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Contents

Soliton Self-wave Number Downshift Compensation by the Increasing Second-Order Dispersion

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Abstract Dynamics of solitons in the frame of the extended nonlinear Schrödinger equation (NSE) taking into account stimulated Raman scattering (SRS) and inhomogeneous second-order dispersion (SOD) is considered. Compensation of soliton Raman self-wave number downshift in media with increasing second-order linear dispersion is shown. Quasi-soliton solution with small wave number spectrum variation, amplitude and extension are found analytically in adiabatic approximation and numerically. The soliton is considered as the equilibrium of SRS and increasing SOD. For dominate SRS soliton wave number spectrum tends to long wave region. For dominate increasing SOD soliton wave number spectrum tends to shortwave region.

Keywords Dynamics of solitons and quasi-solutions • Extended nonlinear Schrödinger equation • Stimulated Raman scattering • Inhomogeneous second-order dispersion

1 Introduction

Interest to solitons is conditioned because of their possibility to propagate on considerable distance keeping the form and transporting the energy and information without losses. Soliton solutions are considered in many different nonlinear models in various areas of physics for investigation of intensive wave fields in dispersive media propagation: optical pulses in fibers, electromagnetic waves in plasma, and surface waves on deep water [1–4].

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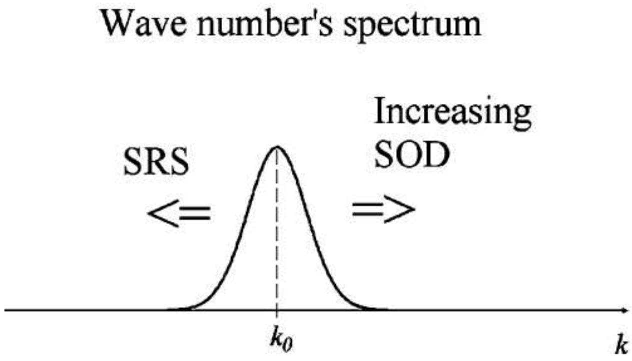

Fig. 1 The equilibrium of the soliton self-wave number downshift (SRS) and wave number upshift by increasing SOD leads to stabilization of soliton wave number spectrum

Propagation of high-frequency wave packets of rather big extension is described by the second-order nonlinear dispersive wave theory. Basic equation of the theory is nonlinear Schrödinger equation (NSE) [5, 6], considering both second-order dispersion (SOD) and cubic nonlinearity. Soliton solution in this case arises as an equilibrium of dispersion dilatation and nonlinear compression of wave packet.

Dynamics of high-frequency wave packets of low extension is described by the third-order nonlinear dispersive wave theory, taking into account third-order terms: nonlinear dispersion [7], stimulated Raman scattering (SRS) [8], and the third-order dispersion (TOD). Basic equation is the third-order nonlinear Schrödinger equation (TNSE) [9–12]. In [13–20] soliton solution in the frame of the TNSE without SRS was found. Such soliton exists as an equilibrium of the TOD and nonlinear dispersion. In [21] stationary kink-waves in the TNSE without TOD were found. This solution exists as an equilibrium of nonlinear dispersion and SRS. Taking into account SRS leads to downshift of soliton spectrum [8] and destroys stability of soliton propagation.

SRS in time presentation, corresponding to delay of nonlinear response, leads to soliton self-frequency downshift [8]. Compensation of the SRS by linear radiation fields from soliton core was considered in [22]. Compensation of the SRS in inhomogeneous media was considered in cases: for media with periodic SOD [23, 24], for media with shifting zero dispersion point (ZDP) of SOD [25], and for dispersion decreasing fiber (DDF) [26].

