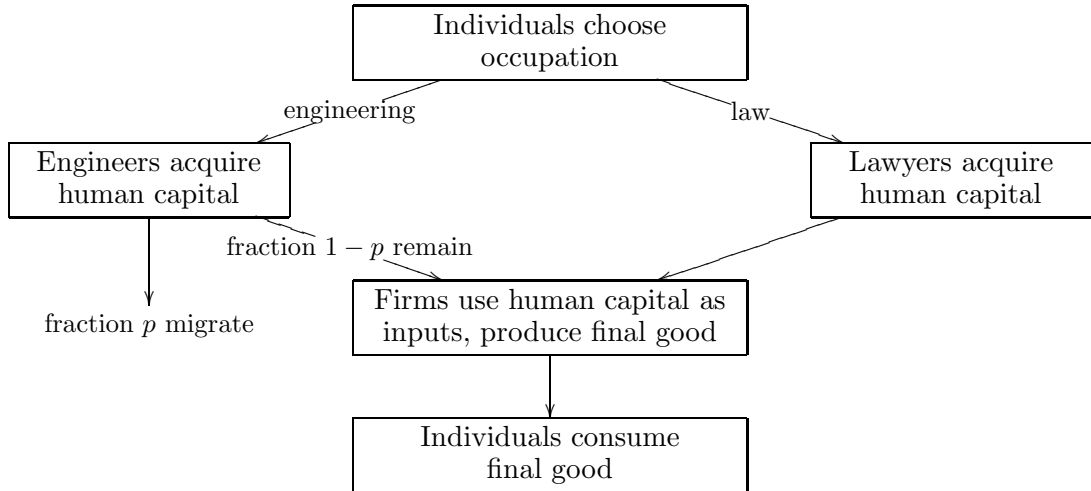
Figure 2: The timing of events in the model with migration

3 The effects of the possibility of migration by engineers

3.1 The open-economy setup

We now assume that there is a prospect of migration for engineers, but not for lawyers whose human capital is specific to their home country. The timing of events is as follows: first, and as before, individuals choose what type of human capital to acquire; second, and again as before, individuals decide how much human capital to acquire; third, a randomly chosen fraction $p \in [0, 1)$ of engineers migrate. A migrant engineer earns a higher foreign wage $\overline{w} > w_M$, where $\overline{w}$ is fixed and is exogenous to the model. The non-migrating engineers and all the lawyers work in the home country for the prevailing wage rates. Figure 2 illustrates the timing of the events in the model with migration.

As before, in the first step individuals choose an occupation that brings them the highest expected welfare. In equilibrium, they must be indifferent between the two occupations. This condition has two implications: first, and as in the closed-economy setting, individuals acquire the same amount of human capital in both occupations; second, the expected incomes of the individuals in both occupations must be equal to each other and, in turn, are equal to θK, as in (17) and (18):

$$p\overline{w} + (1 - p)w_M = w_L = \theta K \tag{27}$$

where, to recall, $w_j, j \in \{M, L\}$ is the domestic wage of an occupation.

Recalling that there is a positive externality of the average level of engineering human capital for firm productivity, the prospect of migration has three effects: first, it induces

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individuals to acquire more human capital (a positive effect); second, it increases the ex-ante (prior to migration) fraction of the individuals who study engineering (another positive effect); third, it results in a fraction of engineers leaving the country, potentially decreasing the ex-post share of engineers in the non-migrating population (a negative effect).

