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HORIZONTAL DIFFERENTIATION AND ECONOMIC GROWTH UNDER NON–HOMOTHETICITY

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Abstract

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KEY – WORDS: Endogenous growth; Horizontal differentiation; Non–homothetic production functions; Variable elasticity of substitution; Price–increasing and Price–decreasing competition

JEL CODES: C62; D43; L16; O33; O41

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1. Introduction

The model of endogenous growth with an expanding variety of products does now represent an important benchmark within the theory of economic growth and development (see Gancia and Zilibotti, 2005). The distinctive feature of this paradigm is to regard innovation as an increase in the set of available products (either consumption goods, or intermediate inputs), which raises consumers' utility or manufacturing productivity.

In Romer (1990) innovation consists in the creation of a new idea for a new variety of intermediate inputs that are employed to produce a single final output. The larger the number of varieties of these inputs, the larger the productivity of all the other inputs that enter the aggregate production function and that are used in conjunction with intermediates, and the higher the growth rate of GDP. In Romer (1990), as well as in most of its successive extensions, the aggregate production function for final output is postulated to be homogeneous, with constant returns to scale, and of the CES/CRRA type.

The first objective of the present paper is to relax the overly restrictive assumption of CES/CRRA aggregate production function.[1] Thus, we study the production behavior of a firm operating in the final output sector by means of a broader class of symmetric, additively separable technologies, not necessarily homothetic. This class of production functions is similar to the class of utility functions already used to different extents in the theory of monopolistic competition since the path-breaking papers by Dixit and Stiglitz (1977) and Krugman (1979), and ultimately allows us to uncover the differences between *price decreasing* and price *increasing competition* (Zhelobodko *et al.*, 2012) when studying the impact of markups on economic growth. Such differences cannot be studied under the more traditional approach based on the use of a CES/CRRA aggregate production function.

The second aim of this paper is to extend Romer (1990) also along another dimension: we allow the final output sector to be a partially-competitive sector, in the following sense. A firm that operates in this sector must devote a certain amount of expenditure to the purchase of intermediate inputs. If, similarly to the Solow (1956) model, this definite level of expenditure is a fixed share of the firm's output, *i.e.* a sort of *"investment"*, then it may be the case that while the other inputs that are used together with intermediates in the final output sector are rewarded according to their own productivity in that sector, this is not true for the capital goods.

[1] As well known, the main implication of the assumption of CES/CRRA aggregate production function is the presence of constant markups. Hence, this hypothesis cannot account for the variability of markups both across countries (Alessandria and Kaboski, 2011; Fieler, 2011) and over the business cycle (Nekarda and Ramey, 2013).

The main results of our paper can be summarized as follows. The presence of a non-homothetic production function in the sector that produces final goods implies that the standard aggregate equilibrium equation for a closed competitive economy, which usually follows automatically from the Euler theorem, might not be checked. We find conditions for the existence of a perfect competition equilibrium in the final output sector. These existence conditions prove to be a generalization of the commonly-used ones. If they exist, equilibria based on perfect competition in the final output sector may be described by a curve (*Relation between Inputs Curve* in the paper), whose form and main properties are explored in detail.

In the second part of the article we focus on the case in which the final output sector is partially-competitive, in the sense specified above. We find that the symmetric balanced growth path (BGP, henceforth) equilibria may be represented by a family of *iso-growth lines*, that is rays departing from the origin of the axes (*E*, *N*) and located in the first quadrant – see Fig. 1 below – where *N* denotes the number of varieties of horizontally differentiated intermediate inputs and *E* represents the level of expenditure of a final output sector firm for the purchase of such varieties of capital goods.

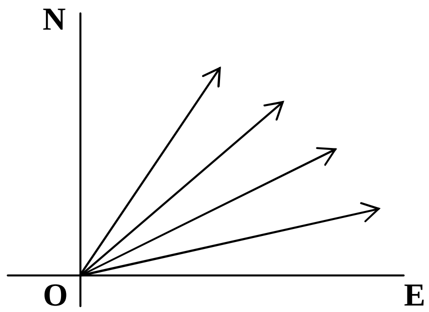

Fig. 1: ISO-GROWTH LINES

Each of the iso-growth lines is characterized, besides its own slope (E/N), by unique values of: the economic growth rate along the corresponding BGP; the output of a generic intermediate firm *j*; the price of a unit of the intermediate input *j*; the operative profit of the firm producing the intermediate input *j*; the interest rate. In this way we can rank different BGPs by means of these rays. In the *'benchmark'* case with perfect competition in the final output sector (Romer, 1990) there would be only one of such rays, as along the BGP the output of a generic intermediate input and its unit price are constants whose equilibrium values are related solely to the model's parameters. The iso-growth lines are correlated among themselves: the output of a generic intermediate firm *j* (x_j) increases with E/N, independently of the type of the final sector production function, while the direction of the changes in the unit price of an intermediate input (p_j), the instantaneous profit of an intermediate firm j (π_j), the interest rate (r), and the BGP growth rate (γ) depend on the properties of the production function in use in the final output sector. This analysis gives us the possibility of studying the long-run relationship between markups and economic growth in two different cases: following Zhelobodko *et al.* (2012) we label these two cases as *price-increasing* and *price-decreasing competition*, respectively. They cannot be examined under the more traditional CES/CRRA production function hypothesis, since under this assumption the markup is constant and independent of the scale of an intermediate firm.

The paper is structured as follows. In Section 2 we extend the Romer (1990)'s model by introducing a non-homothetic, variable elasticity of substitution aggregate production function in the sector that produces final output/consumption goods. Concerning the structure of this sector, we make two alternative assumptions. We start by postulating (as in Romer, 1990) that the final goods sector is perfectly competitive, then we turn our attention to the case of partial competition in the same sector. In Section 3 we study the implications of this specific extension – partial competition in the final output sector – of the Romer (1990)'s model for the long-run correlation between markups and economic growth. Section 4 concludes.

2. Horizontal innovation and economic growth under *non-homothetic* technology and variable elasticity of substitution

In this section we extend the Romer (1990)'s model of endogenous technological change. Our extension allows first of all for a non-homothetic aggregate production function in the sector that produces the homogeneous final output (consumption goods), and hence for a variable elasticity of substitution across the different varieties of intermediates entering such production function as inputs. To do so, we employ a simple (textbook) version of the Romer (1990)'s model.[2]

Each agent has an instantaneous utility (u) which depends solely on her consumption (c) according to the following isoelastic function:[3]

$$u(c) = \frac{c^{1-\varepsilon}}{1-\varepsilon},$$

[2] See Aghion and Howitt (2009, Chap. 3, pp. 74-76).

[3] In order to save notation, we suppress the time-index t wherever this does not cause confusion.

where $\varepsilon > 0$ is the inverse of the intertemporal elasticity of substitution. She discounts her utility over time at a constant rate, $\rho > 0$. This implies that in a BGP equilibrium (in which variables grow at constant exponential rates) the growth rate of per-capita consumption ($\gamma_c \equiv \gamma$) obeys the usual *Euler equation*:

$$\gamma_c \equiv \gamma = \frac{r - \rho}{\varepsilon}, \tag{2.1}$$

with r being the real rate of interest, an endogenous variable.