SRS in space representation, corresponding to nonlocal nonlinear response, leads to soliton self-wave number downshift. On the other hand, inhomogeneous SOD leads to variation of soliton wave number too. In particular, in geometrical optic approximation, velocity of wave number variation in smoothly inhomogeneous media is described by well-known equation $\dot{k} = -\omega'_{\xi}$, where $\omega = \omega(k, \xi)$ is the linear dispersion relation. For inhomogeneous SOD $q(\xi) = -\omega''_{kk}$ velocity of wave number variation is proportional to gradient of dispersion $\dot{k} = q'_{\xi}(k - k_0)^2$ and for $q'_{\xi} > 0$ wave number increases.

Equilibrium of these effects leads to stabilization of the wave number spectrum (Fig. 1). In this chapter soliton dynamics in media with SRS and increasing positive SOD is considered. Quasi-soliton solution with small wave number spectrum variation, amplitude, and extension is found. This soliton exists as the equilibrium of SRS and increasing SOD.

2 Basic Equation

Let us consider the dynamics of the high-frequency wave field $U(\xi,t)exp(i\omega t - ik\xi)$ in the frame of the extension NSE with SRS and inhomogeneous SOD:

$$2i\frac{\partial U}{\partial t} + q(\xi)\frac{\partial^2 U}{\partial \xi^2} + 2\alpha U \mid U \mid^2 + \mu U \frac{\partial \left(\mid U \mid^2\right)}{\partial \xi} = 0, \quad (1)$$

where in consequence of the nonlinear dispersion law $\omega = \omega\left(k, \mid U \mid^2\right)$ the following notation is used: $q = -\partial^2\omega/\partial k^2$ is the SOD, $\alpha = -\partial\omega/\partial\left(\mid U \mid^2\right)$ is the self-phase modulation, and μ is the SRS in space presentation (nonlocal nonlinear response). Equation (1) with the zero conditions at the infinity $U|_{\xi\to\pm\infty} \to 0$ has the following integrals:

- Rate of "mass" change (number of "quantum") wave packet variation

$$\frac{d}{dt}\int_{-\infty}^{+\infty} \mid U \mid^2 d\xi = \int_{-\infty}^{+\infty} \frac{\partial q}{\partial \xi} k \mid U \mid^2 d\xi \quad (2)$$

- Rate of impulse change

$$\frac{d}{dt}\int_{-\infty}^{+\infty} k \mid U \mid^2 d\xi = -\frac{\mu}{2}\int_{-\infty}^{+\infty} \left(\frac{\partial \mid U \mid^2}{\partial \xi}\right)^2 d\xi + \frac{1}{2}\int_{-\infty}^{+\infty} \frac{\partial q}{\partial \xi}\frac{\partial U}{\partial \xi}\frac{\partial U^*}{\partial \xi} d\xi \quad (3)$$

- Rate of the modulus gradient wave field change

$$\frac{d}{dt}\int_{-\infty}^{+\infty} \frac{\partial U}{\partial \xi}\frac{\partial U^*}{\partial \xi} d\xi = -\mu\int_{-\infty}^{+\infty} k\left(\frac{\partial \mid U \mid^2}{\partial \xi}\right)^2 d\xi \quad (4)$$

where $U = \mid U \mid exp(i\varphi)$, $k = \partial\varphi/\partial\xi$ is the local additional wave number of wave packet. Value $\frac{\partial U}{\partial \xi}\frac{\partial U^*}{\partial \xi} = k^2 \mid U \mid^2 + (\frac{\partial |U|}{\partial \xi})^2$ in Eq. (4) corresponds to density of the full energy wave packet: first term corresponds to the density of "kinetic" energy and second to "potential" energy. The right side of Eq. (2) corresponds to "mass" (number of "quanta") variation of the wave packet by inhomogeneous SOD. The right side of Eq. (3) describes impulse variation: first term by SRS and second by inhomogeneous SOD.

