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Opacity and frequency dependence of beta[☆]

Sana Ejaz [a,*], Vladimir Volkov [b,c,a]

[a] University of Tasmania, Australia
[b] Centre for Applied Macroeconomic Analysis, Crawford School of Public Policy, Australian National University, Australia
[c] HSE University, Russia

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ABSTRACT

This paper examines the relationship between opacity and frequency dependence of systematic risk (β), estimated over different horizons using Wavelet Transform, for small and large firms. The findings provide evidence for the frequency-specific nature of opacity and suggest that while opacity is positively related to the frequency dependence of beta for large firms at all frequencies, for small firms the relationship is significant at low (long horizon) and insignificant at higher (short horizon) frequencies.

1. Introduction

Opacity, as defined by Gilbert et al. (2014), is the delay in processing information about the effect of systematic news on firm value. A firm is classified as opaque if the stock's prices adjust to the arrival of new information with a delay and transparent if the prices reflect information immediately Bloomfield and O'Hara (2000). As shown by Gilbert et al. (2014), this delay in adjusting new information by opaque firms leads to lower volatility at high frequency (daily) and lower systematic risk (β). In contrast, transparent assets adjust new information immediately, causing prices to change at high speed, leading to higher volatility and higher beta (high-frequency beta). Gilbert et al. (2014) further provides evidence that as we move toward low frequency (quarterly), information is fully revealed and reflected in the prices of both opaque and transparent firms. Thus, opaque stocks have a high-frequency beta lower than their low-frequency beta, and the opposite applies to transparent stocks. In other words, the difference between low and high-frequency beta for opaque stocks is negative, and for transparent stocks, it is positive. This difference in beta, in turn, may indicate the frequency dependence of systematic risk Bandi et al. (2021).

Although there is abundant literature available on both how information risk and opacity impact the cost of capital and beta of a firm (Barron and Qu, 2014; Barry and Brown, 1985; Barth et al., 2013; Cheynel, 2013; Christensen et al., 2010; Coles et al., 1995; Easley et al., 2002; Francis et al., 2005; Gray et al., 2009; Hughes et al., 2009; Lambert et al., 2007; Riedl and Serafeim, 2011), and the possible reasons of frequency dependence of beta (Roll, 1981; Hawawini, 1983; Handa et al., 1989; Dimson, 1979; Scholes and Williams, 1977; Lo and MacKinlay, 1990; Roll, 1984; Blume and Stambaugh, 1983; Longstaff, 1989; Levhari and Levy, 1977), the role of opacity and information risk in explaining frequency dependence of beta has not been investigated except by Gilbert et al. (2014). It is the only study that specifically relates the opacity of the firm with the frequency dependence of beta.

Gilbert et al. (2014) provide evidence that beta depends upon the underlying frequency of returns and that the beta estimated using daily returns differs from the beta estimated using quarterly returns. In addition, this variation in the values of estimated beta is explained by the opacity of the given firm. Gilbert et al. (2014) argue that at high-frequency, prices of opaque firms adjust new information with a delay; conversely, transparent firms adjust information immediately. Contrarily, at low frequency, information

☆ This research was funded by the University of Tasmania, Hobart Australia.
* Corresponding author.
E-mail addresses: sana.ejaz@utas.edu.au (S. Ejaz), vladimir.volkov@utas.edu.au (V. Volkov).

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is known and adjusted into the prices for both opaque and transparent firms. This information structure and the opaque nature of the firm are responsible for the variation in the values of estimated beta over different investment horizons or, in other words, the frequency dependence of beta.

While Gilbert et al. (2014) provide evidence of a significant relationship between opacity and frequency dependence of beta for large and liquid firms, they do not provide any evidence of a similar relationship for small firms. In addition, while investigating the frequency-dependent behavior of beta, their results are limited to quarterly frequency. They do not consider other frequencies such as monthly and weekly; their measure of frequency dependence of beta is based on the difference between quarterly and daily beta.

