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TREND INFLATION AND TAYLOR PRINCIPLE: DETERMINACY ANALYSES IN NEW KEYNESIAN MODEL WITH CAPITAL ACCUMULATION

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We analyze the effects of positive trend inflation in the framework of the standard New-Keynesian model with Calvo price setting and capital accumulation. We build on the work of Carlstrom and Fuerts (2005) and Ascari and Ropele (2007) who separately considered effects of capital accumulation and trend inflation in the similar context. We show that the simultaneous presence of positive inflation and capital accumulation greatly affects determinacy property of equilibrium in this setup. Namely, we show that in order to maintain stability in addition to actively react to inflation monetary authorities should react to output fluctuations but not to a great extend. Overreaction to output may lead to indeterminacy. We also show that for a large set of plausible parameters standard Taylor rule leads to indeterminacy. We argue that in this case indeterminacy of equilibrium may be an empirically relevant fact and one of the indeterminate equilibrium can resemble the stochastic property of the actual data quite well.

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1. Introduction

Most of the existing New Keynesian general equilibrium literature within Calvo-pricing framework ignores a positive trend in the inflation process. Some papers, e.g. Clarida et al.. (2000) assume log-linearization around zero inflation steady state. The main reason for this assumption is the analytical convenience. But it is obvious that equilibrium inflation even in the developed countries is positive. For example, Schmitt-Grohe and Uribe (2007) used the post-war data for the US GDP deflator and estimated the steady state inflation level at about 4.2%. Moreover, nowadays monetary authorities around the world do not consider price stability as zero inflation: most Central Bank's policy is aimed to inflation level about 2%. And in the long run horizon the steady state inflation converges to Central Bank's target.

A popular technique to eliminate steady state inflation in the equilibrium solution was proposed by Yun (1996). Yun assumed that firms which do not have an opportunity to re-adjust prices simply index them by the steady state inflation. Christiano et al. (2001) proposed indexation by the previous period's inflation. But these assumptions seem to be empirically non-realistic: it was shown that not all firms adjust prices each period. Bils and Klenow (2004) produced an empirical study and showed that many prices remain constant during a long period of time. More importantly, from the theoretical point of view the concept of price indexation is not in line with the idea of "sticky" prices: the menu costs of price adjustment are still significant. Also, there is no such a phenomenon in the state-dependant sticky pricing literature.

The most recent literature deals with the problem in two ways. Some papers propose partial indexation based on the idea that firms face not only menu costs but also information costs. As a result, some fraction of them decides to avoid high information costs and to re-adjust prices by the known inflation of the previous period. But Cogley and Sborne (2008) have shown that assumption of stationary trend inflation eliminates the significance of partial indexation in estimated Phillips curve.

There is also emerging literature which deals with positive trend inflation without any indexation. Ascari (2004) derived the New-Keynesian Phillips curve (NKPC) in the presence of low positive trend in the inflation

process. His major result is that the current inflation depends not only on the expectations of the next period inflation but also on inflation expectations over the longer horizon. He also showed that non-zero trend influences the dynamics of the solution of the general equilibrium model with staggered Calvo and Taylor — type pricing. Cogley and Sborne (2008) estimated modified NKPC and showed that it does not require backward-looking component — the positive trend in inflation process itself generates enough persistence in the model. Finally, Ascari and Ropele (2007) generalized the optimal monetary policy literature in the case of low trend inflation. One of their main results is that the presence of low trend inflation has an impact on the determinacy of rational expectations equilibrium.

There are also recent theoretical works by Hornstein and Wolman (2005), Kiley (2007) and Ascari and Ropele (2009) who analyze Taylor principle implication under low positive trend inflation. The theoretical part of forthcoming paper by Coibion and Gorodnichenko (2008) also deals with the same problem and presents the result that monetary authorities should not respond to output gap variation: better policy is to respond to output growth. They argue that the probability of determinant equilibrium is much higher in the case of policy based on output growth targeting.

This paper follows works by Ascari and Ropele (2007) and Hornstein and Wolman (2005) and aims to fill the gap between the trend inflation literature and sunspot literature. It studies the impact of the trend inflation on equilibrium when the monetary policy is conducted via simple interest rules — the Taylor principle. We analyze the general solution in the model with capital accumulation process.

The idea of "active" monetary policy rule to be used by Central Bank is very popular for almost a decade since the paper by Kerr and King (1996). Active monetary rule is the one which responds to inflation with more than one-to-one changes in interest rate. This type of policy guarantees the uniqueness of the equilibrium solution of the system within some class of models. But this class is not by any mean complete. For example, Duport (2001), Carlstrom and Fuerts (2005) argued that "activeness" of monetary policy rule is not enough to avoid indeterminacy in the presence of pre-determined variables in the model. A combination of endogenous capital accumulation and sticky prices can also lead to multiple equlibria. As a result monetary policy rule which guarantees the uniqueness of the solution is more sophisticated than simpe "active" one. It is worth mentioning that all the research cited above was made within the framework of zero trend inflation.

We analyze the effect of the presence of the trend inflation on the determinacy of the general equilibrium solution. For this purposes we derive and calibrate a standard New-Keynesian model in continuous time with three main blocks: investment-savings curve, modified New-Keynesian Phillips curve and Taylor-type monetary policy rule. As in the most of the existing literature, monetary authorities stabilize inflation and output gap around long-run targets using nominal interest rate. The main findings of the paper are the following. First, there is a significant set of plausible parameters in which the equilibrium is indeterminate under active policy. Second, the case of indeterminacy may have an empirical relevance: the indeterminate equilibrium can resemble the property of the data quite well. Third, the monetary policy should be not "very" active in order to guarantee the determinacy of the solution

The rest of the paper is organized as follows. Section 2 describes the theoretical model with capital accumulation process. Section 3 provides analysis of the model. Then, Section 4 concludes.