Assuming space scales of heterogeneity of both dispersion q and the local wave number k much bigger than the scale of the heterogeneity of the packet envelope $L_{q,k} \gg L_{|U|}$, relations (2)–(4) take forms:

$$N\frac{dk}{dt} = -\frac{\mu}{2}\int_{-\infty}^{+\infty}\left(\frac{\partial \mid U \mid^2}{\partial \xi}\right)^2 d\xi + \frac{1}{2}\left(\frac{\partial q}{\partial \xi}\right)_{\overline{\xi(t)}}\left(W - k^2 N\right), \tag{5}$$

$$\frac{dN}{dt} = \left(\frac{\partial q}{\partial \xi}\right)_{\overline{\xi(t)}} kN, \tag{6}$$

$$\frac{dW}{dt} = \left(\frac{\partial q}{\partial \xi}\right)_{\overline{\xi(t)}} kW, \tag{7}$$

where $N = \int_{-\infty}^{+\infty} \mid U \mid^2 d\xi$ is the number "quantum" of the wave packet ("mass" of the wave packet), $W = \int_{-\infty}^{+\infty} (\frac{\partial |U|}{\partial \xi})^2 d\xi$ is the "potential" energy of the wave packet, and $\overline{\xi(t)} = \frac{1}{N}\int_{-\infty}^{+\infty} \xi \mid U \mid^2 d\xi$ is the "mass" center of wave packet. Taking into account Eqs. (6)–(7) values N and W are connected by the relation

$$N(t)W(t) = const; \tag{8}$$

the equilibrium state of the system (6)–(7) is achieved under conditions:

$$\begin{cases} N = N_0, W = W_0, k = 0, \\ \mu \int_{-\infty}^{+\infty}\left(\frac{\partial \mid U \mid^2}{\partial \xi}\right)^2 d\xi = \left(\frac{\partial q}{\partial \xi}\right)_{\overline{\xi(t)}} W_0. \end{cases} \tag{9}$$

For wave packets with amplitude A and extension Δ, integrals in system (9) can be estimated as $N \approx A^2\Delta, W \approx A^2/\Delta$, and $\int_{-\infty}^{+\infty} (\frac{\partial |U|^2}{\partial \xi})^2 d\xi \approx A^4/\Delta$. In this case parameters of the equilibrium state Eq. (9) are the following: $A = A_*$, $\Delta = \Delta_*$, $k_* = 0$, and $\mu A_*^2 = (q'_\xi)_{\overline{\xi(t)}}$. Increasing dispersion $(q'_\xi)_{\overline{\xi(t)}}$ can stabilize both the additional wave number, the amplitude, and the extension of the wave packet.

3 Adiabatic Approximation

Let us consider the dynamics of localized wave packets with module of envelope $\mid U \mid$ described by the self-similar function

$$\mid U \mid = A(t) f\left(\frac{\xi - \overline{\xi(t)}}{\Delta(t)}\right), \tag{10}$$

where $f(0) = 1$. For such self-similar solution, the relation (8) corresponds to the wave packet of a constant amplitude; therefore the system (5)–(7) can be reduced to

$$\frac{dk}{dt} = -\frac{\mu}{2}\frac{A_0^2}{\Delta^2(t)}\lambda_1 + \frac{1}{2}\left(\frac{\partial q}{\partial \xi}\right)_{\overline{\xi(t)}}\left(\frac{\lambda_2}{\Delta^2(t)} - k^2\right), \tag{11}$$

$$\frac{d\Delta}{dt} = \left(\frac{\partial q}{\partial \xi}\right)_{\overline{\xi(t)}} k\Delta, \tag{12}$$

where $\lambda_1 = \int\limits_{-\infty}^{+\infty}(\frac{\partial f^2}{\partial \eta})^2 d\eta \ \ / \int\limits_{-\infty}^{+\infty} f^2 d\eta$, $\lambda_2 = \int\limits_{-\infty}^{+\infty}(\frac{\partial f}{\partial \eta})^2 d\eta \ \ / \int\limits_{-\infty}^{+\infty} f^2 d\eta$. The equilibrium state of the system (11)–(12) is achieved for wave packets with zero additional wave number $k = 0$, invariable both the extension $\Delta = \Delta_0$ and the amplitude $A = A_0$, propagating in media with linear profile of SOD $(q'_\xi)_{\overline{\xi(t)}} = q' = const$:

$$\mu A_0^2 = \frac{\lambda_2}{\lambda_1} q'. \tag{13}$$

Using replacement $\tau = \frac{1}{2}q't$ and $r = \Delta/\sqrt{\lambda_2}$ the system (11)–(12) is reduced to the form

$$\frac{dk}{d\tau} = \frac{1}{r^2}(1-p) - k^2, \tag{14}$$

$$\frac{dr}{d\tau} = 2kr, \tag{15}$$

where $p = \frac{\lambda_1}{\lambda_2}\frac{\mu}{q'}A_0^2 > 0$. This system has one equilibrium state $\begin{cases} p = 1 \\ k = 0 \end{cases}$. Phase trajectories (14)–(15) are the following:

$$r\left(k^2 + \frac{1-p}{r^2}\right) = r_0\left(k_0^2 + \frac{1-p}{r_0^2}\right), \tag{16}$$

where $r_0 = r(0)$ is the soliton extension at the initial time moment and $k_0 = k(0)$ is the soliton wave number at the initial time moment. Type of the phase plane of Eqs. (14)–(15) depends on the value of parameter p:

1. Weak SRS ($p < 1$). In Fig. 2 the phase plane of Eqs. (14)–(15) for $p = 0.5$ is shown. Direction of movement along trajectories is from left to right. In particular case for wave packets with zero initial wave number $k_0 = 0$, wave number in the first time period increases. Maximum value of the wave number in this case is the following:

$$k_{max} = \left(\frac{k_0^2 r_0}{2\sqrt{1-p}} + \frac{\sqrt{1-p}}{2r_0}\right)^{2/3}.$$

   In the long run wave number tends to zero.

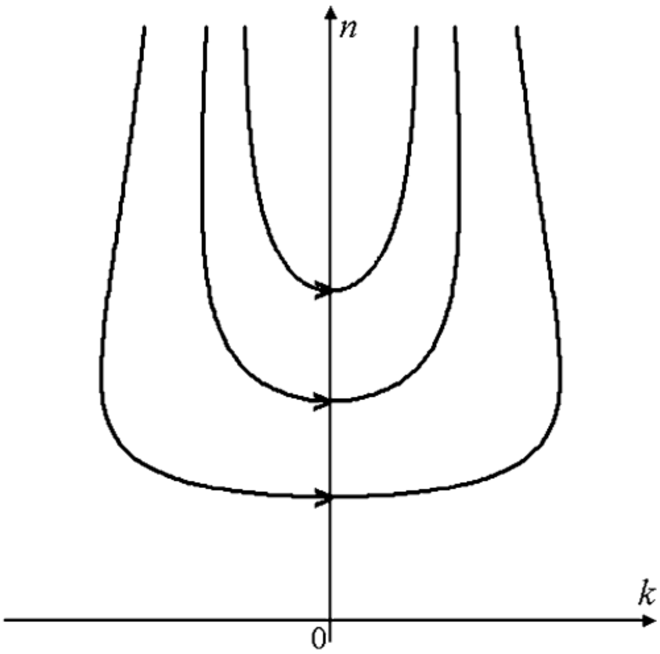

Fig. 2 Phase trajectories (14)–(15) for $p = 0.5$ (weak SRS)