Formally, in the presence of a prospect of migration, total factor productivity (recalling (4)) is

$$A = \left(\theta \frac{(1-p)N_M}{(1-p)N_M + N_L} \right)^\eta \tag{28}$$

As in the closed-economy scenario, a firm's problem is to solve (19), which results in the same ratio of the firm's demand for the two types of human capital as in (20). At the aggregate level, the ratio of the domestically demanded engineers to lawyers is

$$\frac{(1-p)N_M}{N_L} = \frac{\alpha}{1-\alpha} \frac{w_L}{w_M} \tag{29}$$

which enables us to find the equilibrium numbers of workers in both occupations:

$$N_M = \frac{\alpha}{(1-p)w_M} RN \tag{30}$$

$$N_L = \frac{1-\alpha}{w_L} RN \tag{31}$$

where

$$R \equiv \frac{1}{\frac{\alpha}{(1-p)w_M} + \frac{1-\alpha}{w_L}} \tag{32}$$

Next, by setting aggregate profit

$$A \left(\theta(1-p)N_M \right)^\alpha \left(\theta N_L \right)^{1-\alpha} - w_M \theta(1-p)N_M - w_L \theta N_L$$

to zero, substituting the expressions for labor demand (30) and (31), and dividing throughout by θRN, we come up with the following equilibrium wage condition:

$$A \left(\frac{\alpha}{w_M} \right)^\alpha \left(\frac{1-\alpha}{w_L} \right)^{1-\alpha} - 1 = 0 \tag{33}$$

The system of equations (27), (28), (30)-(32), and (33) completely describes the equilibrium. After a series of manipulations, the system can be simplified to the following expression:

$$C_1 \left(\theta \frac{1}{\frac{\alpha}{w_M} + \frac{1-\alpha}{w_L}} \right)^\eta \left(\frac{1}{w_M} \right)^{\alpha+\eta} \left(\frac{1}{w_L} \right)^{1-\alpha} - 1 = 0 \tag{34}$$

From (27), it follows that $w_M = \frac{\theta K - p\overline{w}}{1-p}$ which, along with (27), we substitute into

(34) to get

$$C_1 \left(\frac{\theta}{\frac{\alpha(1-p)}{\theta K - p\overline{w}} + \frac{1-\alpha}{\theta K}} \right)^{\eta} \left(\frac{1-p}{\theta K - p\overline{w}} \right)^{\alpha+\eta} \left(\frac{1}{\theta K} \right)^{1-\alpha} = 1 \qquad (35)$$

The only unknown in the latter equation is the level of human capital θ. Note that by setting $p = 0$, we can verify the equivalence of (35) to the free-market closed-economy equilibrium (24).

For the subsequent analysis, it is convenient to use the logarithmic form of (35):

$$\log C_1 + F(\theta, p) = 0 \qquad (36)$$

where

$$
\begin{aligned}
F(\theta, p) &\equiv \eta \left(\log \theta - \log \left(\alpha G(\theta, p) + \frac{1-\alpha}{\theta K} \right) \right) \\
&+ (\alpha + \eta) \log (G(\theta, p)) - (1 - \alpha) \log (\theta K) \\
G(\theta, p) &\equiv \frac{1-p}{\theta K - p\overline{w}}
\end{aligned}
$$

With the arbitrary values of the model parameters, a closed-form solution for an optimal θ does not exist. Nonetheless, several properties of the solution can be established. First, to render the engineering wage $w_M = \frac{\theta K - p\overline{w}}{1-p}$ meaningful (that is, positive), θ has to exceed a lower bound: $\theta \geq \underline{\theta}(p) = \frac{p\overline{w}}{K}$. Second, it can be shown that $F(\underline{\theta}(p), p) = \infty$, whereas $F(\infty, p) = -\infty$. Therefore, given the continuity of F, a solution to (36) exists. Third, the first derivative of (36) with respect to θ can be shown to be negative, that is:

$$\frac{\partial F(\theta, p)}{\partial \theta} = -\frac{(1-\eta)(\theta K - p\overline{w}) + \left(\alpha + \eta \frac{(1-\alpha)\frac{1}{\theta K}}{\alpha G(\theta, p) + (1-\alpha)\frac{1}{\theta K}} \right) p\overline{w}}{\theta(\theta K - p\overline{w})} < 0 \qquad (37)$$

Thus, a solution to (36) exists, and is unique.

3.2 The repercussions of opening the economy to migration

In this section we inquire under what conditions (if any) a small increase in the probability of migration for engineers brings the domestic economy closer to the socially desirable outcomes in terms of human capital accumulation, the share of engineers in the remaining population, and the welfare of each population group.

