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Method Mathematical Model of Consensus Building in Work of Technical Committees on Standardization

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Summary
Mathematical model of consensus building in work of technical committees on standardization is developed. The model is based on consensus model proposed by DeGroot. Main issues of consensus achievement in development of standards are analyzed in conditions of proposed model. Results of modelling showing dependence of consensus achievement time of number of members of technical committee on standardization are presented.

Key words: *Technical committee on standardization, consensus, regular Markovian chains.*

1. INTRODUCTION

In ISO/IEC 2:2004 Guide "Standardization and related activities - General vocabulary", standard is defined as "document, established by consensus and approved by a recognized body, that provides, for common and repeated use, rules, guidelines or characteristics for activities or their results, aimed at the achievement of the optimum degree of order in a given context". In its turn according to this Guide, "consensus is general agreement, characterized by the absence of sustained opposition to substantial issues by any important part of the concerned interests and by a process

that involves seeking to take into account the views of all parties concerned and to reconcile any conflicting arguments".

As a rule, international, regional and national standards are developed within the frame of correspondent Technical Committees on Standardization (TCS). Thus, achievement of consensus in TCS work is the most important task in standard development. Possibility in principle of consensus achievement on the basis of regular Markovian chains [2, 3] was demonstrated in paper [1]. This approach to the case of TCS work management for consensus achievement was developed in this article.

2. MODEL OF CONSENSUS BUILDING ON THE BASIS OF REGULAR MARKOVIAN CHAINS

Let n is the number of TCS members taking part in discussion and $S(0) = (s_0^1; .. ; s_0^n)$ – vector of initial opinions of TCS members. TCS members (experts) exchange their opinions concerning vector S values during TCS meeting. At that opinion of every expert can be changed depending of credibility degree (level) of one expert to opinion of the other TCS member and also of belief (credibility) degree of the expert to his personal opinion.

Credibility degree of expert #i to opinion of expert #j is given by value $p_{ij,}$ where $0 < p_{ij} < 1$(i=1, .. n; j=1, .. n). It is considered that expert #i believes to himself with some credibility degree p_{ii} ($0 < p_{ii} < 1$). Thereby square credibility matrix $\mathbf{P} = (p_{ij})$ of order n×n is generated. The matrix **P** defines sequential process of TCS member opinions reconciliation. It is also considered that sum of p_{ij} in every row of matrix is equal to 1. At the first step of expert's opinion reconciliation, vector is calculated by formula [1]:

$\mathbf{S}(1) = \mathbf{S}(0)\times\mathbf{P} = (s_0^1, .. ,s_0^n)$.

At k-th step of reconciliations, opinion vector is calculated by formula:

$$\mathbf{S}(k) = (s_k^1 , .. ,s_k^n) = \mathbf{S}(k - 1)\times\mathbf{P} = \mathbf{S}(0)\times\mathbf{P}^k \quad . \qquad (1)$$

Iteration process is stopped at m-th step, if all rows of $\mathbf{P}^m$ matrix become the same. Mathematically it means that credibility matrix **P** has achieved final matrix **F** after m iterations, because final matrix **F** will not change and respectively vector of expert's opinions $\mathbf{S}(m) = \mathbf{S}(0)\times\mathbf{P}^m = (s_m^1 , .. ,s_m^n)$ will not change. This is consistent with common postulate of group dynamics theory that describes processes occuring in social groups [3]: "... finally the groups find points of coincidence which totalize all useful concepts".

First, let us consider an example for n=5. Let credibility matrix **P** for five TCS members be the following:

$$\begin{matrix} 0.0 & 0.6 & 0.0 & 0.3 & 0.1 \\ 0.0 & 0.8 & 0.1 & 0.0 & 0.1 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1 \\ 0.0 & 0.0 & 0.1 & 0.0 & 0.9 \\ 0.0 & 0.0 & 0.0 & 0.1 & 0.9 \end{matrix} \qquad (2)$$

Degrees of expert's credibility to opinions of other experts and his personal opinion are given in rows (i=1, .. ,5) of the matrix, and credibility degrees of other TCS members to expert's opinion are given in columns (j=1, .. ,5) of the matrix. In other words, expert #1 (row 1) does not confide in his personal opinion (element $p_{11} = 0$) and in opinion of expert #3 ($p_{13} = 0$), confide in expert #2 with

credibility level 0.6 (p_{12} = 0.6), confide in expert #4 with credibility level 0.3 (p_{14}= 0.3) and in expert #5 with credibility level 0.1 (p_{15}= 0.1). Other rows of matrix are formed in similar manner.
Let us consider matrix columns. The first columns reflects full non-credibility of all experts to expert #1; opinion of expert #2 is estimated by expert #1 with credibility level 0.6, and by himself with credibility level 0.8 and so on.