The labor-force, L, coincides with population size (hence, per-capita and per-worker variables do coincide) and is constant. If each individual in the population has the same, fixed endowment of human capital (h) which she offers inelastically and cannot increase over time through, for instance, formal education, then the aggregate supply of human capital (H)

$$H \equiv hL$$

is also constant. The total amount of human capital can be allocated either to production (H_1) or to R&D (H_2) activities. Thus,

$$H = H_1 + H_2 .$$

In the BGP equilibrium, H_1 and H_2 are two endogenously determined constants. Final output can be only consumed and is obtained through the following aggregate production function

$$Y = g(H_1) \int_0^N f(x_j) dj , \tag{2.2}$$

where Y is total output (GDP), $j \in [0, N]$ represents varieties of already invented intermediate inputs, and x_j is the quantity of the *j*-th intermediate input employed in the production of final goods. In the *'benchmark'* case (Aghion and Howitt, 2009, Eq. 3.8, p. 74), functions $g(H_1)$ and $f(x_j)$ are, respectively, specified as:

$$g(H_1) = H_1^{1-\alpha} \tag{2.3}$$

$$f(x_j) = x_j^{\alpha} , \tag{2.4}$$

where $0 < \alpha < 1$. We generalize Aghion and Howitt (2009) by assuming that function $f(\cdot)$ is defined on some interval in R_+, is continuous, increasing, and strictly concave, *i.e.*:

$$f^{'}(\cdot) > 0 , \qquad \text{and} \qquad f^{''}(\cdot) < 0 . \tag{2.5}$$

We use similar assumptions for function $g(\cdot)$, too. Let $e_f(x_j)$ denote the elasticity of $f(x_j)$ with respect to x_j

$$e_f(x_j) \equiv \frac{f'(x_j)x_j}{f(x_j)},$$

and $e_g(H_1)$ denote the elasticity of $g(H_1)$ with respect to H_1

$$e_g(H_1) \equiv \frac{g'(H_1)H_1}{g(H_1)}.$$

We can now state the following:

LEMMA 1. *When assumption (2.5) is satisfied, and under the additional hypothesis that* $f(0) \geq 0$, *function* $f(\cdot)$ *is inelastic, that is* $e_f(\cdot) < 1$.

Proof. Function $f(\cdot)$ is differentiable and, according to (2.5), strictly concave. So (see Takayama, 1994, p. 57):

$$f'(x_0)(x-x_0) > f(x) - f(x_0)$$

for all $x, x_0 \in (0, +\infty)$, $x \neq x_0$. If $x < x_0$, then

$$f'(x_0) < \frac{f(x)-f(x_0)}{x-x_0}.$$

The right hand side (RHS) of this inequality decreases with x and, as $x \to x_0$, converges to $f'(x_0)$. Thus, as $x \to 0$, the RHS is strictly greater than $f'(x_0)$. Therefore, $f'(x_0) < \frac{f(x_0)}{x_0}$ takes place for any $x_0 \in (0,+\infty)$. ■

It can be easily showed that the inelasticity of $f(\cdot)$ is equivalent to conclude that $f(x_j)/x_j$ decreases with x_j. In our analysis, two further features of function $f(x_j)$ will also play a fundamental role: the (module of the) elasticity of the first derivative:

$$r_f(x_j) \equiv -\frac{f''(x_j)x_j}{f'(x_j)},$$

which is a measure of the relative concavity of $f(x_j)$, and the elasticity of the second derivative:

$$r_{f'}\left(x_j\right) \equiv -\frac{f'''\left(x_j\right)x_j}{f''\left(x_j\right)}.$$

The importance of $r_f\left(x_j\right)$ and $r_{f'}\left(x_j\right)$ in the context of the present model will become clearer below. At this stage, it suffices to recall that, since the seminal contributions by Pratt (1964), Arrow (1965), and more recently Kimball (1990), similar functions are now extensively used both in risk analysis (where they are, respectively, known as *Arrow-Pratt coefficient of relative risk aversion* and *coefficient of relative prudence*), as well as in industrial organization (and, more specifically, in relation to specific extensions of the well-known monopolistic competition model by Dixit and Stiglitz, 1977[4]). To our knowledge, the present paper is the first attempt at using such tools within a dynamic, endogenous growth framework with horizontal product differentiation.

Similarly to functions $r_f\left(x_j\right)$ and $r_{f'}\left(x_j\right)$, we can also define functions

$$r_g\left(H_1\right) \equiv -\frac{g''\left(H_1\right)H_1}{g'\left(H_1\right)}$$

and

$$r_{g'}\left(H_1\right) \equiv -\frac{g'''\left(H_1\right)H_1}{g''\left(H_1\right)}.$$

From (2.5), it is immediate to see that $r_f\left(x_j\right) > 0$. We follow the *'benchmark'* case[5] in assuming that:

$$0 < r_f\left(x_j\right) < 1. \tag{2.6}$$

Finally, we also hypothesize that the elasticity $e_f\left(x_j\right)$ is bounded away from zero, *i.e.* there exists a $\omega > 0$ such that $e_f\left(x_j\right) > \omega$ for each x_j. We make similar assumptions regarding function $g\left(H_1\right)$. All these hypotheses are evidently satisfied in the *'benchmark'* (Eqs. 2.3 and 2.4).

[4] One recent example of such extensions is represented by Zhelobodko *et al.* (2012), where the so-called *relative love for variety* is a function similar to our $r_f\left(x_j\right)$.

[5] In the *'benchmark'* case: $e_f\left(x_j\right) = \alpha \in (0,1)$, $r_f\left(x_j\right) = 1-\alpha \in (0,1)$, and $r_{f'}\left(x_j\right) = 2-\alpha \in (1,2)$.

2.1. Perfect competition in the final output sector

In this paragraph we follow the canonical assumption that the market for the final output (the numeraire good in the model) is perfectly competitive. In this sector a representative firm maximizes its instantaneous profit function

$$Y - wH_1 - \int_{j=0}^{N} \left(p_j x_j \right) dj \to Max ,$$

taking the common wage rate (w) and the prices of the intermediate inputs – $p_j ,\ j \in [0, N]$ – as given . The FOCs related to the problem are:

$$w = \frac{\partial Y}{\partial H_1} = g^{'}\left(H_1\right) \int_{j=0}^{N} f\left(x_j\right) dj , \tag{2.7}$$

$$p_j = \frac{\partial Y}{\partial x_j} = g\left(H_1\right) f^{'}\left(x_j\right) . \tag{2.8}$$

Eq. (2.8) implies that the price-elasticity of demand (e_d) faced by each producer of intermediates is, in absolute value, equal to:

$$e_d = \frac{1}{r_f\left(x_j\right)} .^{6} \tag{2.9}$$

Notice that now, unlike the *'benchmark'* case, this elasticity crucially depends on x_j. Perfect competition in the final output sector implies that total revenues equal total costs (no profit is available after input remuneration):

$$Y = wH_1 + \int_{j=0}^{N} \left(p_j x_j \right) dj . \tag{2.10}$$

By using (2.2), (2.7) and (2.8), Eq. (2.10) turns into

$$g\left(H_1\right) \int_{j=0}^{N} f\left(x_j\right) dj = g^{'}\left(H_1\right) H_1 \int_{j=0}^{N} f\left(x_j\right) dj + g\left(H_1\right) \int_{j=0}^{N} f^{'}\left(x_j\right) x_j dj .$$

Hence,

[6] Under condition (2.6), the inequality $e_d > 1$ takes place. This corresponds to the familiar property that any (local) intermediate monopolist always produces along the elastic branch of her (inverse) demand curve.

$$e_g\left(H_1\right)+\frac{\int_{j=0}^{N} f'\left(x_j\right)x_j dj}{\int_{j=0}^{N} f\left(x_j\right)dj}=1. \tag{2.11}$$

In the symmetric case ($x_j = x$, $\forall j$) condition (2.11) can be recast as

$$e_g\left(H_1\right)+e_f\left(x_j\right)=1. \tag{2.12}$$

If the aggregate production function in the final output sector were homogeneous of degree one (constant returns to scale, CRS, in the two rival inputs H_1 and x_j), then Eqs. (2.10)-(2.12) would follow automatically from the Euler theorem.[7] In the non-CRS case, instead, if it has a solution Eq. (2.12) defines a curve in the plane $\left(x_j, H_1\right)$ that describes some relation between inputs (the amount of a generic intermediate input, x_j, on the one hand, and labor, H_1, on the other). Henceforth, for the sake of simplicity and abbreviation, in the remainder of the paper we shall label curve (2.12) as the *Relation-between-Inputs* (RbI) *curve*. In a moment we shall analyze more deeply the properties of the RbI curve (2.12) in a point.