Proposition 2 *An increase of the probability of migration from zero to a small positive value increases human capital accumulation. Formally, $\left. \frac{d\theta}{dp} \right|_{p=0} > 0$.*

The prospect of migration *only* of engineers increases human capital accumulation of *both* engineers and lawyers. Since individuals are free to choose their occupation, an

increase of the expected returns to human capital in engineering must be mirrored by an equivalent increase in the returns to human capital in law which, in turn, implies an increased human capital accumulation by lawyers. Thus, an increase in p brings the levels of human capital in both occupations closer to their socially desirable level.

Proof. $\frac{d\theta}{dp}$ in the vicinity of $p = 0$ can be computed from (36), using the implicit function theorem:

$$\frac{d\theta}{dp} = -\frac{\partial F/\partial p}{\partial F/\partial \theta} \tag{38}$$

Below, we evaluate the two terms on the right-hand side of (38) at $p = 0$. Note that at $p = 0$, the amount of human capital θ that individuals acquire is equal to that in the closed-economy market equilibrium θ_1. From (37), we have that

$$\left.\frac{\partial F}{\partial \theta}\right|_{p=0} = -\frac{1-\eta}{\theta_1} \tag{39}$$

$$\frac{\partial F}{\partial p} = \frac{\partial G(\theta, p)}{\partial p}\left[-\eta\frac{\alpha}{\alpha G(\theta, p) + \frac{1-\alpha}{\theta K}} + (\alpha + \eta)\frac{1}{G(\theta, p)}\right]$$

$$\left.\frac{\partial F}{\partial p}\right|_{p=0} = \frac{(\overline{w} - \theta_1 K)}{\theta_1 K}(\alpha + \eta - \eta\alpha) \tag{40}$$

Recall from (18) that $\theta_1 K$ is the closed-economy wage and thus, by assumption, we have that $\overline{w} - \theta_1 K > 0$. Therefore, from (39) and (40) it directly follows that $\left.\frac{d\theta}{dp}\right|_{p=0} = \frac{(\overline{w} - \theta_1 K)}{K}\frac{\alpha + \eta - \eta\alpha}{1-\eta}$ is always positive. ∎

We next calculate the effect of a marginal increase in the prospect of migration on the share of engineers in the remaining population. From the definition of this share, we have that

$$s(p, \theta) \equiv \frac{(1-p)N_M}{(1-p)N_M + N_L} = \frac{s_M(p, \theta)}{s_M(p, \theta) + s_L(\theta)}$$

where (cf. (27), (30), (31))

$$s_M(p, \theta) \equiv \frac{(1-p)N_M}{RN} = \frac{\alpha}{w_M} = \frac{\alpha(1-p)}{\theta K - p\overline{w}}$$

$$s_L(\theta) \equiv \frac{N_L}{RN} = \frac{1-\alpha}{w_L} = \frac{1-\alpha}{\theta K}$$

The full derivative of $s(p, \theta)$ with respect to p is

$$\frac{ds}{dp} = \frac{\partial s}{\partial p} + \frac{\partial s}{\partial \theta}\frac{d\theta}{dp} \tag{41}$$

Before computing this full derivative, we establish the following result.

Lemma 3 *In the vicinity of $p = 0$, the change of human capital θ has no effect on the fraction of engineers in the population.*

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Proof.

$$\frac{\partial s}{\partial \theta} = \frac{\frac{\partial s_M}{\partial \theta} s_L - \frac{\partial s_L}{\partial \theta} s_M}{(s_M + s_L)^2}$$

The numerator of this expression, in the vicinity of $p = 0$, is

$$-\frac{\alpha K}{(\theta K)^2}\frac{1-\alpha}{\theta K} + \frac{1-\alpha}{\theta^2 K}\frac{\alpha}{\theta K} = 0$$

which implies that $\left.\frac{\partial s}{\partial \theta}\right|_{p=0} = 0$ ■

An immediate implication of this result is that in the vicinity of $p = 0$, the full derivative of $s(p, \theta)$ with respect to p is equal to its partial derivative with respect to p. This enables us to state the following proposition.