After m iterations, that is after m reconciliations, the final matrix **F**, supporting consensus, is equal to:

$$\begin{matrix} 0.0 & 0.0 & 0.0 & 0.1 & 0.9 \\ 0.0 & 0.0 & 0.0 & 0.1 & 0.9 \\ 0.0 & 0.0 & 0.0 & 0.1 & 0.9 \\ 0.0 & 0.0 & 0.0 & 0.1 & 0.9 \\ 0.0 & 0.0 & 0.0 & 0.1 & 0.9 \end{matrix} \qquad (3)$$

As follows from Markovian chains theory [2], necessary and sufficient condition of convergence of initial matrix **P** to final matrix **F** (i.e. necessary and sufficient condition of consensus achievement) for any source opinion vector is regularity of matrix **P**. In other works, it is necessary and sufficiently that sums on rows of matrix **P** are equal to 1 and at that strict inequalities $0 < p_{ij} < 1$ took place for every p_{ij}. In terms of TCS activities, it is important, that some TCS members had their personal opinion and had credibility to opinions of some colleagues.

Thus, having any initial opinions of TCS members, if matrix **P** is regular (that is there are non-ambitious experts with expressed opinion in TCS), then consensus is achievable for finite number of iterations (sessions of TCS). This conclusion follows from property of convergence of regular Markovian matrix **P** to final matrix **F**.

Therefore, control (both from national standardization body and TCS head) for consensus achievement within the frames of considered model consists in avoidance of situations where consensus is principally unachievable or where achievement of consensus requires significant time expenditures.

Since convergence of matrix **P** to final matrix does not depend on vector **S**(0) of initial opinions, it is obvious that one should control credibility matrix **P**.
Let us check performance capability of proposed model by analysis of negative situations related to consensus building.

1. Domination. If there is an expert with high self-rating (without loss of generality let it be expert #1, that is p_{11} = 1), then his opinion does not change in the process of reconciliations (iterations), because exactly element p_{11} does not change in final matrix **F** and remains equal to 1. The conclusion is obvious: it is difficult to make dominant expert change his mind. Therefore presence of ambitious member in TCS must be abolished, because exactly opinion of this expert will dominate. For this reason, for example, representatives of regulatory bodies must be included in TCS only as ordinary members of committees. In addition presence of dominant expert in TCS can significantly retard achievement of consensus. Let us consider a simple example.
Let initial credibility matrix $\mathbf{P_1}$ is as follows:

$$\begin{matrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.1 & 0.4 & 0.3 & 0.2 \\ 0.3 & 0.2 & 0.5 & 0.0 \\ 0.1 & 0.2 & 0.1 & 0.6 \end{matrix}$$

In this case convergence of the matrix to final matrix (that is consensus) is achieved only after fifteen iterations (m=15).
At the same time, if ambitious leader is replaced by "normal" expert, then matrix $\mathbf{P_1}$ is replaced by matrix $\mathbf{P_2}$:

0.6 0.2 0.1 0.1
0.1 0.4 0.3 0.2
0.3 0.2 0.5 0.0
0.1 0.2 0.1 0.6

Here consensus is achieved in nine iterations.

2. Presence of several leaders. Situation, when where are several leaders in TCS, is characterized by matrix **P**, with several 1.0 values on main diagonal. For example, in case of two leaders, matrix **P** looks as follow:

1 0 0……0
0 1 0……0
p_{31} p_{32} p_{33}….p_{3n}
……..

Matrices of this kind and correspondent Markovian chains are called reducible [2]. Since multiplication of reducible matrices generates reducible matrix [2], then it is obvious that consensus is not ever achievable in such situation (for any n>2).

Similar conclusions are reported in literature on group dynamics: "work of group is often paralyzed by personalities and fractions which stay on strictly opposite positions" [3]. Presence of several leaders in TCS distinguishes this situation from the previous in principle. Presence of one leader in group provides achievement of consensus, although of a low quality, whereas presence of several leaders in TCS means principal impossibility of consensus. In this case task of TCS head is to avoid during formation of TCS inclusion of several ambitious leader which paralyze work of TCS.

3. Global domination. If **all** experts in TCS have high self-rating (that is for all i we can consider p_{ii} =1), then credibility matrix **P** becomes unity matrix **E** (units on main diagonal, all other matrix elements equals to 0). Since for any number of iterations (sessions in TCS) $\mathbf{P}^m = \mathbf{E}^m = \mathbf{E}$, then matrix **P** does not converge to final matrix, therefore consensus is unachievable in principle.