Concerning the RbI curve (2.12), two things are worth emphasizing at this stage. The first is that the form of the RbI curve depends on the functions $g\left(H_1\right)$ and $f\left(x_j\right)$, and more precisely on the elasticities, $e_g\left(H_1\right)$ and $e_f\left(x_j\right)$, of these functions and the elasticities of their derivatives, $r_g\left(H_1\right)$ and $r_f\left(x_j\right)$. Secondly, when removing the traditional assumption of homogeneity of degree one (CRS) of the aggregate production function, but continuing to assume perfect competition in the final output sector, we have to make sure that the representative final-output-sector firm employs bundles of inputs (labor and intermediates) that explicitly satisfy Eq. (2.11) or, in the symmetric case, Eq. (2.12). What we are going to show now is that for the set of these feasible combinations of productive inputs to be nonempty, and therefore for the RbI curve to exist, the functions $g\left(H_1\right)$ and $f\left(x_j\right)$ entering the separable production function (2.2) have to be in some specific relation to each other.

To show this, let (α, β) be the range of values of $e_g\left(H_1\right)$, and (γ, δ) be the range of values of $e_f\left(x_j\right)$. Eq. (2.10) is defined for any $H_1 > 0$ if $(\alpha, \beta) \subset (1-\delta, 1-\gamma)$ and for any

[7] In particular, in the *'benchmark'* case: $e_g\left(H_1\right)=1-\alpha$ and $e_f\left(x_j\right)=\alpha$.

$x_j > 0$ if $(\gamma, \delta) \subset (1-\beta, 1-\alpha)$. Hence, for the RbI curve (2.12) to be defined for all $H_1 > 0$ and $x_j > 0$, the following system of inequalities has to be satisfied:

$$1-\delta \le \alpha, \qquad \beta \le 1-\gamma, \qquad 1-\beta \le \gamma, \qquad \delta \le 1-\alpha,$$

which implies

$$\alpha + \delta = \beta + \gamma = 1.$$

All in all, if (α, β) and (γ, δ) are, respectively, the ranges of values of the elasticities $e_g(H_1)$ and $e_f(x_j)$, then (2.10) is defined for $H_1 \in g^{-1}\left[\left(\alpha', \beta'\right)\right]$ and for $x_j \in f^{-1}\left[\left(\gamma', \delta'\right)\right]$ where

$$\alpha' = \text{Max}\{\alpha, 1-\delta\}, \qquad \beta' = \text{Min}\{\beta, 1-\gamma\}$$

$$\gamma' = \text{Max}\{\gamma, 1-\beta\}, \qquad \delta' = \text{Min}\{\delta, 1-\alpha\},$$

and where $g^{-1}[\cdot]$, $f^{-1}[\cdot]$ are inverse images of intervals. The reader can easily check that:

$$\alpha' + \delta' = \beta' + \gamma' = 1.$$

Evidently, the last condition generalizes the *'benchmark'* case. We now provide two examples of the theory exposed so far. These two examples are designed in such a way to represent possible deviations from the *'benchmark'* case.

EXAMPLE 1. Let the aggregate production function be

$$Y = AH_1^{\beta} \int_0^N f\left(x_j\right) dj, \tag{2.13}$$

with $0 < \beta < 1$ and $A > 0$. Evidently, $e_g(H_1) = \beta$ for all $H_1 > 0$. Let the elasticity $e_f(x_j)$ have the range of values (γ, δ). If $\beta \in (1-\delta,\ 1-\gamma)$ then the RbI curve (2.12) is defined for all $H_1 > 0$. Moreover, if $e_f(x_j)$ increases, then equation

$$e_f\left(x_j\right) = 1-\beta, \tag{2.14}$$

has a unique solution. This means that the RbI curve (2.12) takes the form of a straight line, $x_j = \text{constant}$. But, if $\beta < 1-\delta$ or $\beta > 1-\gamma$, then the set of all the feasible combinations of productive inputs that satisfy (2.12) is empty, which means that no equilibrium with perfect competition in the final output sector may exist under the production function (2.13). In particular, a production function of the type $Y = AH_1^{\beta}\int_0^N x_j^{\alpha}dj$ is compatible with the existence of an equilibrium with perfect competition in the final output sector only when the benchmark condition

$$\alpha + \beta = 1$$

is met.

EXAMPLE 2. Let now the production function be of the type (2.13), but with

$$f\left(x_j\right) = Cx_j^{\gamma} + Dx_j^{\delta}, \qquad 0 < \gamma < \delta < 1, \qquad C, D > 0.$$

Then

$$e_f\left(x_j\right) = \frac{\gamma Cx_j^{\gamma} + \delta Dx_j^{\delta}}{Cx_j^{\gamma} + Dx_j^{\delta}},$$

and

$$e_f^{'}\left(x_j\right) \equiv \frac{de_f\left(x_j\right)}{dx_j} = \frac{\left(\gamma - \delta\right)^2 CDx_j^{\gamma+\delta-1}}{\left(Cx_j^{\gamma} + Dx_j^{\delta}\right)^2} > 0, \tag{2.15}$$

i.e. the elasticity of $f\left(x_j\right)$ with respect to x_j increases with x_j. Moreover:

$$e_f\left(x_j\right) = \frac{\gamma C + \delta Dx_j^{\delta-\gamma}}{C + Dx_j^{\delta-\gamma}} \underset{x_j \to 0}{\longrightarrow} \gamma,$$

$$e_f\left(x_j\right) = \frac{\gamma Cx_j^{\gamma-\delta} + \delta D}{Cx_j^{\gamma-\delta} + D} \underset{x_j \to +\infty}{\longrightarrow} \delta.$$

Thus, for any $x_j > 0$, the range of values of the elasticity $e_f\left(x_j\right)$ is the interval $(\gamma,\ \delta)$. If $\beta \in \left(1-\delta,\ 1-\gamma\right)$, then Eq. (2.14) gives

$$x_j = \left[\frac{\left(1-\gamma\right)-\beta}{\beta-\left(1-\delta\right)} \cdot \frac{C}{D}\right]^{\frac{1}{\delta-\gamma}}, \tag{2.16}$$

and the RbI curve (2.12) is the straight line (2.16). But if $\beta < 1-\delta$ or $\beta > 1-\gamma$ then the set (2.12) is empty, implying that no equilibrium with perfect competition in the final output sector may occur under a production function of the type (2.13), with $f\left(x_j\right) = Cx_j^{\gamma} + Dx_j^{\delta}$.

On the whole, if the set of all the feasible combinations of productive inputs that satisfy (2.12) is nonempty, implying that the RbI curve (2.12) does exist, then the shape of this curve is determined by the properties of the functions $f(x_j)$ and $g(H_1)$, as the following lemmas and corollaries are going to clarify in more detail.

LEMMA 2. *Assume that the RbI curve (2.12) does exist. Then, it has a positive slope, i.e.* $\frac{dH_1}{dx_j} > 0$, *iff one of the two elasticities,* $e_g(H_1)$ *and* $e_f(x_j)$, *increases and the other decreases with respect to its own arguments. It has a negative slope, i.e.* $\frac{dH_1}{dx_j} < 0$, *iff both elasticities increase or both decrease with respect to their own arguments.*

Proof. Consider the left hand side (LHS) of Eq. (2.12) as a function of two variables (H_1 and x_j) and apply the implicit function theorem. It is possible to obtain:

$$\frac{dH_1}{dx_j} = -\frac{e_f^{'}(x_j)}{e_g^{'}(H_1)}, \qquad\qquad e_g^{'}(H_1) \equiv \frac{de_g(H_1)}{dH_1}. \tag{2.17}$$

Hence, inequalities $\frac{dH_1}{dx_j} > (<) 0$ are equivalent to inequalities

$$e_f^{'}(x_j)\, e_g^{'}(H_1) < (>) 0, \tag{2.18}$$

from which the Lemma follows immediately. ■

Lemma 2 is important because it suggests that the RbI curve can be either positively or negatively sloped. In the first case, the relation between H_1 and x_j is positive, implying that the inputs are complementary for each other in the production of final output; in the second case, instead, the relation between H_1 and x_j is negative, implying that now the same two inputs are substitutes for each other.