Proposition 3 *With an increase in the prospect of migration of engineers from $p = 0$ to a small positive value, their share in the population that remains in the home country increases if $\alpha \in (0, 1)$.*

There are two effects of an increase in p on the share of the remaining engineers. The ex-post effect is negative: a higher probability of migration means a lower probability of staying in the home country. The ex-ante effect is positive: an increased prospect of migration (a prospect of increased earnings) induces more individuals to study engineering. Proposition 3 states that the ex-ante effect is stronger than the ex-post effect, which means that the higher the prospect of migration, the closer the share of engineers in the non-migrating population to the socially desirable share. The Proposition is valid only for the interior values of α: with $\alpha = 0$ or with $\alpha = 1$, one of the two occupations is virtually non-existent, and thus the share of population in a given occupation cannot change.

Proof. From (41) and Lemma 3, $\left.\frac{ds}{dp}\right|_{p=0} = \left.\frac{\partial s}{\partial p}\right|_{p=0}$. This latter expression is equal to

$$\frac{\partial s}{\partial p} = \frac{\frac{\partial s_M}{\partial p} s_L}{(s_M + s_L)^2}$$

For $\alpha \in (0, 1)$, we have that $s_L > 0$, and that

$$\frac{\partial s_M}{\partial p} = \frac{\alpha(\overline{w} - \theta K)}{(\theta K - p\overline{w})^2} > 0$$

where the latter inequality follows from the assumption that the foreign wage $\overline{w}$ is higher than the domestic expected wage, which in turn is equal to θK (cf. (27)). Thus, we have that $\left.\frac{ds}{dp}\right|_{p=0} = \left.\frac{\partial s}{\partial p}\right|_{p=0} > 0$. ■

We now turn to analyze the effects of a marginal increase in the prospect of migration on the welfare of all the population groups. Ex-post, after the migration "lottery" has been played, there are three such groups: migrating engineers, non-migrating engineers, and lawyers. From (27), it follows that the three groups are ranked as follows: migrant

engineers are the most well-off, lawyers are in the middle, and the engineers who stay at home are the least well-off. Indeed, migrating engineers (winners of the "lottery") must be better off than non-migrating engineers; and the expected payoff from acquiring human capital in engineering, which is a linear combination of the payoffs of the two groups of engineers, is equal to the lawyers' payoff. Below, we analyze the payoff of each group in detail.

The welfare of a lawyer is (cf.(16)) $W_L = w_L\theta - \frac{K}{2}\theta^2$; from (27), it follows that this welfare is equal to $\frac{K}{2}\theta^2$. From Proposition 2, we know that θ is increasing with p, and therefore so does lawyers' welfare.

The welfare of a migrating engineer is $W_M^1 = \overline{w}\theta - \frac{K}{2}\theta^2$. Since from Proposition 2, $\frac{d\theta}{dp}\big|_{p=0} > 0$, we have that in the vicinity of $p = 0$, $\frac{dW_M^1}{dp} = \frac{dW_M^1}{d\theta}\frac{d\theta}{dp}$ has the same sign as $\frac{dW_M^1}{d\theta}$. From (27) and the fact that $\overline{w} > w_M$, it follows that $\frac{dW_M^1}{d\theta} = \overline{w} - K\theta > 0$. Therefore, engineers that ex-post are able to migrate benefit from an increasing prospect of migration since they have an increased ex-ante incentive to acquire human capital that yields high returns abroad.

The effect of a marginal increase in p on the welfare of non-migrating engineers is non-trivial, and we next turn to analyze this effect.

Proposition 4 *An increase in the probability of migration from $p = 0$ to a small positive value increases the welfare of non-migrating engineers if and only if the externality effect of engineering is sufficiently high: $\frac{\eta}{1-\eta} > 1 - \alpha$.*

An increasing prospect of migration induces all the individuals to accumulate more human capital, which increases total factor productivity such that if the inequality in Proposition 4 holds, even the losers of the migration "lottery" are better off compared to how they would have fared in the closed economy.

