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Spin-plasmons in topological insulator

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ABSTRACT

Collective plasmon excitations in a helical electron liquid on the surface of strong three-dimensional topological insulator are considered. The properties and internal structure of these excitations are studied. Due to spin-momentum locking in helical liquid on a surface of topological insulator, the collective excitations should manifest themselves as coupled charge- and spin-density waves.

1. Introduction

In recent years, topological insulators with a non-trivial topological order, intrinsic to their band structure, were predicted theoretically and observed experimentally (see [1] and references therein). Three-dimensional (3D) realizations of "strong" topological insulators (such as Bi_2Se_3, Bi_2Te_3 and Sb_2Te_3) are insulating in the bulk, but have gapless topologically protected surface states with a number of unusual properties [2]. These states obey two-dimensional Dirac equation for massless particles, similar to that for electrons in graphene [3], but related to real spin of electrons, instead of sublattice pseudospin in graphene.

The consequence of that is a spin-momentum locking for electrons on the surface of strong 3D topological insulators, i.e., spin of each electron is always directed in the surface plane and perpendicularly to its momentum [1,4]. The surface of topological insulator can be chemically doped, forming a charged "helical liquid".

Collective excitation of electrons in such helical liquid were considered in [5], where relationships between charge and spin responses to electromagnetic field were derived. It was shown that charge-density wave in this system is accompanied by spin-density wave. Application of spin-plasmons to create "spin accumulator" was proposed in [6]. Also the surface plasmon-polaritons under conditions of magnetoelectric effect in 3D topological insulator were considered [7].

In the present article we consider the properties and internal structure of spin-plasmons in a helical liquid. Within the random-phase approximation, we derive plasmon wave function and calculate amplitudes of charge- and spin-density waves in the plasmon state.

2. Wave function of spin-plasmon

Low-energy effective Hamiltonian of the surface states of Bi_2Se_3 in the representation of spin states $\{|\uparrow\rangle,|\downarrow\rangle\}$ is $H_0=v_F(p_x\sigma_y-p_y\sigma_x)$ for a surface in the xy plane, where the Fermi velocity $v_F\approx 6.2\times 10^5$ m/s [2]. Its eigenfunctions can be written as $e^{i\mathbf{p}\cdot\mathbf{r}}|f_{\mathbf{p}\gamma}\rangle/\sqrt{S}$, where S is the system area and $|f_{\mathbf{p}\gamma}\rangle=(e^{-i\phi_\mathbf{p}/2},i\gamma e^{i\phi_\mathbf{p}/2})^T/\sqrt{2}$ is the spinor part of the eigenfunction, corresponding to electron with momentum $\boldsymbol{p}$ (its azimuthal angle in the xy plane is $\phi_\boldsymbol{p}$) from conduction ($\gamma=+1$) or valence ($\gamma=-1$) band. Many-body Hamiltonian of electrons populating the s urface of topological insulator is $H=\sum_{\mathbf{p}\gamma}\xi_{\mathbf{p}\gamma}a^+_{\mathbf{p}\gamma}a_{\mathbf{p}\gamma}+(1/2S)\sum_\mathbf{q}V_q\rho^+_\mathbf{q}\rho_\mathbf{q}$, where $a_{\boldsymbol{p}\gamma}$ is the destruction operator for electron with momentum $\boldsymbol{p}$ from the band γ, $\xi_{\boldsymbol{p}\gamma}=\gamma v_F|\boldsymbol{p}|-\mu$ is its energy measured from the chemical potential μ, $V_q=2\pi e^2/\varepsilon q$; $\rho^+_\mathbf{q}=\sum_{\mathbf{p}\gamma\gamma'}\langle f_{\mathbf{p}+\mathbf{q},\gamma'}|f_{\mathbf{p}\gamma}\rangle a^+_{\mathbf{p}+\mathbf{q},\gamma'}a_{\mathbf{p}\gamma}$ is the charge density operator for helical liquid.

The creation operator for spin-plasmon with wave vector $\boldsymbol{q}$ can be presented in the form:

$$Q^+_\mathbf{q}=\sum_{\mathbf{p}\gamma\gamma'}C^{\gamma'\gamma}_{\mathbf{p}\mathbf{q}}a^+_{\mathbf{p}+\mathbf{q},\gamma'}a_{\mathbf{p}\gamma} \quad (1)$$