LEMMA 3. *The condition for the elasticity* $e_f(x_j)$ *to increase (decrease), i.e. for the inequality* $e_f^{'}(x_j) > (<) 0$ *to be checked, is:*

$$1 - r_f(x_j) - e_f(x_j) > (<) 0. \tag{2.19}$$

Proof. The derivative of the elasticity $e_f(x_j)$ with respect to x_j is:

$$e_f^{'}(x_j) = \frac{f^{''}(x_j)x_j f(x_j) + f^{'}(x_j) f(x_j) - \left[f^{'}(x_j)\right]^2 x_j}{\left[f(x_j)\right]^2} = \left[-r_f(x_j) + 1 - e_f(x_j)\right]\frac{f^{'}(x_j)}{f(x_j)}, \tag{2.20}$$

which leads to (2.19). ■

Conditions of the type of (2.19) have appeared only recently in economic literature. As an example, Mrazova and Neary (2013) in analyzing firms' behavior introduce the so-called *superconvex function* which is defined in a way similar to $1-r_f(\cdot)-e_f(\cdot)\le 0$. Levine (2012) defines the so called *moderately concave function* that satisfies an inequality similar to $1-r_f(\cdot)-e_f(\cdot)\ge 0$.

LEMMA 4. *The condition for* $r_f(x_j)$ *to increase (decrease), i.e. for the inequality* $r_f^{'}(x_j)>(<)0$ *to be checked, is:*

$$1-r_{f'}(x_j)+r_f(x_j)>(<)0. \tag{2.21}$$

Proof. The derivative of $r_f(x_j)$ with respect to x_j is

$$r_f^{'}(x_j)=-\frac{f^{''}(x_j)}{f^{'}(x_j)}-\frac{x_j f^{'''}(x_j)}{f^{'}(x_j)}+x_j\left[\frac{f^{''}(x_j)}{f^{'}(x_j)}\right]^2=\left[1-r_{f^{'}}(x_j)+r_f(x_j)\right]\left[-\frac{f^{''}(x_j)}{f^{'}(x_j)}\right], \tag{2.22}$$

which leads to (2.21). ■

COROLLARY 1. *If (2.6) is checked and* $r_f(x_j)$ *increases with* x_j*, then:*

$$r_{f^{'}}(x_j)<2. \tag{2.23}$$

Moreover, if both $r_f(x_j)$ *and* $e_f(x_j)$ *increase with* x_j*, then:*

$$r_{f^{'}}(x_j)+e_f(x_j)<2. \tag{2.24}$$

Proof. Eqs. (2.21) and (2.6) imply (2.23). To obtain (2.24), we can use the proper version of inequalities (2.19) and (2.21). ■

COROLLARY 2. *Assume that the RbI curve (2.12) does exist. Then, it has a positive (negative) slope, i.e.* $\frac{dH_1}{dx_j}>0\ (<)0$*, iff, respectively:*

$$\left[e_f(x_j)-r_g(H_1)\right]\left[e_g(H_1)-r_f(x_j)\right]<(>)0. \tag{2.25}$$

Proof. Follows from Lemma 2, Lemma 3 and Eq. (2.12). ■

While Corollary 2 provides alternative (see Lemma 2) conditions for the presence of complementarity/substitutability between inputs, the following Lemma 5 provides necessary and sufficient conditions for the RbI curve (2.12) to have a definite behavior in a point.

LEMMA 5. *Assume that the RbI curve (2.12) does exist. Then, it is convex downwards (convex upwards) iff the following pair of corresponding inequalities is checked:*

$$\left[1-r_f\left(x_j\right)-e_f\left(x_j\right)\right]\left[1+2\frac{e_f\left(x_j\right)}{r_f\left(x_j\right)}\right]+\left[1-r_{f'}\left(x_j\right)+r_f\left(x_j\right)\right]\geq(\leq)0,$$

$$\left[1-r_g\left(H_1\right)-e_g\left(H_1\right)\right]\left[1+2\frac{e_g\left(H_1\right)}{r_g\left(H_1\right)}\right]+\left[1-r_{g'}\left(H_1\right)+r_g\left(H_1\right)\right]\geq(\leq)0.$$

Proof. For the function $e_g\left(H_1\right)+e_f\left(x_j\right)$, *i.e.* the LHS of Eq. (2.12), the Hessian matrix is:

$$\mathbf{H}=\begin{pmatrix} e_f^{''} & 0 \\ 0 & e_g^{''} \end{pmatrix}.$$

Hence, the necessary and sufficient condition for the RbI curve (2.12) to be convex downwards (convex upwards) is $e_f^{''}\leq 0$, $e_g^{''}\leq 0$ ($e_f^{''}\geq 0$, $e_g^{''}\geq 0$).[8]

By use of (2.20) and (2.22) one obtains:

$$e_f^{''}(\cdot)=\frac{\left[1-r_f(\cdot)-e_f(\cdot)\right]\left[f^{''}(\cdot)f(\cdot)-2f^{'}(\cdot)^2\right]+\left[1-r_{f'}(\cdot)+r_f(\cdot)\right]f^{''}(\cdot)f(\cdot)}{f(\cdot)^2},$$

which has the same sign as

$$-\left\{\left[1-r_f(\cdot)-e_f(\cdot)\right]\left[1+2\frac{e_f(\cdot)}{r_f(\cdot)}\right]+\left[1-r_{f'}(\cdot)+r_f(\cdot)\right]\right\}.$$

Similarly, $e_g^{''}(\cdot)$ has the same sign as

$$-\left\{\left[1-r_g(\cdot)-e_g(\cdot)\right]\left[1+2\frac{e_g(\cdot)}{r_g(\cdot)}\right]+\left[1-r_{g'}(\cdot)+r_g(\cdot)\right]\right\}.$$

This leads to the statement of the Lemma. ■

Lemma 5 and Corollary 2 identify necessary and sufficient conditions under which the RbI curve is:

[8] Similar pairs of strict inequalities provide necessary and sufficient conditions for strict convexity downwards (strict convexity upwards).

- Negatively sloped and convex downwards;
- Negatively sloped and convex upwards;
- Positively sloped and convex downwards;
- Positively sloped and convex upwards.

Such identification is important because it conveys precise information not only about the relationship of complementarity/substitutability between inputs (that is, on whether the RbI curve is negatively or positively sloped), but also about the ratio at which such relationship of complementarity/substitutability takes place in a point (that is, on whether in that point the RbI curve is concave or convex). To be more concrete, it suggests not only whether in a given point a certain percentage change in the amount employed of one of the two inputs is associated to a percentage change in the same or opposite direction in the amount employed of the second input, but also in which amount (*i.e.*, whether less or more than proportionally) the change in the quantity employed of the second input takes place following the initial percentage change in the quantity employed of the first input.

COROLLARY 3.

1. If in a point $\left(x_j, H_1\right)$ the following inequalities are checked:

$$r_f'\left(x_j\right)>0, \qquad e_f'\left(x_j\right)>0, \qquad r_g'\left(H_1\right)>0, \qquad e_g'\left(H_1\right)>0$$

then the RbI curve (2.12) has a negative slope and is convex downwards in the point.

2. If in a point $\left(x_j, H_1\right)$ the following inequalities are checked:

$$r_f'\left(x_j\right)<0, \qquad e_f'\left(x_j\right)<0, \qquad r_g'\left(H_1\right)<0, \qquad e_g'\left(H_1\right)<0,$$

then the RbI curve (2.12) has a negative slope and is convex upwards in the point.