This conclusion is confirmed by large number of observations over work of different groups: the more ambitious members are in the TCS, the more complex is achievement of consensus in group. For example, there is noted in paper [3] that "presence of natural-born leaders, which hog the cover, disturbs efficient work of group most of all". For this reason, domination must be stopped by TCS head, becomes domination excludes achievement of consensus.

4. Transfer of responsibility. Let us consider the situation where each expert declines responsibility for decision making, fully confiding in opinion of another TCS member. This is conformal behaviour, where experts espouse the opinion of group, considering this opinion is right and his personal estimate is wrong. The following matrix **P** corresponds to this case:

0 1 0 .. 0 0
0 0 1 .. 0 0
……
0 0 0 .. 1 0
1 0 0 .. 0 0

It is easily seen that matrix **P** illustrates situation where expert #1 fully confides in expert #2, #2 fully confides in expert #3 and so on, the last (by number) expert fully confides in expert #1. It is shown in Markovian chains theory that transfer matrix of this type does not converge to final matrix. Therefore, consensus is not achievable for such group. In fact, it is sufficient to have only two "irresponsible" experts in the group to make achievement of consensus impossible.

It may sound strange that experts decline responsibility for decision making and shift decision of other members of TCS. However analysis of group dynamics shows that it is a wide-spread phenomenon, because "group activity allows to shield oneself behind somebody else, to shift responsibility, ... Special kind of people appears which can be called deadheads" [4].

5. Coalitions. Another case of impossibility of consensus achievement is formation of coalitions in TCS. We shall illustrate this on the example of TCS consisting of 4 members which formed two coalitions. Expert #1 confides only in himself and in expert #2, expert #2 confides only in himself and in expert #1, respectively expert #3 confides only in himself and in expert #4, expert #4 confides only in himself and in expert #3. One of the correspondent initial credibility matrix **P** is the following:

0.7 0.3 0.0 0.0
0.4 0.6 0.0 0.0
0.0 0.0 0.5 0.5
0.0 0.0 0.5 0.5

It can be seen by direct verification (that is by multiplication of matrix on itself) that only non-zero values are changed. This means that coalitions in group are stable, and consensus in group is unachievable.

As noted above, matrices of this kind and correspondent Markovian chains are reducible, so consensus in this situations cannot be achieved (for any n>2).
Similar conclusions are reported in literature on group dynamics [3]: "quite often there are more small formations within the group, between which various coalitions and alliances exist. This complicates a process of consensus building" [3]. In this situation task of TCS head is to remove formed coalitions in TCS by selection of compromise decisions.

6. Number of discussions in TCS. From practical point of view it is important to estimate convergence time of opinion matrix **P** to final matrix $\mathbf{F} = \mathbf{P}^m$. This time is determined by number m of iterations (discussions within the frame of TCS) necessary to build consensus. Value m can be estimated from the following considerations: determinant of matrix **F** is equal to zero, because all rows of this matrix are equal. If assign error ε in estimation of matrix **F** determinant close to zero, then after elementary transformation one can take an assessment of m [5]:

$$m > ln\,\varepsilon / ln\,\det\mathbf{P}, \qquad (4)$$

where det**P** is determinant of matrix **P**.

Analysis of formula (4) shows that the more accuracy in estimation of matrix **F** is required, the less value ε, and therefore the greater value m, that is the greater number of iterations (reconciliations).
If source relations of credibility in TCS are close, (this means that rows of matrix **P** are similar to each other, and matrix **P** is from the beginning close to matrix **F)**, then at given accuracy ε, number of iterations for convergence of matrix **P** to matrix **F** and building of consensus is small. Determinant of matrix **P**, close to 1, shows that number of iterations m (TCS sessions) is very large. This statement is close to statement 2.

It is possible to estimate roughly number of iterations required for consensus building on the basis of formula (4):

- for case where credibility relations in group are rather close, which means that determinant of matrix **P** is close to zero (for example, det**P** = 0.0005), two discussions are sufficient for consensus achievement (at ε = 0.0001);

For "average" initial coincidence of opinions (det**P** = 0.3), at least eight discussions are necessary to achieve consensus:

- "bad" initial coincidence of opinions (det**P** = 0.5 or greater) can require more than hundred iterations, which is near-impossible.

3. ESTIMATION OF INFLUENCE OF NUMBER OF TCS MEMBERS ON CONSENSUS ACHIEVEMENT TIME BY MODELLING

Influence of number n of TCS members on time of consensus achievement was studied by statistical modelling.