2. The model

To analyze the consequences of positive trend inflation, we adopt the framework of Blanchard and Kiyotaki (1987), developed in Woodford (2003) and Gali (2008) and add capital accumulation process. The model is the standard version of sticky prices general equilibrium model. The model economy is composed of continuum of infinitely-living consumers-producers of one final and continuum of intermediate goods. We use standard functional forms for preferences and technology and assume perfectly competitive labor market.

2.1 Households

The representative infinitely-lived household seeks to maximize the objective function:

$$\int_0^\infty \left(\ln C_t + b \ln \frac{M_t}{P_t} - \frac{L_t^{1+s}}{1+s} \right) \exp(-\theta t)\, dt \qquad (1)$$

where C_t is the consumption of the single final good, and L_t denotes hours of work or employment, θ is a discount factor and s is the inverse of the labor supply elasticity. M_t denotes holding of money and instantaneous utility function is increasing and concave in real balances $\frac{M_t}{P_t}$. The instantaneous utility function is additively separable in consumption, labor and money.

The budget constraint takes the form

$$\left(\frac{\dot{B}}{P_t}+\frac{\dot{M}}{P_t}+\dot{K}\right)\leq W_t L_t - C_t + (r+\delta)K_t + \frac{i_t B_t}{P_t} + T_t \tag{2}$$

where i is an interest rate, B_t is a quantity of one-period, nominally riskless discount bonds, K_t is aggregate capital in the economy, W_t is real wage, $r+\delta$ is real user cost of capital and T_t represents lump-sum additions or subtractions to the income, expressed in real terms. The maximization of (1) subject to (2) leads to the following solution:

$$W_t = L_t^s C_t \tag{3}$$

$$\frac{\dot{C}}{C_t} = i_t - \pi_t - \theta \tag{4}$$

$$\frac{M_t}{P_t} = C_t i_t^{-1} b \tag{5}$$

$$r_t = i_t - \pi_t \tag{6}$$

where $\pi_t = \frac{\dot{P}_t}{P_t}$ is the inflation rate.

2.2 Firms

The economy produces a single final good and a continuum of intermediate goods indexed by j where j is distributed over the unit interval ($j \in [0,1]$). The production of final good is perfectly competitive and is subject to the following production function

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}} \tag{7}$$

where Y_t is the quantity of the final good produced, $Y_{t,j}$ is the quantity of intermediate good produced by the firm j and η stands for elasticity of substitution between different inputs. As it was mentioned above, the final good is consumed by the household.

Perfect competition in a final good's sector ensures that the demand which faces the firm j is given by

$$\frac{Y_{t,j}}{Y_t} = \left(\frac{P_{t,j}}{P_t}\right)^{-\eta} \tag{8}$$

$P_{t,j}$ is a price of intermediate good j and P_t is an aggregate price which takes the following form

$$P_t = \left(\int_0^1 P_{t,j}^{\frac{1}{\eta-1}} dj\right)^{\eta-1} \tag{9}$$

There is monopolistic competition in the markets for intermediate goods: each intermediate good is produced by a single firm subject to the constant return to scale production function

$$Y_{t,j} = K_{t,j}^{\alpha} L_{t,j}^{1-\alpha} \tag{10}$$

where $L_{t,j}$ is the labor-input and $K_{t,j}$ is the capital input for the production of the firm $Y_{t,j}$ and in the equilibrium

$$L_t = \int_0^1 L_{j,t} dj \tag{11}$$

$$K_t = \int_0^1 K_{j,t} dj \tag{12}$$

The aggregate capital accumulation process is given by

$$\dot{K} = Y_t - C_t - \delta K_t \tag{13}$$

where δ is depreciation rate.

As in Calvo (1983), firms are not allowed to change their prices unless they receive a random "price change signal". It is expressed by exogenous Poisson process with arrival rate ψ and expected time between price changes $\frac{1}{\psi}$. At the moment of realization t_0 the probability that the firm will

not have an opportunity to readjust prices during the time period Δt is given by

$$\psi \int_{t_0}^{t_o+\Delta t} \exp\left(-\psi(\tau - t_0)\right) d\tau = 1 - \exp\left(-\psi \Delta t\right)$$

and is independent of t_0.

A firm chooses its reset price $X_{j,t}$ to maximize the present value of all future profits subject to Calvo pricing scheme

$$\int_{t_0}^{\infty} \left[\frac{Y_{j,t} X_{j,t} - C(Y_{j,t})}{P_t} \right] \frac{U'(C_t)}{U'(C_{t_0})} \exp\left(-\psi\left(t - t_0\right)\right) \exp\left(-\theta(t - t_0)\right) dt \qquad (14)$$

and

$$C(Y_{j,t}) = MC_t Y_{j,t} \qquad (15)$$

$$\frac{\partial Y_t}{\partial K_t} = \frac{(r_t + \delta)}{MC_t} \qquad (16)$$

$$\frac{\partial Y_t}{\partial L_t} = \frac{W_t}{P_t} \frac{1}{MC_t} \qquad (17)$$

where $C(Y)$ is a cost function and MC_t is real marginal cost which in case of constant return to scale production function equal to average cost of production.

The maximization problem leads to the following solution

$$X_{j,t} = \frac{\eta}{\eta - 1} \frac{\int_t^{\infty} e^{-(\theta+\psi)(\tau - t)} P_\tau^{\eta} MC_\tau (1/C_\tau) d\tau}{\int_t^{\infty} e^{-(\theta+\psi)(\tau - t)} P_\tau^{\eta-1} (1/C_\tau) d\tau} \qquad (18)$$

where $X_{j,t}$ denotes a reset price. Using definition of $X_{j,t}$ one can rewrite (9) as a function of reset-price:

$$P_t^{1-\eta} = \psi \int_{-\infty}^{t} e^{-\psi(t-\tau)} X_{j,\tau}^{1-\eta} d\tau \qquad (19)$$

2.3 Policy

Fiscal policy is conducted with lump-sum taxes and subsidies so that $B_t = 0$. Monetary policy is conducted using simple Taylor-type interest rule. Namely, monetary authorities target inflation and output to their steady state levels.

$$i_t - i^* = (1 + a_1)\left(\pi_t - \pi^*\right) + a_2 \left(\ln Y_t - \ln Y^*\right) \qquad (20)$$

where i^*, π^*, Y^* are non-stochastic steady states levels of interest rate, inflation and output.