Proof. It follows from Lemma 5, Lemma 3 and Lemma 4. ■

EXAMPLE 3. Let:

$$g\left(H_1\right)=AH_1^{\alpha}+BH_1^{\beta}, \qquad 0<\alpha<\beta<1, \qquad A, B>0$$

$$f\left(x_j\right)=Cx_j^{\gamma}+Dx_j^{\delta}, \qquad 0<\gamma<\delta<1, \qquad C, D>0.$$

We have already said that Eq. (2.10) is defined for any $H_1\in(0,+\infty)$ and $x_j\in(0,+\infty)$ if

$$\alpha+\delta=\beta+\gamma=1.$$

According to (2.15):

$$e_f^{'}\left(x_j\right)>0\,, \qquad\qquad e_g^{'}\left(H_1\right)>0\,.$$

Hence, from *Lemma 2*, the RbI curve (2.12) has a negative slope, meaning that H_1 and x_j are substitutes for each other. Moreover, we can compute:

$$r_f\left(x_j\right)=\frac{C\gamma\left(1-\gamma\right)x_j^{\gamma-1}+D\delta\left(1-\delta\right)x_j^{\delta-1}}{C\gamma x_j^{\gamma-1}+D\delta x_j^{\delta-1}}\,,$$

$$r_f^{'}\left(x_j\right)=\frac{CD\gamma\delta x_j^{\gamma+\delta-3}\left(\gamma-\delta\right)^2}{\left(C\gamma x_j^{\gamma-1}+D\delta x_j^{\delta-1}\right)^2}\,.$$

Because $r_f^{'}\left(\cdot\right)>0$ and, similarly, $r_g^{'}\left(\cdot\right)>0$, Corollary 3 implies that function $H_1(x_j)$ corresponding to the RbI curve is convex, which means that a 1% decrease in the amount employed of one of the two inputs leads to a more than 1% increase in the amount employed of the other input. In an explicit form, function $H_1(x_j)$ can be derived as:

$$H_1=\left(\frac{1-\alpha-\gamma}{\beta+\delta-1}\cdot\frac{AC}{BD}\right)x_j^{-\left(\frac{\delta-\gamma}{\beta-\alpha}\right)}.$$

2.2. Partial competition in the final output sector

The main point of the last paragraph was that if we assume perfect competition in the final output sector but abandon the traditional assumption that this sector employs a constant returns to scale aggregate production function, then the Euler theorem ceases to be checked automatically. In this case, it would still be possible to meet the Euler theorem's equation only if a RbI curve existed, so that only bundles of inputs explicitly satisfying, in the symmetric equilibrium, Eq. (2.12) can be employed by a representative final-output-sector firm. In turn, we also showed that for the RbI curve (and, therefore, for an equilibrium with perfect competition in the final output sector) to exist, the functions $g\left(H_1\right)$ and $f\left(x_j\right)$ entering the separable technology (2.2) have to be in some specific relation to each other.

In what follows we analyze the predictions of the most general possible equilibrium model in which the Euler theorem may or may not take place. In the specific case we are going to analyze this occurs because of the presence of partial competition in the final output sector. The hypothesis that the final output sector might be partially competitive represents an important departure from the traditional horizontally differentiation-based endogenous growth theory which generally assumes perfect competition in both the labor-input and final output markets.

In particular, we interpret the presence of partial competition in the final output sector in the following sense. Consider a generic firm *i* operating in this sector. Such firm purchases labor ($H_{1,i}$) from a competitive labor-market at a given price (equal to the marginal productivity of this factor input), and makes some specified expenditures, E_i (equal, for example, to a fixed share of the final output it produces), to purchase intermediate inputs, x_j, $j \in [0; N]$. So, while labor is rewarded according to its marginal product in the final output sector, this is not necessarily true for the intermediate goods. It is possible to show that the following first order conditions hold in the symmetric case (see *Notes to the Referees* – not intended for publication):

$$\frac{E}{N} = x$$

$$e_g(H_1) + \frac{1}{\lambda} e_f(x) = 1$$

$$\frac{1}{\lambda} g(H_1) f'(x) = p$$

$$g'(H_1) N f(x) = w,$$

where λ is the Lagrange multiplier in the problem of a generic final output producer *i*; p is the price that firm *i* pays for one unit of any intermediate *j*; and w is the labor wage rate.

EXAMPLE 4. Let us consider the case in which $N = 1$ and $Y = H_1^a x^b$, where $0 < a,\ b < 1$ are parameters. Thus, there exists only one producer of intermediates. If $a + b \neq 1$, then no perfect competition equilibrium can exist in the final output sector because Eq. (2.12) is not met. Assume that labor is paid at its marginal product and let the quantity of labor employed be chosen by a generic final output firm, whose output is Y.[9] Then, labor receives a fraction $aH_1^a x^b = aY$ of output, while the producer of the intermediate input receives $E = (1-a) H_1^a x^b = (1-a) Y$, where $(1-a) \neq b$. If the price of a generic intermediate input is *p*, then $E = px$. Hence, the intermediate producer faces the following direct demand function:

$$x = \left[(1-a) H_1^a p^{-1} \right]^{\frac{1}{1-b}} \tag{2.26}$$

or, alternatively, the following inverse demand function:

[9] We do not touch here the issue of unemployment. For instance, it may be the case that the whole labor supply is engaged by identical firms, whose number varies to ensure full employment of labor. Alternatively, one may think of an economy in which the rest of the labor-supply (in excess with respect to firms' labor demand) is self-employed in a sector which is outside the model.

$$p = (1-a) H_1^a x^{b-1}.$$

If the intermediate monopolist produces with a one-to-one technology (one unit of the unique intermediate good is obtained from one unit of final output), then her problem is:

$$(1-a) H_1^a x^b - x \to \text{Max},$$

and the FOC is

$$b(1-a) H_1^a x^{b-1} = 1. \tag{2.27}$$

The equilibrium quantity of the intermediate input being produced is

$$x = \left[\frac{1}{b(1-a) H_1^a} \right]^{\frac{1}{b-1}}.$$

Comparison of the last equation with (2.26) shows that $p = \frac{1}{b}$, where $\frac{1}{b}$ is the markup over the marginal cost of production. The total expenditure made on the purchase of the intermediate input will be: $E = px = \frac{x}{b}$. As a consequence, in equilibrium $x = Eb$. From (2.27):

$$(1-a) b^b H_1^a E^{b-1} = 1,$$

which ultimately leads to

$$H_1 = (1-a)^{-\frac{1}{a}} b^{-\frac{b}{a}} E^{\frac{1-b}{a}}.$$

The larger the expenditure devoted to the purchase of the intermediate good, E, and the larger the amount of labor being hired, H_1.

2.3. The intermediate-inputs sector

We now focus on the structure of the intermediate sector. This sector is monopolistically competitive, and each intermediate input j is produced one to one with forgone consumption (final output). Under symmetry ($x_j = x$, $\forall j$), the flow of instantaneous profits of a generic local intermediate monopolist, measured in units of final good, is:

$$\Pi = \underset{x}{\text{Max}} \left(px - x \right) = \underset{x}{\text{Max}} \left[g(H_1) f'(x) x - x \right]. \tag{2.28}$$

The existence of a solution in (2.28) is guaranteed by the condition of strict concavity of the profit function written in square brackets, which implies:

$$2 f''(x) + f'''(x) x < 0.$$

The last inequality can be also recast as

$$r_{f'}(x) < 2.^{10} \tag{2.29}$$

The profit-maximizing quantity (x) produced by each intermediate firm is implicitly defined by

$$f'(x) + f''(x)x = \frac{1}{g(H_1)}. \tag{2.30}$$

Eqs. (2.8) and (2.30) imply that each intermediate producer sets a profit-maximizing price (p) equal to:

$$p = g(H_1) f'(x) = \frac{1}{1 - r_f(x)}. \tag{2.31}$$

Since the unit cost of production of each intermediate producer is equal to one, the optimal markup (m) on the marginal cost is

$$m = \frac{1}{1 - r_f(x)}. \tag{2.32}$$

Eq. (2.32) represents one of the main novelties of the approach proposed here. In fact, comparison with the Romer (1990)'s model reveals that, unlike that framework in which $m = 1/\alpha$ (so that the mark-up rate is completely independent of x),[11] in our model the gross mark-up of price over the marginal cost of production is strictly dependent on the amount of output that any intermediate firm decides to produce at equilibrium. Therefore, any change in x (due, for instance, to a different amount of labor, H_1, that the representative final output firm decides to employ in production, see Eq. 2.30), would lead to a change in the optimal mark-up rate. This outcome rests on the fact that in our model the elasticity of substitution between two generic varieties of intermediate inputs is variable, rather than constant.

A sufficient condition for the existence of an economically meaningful solution to (2.30) is that the following two equations

$$\lim_{x \to 0} \left[f'(x) + f''(x)x \right] = +\infty, \tag{2.33}$$

$$\lim_{x \to \infty} \left[f'(x) + f''(x)x \right] = 0. \tag{2.34}$$

are checked.[12]

[10] Concerning this inequality, see Corollary 1.