Our study contrarily, extends beyond Gilbert et al. (2014) and investigates the relationship between opacity and frequency dependence of beta over multiple frequencies. In contrast to Gilbert et al. (2014), instead of using time domain techniques (Capital Assets Pricing Model (CAPM)) to estimate beta, we present a new perspective using wavelet analysis, which allows us to decompose data into several time scales without imposing any assumptions on the return series. In addition, unlike Gilbert et al. (2014), our study divides the sample based on both size and opacity to conduct a thorough investigation of the size and opacity dynamics in relation to the frequency dependence of beta. This study helps to deepen the understanding of the true nature of the relationship between opacity and frequency dependence of beta over different time scales. The results therefore should be of interest to both individual investors with either short or long investment horizons, and to long-term investors such as superannuation funds and insurance companies.

2. Data

The sample includes US-based common stocks traded on the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), and NASDAQ, and it covers a period of 32 years from 1991 through 2023 inclusive. Stock returns data for the year is compiled from the Data Stream daily database. Following the literature, REITs, American Depository Receipts (ADRs), closed-end funds, and other securities are excluded. As a common practice, regulated utilities and financial firms are also excluded. Financial firms are excluded because the assets and liabilities of these firms do not have the same meaning as non-financial firms and the high leverage that is normal for these firms most likely indicates distress in non-financial firms. Moreover, the CRSP daily value-weighted index is used as a proxy for the market portfolio, which consists of all securities in the CRSP database, excluding ADRs.

Following Gilbert et al. (2014), arithmetic (multiplicative) returns have been calculated and used for empirical estimations and statistical testing instead of logarithmic (additive) returns. With logarithmic (additive) returns, market betas estimated across all the different frequencies should be identical, provided there are no estimation errors. In addition, the excess return of security i in year t is calculated as the return of stock i in year t minus the return of the risk-free security in year t. The risk-free rate of return to calculate excess returns for securities and market is taken from the Kenneth French data library[1]. Following Gilbert et al. (2014), we omit stocks that do not have at least 75% of the total observations in each year. In addition, for the calculation of abnormal accrual variance, data on receivables, revenue, total assets, plant property, and equipment is also obtained from Data Stream.

3. Methodology

We use the Wavelet Transform proposed by Gencay et al. (2003) to estimate horizon-specific systematic risk. The Wavelet Transform is based on two sets of functions, known as wavelet functions and scaling functions, representing high pass and low pass filters, respectively. The decomposition of a time series into its different frequency components is achieved by successive high-pass and low-pass filtering of the time domain data series. One of the simplest examples of Wavelet Transform is the Haar Wavelet, which can perform a multi-scale decomposition of return series such that the sum of decomposed components equals the original returns series. Following Gencay et al. (2003), the Haar Wavelet Transform[2] is used to obtain a multi-scale decomposition of daily company and market returns. Using these decomposed frequency-specific company and market returns in Eq. (1), β for individual stocks is estimated over six different scales. The interpretation of scales is such that scale one and scale two represent returns over a horizon of two to four and four to eight days respectively; scale three and scale four are associated with returns over a time interval of eight to sixteen and sixteen to thirty-two days respectively; scale five and scale-six is associated with returns over a horizon of thirty-two to sixty-four and sixty-four to one twenty-eight days respectively. Following existing literature, Gilbert et al. (2014), a five-year return window is used to estimate betas over different frequencies.

$$\widehat{\beta}_J = \widehat{E}\frac{[\widehat{x^{l,(J)}_{k*2^J}}, \widehat{x^{m,(J)}_{k*2^J}}]}{v[\widehat{x^{m,(J)}_{k*2^j}}]}, \tag{1}$$

in which, $\widehat{\beta}_J$ is the horizon-specific β, $\widehat{x^{l,(J)}_{k*2^J}}$ are horizon-specific company returns and $\widehat{x^{m,(J)}_{k*2^j}}$ are horizon-specific market returns estimated using Wavelet Transforms.

[1] https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[2] Kindly refer to Appendix for a detailed discussion of Haar wavelet transform.

Furthermore, the estimated individual stock beta is then used to calculate the difference between low (scale six) and high (scale one) frequency beta, which is defined as

$$\Delta\beta_i = \beta_{i,L} - \beta_{i,H}, \tag{2}$$

in which, $\beta_{i,L}$ is the β of the company i at low frequency and $\beta_{i,H}$ is the beta at high frequency.

While prior literature demonstrates the importance of time scale issues and horizons, standard methods of estimating beta do not allow us to estimate horizon-specific systematic risk and it does not accommodate the multi-horizon nature of beta. On the other hand, wavelet analysis is a natural tool to investigate horizon-specific properties of beta as it allows us to decompose returns on a scale-by-scale basis. It deals with the limitations of the standard OLS method and accommodates the multi-horizon nature of systematic risk. Thus, the main advantage of using wavelet analysis is the ability to decompose the data into several time scales (investment horizons).