Proof. Drawing again on (27), the welfare of a non-migrating engineer is

$$W_M^0 = w_M\theta - \frac{K}{2}\theta^2 = \left(\frac{K}{1-p} - \frac{K}{2}\right)\theta^2 - \frac{p\overline{w}}{1-p}\theta$$

Our goal is to determine the sign of $\frac{dW_M^0}{dp}$ in the vicinity of $p = 0$. To this end, we need to find

$$\frac{dW_M^0}{dp} = \frac{\partial W_M^0}{\partial\theta}\frac{d\theta}{dp} + \frac{\partial W_M^0}{\partial p}$$

The partial derivatives are:

$$\frac{\partial W_M^0}{\partial\theta} = 2\left(\frac{K}{1-p} - \frac{1}{2}K\right)\theta - \frac{p\overline{w}}{1-p} =|_{p=0} \theta_1 K$$

$$\frac{\partial W_M^0}{\partial p} = \frac{\theta^2 K}{(1-p)^2} - \frac{\overline{w}\theta}{1-p} - \frac{p\overline{w}\theta}{(1-p)^2} =|_{p=0} -\theta_1(\overline{w} - \theta_1 K)$$

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Using these derivatives and the expression for $\left.\frac{d\theta}{dp}\right|_{p=0}$ from the proof of Proposition 2, we find that the derivative of the welfare of the non-migrating engineers with respect to the migration probability is

$$\left.\frac{dW_M^0}{dp}\right|_{p=0} = \theta_1 \left(\overline{w} - \theta_1 K\right) \left[\frac{\alpha + \eta - \alpha\eta}{1 - \eta} - 1\right] \tag{42}$$

Since $\theta_1 K$ is the domestic (ex-ante) wage, we always have that $\overline{w} - \theta_1 K > 0$ and therefore, (42) is positive if and only if $\frac{\alpha + \eta - \alpha\eta}{1-\eta} - 1 > 0$, or, upon rearranging, if and only if $\frac{\eta}{1-\eta} > 1 - \alpha$. $\blacksquare$

If there was no positive externality of engineering ($\eta = 0$), (42) would have been unambiguously negative: with constant total factor productivity, the losers of the migration "lottery" must be worse off ex-post than those who never played the "lottery" to begin with (that is, engineers living in a closed economy). On the other hand, if engineering was the only productive input ($\alpha = 1$), the welfare effect of a marginal increase from zero in the migration probability would be positive at *any* value of $\eta \in (0, 1)$. This is akin to the result of Stark and Wang (2002).

4   Conclusion

In a one-sector, two-input model with endogenous human capital formation, one of the two inputs of production (engineering) exerts a positive externality on total factor productivity, while the other (law) does not. We show that a (marginal) prospect of migration for engineers increases human capital accumulation in both sectors (engineering and law), and leads to an increase in the number of engineers who remain in the home country. Since these two effects move the home economy away from the (inefficient) free-market equilibrium towards the social optimum, they are both socially desirable. We also show that if the externality effect of engineers is sufficiently powerful, all the individuals will be better off when there is a prospect of migration, including the engineers who lose the migration "lottery" and the individuals who practice law.

Receiving (destination) countries often select the type of professionals that they admit rather than open their arms or gates to migrants of all types. When the receiving country accepts, for example, only engineers, computer programmers, or natural scientists, the home country need not lose, either absolutely or in comparison with a receiving country with an open-to-all migration policy. Indeed, when the said selection is tantamount to a small probability of migration, and the type selected is the one that confers a productive externality in the sending country, that country stands to gain.