[11] See Aghion and Howitt (2009, Section 3.2.2, pp. 74-75).

[12] Notice that conditions (2.29), (2.33) and (2.34) may straightforwardly be interpreted in the light of some properties of an intermediate firm's revenue function, $R(x) = px = g(H_1) f'(x)x = \delta f'(x)x$, where δ is taken as an exogenous constant by any generic producer of intermediate inputs. Given this function, inequality (2.29) can be understood as a condition of concavity of $R(x)$, while (2.33) and (2.34) are Inada conditions applied to $R(x)$.

EXAMPLE 5. For the function

$$f(x) = Cx^{\gamma} + Dx^{\delta}, \qquad 0 < \gamma < \delta < 1, \qquad C, D > 0,$$

which was already used in Examples 2 and 3, we have

$$0 < r_f < 1 \qquad \text{and} \qquad r_f^{'} > 0.$$

Hence, by Corollary 1, inequality (2.29) is satisfied and problem (2.28) has a solution. Evidently,

$$f^{'}(x) + f^{''}(x)x = C\gamma^2 x^{\gamma-1} + D\delta^2 x^{\delta-1}.$$

This expression converges to 0 as $x \to \infty$, and diverges to infinity as $x \to 0$. Thus, conditions (2.33) and (2.34) are checked. Eqs. (2.31) and (2.32) turn into the following expression for the variable mark-up:

$$p = m = \frac{C\gamma x^{\gamma-1} + D\delta x^{\delta-1}}{C\gamma^2 x^{\gamma-1} + D\delta^2 x^{\delta-1}}.$$

Eq. (2.30) defines production (x) of each intermediate firm as a function of labor (H_1). If (2.29) is met, then the LHS of Eq. (2.30) decreases and variables H_1 and x change in the same direction: in particular, an increase in H_1 leads to an increase in x.

In the benchmark case (*i.e.*, Eqs. 2.3 and 2.4), Eq. (2.30) turns into:

$$\alpha^2 x^{\alpha-1} = H_1^{\alpha-1},$$

which provides a linear relation between H_1 and *x*:

$$x = (\alpha)^{\frac{2}{1-\alpha}} H_1.$$

In our more general case, instead, by using Eq. (2.31) we can write the equilibrium value of the instantaneous profit of a generic local monopolist as:

$$\Pi = \left[\frac{1}{1 - r_f(x)} - 1\right] x,$$

or,

$$\Pi = \frac{r_f(x)x}{1 - r_f(x)}. \tag{2.35}$$

2.4. The Research and Development (R&D) sector

There is an extremely large number of infinitely small firms undertaking R&D activity, hence this sector is perfectly competitive. These firms produce ideas indexed by 0 through an upper bound $N_t \geq 0$, which measures the total stock of knowledge available to a society at any given date $t \geq 0$. New ideas allow for the production of new varieties of intermediate goods that, in turn, can be used as inputs in the production of final output. Ideas are patented and partially excludable, but *non-rival* and indispensable for manufacturing capital goods. With access to the available stock of knowledge, N, the representative research firm uses only labor to develop new ideas, whose dynamics is provided by the following equation:

$$\dot{N} = \eta H_2 N, \tag{2.36}$$

where η is a strictly positive productivity parameter and H_2 is the amount of labor devoted to R&D. Eq. (2.36) suggests that inventing the latest design for a new intermediate good requires a labor-input equal to $H_2 = 1/\eta N$, which decreases with N (there exists a positive intertemporal spillover coming from past R&D). The same equation (2.36) implies the presence, through the term $H_2 = H - H_1$, of a strong scale effect of population size on economic growth. Although we know that this effect is rejected on empirical grounds (Jones 2005), we continue to use (2.36) in the present paper because it is not one of the aims of our analysis to propose alternative solutions to the removal of such effects.[13] In other words, our extension of the Romer (1990)'s model of endogenous technological change has nothing to say about possibly new ways of removing strong scale effects in that context.

Eq. (2.36) implies:

$$H_2 = \frac{\gamma}{\eta},$$

where $\gamma \equiv \frac{\dot{N}}{N}$ will be shown in a moment to be the real per capita income growth rate of the economy. The instantaneous flow of profit from R&D activity is

$$\frac{\dot{N}\Pi}{r} - wH_2,$$

where *w* is the wage rate accruing to one unit of labor devoted to R&D activity and *r* is the real interest rate. Hence, free entry in the R&D sector leads to

$$r = \frac{\dot{N}\Pi}{wH_2},$$

or, equivalently, by use of (2.36), to

[13] Li (2000) and Peretto and Smulders (2002) were among the first to propose theoretical solutions to the removal of scale effects in R&D-based growth models. See also Laincz and Peretto (2006) for a survey.

$$r = \frac{\Pi \eta N}{w}. \tag{2.37}$$

Since labor is perfectly mobile across sectors, at equilibrium the wage rates accruing to one unit of labor employed, respectively, in the production of final output and in the discovery of new ideas need to be the same, and equal to *w*. In turn, this common wage rate is equal to the marginal productivity of labor in the final output sector. In the symmetric case, *w* will be equal to (see Eq. 2.7):

$$w = \frac{\partial Y}{\partial H_1} = g^{'}\left(H_1\right) N f\left(x\right). \tag{2.38}$$

By using Eqs. (2.35), (2.37) and (2.38), we can re-write the real interest rate as:

$$r = \frac{\eta r_f\left(x\right) x}{g^{'}\left(H_1\right)\left[1 - r_f\left(x\right)\right] f\left(x\right)}. \tag{2.39}$$

Eq. (2.39), the Euler equation (2.1), and the labor market clearing condition $H_1 = H - H_2 = H - \frac{\gamma}{\eta}$ imply:

$$\left(\varepsilon\gamma + \rho\right) g^{'}\left(H - \frac{\gamma}{\eta}\right) = \frac{x r_f\left(x\right)\eta}{\left[1 - r_f\left(x\right)\right] f\left(x\right)}, \tag{2.40}$$

where ε is the parameter of the instantaneous utility function and ρ is the time-discount rate.

In the benchmark case (Eqs. 2.3 and 2.4), we have $r_f\left(x\right) = 1 - \alpha$. This, using (2.30) and (2.40) leads to:

$$\varepsilon\gamma + \rho = \alpha\left(\eta H - \gamma\right),$$

and therefore (see Aghion and Howitt, 2009, Chap. 3, p. 76),

$$\gamma = \frac{\alpha\eta H - \rho}{\alpha + \varepsilon}.$$

Along the BGP, it is possible to show that:

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{N}_t}{N_t} = \frac{\dot{C}_t}{C_t} = \frac{\dot{w}_t}{w_t} \equiv \gamma = \frac{\alpha\eta H - \rho}{\alpha + \varepsilon}.$$

In the model, economic growth occurs through proportional increases in the number of available ideas (that is, varieties of intermediate goods, *N*) and, hence, in the expenditure for the purchase of the existing varieties of intermediate goods, $E \equiv \int_0^N \left(p_j x_j\right) dj$. Each symmetric BGP equilibrium (in which $p_j = p$ and $x_j = x$, $\forall j$) moves along one of what we may call *iso-growth lines*, which are the rays departing from the origin of the axes (E, N) and located in the first quadrant – see Fig. 1. To different iso-growth lines (different E/N ratios), we can associate different values of x, p, and hence m and γ, which remain unchanged along the same iso-growth line and, therefore, along the corresponding symmetric BGP equilibrium. In this way we can rank different BGPs by means of these rays. In the '*benchmark*' case with perfect competition there is only one of such rays, as along the BGP x and p are constants whose equilibrium values are related solely to the model's parameters. Clearly, this is no longer true in our model, where p depends on x.

We are interested to know how (*i.e.*, in which direction) the equilibrium values of x, p, and γ do change when different equilibrium BGPs (with different E/N ratios) are compared to each other. This is what we do in the following five propositions.