2.4 Non-stochastic steady state

Equilibrium is given by 3–13 and 15–20. Non-stochastic steady-state is derived from the assumption that $\dot{C} = 0$ and $\dot{X} = \dot{P} = \pi^*$. The steady states levels of variables i^*, MC^*, $\left(\frac{Y}{K}\right)^*$ and $\left(\frac{X}{P}\right)^*$ are given by

$$i^* = \theta + \pi^* \qquad (21)$$

$$\left(\frac{X}{P}\right)^* = \left(\frac{\psi}{\psi - \pi(\eta - 1)}\right)^{\frac{1}{\eta - 1}} \qquad (22)$$

$$(MC)^* = \left(\frac{\eta - 1}{\eta}\right)\left(\frac{\theta + \psi - \pi\eta}{\theta + \psi - \pi(\eta - 1)}\right)\left(\frac{X}{P}\right)^* \qquad (23)$$

$$\left(\frac{Y}{K}\right)^* = \frac{\theta + \delta}{\alpha}\left(\frac{1}{MC}\right)^* \qquad (24)$$

$$\left(\frac{C}{Y}\right)^* = 1 - \frac{\delta}{\left(\frac{Y}{K}\right)^*} \qquad (25)$$

As shown in Ascari and Ropele (2007) the presence of steady-state inflation lowers Y^*.

2.5 Log-linearization

Standard log-linearization procedure of equations 3, 4, 6 and 15–21 around non-stochastic steady-state, described in the previous section leads to the following system

$$\dot{x}_t = (\psi + \theta - \eta\pi^*)(xp_t - mc_t) + \pi^*(1-\eta)z_t \tag{26}$$

$$\dot{p}_t = (\psi + (1-\eta)\pi^*)(xp_t) \tag{27}$$

$$\dot{c}_t = i_t - \pi_t \tag{28}$$

$$i_t = (1+a_1)\pi_t + a_2 y_t \tag{29}$$

$$\dot{k} = \left(\frac{Y}{K}\right)^* y_t - \left(\frac{C}{K}\right)^* c_t - \delta k_t \tag{30}$$

$$r_t = i_t - \pi_t \tag{31}$$

$$y_t - l_t = w_t - mc_t \tag{32}$$

$$y_t - k_t = \frac{r_t}{\theta + \delta} - mc_t \tag{33}$$

$$mc_t = \alpha r_t + (1-\alpha)w_t \tag{34}$$

$$w_t = c_t + sl_t \tag{35}$$

$$\dot{z}_t = -\pi_t + (\theta + \psi - \pi^*(\eta - 1))z_t + \frac{\dot{c}}{\eta - 1} \tag{36},$$

where $z_t = \left(\theta + \psi - \pi^*(\eta-1)\right)\int_t^\infty \exp-\left(\theta + \psi - \pi^*(\eta-1)\right)\left(p_\tau - p_t - \frac{c_\tau - c_t}{\eta - 1}\right)d\tau$

and the lower case letters denote logs of initial values in deviations from their steady-state levels except interest rates and inflation which are deviations from steady-state in levels. For nominal variables (x and p) small case stands for log deviations from balanced-growth path levels deflated by steady state inflation level. For example, $x_t = \log[X_t \exp(-\pi^* t)]$.

New-Keynesian Phillips curve is derived from 26–27 and takes the following form, analogously to Ascary (2004) in discrete time:

$$\dot{\pi}_t = (\theta - \pi^*)\pi_t - \varphi mc_t - \varphi'\pi^*(\eta - 1)z_t \tag{37}$$

$$\varphi' = \psi - \pi^*(\eta - 1) \quad \varphi = \varphi'\left(\varphi' + \theta - \pi^*\right)$$

As in Ascary (2004) today's inflation depends not only on expectations of its change today which is the case of convenient Philips curve but on expectations of all path of future inflation

The equilibrium solution of (27–37) results in the following system:

$$AX = BY$$

$$\text{where } Y_t = \begin{bmatrix} k \\ c \\ \pi \\ z \end{bmatrix}_t \quad X_t = \begin{bmatrix} l \\ w \\ r \\ i \\ mc \\ y \end{bmatrix}_t$$

$$A = \begin{bmatrix} -s & 1 & 0 & 0 & 0 & 0 \\ -(1-\alpha) & 0 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & \frac{-1}{\theta+\delta} & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -a_2 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1+a_1 & 0 \end{bmatrix}$$

Given equations $\dot{Y} = CY + DX = JY$ (38)
where

$$C = \begin{bmatrix} -\delta & -\left(\frac{C}{K}\right)^* & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & \theta - \pi^* & -\pi^*(\eta-1)\varphi' \\ 0 & 0 & \frac{a_1}{\eta-1} - 1 & \varphi' + \theta \end{bmatrix}$$

$$D=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \left(\frac{Y}{K}\right)^{*} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\varphi & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{a_2}{\eta-1} \end{bmatrix} \text{ and } J=\left(C+DA^{-1}B\right).$$

3. Analysis of determinacy

It is considered that an adequate monetary policy rule should imply a unique equilibrium. The logic is the following. If the policy conducted does not lead to unique equilibrium random shocks which are not connected with fundamentals of the economy may affect its dynamics. In other words, unnecessary fluctuations may be introduced to the economy and may increase its volatility. So, it may be desired that optimal monetary policy should not give rise to indeterminacy of equilibrium.

3.1 Methodology

According to Blanchard and Kahn (1980) the system of equations described above has a unique solution if and only if the number of eigenvalues of matrix *J* with negative real part is equals to the number of predetermined variables. With the presence of capital accumulation process capital is the only predetermined variable in the model. So, the equilibrium of the model is unique if and only if one eigenvalue of matrix *J* has a negative real part. If the number of such eigenvalues (k) is greater than one, the equilibrium is indeterminate and the dimension of equilibrium space equals to $k-1$.