Modelling was performed in several stages.

At the first stage evels of changes of number n of TCS members were selected:

1-st	5
2-nd	10
3-rd	20
4-th	30
5-th	40
6-th	50

On the second stage probability p_{ii}, describing credibility degree to himself (authoritarian degree), was selected.

The following levels were selected for p_{ii}:

1-st	0.20 - 0.30
2-nd	0.45 - 0.55
3-rd	0.65 - 0.75
4-th	0.85 - 0.95
5-th	0.90 - 1.00

Number m of TCS sessions before achievement of consensus, at which condition $\det\mathbf{P}^m < \varepsilon$ is fulfilled, was used as dependent variable.

The third stage is stage of modelling. Modelling of diagonal elements p_{ii} of matrix **P** was performed for every level n using uniform distribution law. Boundaries of correspondent level of p_{ii} factor were used as parameters of uniform distribution.

In every row of matrix **P** probabilities p_{ij} ($i \neq j$) were modelled on uniform distribution law with parameters 0 and 1 and consequent normalization so that sum of probabilities within one row was equal to 1, which is required for provision of regularity of matrix **P**.

To obtain robust inference in relation of number n, 100 runs of modelling were performed 100 at every fixed level of factors n and p_{ii} [6].

Results of modelling are presented in Figure 1. We can observe power-law growth of number of reconciliations with increase of number of TCS members. In addition it is seen that rate of growth of number of reconciliations in the model significantly depends on authoritarian level.

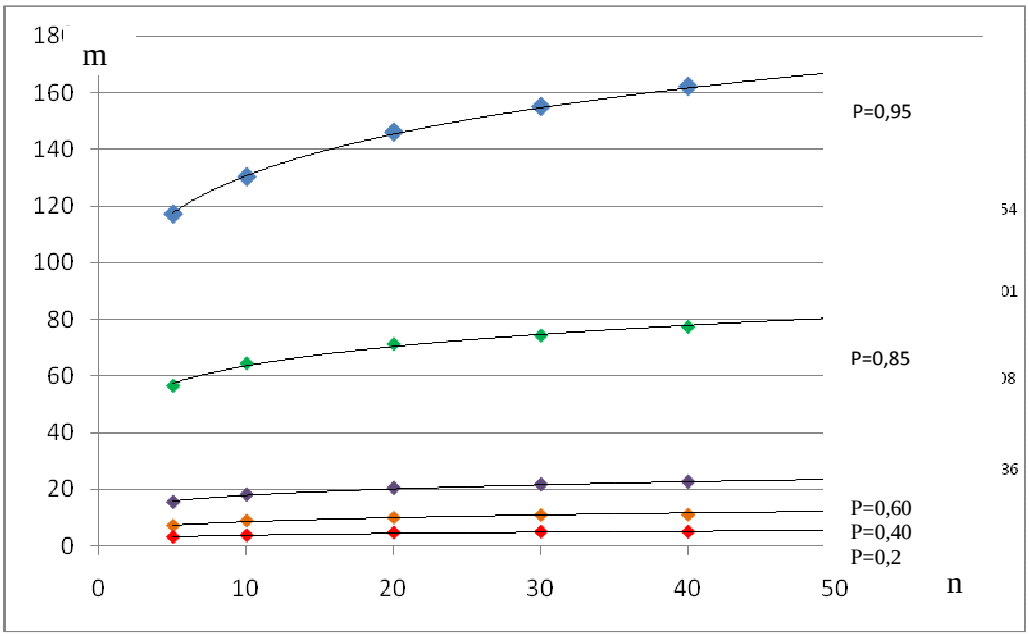

Figure 1. Dependence of number of reconciliations on number of TCS members

Thus, results of modelling show that increase of number of TCS members leads to increase of consensus achievement time. Growth of consensus achievement time is particular significant in cases when there are authoritarian members in TCS. In case when there are no clear leader in TCS, growth of number of TCS members is not significant.

Results of modelling in large correspond to results of study [7], which show (usin other modelling techniques) that increase of ministry size above 20 leads to deterioration of consensus.

4. CONCLUSION

Performance capability of model of consensus building in work of technical committees on standardization (or correspondent work groups) was demonstrated in this paper. The model is based on regular Markovian chains. It was shown that within the frames of sufficiently general assumptions it is possible to model main negative processes prohibiting consensus achievement in TCS work. Therefore, it is possible to control these processes with goal of TCS efficiency growth. Results of statistical modelling of consensus achievement process show that it is inadvisable to increase number of TCS members, because this leads to growth of consensus achievement time.

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