PROPOSITION 1. *If (2.29) is met, then output produced by any intermediate firm increases with E/N, i.e. x moves clockwise in Fig. 1.*

Proof: Evidently, $\frac{E}{N} = px$. Since $px = \frac{x}{1-r_f(x)}$, x increases in E/N iff

$$\left(\frac{x}{1-r_f(x)}\right)' > 0,$$

i.e., iff the following inequality is checked:

$$1 - r_f(x) + x r_f'(x) > 0. \tag{2.41}$$

Taking into account that

$$x r_f'(x) = r_f(x) - r_f(x) r_{f'}(x) + r_f^2(x), \tag{2.42}$$

inequality (2.41) is equivalent to

$$1 - r_f(x) r_{f'}(x) + r_f^2(x) > 0,$$

which, in turn, is equivalent to

$$r_{f'}(x) < r_f(x) + \frac{1}{r_f(x)}. \tag{2.43}$$

Inequality (2.43) follows from (2.29). Indeed:

$$r_f(x) + \frac{1}{r_f(x)} \geq 2 > r_{f'}(x).$$

Hence, x increases with E/N. ■

PROPOSITION 2. *Inequality (2.29) is a necessary and sufficient condition for the profit (Π) of a generic intermediate firm to increase with output, x.*

Proof: By using Eq. (2.42), we have

$$\Pi' = \left(\frac{r_f x}{1-r_f}\right)' = \frac{x r_f' + r_f - r_f^2}{\left(1-r_f\right)^2} = \frac{r_f\left(2-r_{f'}\right)}{\left(1-r_f\right)^2}.$$

Hence, $\Pi' > 0$ iff (2.29) holds. ■

PROPOSITION 3. *Along the BGP equilibrium, output of a generic intermediate firm (x) and the growth rate (γ) do change in the same direction.*

Proof: From (2.40) it follows that $\frac{d\gamma}{dx} > 0$ iff the following two functions have the same sign:

$$\varepsilon g'\left(H - \frac{\gamma}{\eta}\right) - \frac{1}{\eta}(\varepsilon\gamma + \rho)\, g''\left(H - \frac{\gamma}{\eta}\right) \tag{2.44}$$

and

$$\left(r_f' x + r_f\right)\left(1-r_f\right) f - r_f x\left(f' - r_f' f - r_f f'\right). \tag{2.45}$$

If the growth rate γ is positive, then function (2.44) is also positive. At the same time, function (2.44) can also be recast as

$$r_f' x f + r_f\left(1-r_f\right)\left(f - x f'\right),$$

which is positive since, for the inelastic function f, we have $f - x f' > 0$. Hence, $\frac{d\gamma}{dx} > 0$. ■

PROPOSITION 4. *Along the BGP, price (p) increases (respectively, decreases) with respect to output (x) iff $r_f\left(x\right)$ increases (respectively, decreases).*

Proof: It is a direct consequence of Eq. (2.31). ■

In Proposition 4, the cases of increasing $r_f\left(x\right)$ and decreasing $r_f\left(x\right)$ can be referred to as the cases of *price-increasing* and *price-decreasing competition,* respectively (see Zhelobodko *et al.*, 2012).

PROPOSITION 5. *If (2.29) is met, then the relationship between markup (m) and economic growth (γ):*

- *Is positive under price-increasing competition [increasing $r_f\left(x\right)$];*
- *Is negative under price-decreasing competition [decreasing $r_f\left(x\right)$].*

Proof: Evidently, the markup rate changes in the same direction as $r_f(x)$ while $\gamma(x)$ increases. Thus, m increases (decreases) with γ in the case of price-increasing (price-decreasing) competition. ■

This is probably the most important economic implication of our paper. The next section is devoted to a more detailed discussion of our results.

3. Markups, factor-shares and long-run economic growth: A discussion of the results

In the *benchmark* model, in which there exists no ambiguity in the correlation between competition and economic growth (a decrease in the degree of product market competition is always associated to a lower economic growth rate along the BGP equilibrium), the constant parameter (α) that determines the price-elasticity of demand faced by an intermediate local monopolist, hence the level of the equilibrium mark-up in the monopolistically-competitive sector,[14] also determines the share of intermediate inputs in aggregate income (Y). This is clearly a weakness of the *benchmark* model. The approach proposed in the present paper prevents the problem of relating a firm's degree of market power to an ambiguous parameter (*i.e.*, a parameter that measures different things at the same time) by linking the mark-up not to a constant (α), but rather to the amount of output (x) produced by a generic monopolistically-competitive intermediate firm, where x is in turn an endogenous variable (see Eq. 2.30). In other words, in our model the mark-up (m) is no longer constant, but variable. This is a direct consequence of our choice of working with a general (non-homothetic) aggregate production function (Eq. 2.2).

Besides this, we believe that the approach taken in our paper has another attractive feature. This resides in the result that the relationship between markup (m) and economic growth (γ) can in principle be ambiguous in the presence of partial competition in the final output sector (Proposition 5).

Since Schumpeter (1942) the analysis of the relationship between market structure (markups), innovation and long-run productivity growth has attracted the interest of several theoretical as well as applied economists. Schumpeter (1942) was, in fact, among the first to recognize that more market power spurs innovation and long-run economic growth. Even

[14] "*…The monopoly pricing problem…is that of a firm with constant marginal cost that faces a constant elasticity demand curve. The resulting monopoly price is a simple markup over marginal cost, where the markup is determined by the elasticity of demand…*" (Romer, 1990, pp. S86-S87).

though the *Schumpeterian hypothesis* is now shared by many theoretical works (Dasgupta and Stiglitz, 1980 and Vives, 2008, are just two notable examples), there now exists a recent empirical literature (see Cette *et al.*, 2013 for a review of this firm-level and macro-based research line) supporting the belief that competitive pressure encourages innovative activity and, thus, may play a positive role in fostering productivity growth. In order to account for this evidence, the basic *Schumpeterian-growth paradigm* (Aghion and Howitt, 1992) has been re-formulated and extended along different directions. A first strand of the literature (Aghion *et al.*, 1997a; 1999) has emphasized the importance of agency issues: intensified product market competition forces managers to speed up the adoption of new technologies in order to avoid loss of control rights due to bankruptcy. This *disciplining effect* of competition can, hence, cause higher economic growth rates in the future. An alternative approach, introduced by Aghion and Howitt (1996), has shown that more competition between new and old production lines (parameterized by increased substitutability between them) can make workers more adaptable in switching to newer ones. Holding the fixed supply of these workers constant, the consequence is an increase in the flow of workers into newly discovered products, which enhances the profitability of research (and, hence, economic growth) by reducing the cost of implementing a successful innovation. According to these two research lines there would be an unambiguously positive relationship between product market competition and economic growth. Aghion *et al.* (1997b; 2001; 2005) have extended the basic Schumpeterian growth model by introducing a less radical *step-by-step* hypothesis: they assume that a firm which is currently *m* steps behind the technological leader in the same sector must first catch up with the leader before becoming a leader itself. Under this assumption, they find that the relationship between competition and innovation/economic growth may also be nonlinear in that when competition is low, an increase will raise innovation through the *escape competition effect* on neck-and-neck firms, but when it becomes intense enough it may lower innovation through the traditional *Schumpeterian effect* on laggards.[15] In a more recent paper, Aghion *et al.* (2009), by developing an industry model, have claimed that a firm's innovative response to increased competition is nonlinear and dependent on how far that firm is from the world technology frontier.

Starting from Aghion and Howitt (1992), all the subsequent works by Aghion and co-authors mentioned above have relied on the assumption that technological progress takes the form of innovations that improve the quality of the existing intermediate inputs (*quality-*

[15] For a detailed description of the *Schumpeterian* and *Escape Competition Effects*, see Aghion *et al.* (2013).

ladder growth models). In this framework the general conclusion is that, depending on the underlying model's assumptions, the relationship between competition and the long-run rate of economic growth may be either always positive, or always negative, or else non-monotonic (at first positive then, after a given threshold, negative).