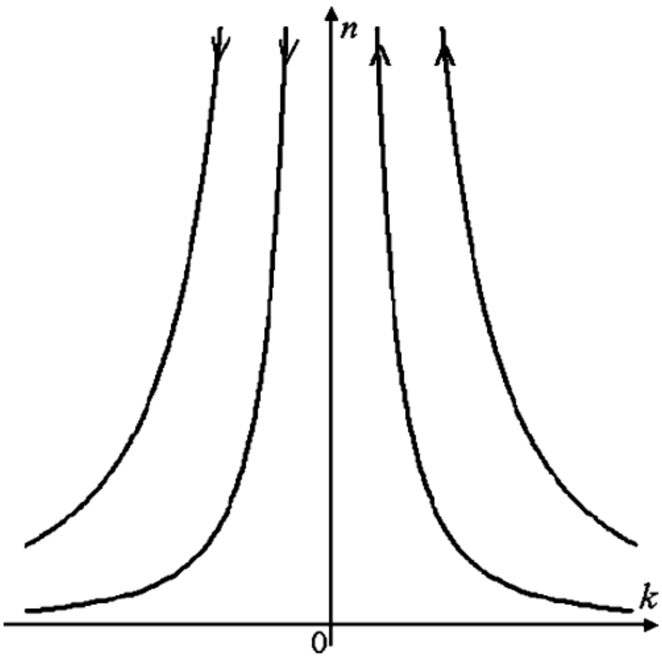

Fig. 3 Phase trajectories (14)–(15) for $p = 1$ (critical SRS)

2. Critical SRS $(p = 1)$. In Fig. 3 the phase plane of Eqs. (14)–(15) for $p = 1$ is shown. Direction of movement along trajectories is from right to left. For $p = 1$ time variation of wave number from Eq. (14) is the following: $k = \frac{k_0}{1+k_0\tau}$. In particular for positive initial wave number $k_0 > 0$ wave number tends to zero. It corresponds to stabilization of SRS and increasing SOD. For negative initial wave number $k_0 < 0$ wave number tends to long wave region. It corresponds to domination of SRS.
3. Strong SRS $(p > 1)$. In Fig. 4 phase plane of Eqs. (14)–(15) for $p = 2$ is shown. Direction of movement along trajectories is from right to left. In this case wave number is decreased monotonically $\frac{dk}{d\tau} < 0$. It corresponds to domination of SRS.

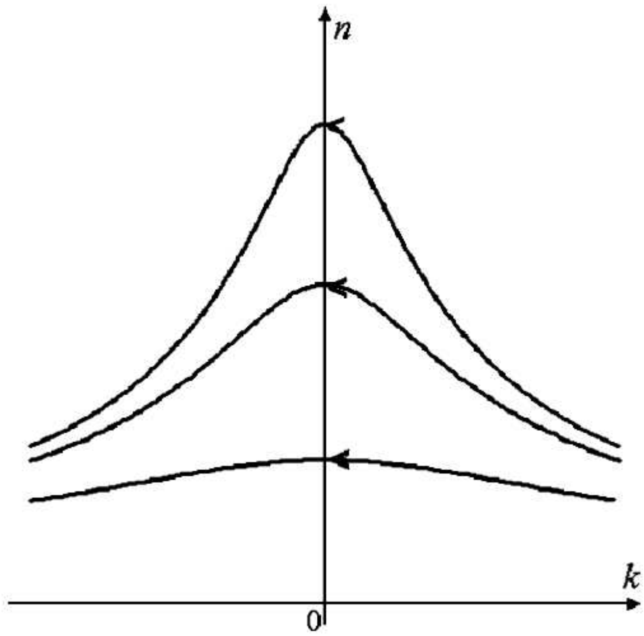

Fig. 4 Phase trajectories (14)–(15) for $p > 1$ (strong SRS)

4 Numerical Results

Let us consider numerically the initial-value problem of dynamics of soliton-like wave packets

$$U(\xi, t = 0) = \frac{A_0}{cosh(\xi/\Delta)} \quad (17)$$

in the frame of Eq. (1) for $\alpha = 1$, $q(\xi) = 1 + \xi/20$, $A_0 = 1$, and different μ. For sech-like profile Eq. (17) we have $\lambda_1/\lambda_2 = 8/5$ and parameter p from adiabatic approximation is the following: $p = \frac{8}{5}\frac{\mu}{q'}A_0^2$. In the particular case, for the value of SOD gradient $q' = 1/20$ and for the initial soliton amplitude $A_0 = 1$ we have $p = 32\mu$. Equilibrium of SRS and increasing SOD is achieved under condition $p = 1$, corresponding to parameter of SRS $\mu = 1/32$.