Financial securities markets are complex systems, which consist of heterogeneous investors that trade in the stock market and make decisions over different time horizons depending on their savings and consumption needs. Therefore, the level of systematic risk for all these heterogeneous investors varies with their investment. For instance, stock traders operate based on every minute, every hour, every day, every month, or even every year. Thus, because of the different decision-making time scales among traders, the true dynamic structure of the relationship between the frequency dependence of beta and opacity will vary over different time scales associated with those different horizons. Economists and financial analysts have long recognized that there are several time periods in decision-making, whereas economic and financial analyses have been restricted to at most two time scales (the short run and the long run) because of the lack of analytical tools to decompose data into more than two time scales. However, unlike previous studies, this paper uses wavelets to produce an orthogonal decomposition of systematic risk over several different time scales. In particular, this feature of time scale decomposition enables us to examine the relationship between the frequency dependence of beta and opacity at different investment horizons.

Moreover, to conduct a thorough investigation of the size and opacity dynamics concerning the frequency dependence of beta, the entire sample is divided into four portfolios formed based on size and $\Delta\beta$. Where $\Delta\beta$ is calculated with the help of Eq. (2). To construct portfolios, a bi-variate independent-sort procedure is used. In bi-variate independently sorted portfolios, the securities are ranked separately based on any two attributes, and then, the intersection of two independently formed groups is used to form portfolios.

Breakpoints are calculated each year based on median size and median $\Delta\beta$. Where based on size breakpoints, securities are ranked large and small, and using $\Delta\beta$ breakpoints, the same universe of stocks is then ranked independently and divided into transparent and opaque. Moreover, breakpoints of year $t-1$ are used to form portfolios for year t. Each year, stocks having a value of size greater(smaller) than size breakpoints are categorized as large(small), and stocks with a value of $\Delta\beta$ greater (less) than $\Delta\beta$ breakpoints are grouped as opaque(transparent). Once all stocks are grouped into small, large, opaque, and transparent, four portfolios are then constructed. Portfolios are constructed such that small-transparent portfolios have stocks at the intersection of both small and transparent groups. The same is true for small-opaque, large-transparent, and large-opaque portfolios. Furthermore, these portfolios are re-balanced at the end of each year and value-weighted returns are estimated for each portfolio.

Furthermore, panel data regression with time and firm fixed effects are used to investigate whether opacity explains the observed difference in betas across different frequencies. The underlying proposition of this relationship is that opaque firms have higher $\Delta\beta$ (β_L - β_h) and transparent firms have lower $\Delta\beta$. The choice of panel data regression is guided by the availability of data on the relevant frequency and is consistent with existing literature Gilbert et al. (2014). The dependent variable is $\Delta\beta$ and the independent variable is opacity measured using AAV (abnormal Accruals Variance).

AAV is measured at an annual frequency (Gilbert et al., 2014; Jones, 1991). Other proxies that have been used in the existing literature include co-variance of changes in earnings with stock returns Barth et al. (2013), managerial discretion (Hambrick and Abrahamson, 1995), analyst following Lang et al. (2012), forecast accuracy Lang and Lundholm (1996) and choice of auditor Teoh and Wong (1993). However, this study chooses AAV as the main proxy because it is extensively used in the existing literature and its use is consistent with the recent literature on the subject Gilbert et al. (2014)[3].

AAV is measured as the five-year rolling variance of discretionary accruals (DA). Theoretically, total accruals can be decomposed into discretionary and non-discretionary Accruals as follows:

$$TA_{i,t} = DA_{i,t} + NDA_{i,t} \tag{3}$$

$DA_{i,t}$ and $NDA_{i,t}$ refer to discretionary and non-discretionary accruals respectively. In addition, the non-discretionary accruals are assumed to be explained by the model in Eq. (4) while the residuals represent discretionary accruals. The model borrows from Dechow et al. (1995) also known as the Modified Jones model and is given in Eq. (4).