The intuition behind the fact that the presence of the steady inflation can affect the determinacy of equilibrium is easily seen by looking at derived Philips curve. When inflation is greater then the discount factor the slope of Phillips curve (37) changes and become negative and thus Blanchard-Kahn condition may be violated: there is a large chance that inflation process may become stationary autoregression under virtually any expectation formation mechanism and this fact may bring about multiplicity of equlibria.

The solution of the system (38) even if can be derived analytically is too complicated to provide intuition. That is why the solution of the model was obtained using numerical methods with the standard calibration of parameters.

3.2 Calibration of basic parameters

Models parameters are calibrated according to values accepted as plausible in the business cycles literature: α, the Cobb-Douglas parameter of production function, is fixed at 1/3; δ, depreciation of capital rate, is set to 0.08 per annum and θ, consumer's discount factor, is assumed to be 0.02 per annum. For the analytical simplicity, consumer's instantaneous utility function is linear with respect to labor: in this case *s* equals 0 which means that labor supply is infinitely elastic as for example in Hansen's indivisible labour model (1985).

According to the recent empirical findings by Bils and Klenow (2004) and Nakamura and Steinsson (2008) we set the Poisson parameter of Calvo scheme (ψ) to correspond to opportunity for firms to change their prices every 6 and 12 month. The same logic was used in Coibon and Gorodnichenko (2008). The shorter time period was proposed in the first paper and longer time horizon corresponds with recent paper by Nakamura and Steinsson. We also adopt two different values of firm's steady-state markups: 10% and 20%. This means that corresponding values of η equals to 11 and 6. To illustrate how the increase in steady state inflation influences the determinacy of equilibrium we analyze the steady state inflation level of 2%, 4% and finally 7%.

Finally, the solution of the system (38) was obtained according to the calibration of basic parameters presented in Table 1.

*Table 1. Ca*libration parameters

Parameter	Definition	Value
π^{*}	Steady-state inflation rate	2%; 4%; 7%
ψ	Inverse of average time of price fixity	1; 2
η	Elasticity of substitution among intermediate goods	11 ; 6
α	Capital share in output	1/3
δ	Depreciation rate	0.08
s	Inverse of labor supply elasticity	0
θ	Discount factor	0.02

3.3 Analysis of indeterminacy regions and policy implications

The results of solution are presented on Figures 1–4. The axes are a1 and a2 values — the monetary policy rule coefficients in equation (20). The shaded area indicates the region with one eigenvalue with negative real part. This is the region of determinant equilibrium. The white area is the region with two eigenvalues with negative real part. It is the indeterminacy region. It should be stressed that the indeterminacy is what is called "real indeterminacy", i.e. not only is the dynamics of nominal variable such as inflation indeterminate but also the dynamics of real variables such as output or consumption.

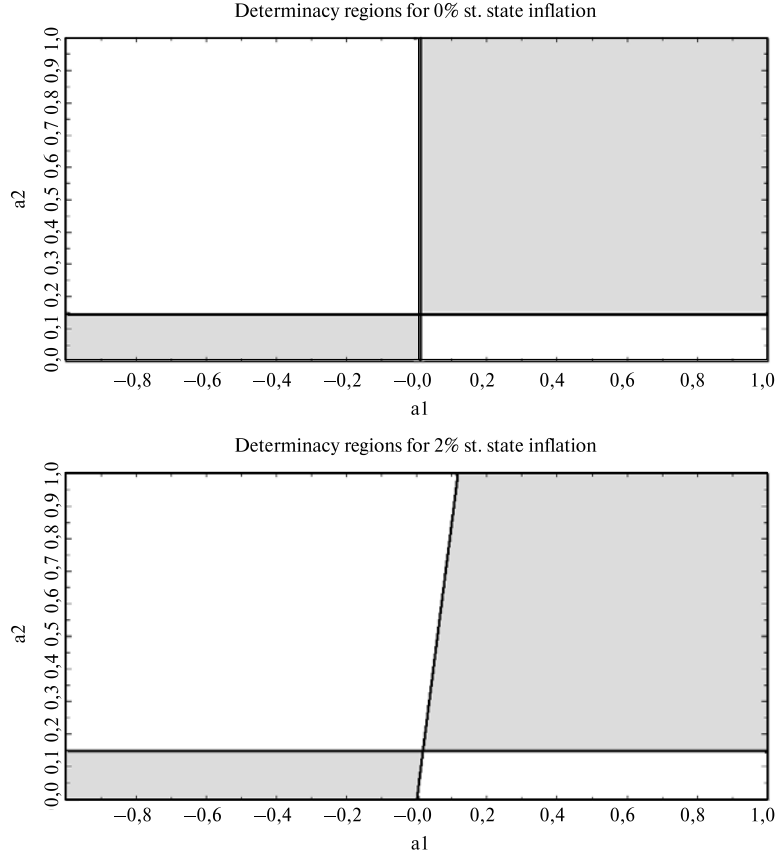
To compare and illustrate how our results corresponds to the main finding by Duport (2001) and Carlstrom and Fuerts (2005) first of all we analyze the case of zero-trend inflation. The first graph on Figures 1—4 represents eigenvalues regions for this case and we are interested in colored zone — determinacy. One can see that monetary policy which responds only to inflation variation leads to indeterminate equilibrium. To guarantee the uniqueness of equilibrium monetary authorities should also respond to variations in output. This result perfectly corresponds with one presented in papers stated above.

From Figures 1–4 one can see that monetary policy parameter's regions which guarantee the determinacy of equilibrium become smaller with an increase of steady state inflation. Higher firm's mark-ups lead to broader range of possibilities for monetary authorities to respond to variation in output and inflation which leads to a unique equilibrium.