This operator should obey the equation of motion $[H,Q^+_\mathbf{q}]=\Omega_qQ^+_\mathbf{q}$, where Ω_q is the plasmon frequency. We can get solution of this equation in the random phase approximation at $T=0$ (similarly to [8]):

$$C^{\gamma'\gamma}_{\mathbf{p}\mathbf{q}}=\frac{|n_{\mathbf{p}\gamma}-n_{\mathbf{p}+\mathbf{q},\gamma'}|\langle f_{\mathbf{p}+\mathbf{q},\gamma'}|f_{\mathbf{p}\gamma}\rangle N_\mathbf{q}}{\Omega_q+\xi_{\mathbf{p}\gamma}-\xi_{\mathbf{p}+\mathbf{q},\gamma'}+i\delta}, \quad (2)$$

where $n_{\boldsymbol{p}+}=\Theta(p_F-|\boldsymbol{p}|)$ and $n_{\boldsymbol{p}-}$ are occupation numbers for electron-doped helical liquid ($p_F=\mu/v_F$ is the Fermi momentum).

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The plasmon frequency is determined in this approach from the equation $1-V_q\Pi(q,\Omega_q)=0$, where

$$\Pi(q,\omega)=\sum_{\mathbf{p}\gamma\gamma'}|\langle f_{\mathbf{p}+\mathbf{q},\gamma'}|f_{\mathbf{p}\gamma}\rangle|^2\frac{n_{\mathbf{p}\gamma}-n_{\mathbf{p}+\mathbf{q},\gamma'}}{\omega+\xi_{\mathbf{p}\gamma}-\xi_{\mathbf{p}+\mathbf{q},\gamma'}+i\delta} \quad (3)$$

is the polarization operator of the helical liquid, different from that for graphene [3] only by degeneracy factor. The factor $N_{\boldsymbol{q}}$ in (2) can be determined from the normalization condition

$$\langle 0|[Q_{\mathbf{q}},Q^+_{\mathbf{q}'}]|0\rangle=\delta_{\mathbf{q}\mathbf{q}'}\sum_{\gamma\gamma'}D_{\gamma'\gamma}=\delta_{\mathbf{q}\mathbf{q}'},$$

$$D_{\gamma'\gamma}=\sum_{\mathbf{p}}|C^{\gamma\gamma'}_{\mathbf{p}\mathbf{q}}|^2(n_{\mathbf{p}\gamma}-n_{\mathbf{p}+\mathbf{q},\gamma'}) \quad (4)$$

($|0\rangle$ is the ground state), so that $|N_{\mathbf{q}}|^{-2}=-S[\partial\Pi(q,\omega)/\partial\omega]|_{\omega=\Omega_q}$. The quantities $D_{\gamma'\gamma}$ in (4) can be considered as total weights of intraband (D_{++}) and interband ($D_{+-}+D_{-+}=1-D_{++}$) electron transitions, contributing to the plasmon wave function (1). Note that all these formulas are also applicable to the case of graphene.

Spin-plasmon dispersion Ω_q and contribution of intraband transitions into its wave function are plotted in Fig. 1 at various $r_s=e^2/\varepsilon v_F$, where ε is the dielectric susceptibility of surrounding 3D medium. For Bi_2Se_3, $r_s\approx 0.09$ with $\varepsilon\approx 40$ for dielectric half-space [5] (for such small r_s, the corresponding dispersion curve approaches very closely to the upper bound $\omega=v_Fq$ of the intraband continuum). The results for suspended graphene with rather large $r_s=8.8$ (for $v_F\approx 10^6$ m/s, $\varepsilon=1$ and with the degeneracy factor 4 incorporated into r_s) are also presented for comparison. It is seen that the undamped spin-plasmon consists mainly of intraband transitions. When the dispersion curve enters the interband continuum, the spin plasmon becomes damped and inter- and intraband transitions contribute almost equally to its wave function.

3. Charge- and spin-waves

The helical liquid in the state $|1_{\mathbf{q}}\rangle=Q^+_{\mathbf{q}}|0\rangle$ with one spin-plasmon of wave vector $\boldsymbol{q}$ has a distribution of electron–hole excitations (2), shifted towards $\boldsymbol{q}$. Due to the spin-momentum locking, the system acquires a total nonzero spin polarization, perpendicular to $\boldsymbol{q}$. A similar situation occurs in the current-carrying state of the helical liquid, which turns out to be spin-polarized [4].