$$TAR_{i,t} = \alpha_0 + \beta_1(\Delta Rev_{i,t} - \Delta Rec_{i,t})/TA_{i,t-1} + PPE_{i,t}/TA_{i,t-1} + \epsilon_{i,t} \tag{4}$$

in which $\Delta \mathrm{Rev}_{i,t}$ is the change in revenue (sales), $\Delta \mathrm{Rec}_{i,t}$ is the change in receivables, and $\mathrm{PPE}_{i,t}$ is the value of property, plant, and equipment for firm i in year t. The residuals $\epsilon_{i,t}$ from the regression in Eq. (4) is the measure for $\mathrm{DA}_{i,t}$. Thus, the measure of AAV is obtained by taking the five-year rolling variance of the $\mathrm{DA}_{i,t}$.

[3] For robustness, managerial discretion was also used as a proxy of opacity. Results are found to be consistent with AAV.

The empirical literature suggests other factors that may significantly influence a firm's risk–return relationship. Therefore, the panel regression analysis has included such factors as control variables. These include size (Sz) measured as the natural log of the market value of equity (MCap) of the firm i in year $t-1$ (i.e., $\mathrm{Sz}_{i,t-1} = \ln(\mathrm{Mcap}_{i,t-1})$), average returns measured as yearly average of scale two returns, and the difference of returns (i.e., $\Delta \mathrm{R}_{i,t-1}$) measured as scale six(scale five, scale four) returns minus scale one returns. Similarly, Amihud's illiquidity measure-Illiquidity (ILLQ)- is used to control for firm-level differences in liquidity. This measure is calculated based on yearly frequency.

Volume Turnover (TO) for each stock has also been used as a control variable for firm-level differences in liquidity. TO is the natural log of the monthly average volume turnovers over 12 months in any sample year. Furthermore, before 2001, the stock prices were quoted in fractions. However, after 2001 as a result of "decimalization", stocks were now required to be quoted in decimals. This change led to a tighter bid–ask spread and reduced transaction costs. To capture the effects of decimalization[4] on the relationship between opacity and frequency dependence of beta, an interaction term $\mathrm{D}\times Opacity_{i,t-1}$ has been added to the panel regression.

The empirical specification for the panel regression is given as follows:

$$\Delta\beta_{it} = \alpha + \gamma_1 Opacity_{i,t-1} + \gamma_2 D \times Opacity_{i,t-1} + \gamma_3 Controls_{i,t-1} + \mu_t + \mu_j + \epsilon_{i,t} \tag{5}$$

Where α is the intercept term, γ_1, γ_2 and γ_3 are the estimated coefficients from the panel regression, D is the dummy variable which takes a value of 1 if the year is < 2001 and 0 otherwise, μ_t, μ_j represent time and firm fixed effects and $\epsilon_{i,t}$ is the random error term with zero mean and constant variance.

4. Empirical results

This section provides empirical evidence and documents a frequency-specific relationship between opacity and frequency dependence of beta, where the relationship between opacity and $\Delta\beta$ holds true for large firms at all frequencies. For small firms, it is significant at low and insignificant at high frequency.

We begin with the empirical analysis by examining whether market betas for small, large, opaque, and transparent firms change as a function of the frequency of underlying returns. Drawing upon the findings of prior research, it is predicted that as we transition from high to low frequency, the market beta of small(opaque) firms will rise and that of large (transparent) firms will fall. This is because large firms are generally more intensively traded, are followed by more analysts, and release information timely. Frequent trading, extensive analyst coverage, and timely release of information, often lead to the transparency of firms. According to Gilbert et al. (2014), prices of transparent firms adjust to new information immediately. Immediate adjustment of prices to new information will lead to higher fluctuations in stock prices at high frequency and, in turn, higher volatility. This high volatility will result in a decreasing beta for large firms whereas the opposite applies to small firms.