In Fig. 5 numerical results of distributions of module of wave packet envelope $\mid U \mid$ on ξ with $\mu = 1/32$ at different time moments are shown as example. Wave packet propagates with keeping of soliton-like form with small amplitudes of radiation fields. This gives the possibility of using adiabatic approximation for the description of the soliton dynamics in the frame Eq. (1). In Fig. 6 distributions of module of wave number spectrum $\mid U_k \mid$, where $U_k(k,t) = \int_{-\infty}^{+\infty} U(\xi,t)e^{-ik\xi}d\xi$, on k with $\mu = 1/32$ at different time moments are shown. For $\mu = 1/32$ maximum of the wave number spectrum is varying slightly. It corresponds to the equilibrium of self-wave number downshift by SRS and increasing SOD. Deviation of parameter μ leads to disturb the dynamical equilibrium of SRS and inhomogeneous dispersion. In Fig. 7 numerical results of maximum of modulus of wave number spectrum $k_{max}(t) = k(\mid U_k \mid = max)$ as function t for different values μ are shown. Curves for

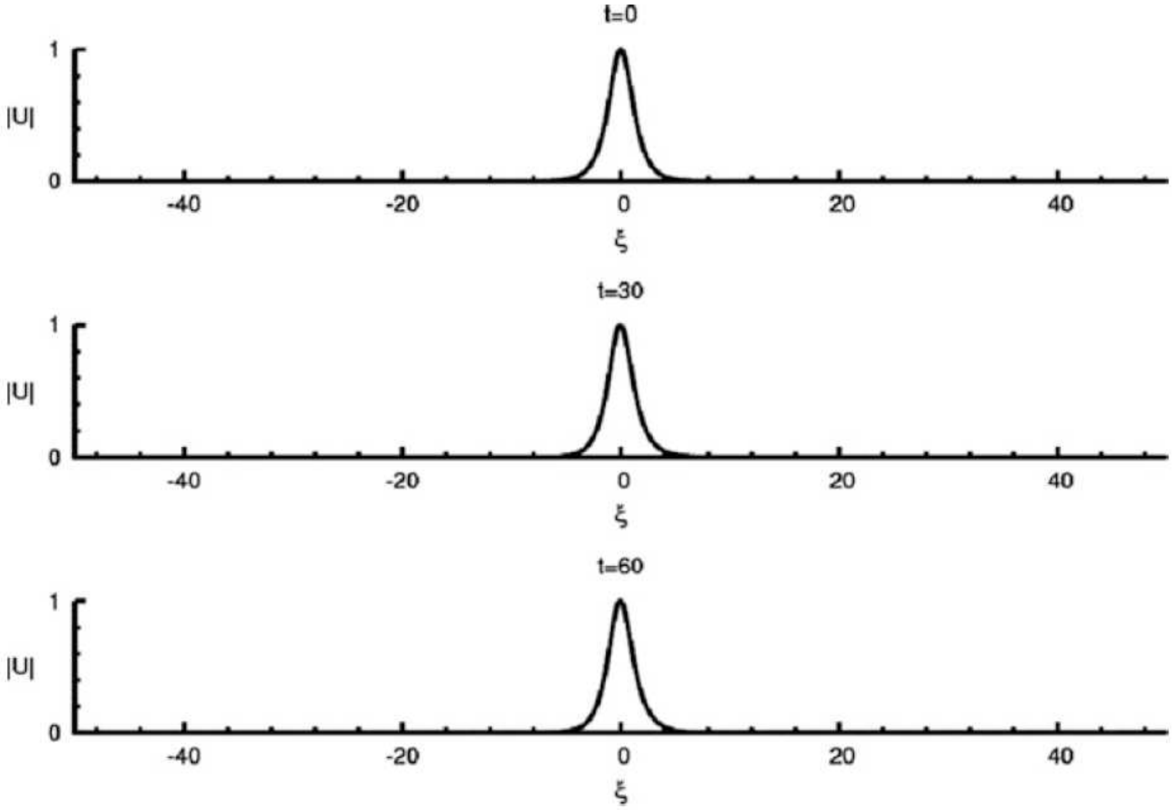

Fig. 5 Numerical results of distributions of module of wave packet envelope $| U |$ on ξ for $\mu = 1/32$ at different time moments