The result, that active monetary policy rule in response to inflation does not provide a determinant equilibrium, obtained for zero trend inflation holds. In absence of the policy reaction to output fluctuations the economy is guaranteed to be in sunspot equilibrium. According to results presented on Figures 1–4, if trend inflation is greater than discount rate monetary authorities should react not too active to changes in output. The idea that monetary authorities still should react to output gap contradicts findings of Coibon and Gorodnichenko (2008), but they did not include capital accumulation process in the theoretical model. Also, our result about the reaction to output extends the analysis of Carlstrom and Fuerts (2005) who argued that the equilibrium is determinant once the reaction to output parameter is greater than some value (as graphs for zero inflation case clearly show). We argue that if the reaction is parameter is too large then either

equilibrium becomes indeterminate again, so the policy should not overreact to fluctuations of output.

For most parameter calibrations stated above the "classical" Taylor rule (a1 = 0.5, a2 = 0.5) for trend inflation greater than 2% does not lead to a unique equilibrium. This means that the policy based on Taylor principle does not necessarily lead to determinant equilibrium. And as it was stated above good policy should not lead to indeterminacy of equilibrium. This finding extends to all the results from previous literature: Taylor principle breaks down in presence of non-zero trend inflation.



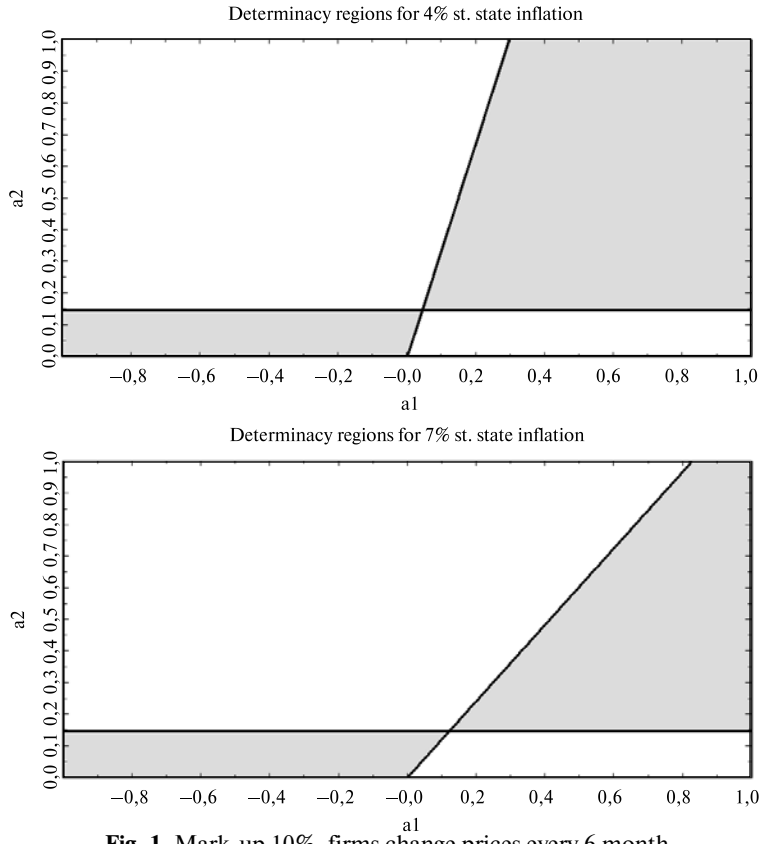
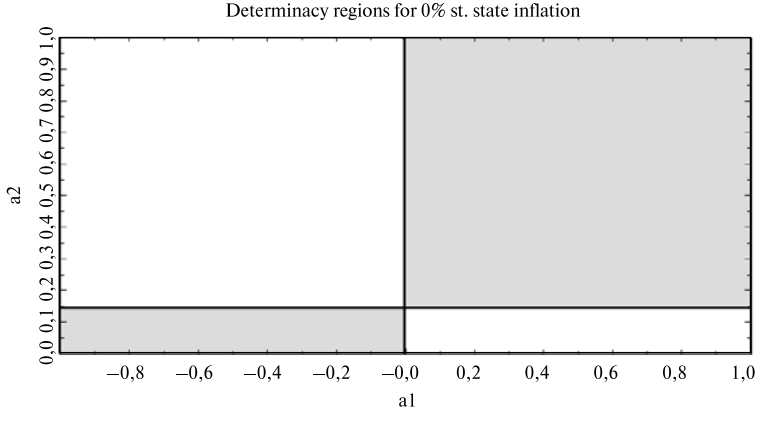
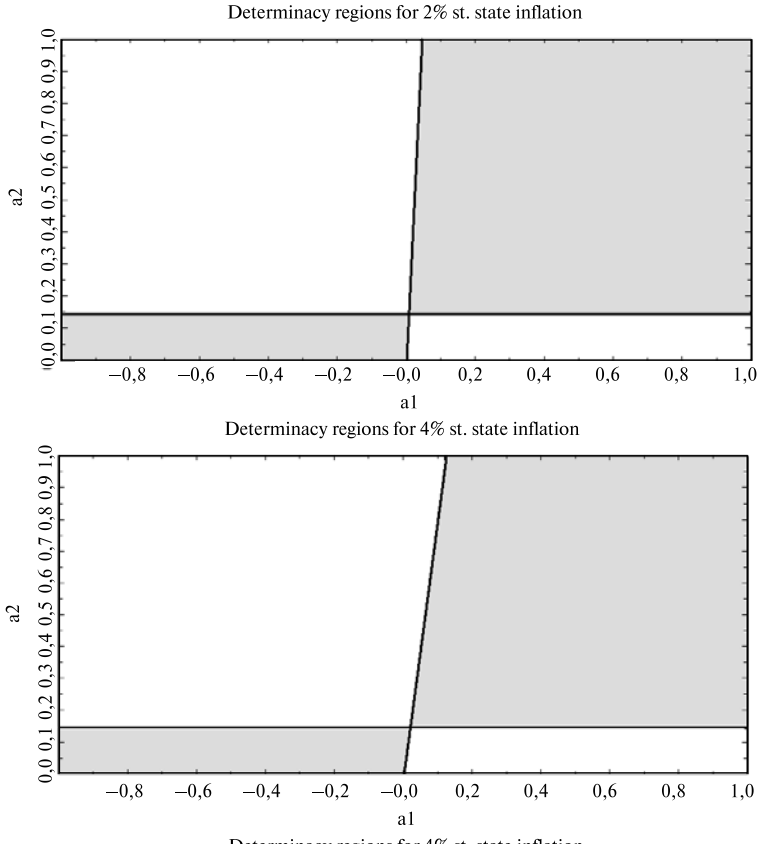