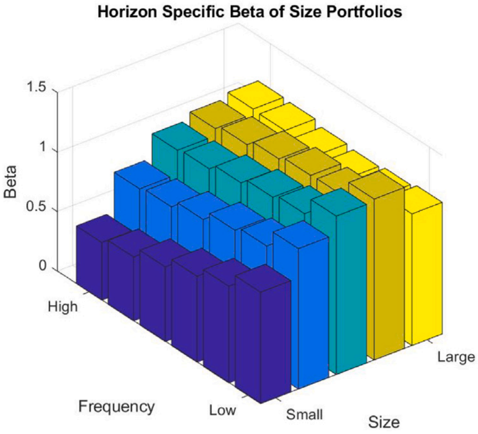
Five portfolios based on size and five based on $\Delta\beta$ are formed and rebalanced yearly from 1991 to 2023. The maximum number of stocks in each size portfolio is 1337 and the minimum is 603, with the average number of stocks being 914. The maximum, minimum, and average number of stocks for $\Delta\beta$ portfolios are 909, 491, and 710, respectively. Furthermore, value-weighted daily returns of these portfolios are calculated using individual daily returns recorded from the Data Stream. Implementing Wavelet Transform on a scale of 1 to 6, using the past five years' portfolio and market returns, frequency-specific beta is estimated for all ten portfolios. Three-dimensional graphs of average horizon-specific beta (averaged across time) for both size and $\Delta\beta$ portfolios are given in Figs. 1(a) and 1(b).

As seen from Fig. 1 , the systematic risk of opaque and small firms increases and that of transparent and large firms decreases monotonically as we move from high to low frequency. These findings broadly support evidence from previous studies (Gilbert et al., 2014; Hawawini, 1983; Bandi et al., 2021). Moreover, the horizon-specific systematic risk varies not only across the delta beta and size dimension but also across the frequency dimension, and the change across both dimensions is significant. Small and opaque stocks are sensitive to market fluctuations at low frequencies and large and transparent stocks are the opposite. As the systematic risk at low-frequency capture fluctuations over a horizon of 64 to 128 days(i.e., longer than two months), the results suggest that small and opaque stocks are relatively more affected by the long-run dynamics.

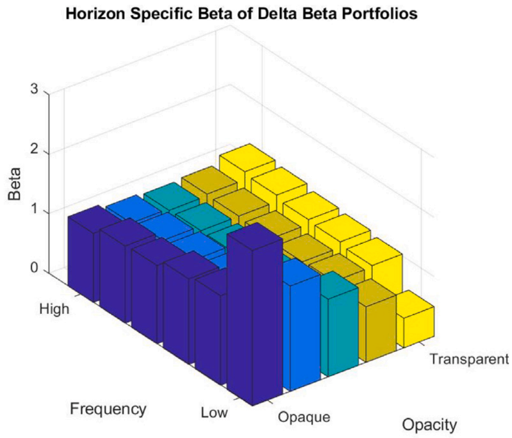
Furthermore, Table 1 reports summary statistics for the stocks included in the five-size sorted portfolios. Table 1 shows that as the market capitalization (size) increases, the delta beta of the firms in portfolios decreases. Based on evidence from Gilbert et al. (2014), a small value of delta beta indicates transparency and large value opacity. Therefore, this means that, in general, small firms tend to be opaque and large firms transparent. Even though both Tables 1 and Fig. 1(a) indicate that small firms are opaque and large firms transparent, it can be seen from the 1st and 99th percentiles of $\Delta\beta$ given in Table 1, that there is substantial $\Delta\beta$ variation within all size-based portfolios. Portfolio 1 (small) includes firms with significant negative $\Delta\beta$ and portfolio 5 (large) with significant positive $\Delta\beta$. This variation confirms the existence of firms that are small(large) and transparent (opaque) at the same time.

Therefore, keeping in view the existence of firms that are both small and transparent (large and opaque) at the same time, the entire sample is divided into four median portfolios double sorted based on size and $\Delta\ \beta$ (i.e., small transparent [ST], small

[4] The sample includes the significant event of the implemented rule of "decimalization" in 2001 when the U.S. Securities and Exchange Commission (SEC) required stocks to be quoted in decimals, such as $0.01 increments. This shift had impact on the financial markets and we conjecture if decimalization had impact on the link between opacity and $\Delta\beta$.