Introducing one-particle spin operator as $\boldsymbol{s}=\boldsymbol{\sigma}/2$, we can calculate its average value in the one-plasmon state $\langle\boldsymbol{s}\rangle=\langle 1_{\boldsymbol{q}}|\boldsymbol{s}|1_{\boldsymbol{q}}\rangle$ as

$$\langle\mathbf{s}\rangle=\sum_{\mathbf{p}\gamma\gamma'\tau}[\langle f_{\mathbf{p}+\mathbf{q},\gamma'}|\mathbf{s}|f_{\mathbf{p}+\mathbf{q},\tau}\rangle C^{\tau\gamma}_{\mathbf{p}\mathbf{q}} - C^{\gamma'\tau}_{\mathbf{p}\mathbf{q}}\langle f_{\mathbf{p}\tau}|\mathbf{s}|f_{\mathbf{p}\gamma}\rangle](C^{\gamma\gamma'}_{\mathbf{p}\mathbf{q}})^*(n_{\mathbf{p}\gamma}-n_{\mathbf{p}+\mathbf{q},\gamma'}) \quad (5)$$

If $\boldsymbol{q}$ is parallel to $\boldsymbol{e}_x$, only the y-component of $\langle\boldsymbol{s}\rangle$ is nonzero. Its dependence on q at various r_s is plotted in Fig. 2(a).

Charge- and spin-density waves, accompanying spin-plasmon with the wave vector $\boldsymbol{q}$, can be characterized by corresponding spatial harmonics of charge- and spin-density operators: $\rho^+_{\mathbf{q}}$ and $\mathbf{s}^+_{\mathbf{q}}=\sum_{\mathbf{p}\gamma\gamma'}\langle f_{\mathbf{p}+\mathbf{q},\gamma'}|\mathbf{s}|f_{\mathbf{p}\gamma}\rangle a^+_{\mathbf{p}+\mathbf{q},\gamma'}a_{\mathbf{p}\gamma}$. Using, similarly to [9], the

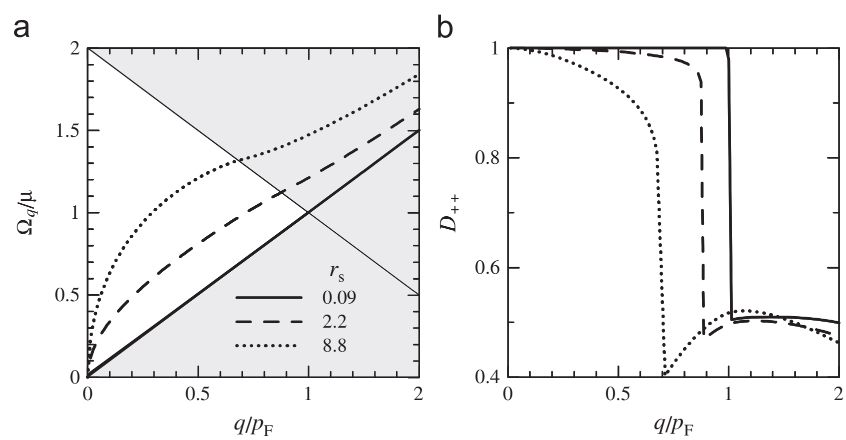

Fig. 1. Dispersions of spin-plasmon (a) and contributions D_{++} of intraband transitions into its wave function (b) at various r_s. Continuums of intraband ($\omega<v_Fq$) and interband ($\omega+v_Fq>2\mu$) single-particle excitations are shaded in (a).

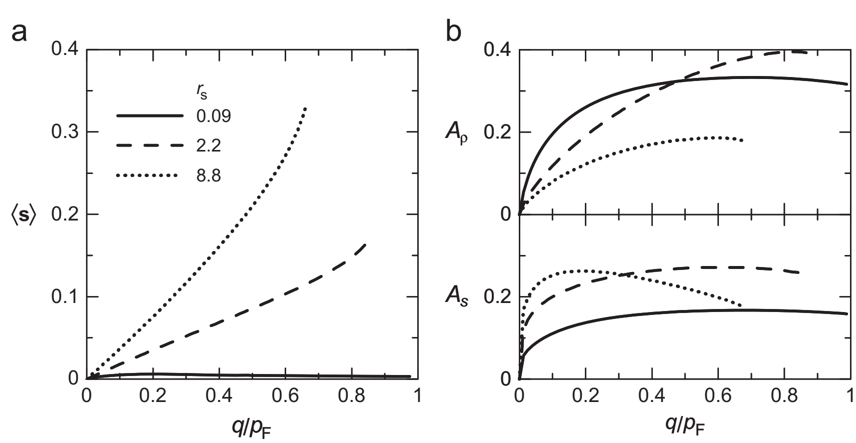

Fig. 2. Total spin polarization $\langle\boldsymbol{s}\rangle$ of the helical liquid in the one-plasmon state (a) at various r_s and normalized amplitudes A_ρ and A_s of charge- and spin-density waves respectively in the many-plasmon state (b).