Fig. 1. Mark-up 10%, firms change prices every 6 month





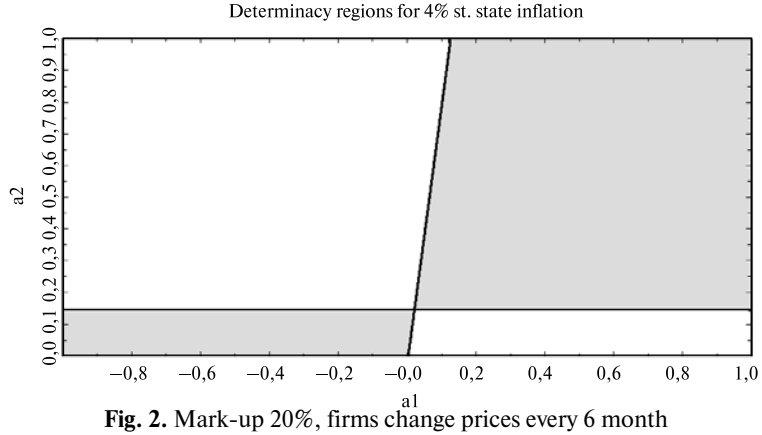
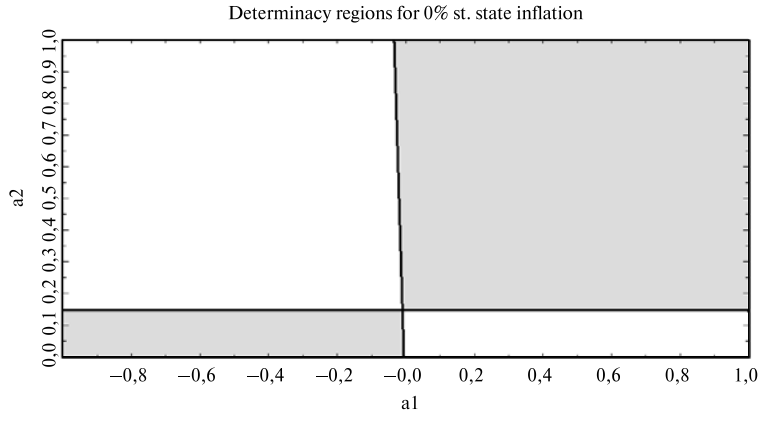
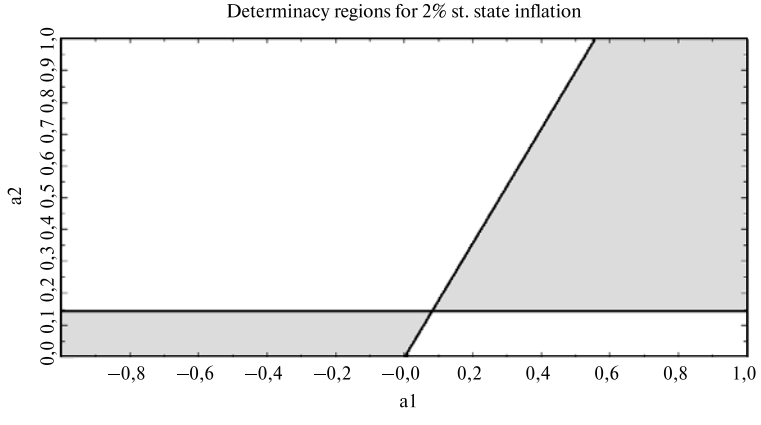
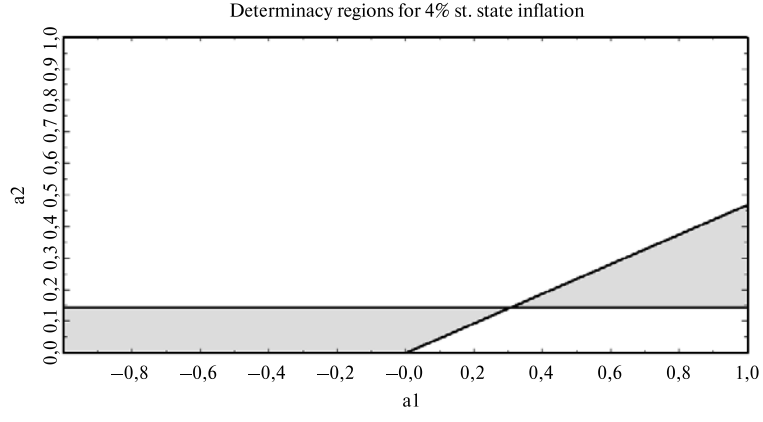