Using an *expanding-variety* (horizontal differentiation) growth model with purposeful human capital accumulation where the (constant) monopolistic markup is disentangled from the shares of factor-inputs in GDP, Bucci (2013) has explained not only why in some (typically OECD) countries the correlation between product market competition and economic growth may be (depending on the country) positive or negative, but also and at the same time why in other (notably, non-OECD) countries the same correlation seems definitely negative. His analysis reveals that an important role in this regard is played by whether a rise in the number of available input-varieties that are combined within the same production process can result (as it is more likely to occur in OECD, as opposed to non-OECD, countries) in a simultaneous escalation of production-complexity. In other words, Bucci (2013) finds that the observed ambiguity in the sign of the correlation between product market competition and economic growth can ultimately be explained by the presence or absence of an *increasing production-complexity effect* related to the expansion in the number of intermediate varieties employed in the same production process.

Unlike Bucci (2013), in the present paper our explanation of why the (variable) markup and the horizontal differentiation-driven economic growth rate are positively correlated in some countries and negatively correlated in others is founded, instead, on whether in a given country function $r_f\left(\cdot\right)$ is increasing or decreasing in x. This result is new and definitely worth of further analyses.

4. Concluding Remarks

The *horizontal innovation paradigm* has now gained a prominent role within the dynamic, general equilibrium, economic development and growth theories. In this paper an attempt has been undertaken to depart from one of the most standard assumptions of this theoretical paradigm: homotheticity in the production function used in the final output sector. The use of a more general aggregate production function allows us to look at the benchmark case (Romer, 1990) from a new perspective and to find and analyze relations more general than those we know since the 1990s.

Under definite conditions, we find that an equilibrium with perfect competition in the final output sector is possible under a non-homothetic production function. We also establish conditions under which the general equilibrium is unique. The equilibrium is characterized by specific values for the amount of output and the price of a given variety of intermediate inputs (as in the benchmark Romer's model), but also by a definite size (namely, amount of labor) of a firm operating in the final output sector. We also find conditions under which either multiple equilibria or no equilibrium at all may exist.

In a further extension of the model, we assumed that a firm in the final output sector devotes a given level of expenditure to the purchase of intermediate inputs (partial competition in the final output sector). As in Solow (1956), we can interpret this definite level of expenditure as a fixed share of the final output produced by a firm, *i.e.* as a sort of *"investment"*. We postulated that this expenditure for investment is a control variable for a final output sector firm. This modeling strategy ultimately allows us to compare different balanced growth paths and to study the effects of *price-increasing* and *price-decreasing* competition (Zhelobodko *et al.*, 2012) in a dynamic framework. To have an intuition of our results, suppose that the gross markup depends on the amount of output (x) that any intermediate firm decides to produce at equilibrium. Related to a rise in the scale x of an intermediate firm one can associate two opposite effects: the first is positive (producing more means to learn producing more efficiently – we may label this as *specialization effect*), while the second is negative (producing more may also imply the emergence of diseconomies of scale due to co-ordination problems within the firm – we may label this as *complexity effect*).[16]

In our model, function $r_f(\cdot)$ modulates the tension between specialization and complexity following an increase in the scale of production at the level of an intermediate firm. In this respect, two cases are possible. If $r_f(\cdot)$ increases when x rises, then the positive specialization effect prevails over the negative complexity effect and this allows the producer of a given intermediate input to charge higher prices, as production is now more efficient. This is the case of *price increasing competition* in Zhelobodko *et al.* (2012). In this case, at equilibrium we observe that a higher $r_f(\cdot)$ is accompanied by a higher growth rate, γ, and by an increase in the size (amount of labor employed) of a firm operating in the final output sector.

[16] Aghion and Howitt (1998, p. 407) were among the first to introduce the opposing *specialization* and *complexity effects* in relation to an increase in scope, N (rather than in scale, x). For a more recent analysis of these two effects within a horizontal differentiation-driven growth model, see Bucci (2013). We believe that, from the point of view of the internal organization of a firm, an increase in scope or in scale may produce comparable (positive and negative) effects.

Similarly, if $r_f(\cdot)$ decreases when *x* rises, then the negative complexity effect prevails over the positive specialization effect and this forces an intermediate firm to decrease the price for its own good, as production is now less efficient. This is the *price decreasing competition* case in Zhelobodko *et al.* (2012). In this case, at equilibrium a lower $r_f(\cdot)$ is accompanied by a higher growth rate, γ, and by a decrease in the size (labor-input) of a firm operating in the final output sector.

We have interpreted such results in the light of the huge debate (both empirical and theoretical) initiated by Schumpeter (1942) about the long-run effects of a change in the degree of product market competition on economic growth. Indeed, in the model product market competition can be measured in two alternative ways: (i) By the value of the ratio N/E (the larger this ratio, the lower the monopolistic profits in the intermediate sector, and the larger the degree of product market competition in that sector), and (ii) By the magnitude of the markup (the larger the markup and the lower the degree of product market competition in the intermediate sector). If we measure product market competition by the ratio N/E, we find that in the long-run the correlation between product market competition and economic growth is always negative, as postulated by Schumpeter (1942) – N/E is always negatively correlated with economic growth. The intuition is straightforward: as monopoly profits represent the main engine of the R&D activity and, hence, economic growth, a rise in N/E by reducing such profits leads ultimately to a lower growth rate of the economy. However, if we measure product market competition by the magnitude of the markup, we find an ambiguous correlation between competition and economic growth in the long-run. This correlation would be positive under *price decreasing competition* (in this case a decrease in the markup is followed by an increase in the growth rate of the economy), while it would be negative under *price increasing competition* (in this case an increase in the markup is followed by an increase in the growth rate of the economy).

All in all, when we measure the degree of competition in the intermediate sector by the magnitude of the (variable) markup, our model suggests that the ultimate reason why in some countries we may observe an unambiguously positive correlation between competition and economic growth while in others we simultaneously observe the exact opposite depends on whether across countries function $r_f(\cdot)$ is, respectively, decreasing or increasing in x. We believe that this result represents a nice complement to other existing theories that already explain the presence of an ambiguous cross-country relation between product market competition and economic growth.

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NOTES TO THE REFEREES
(NOT INTENDED FOR PUBLICATION)

PARTIAL COMPETITION IN THE FINAL OUTPUT SECTOR

In parallel with the version of the model displaying perfect competition in the final output sector, in these notes we consider a more general version of the model. In the final output sector there is now partial competition, in the following sense: the labor/human capital input employed to produce final goods is paid at its marginal product while intermediate inputs are paid proportionally to their marginal products.

Let the final-output-sector firm take factor prices, w and p_j ($j \in [0, N]$), and expenditure E devoted to the purchase of intermediate goods as given. This firm chooses labor, H_1, and each of the intermediate goods, x_j, by solving the following problem:

$$Y - wH_1 \to Max$$

s.t.

$$\int_{j=0}^{N} \left(p_j x_j \right) dj = E .$$

The Lagrangian is

$$\tilde{L} = g\left(H_1\right) \int_{j=0}^{N} f\left(x_j\right) dj - wH_1 - \lambda \left[\int_{j=0}^{N} \left(p_j x_j \right) dj - E \right], \qquad \lambda > 0 .$$

The FOCs are

$$w = \frac{\partial Y_i}{\partial H_1} = g'\left(H_1\right) \int_{j=0}^{N} f\left(x_j\right) dj ,$$

$$\lambda p_j = g\left(H_1\right) f'\left(x_j\right). \tag{A1}$$

Similarly to (2.8) in the main text, Eq. (A1) implies that the (module of the) price elasticity of demand for an intermediate good is equal to: $e_d = \dfrac{1}{r_f(x_j)}$.

In the symmetric case, when $x_j = x$, it is evident that

$$x_j = x = \frac{E}{Np} .$$

Clearly, there is some value *E*, for which $\lambda = 1$, *i.e.* the equilibrium in the version with perfect competition in the final output sector coincides with the equilibrium in the extended version of the model with partial competition in the final output sector. In this equilibrium, labor receives the maximum wage, given the prices for intermediate inputs.

In particular, one can assume (as in the Solow model) that the expenditure E is a constant share $0 < s < 1$ of the final output of a firm producing final output:

$$E = sg\left(H_1\right) \int_{j=0}^{N} f\left(x_j\right) dj .$$

In this case, in the symmetric equilibrium:

$$\frac{E}{N} = sg\left(H_1\right) f\left(x_j\right). \quad \blacksquare$$