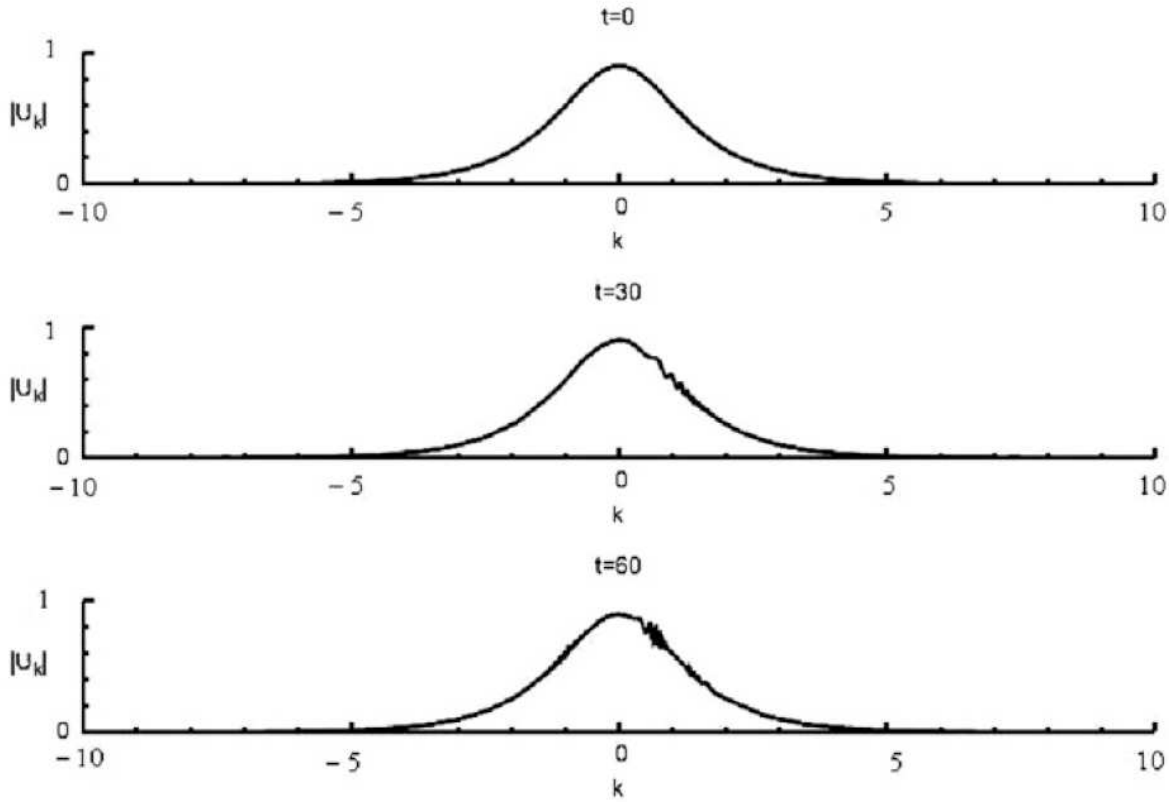

Fig. 6 Numerical results of distributions of module of wave number spectrum $| U_k |$ on k for $\mu = 1/32$ at different time moments

