

The technology for solving the boundary value problems for systems of functional-differential equations of point type

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A class of nonlinear functional-differential equations, including equations with deviating argument of various types with time-lag and advance, as well as combine both of these elements (see. e.g., [1, 2]) is considered.

The proposed technology for solving boundary value problems is based on the Ritz method and spline collocation approaches. To solve the problem the system trajectories are discretized on the grid with a constant step and it is formulated the generalized residual functional, including both weighted residuals of the original differential equation and residuals of boundary conditions. To evaluate the derivatives of the system trajectories we use a technique of "spline-differentiation", based on two methods of spline approximation: using cubic splines and using a special type of splines, which second derivatives at the edges are controlled by the optimized parameters.

To solve the finite-dimensional optimization problems, in general non-convex and ravine, it is implemented a set of algorithms of local optimization (BFGS quasi-Newton method, two versions of the Powell method, the Barzilai-Borwein method, variant of trust regions method, stochastic search techniques in subspaces of dimension 3, 4 and 5) and global optimization algorithms (random multistart, curvilinear search technique, the tunneling method, "parabola" method and others).

The proposed technology includes the algorithm of increase the accuracy of approximations by increasing the number of points of the sampling grid (see. e.g., [3]), the algorithms of the functional derivative estimation by finite differences schemes with accuracy degrees from one to six, methodology for improving the accuracy of the spline-differentiation.

The report examines a small collection of test problems, created by the

traditional techniques ([4]), and the results of computational experiments are carried out for all the problems generated from the collection.

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