In the setting developed in this paper, when the externality effect is powerful enough, the prospect of selective migration for a heterogeneous workforce penalizes neither the workers who, in spite of responding to the opportunity to migrate do not in the end

take it up, nor the workers for whom there is no such opportunity. In the context of the strong and rising interest in the topic of equality of opportunity in modern welfare economics and social choice theory (Roemer 1998, 2002), this latter result is quite telling. The equality of opportunity premise is that regardless of type, all members of a society should be allowed to compete on equal terms and enjoy the same access to rewarding opportunities for their hard-earned skills. The expansion of options for individuals to choose and pursue is a cherished goal. A configuration in which individuals of only one type have an opportunity to migrate and reap higher returns to their acquired skills could thus be deemed orthogonal to the basic tenet of the equality of opportunity concept. This paper presents an example of a case where unequal access to rewarding opportunities and an improvement throughout of welfare need not be incompatible.

Appendix

Proof of Lemma 1

Consider a function $f(x) \equiv x^{\alpha+\eta}(1-x)^{1-\alpha}$, with $x \in [0,1]$. It is straightforward to show that the maximum of $f(x)$ is attained at $x_0 = \frac{\alpha+\eta}{1+\eta}$, that is, $f(\frac{\alpha+\eta}{1+\eta}) > f(x)$ for any $x \neq \frac{\alpha+\eta}{1+\eta}$. Also, we can rewrite C_0 as

$$C_0 = \left(\frac{\alpha+\eta}{1+\eta}\right)^{\alpha+\eta} \left(\frac{1-\alpha}{1+\eta}\right)^{1-\alpha} = f\left(\frac{\alpha+\eta}{1+\eta}\right)$$

while

$$C_1 \equiv \alpha^{\alpha+\eta}(1-\alpha)^{1-\alpha} = f(\alpha)$$

Note that $\frac{\alpha+\eta}{1+\eta} \neq \alpha$ if and only if $\eta \in (0,1)$ and $\alpha < 1$. Therefore, $C_0 = f(\frac{\alpha+\eta}{1+\eta}) > f(\alpha) = C_1$ if and only if $\eta \in (0,1)$ and $\alpha < 1$. ($C_0 = C_1$ if $\eta = 0$, or if $\alpha = 1$). ∎

Proof of Lemma 2

Let $g(x) \equiv (1+x)^{\frac{1+x}{1-x}} - \frac{1}{2}(1+x)^{\frac{2}{1-x}}$, with $x \in [0,1]$. It is straightforward to verify that $g(0) = \frac{1}{2}$. Our objective is to show that $g(x) > \frac{1}{2}$ for $x \in (0,1)$. Since $g(x)$ is continuously differentiable on $(0,1)$, it is sufficient to show that $\frac{dg(x)}{dx} > 0$ for any $x \in (0,1)$.

Using the fact that

$$g(x) = \exp\left(\frac{1+x}{1-x}\log(1+x)\right) - \frac{1}{2}\exp\left(\frac{2}{1-x}\log(1+x)\right)$$

we compute the derivative of this function:

$$\frac{dg(x)}{dx} = \left[\frac{2}{(1-x)^2}\log(1+x) + \frac{1}{1-x}\right](1+x)^{\frac{1+x}{1-x}}$$

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$$- \frac{1}{2} \left[\frac{2}{(1-x)^2} \log(1+x) + \frac{2}{(1-x)(1+x)} \right] (1+x)^{\frac{2}{1-x}}$$

Upon dividing both sides of the last equation by $(1+x)^{\frac{1+x}{1-x}} > 0$, we obtain

$$
\begin{aligned}
\left[(1+x)^{-\frac{1+x}{1-x}} \right] \frac{dg(x)}{dx} &= \left[\frac{2}{(1-x)^2} \log(1+x) + \frac{1}{1-x} \right] \\
&\quad - \left[\frac{1}{(1-x)^2} \log(1+x) + \frac{1}{(1-x)(1+x)} \right] (1+x) \\
&= \frac{2}{(1-x)^2} \log(1+x) + \frac{1}{1-x} - \frac{1+x}{(1-x)^2} \log(1+x) - \frac{1}{1-x} \\
&= \frac{1}{1-x} \log(1+x) > 0
\end{aligned}
$$

for any $x \in (0,1)$. ∎

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