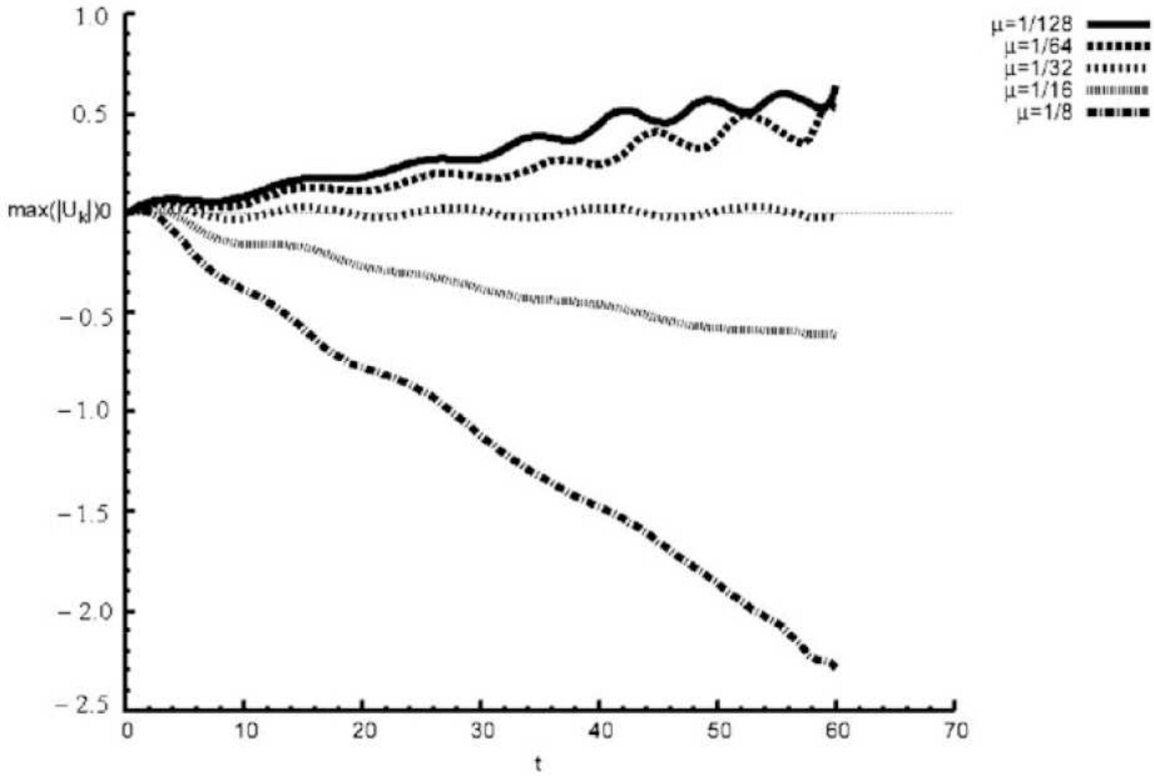

Fig. 7 Numerical results of maximum of modulus of wave number spectrum $k_{max}(t)$ for different values μ as a function of t

$\mu = 1/128$ and $\mu = 1/64$ correspond to domination of SOD, $\mu = 1/32$—dynamical equilibrium of SRS and inhomogeneous dispersion, $\mu = 1/16$ and $\mu = 1/8$—domination of SRS.

Adiabatic approximation is in a good agreement with numerical results for regime of the equilibrium SRS and increasing SOD and for regime of the domination SRS. Regime of the domination increasing SOD from adiabatic approximation corresponds to numerical results only for initial time period.

5 Conclusion

Dynamics of solitons envelope in the frame of the expanded NSE taking into account SRS and inhomogeneous SOD is considered both analytically in adiabatic approximation and numerically. Compensation of the SRS by the increasing SOD under condition $\mu_* \approx q'_\xi / A_0^2$ is shown. In this case soliton propagates with unvariable both additional wave number, amplitude, and extension. For strong SRS $\mu > \mu_*$ soliton wave number downshift. For weak SRS $\mu < \mu_*$ soliton wave number upshift.

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