unitary transformation, inverse with respect to (1), we can write:

$$\rho^{+}_{\mathbf{q}} = SN^{*}_{\mathbf{q}}\Pi(q,\Omega_q)Q^{+}_{\mathbf{q}} + \tilde{\rho}^{+}_{\mathbf{q}}, \quad (6)$$

$$\mathbf{s}^{+}_{\mathbf{q}} = SN^{*}_{\mathbf{q}}\mathbf{\Pi}_s(q,\Omega_q)Q^{+}_{\mathbf{q}} + \tilde{\mathbf{s}}^{+}_{\mathbf{q}}, \quad (7)$$

where the operators $\tilde{\rho}^{+}_{\mathbf{q}}$ and $\tilde{\mathbf{s}}^{+}_{\mathbf{q}}$ are the contributions of single-particle excitations and are dynamically independent on plasmons. Here the crossed spin-density susceptibility of the helical liquid [5] has been introduced:

$$\mathbf{\Pi}_s(q,\omega) = \sum_{\mathbf{p}\gamma\gamma'} \langle f_{\mathbf{p+q},\gamma'} | f_{\mathbf{p}\gamma} \rangle \langle f_{\mathbf{p}\gamma} | \mathbf{s} | f_{\mathbf{p+q},\gamma'} \rangle \frac{n_{\mathbf{p}\gamma} - n_{\mathbf{p+q},\gamma'}}{\omega + \xi_{\mathbf{p}\gamma} - \xi_{\mathbf{p+q},\gamma'} + i\delta} \quad (8)$$

The average values of $\rho^{+}_{\mathbf{q}}$ and $\mathbf{s}^{+}_{\mathbf{q}}$ in the $n_{\boldsymbol{q}}$-plasmon state $|n_{\mathbf{q}}\rangle = [(Q^{+}_{\mathbf{q}})^{n_{\mathbf{q}}}/\sqrt{n_{\mathbf{q}}!}]|0\rangle$ vanish, therefore we consider their mean squares in $|n_{\boldsymbol{q}}\rangle$ after subtracting their background values in $|0\rangle$, i.e.,

$$\langle \rho_{\mathbf{q}}\rho^{+}_{\mathbf{q}} \rangle \equiv \langle n_{\mathbf{q}} | \rho_{\mathbf{q}}\rho^{+}_{\mathbf{q}} | n_{\mathbf{q}} \rangle - \langle 0 | \rho_{\mathbf{q}}\rho^{+}_{\mathbf{q}} | 0 \rangle = n_{\mathbf{q}} S^2 |N^{*}_{\mathbf{q}}\Pi(q,\Omega_q)|^2, \quad (9)$$

$$\langle s^{\perp}_{\mathbf{q}}(s^{\perp}_{\mathbf{q}})^{+} \rangle \equiv \langle n_{\mathbf{q}} | s^{\perp}_{\mathbf{q}}(s^{\perp}_{\mathbf{q}})^{+} | n_{\mathbf{q}} \rangle - \langle 0 | s^{\perp}_{\mathbf{q}}(s^{\perp}_{\mathbf{q}})^{+} | 0 \rangle = n_{\mathbf{q}} S^2 |N^{*}_{\mathbf{q}}\Pi^{\perp}_s(q,\Omega_q)|^2 \quad (10)$$

(only the in-plane transverse component $s^{\perp}$ of the spin $\boldsymbol{s}$ is nonzero in these averages). The normalized amplitudes $A_\rho(q) = [\langle \rho_{\mathbf{q}}\rho^{+}_{\mathbf{q}} \rangle / n_{\mathbf{q}} S\rho]^{1/2}$ and $A_s(q) = [\langle s^{\perp}_{\mathbf{q}}(s^{\perp}_{\mathbf{q}})^{+} \rangle / n_{\mathbf{q}} S\rho]^{1/2}$ of charge- and spin-density waves are plotted in Fig. 2(b) ($\rho = p_F^2/4\pi$ is the average electron density). The "continuity equation" for density and transverse spin, following from the spin-momentum locking [5], requires that $\Omega_q A_\rho(q) = 2v_F q A_s(q)$, in agreement with our results.

4. Conclusions

We have considered microscopically spin-plasmons in helical liquid in the random phase approximation. The developed quantum-mechanical formalism can be applied for a number of problems in spin-plasmon optics.

We calculated the average spin polarization, acquired by the helical liquid in a spin-plasmon state, as well as mean-square amplitudes of charge- and spin-density waves, arising in this state. Coupling between these amplitudes, caused by spin-momentum locking, was demonstrated. The interconnection between charge- and spin-density waves can be applied for constructing various spin-plasmonic and spintronic devices.

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