Fig. 2. Mark-up 20%, firms change prices every 6 month







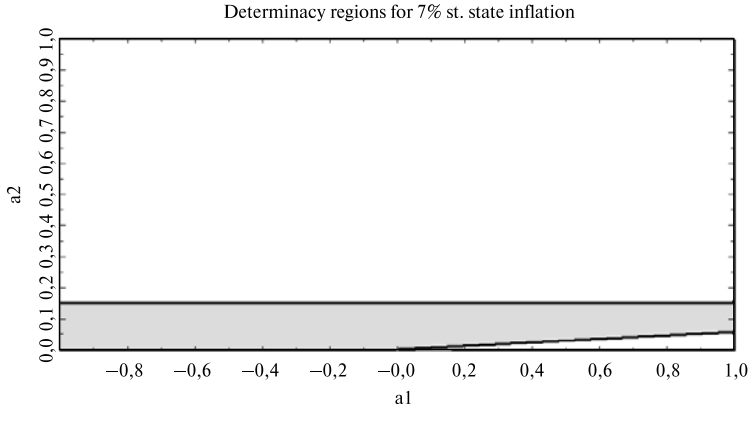
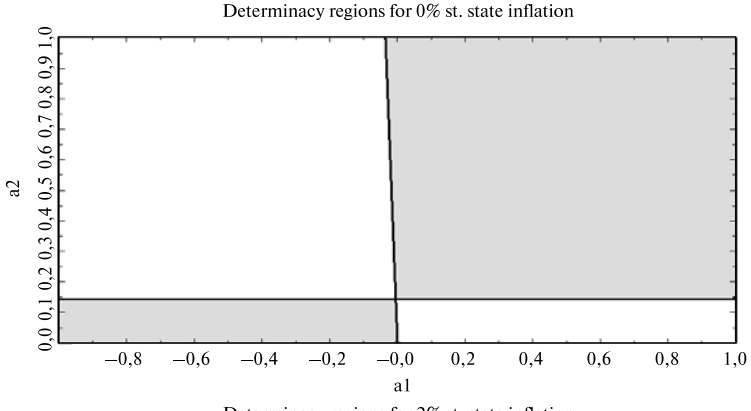
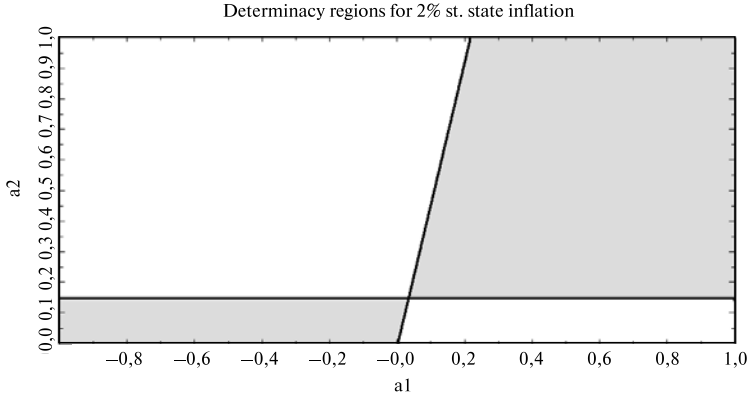


Fig. 3. Mark-up 10%, firms change prices every 12 month





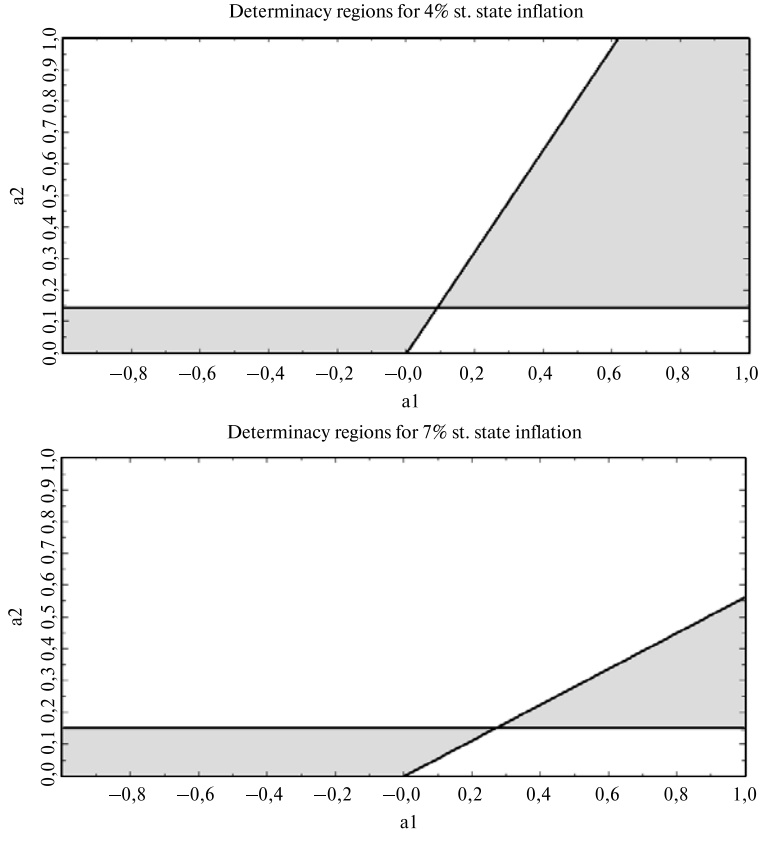


Fig. 4. Mark-up 20%, firms change prices every 12 month

Indeterminacy leads to different equilibrium paths for the same fundamentals. So in the indeterminate case there may exist equilibrium which is closer to empirical results that in determinacy case.

In order to demonstrate our point we expand the model by allowing for 2 types of fundamental uncertainty: technology and monetary policy shocks. Technology shock is introduced into production function (9) which is changed to

$$Y_{t,j} = K_{t,j}^{\alpha}(A_t L_{t,j})^{1-\alpha} \qquad (9')$$

and the process for shock is

$$\dot{a}_t = -\rho_a a_t ,$$

where $a=\ln A$.

The monetary policy shock is introduced into the policy rule (19) which is changed to

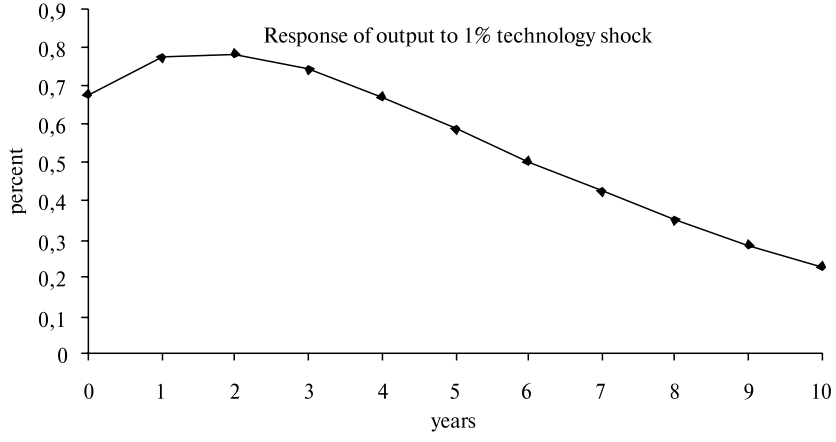
$$i_t - i^* = (1+a_1)(\pi_t - \pi^*) + a_2(\ln Y_t - \ln Y^*) + m_t$$

and the process for shock is

$$\dot{m}_t = -\rho m_t$$

The third non-fundamental shock can be chosen arbitrarily and constructed for example as an inflation forecast error shock and may be any i.i.d. random variable. This shock may be independent of fundamental shocks or at the other extreme may simply be a linear compilation of them in which case there is no non-fundamental uncertainty. It is worth mentioning that the equilibrium is still indeterminate even in this case.

The graphs below present impulse response of output to two fundamental shocks for the equilibrium in which inflation forecast error is equal to $0.3a_t - 2.25m_t$ for the following parameterization: $\pi^* = 4\%; \eta = 11; \psi = 1$. We set autoregression parameter for both shock to 0.35 which corresponds to half-life of a shock equal to two years.



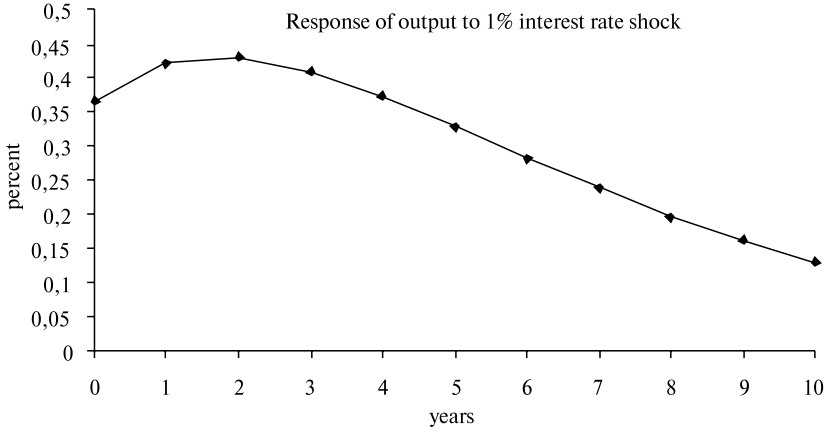


Fig. 5. Impulse response to fundamental shocks

Those impulse response functions look qualitatively alike the impulse response function often found in empirical studies. Of course, this can only be considered as a hint on the empirical relevance of the model. The thorough answer requires careful examination of the data from some country and calibration and simulation of the model based on this data. The final version of this paper will include this analysis as well.

4. Conclusions

This paper investigates the influence of positive trend inflation on the equilibrium determinacy in the typical New-Keynesian model with capital accumulation process. In this model the monetary policy is conducted using simple interest rates rule. The recent findings in literature that "active" monetary policy rule does not guarantee the uniqueness of the equilibrium correspond to our results. We show that presence of capital accumulation affects the indeterminacy region. In short, the result of Carlstrom and Fuerts (2005) that determinacy is restored under policy which is active enough in response to output fluctuations does not hold. We show that if the reaction is larger than some value of this parameter the equilibrium is also indeterminate. In other words, the response to output should be active; but not too active.

Monetary policy rule's parameters which lead to determinacy depend on the level of trend inflation (inflation in the steady state). Higher levels of trend inflation reduce the space of parameters value suitable for the unique solution of the model. We have also shown that a "classical" Taylor principle leads to indeterminate equilibrium for a wide range of plausible values of model's parameters.

Finally, we add some fundamental disturbances to the model and find out that the reaction of output to these shocks looks alike the empirical results of most of the literature. But to justify the empirical relevance the country's case study will be helpful. So, the next step will be devoted to the comparison of models simulation results with the empirical result for the particular country with inflation target of Central Bank greater than 2%.

It would be also very interesting to find out how the model's results change if monetary authorities conduct policy using interest rate smoothing in their rule. It is not obvious that the model will provide the same results as Coibon and Gorodnichenko (2008): interest rate smoothing increases sufficiently the probability of determinate equilibria. And it is also interesting to show to what measure of output should react monetary authorities. In other words, what variable (output gap or output growth or maybe even consumption) in monetary policy rule provide policymakers with wider range of possibilities of reaction which guarantee the uniqueness of equilibrium.

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Громова, Е. Инфляция и правила денежной политики: анализ множественности равновесий в новокейнсианской модели с накоплением капитала : препринт WP12/2011/01 [Текст] / Е. Громова, К. Сосунов ; Высшая школа экономики. — М.: Изд. дом Высшей школы экономики, 2011. — 28 с. — 150 экз. (на англ. яз.).

В работе анализируется влияние ненулевой трендовой инфляции в неокейнсианской модели с ценообразованием по Кальво и накоплением капитала. Мы опираемся на работы Дюпорта (2001) и Аскари, Рупела (2007), которые рассматривали эти эффекты по отдельности. Мы показываем, что одновременное присутствие положительной инфляции и накопления капитала значительно изменяет свойство единственности равновесия: для его поддержания денежные власти должны не только активно реагировать на инфляционные шоки, но и значительно менять процентную ставку при колебаниях выпуска. С другой стороны, слишком большая реакция на выпуск также может привести к неединственности равновесия. Мы также показываем, что для большого набора разумных параметризаций стандартное правило Тейлора влечет множественность равновесий. Мы также рассматриваем ряд альтернативных правил денежной политики.

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(*на английском языке*)

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