(a)



(b)

Fig. 1. Horizon specific β of portfolios based on size and $\Delta\beta$: This figure shows horizon specific β of portfolios based on size and $\Delta\beta$. Where figure(a) shows a three-dimensional plot of horizon specific β for quintile portfolios that are sorted based on size and figure(b) shows a three-dimensional plot of horizon specific β for quintile portfolios that are sorted based on $\Delta\beta$ for the sample period of 1991 to 2023. Quintile portfolios are constructed using market capitalization and $\Delta\beta$ of individual stocks. Where portfolio one in size-based portfolios consists of small and portfolio five comprised of large stocks. Moreover portfolio one of $\Delta\beta$ portfolios consists of Transparent and portfolio five comprised of Opaque stocks. Using past daily returns of 60 months, horizon-specific β is then estimated for all 10 portfolios with the help of the below-given equation.

$$\widehat{\beta}_J = \widehat{E}\,\frac{[\widehat{x^{l,(J)}_{k*2^J}}, \widehat{x^{m,(J)}_{k*2^J}}]}{\upsilon[\widehat{x^{m,(J)}_{k*2^J}}]}.$$

opaque [Sop], large transparent [LT], and large opaque [Lop]). Table 2 reports the summary statistics (pooled averages) of the stocks included in the mentioned portfolios. In general, while $\Delta\ \beta$ takes a negative value for both small and large transparent, it is positive for opaque firms. Overall, variations in values of $\Delta\ \beta$ with the changing scale are observed for constituents of all four portfolios. This provides evidence for the frequency-specific nature of $\Delta\ \beta$.

In addition, this section estimates horizon-specific beta and the difference in beta across different frequencies for all four portfolios. The plots are given in Fig. 2. Figs. 2(a) and 2(b), based on wavelet beta, reveal that as we move from high to low frequency, there has been a marked increase in the magnitude of systematic risk for opaque firms, both large and small, whereas a considerable decrease for large transparent firms. What stands out is the general pattern of systematic risk for small transparent firms. Where, in contrast to theory and expectation, instead of decreasing, the beta value significantly increased as we transitioned from high to low frequency.

Furthermore, Table 3 reports β and $\Delta\beta$ for all four portfolios across different frequencies. Based on empirical evidence provided by Gilbert et al. (2014), a negative $\Delta\beta$ for transparent and positive $\Delta\beta$ for opaque firms is expected. Consistent with the expectation,

Table 1
Summary statistics of size portfolio constituents: This table reports summary statistics for the constituents of five portfolios based on size. The portfolios are re-balanced annually, and β is estimated for different scales with the help of a wavelet transform using past five-year returns. Where β_1 corresponds to scale one and β_6 to scale six β, $\Delta\beta_1$ = scale six minus scale one β; $\Delta\beta_2$ = scale five minus scale one β and $\Delta\beta_3$ = scale four minus scale one β and Mcap represents market capitalization of the Constituents of the size portfolios.

Summary statistics for constituents of size portfolio					
	Small	Q2	Q3	Q4	Large
	Small	Q2	Q3	Q4	Large
# of firms	914	914	914	914	914
β_1	0.40	0.71	0.96	1.04	1.08
β_2	0.45	0.75	0.98	1.08	1.12
β_3	0.53	0.82	1.04	1.14	1.14
β_4	0.62	0.91	1.10	1.18	1.16
β_5	0.72	0.96	1.14	1.18	1.16
β_6	0.84	1.10	1.30	1.34	1.24
$\Delta\beta_1$	0.44	0.39	0.34	0.30	0.15
$\Delta\beta_2$	0.32	0.24	0.18	0.14	0.08
$\Delta\beta_3$	0.22	0.19	0.14	0.14	0.08
Mcap ($mil)	17	80	245	748	9862
$\Delta\beta_1$:1st	−1.63	−1.36	−1.21	−1.05	−0.92
50th	0.45	0.37	0.31	0.27	0.13
99th	2.48	2.25	1.98	1.79	1.30

Table 2
Summary statistics for the Constituents of double sorted portfolios based on $\Delta\beta$ and size: This table reports summary statistics for the constituents of size and $\Delta\beta$ portfolios. Where β_1, β_2, β_3, β_4, β_5 and β_6 are the β estimated using scale one, scale two, scale three, scale four, scale five and scale six returns respectively, $\Delta\beta_1$ is a difference of scale six and scale one β, $\Delta\beta_2$ difference of scale five and scale one β, $\Delta\beta_3$ difference of scale four and scale one β, ST is a portfolio which comprised of firms that are both small and transparent, SOp is small Opaque, LT large Transparent and LOp large Opaque.

Summary Statistics for constituents of double sorted portfolios based on $\Delta\beta$ and size				
	ST	SOp	LT	LOp
	ST	Sop	LT	Lop
# Firms	476	699	804	576
Mcap($mil)	106	83	6922	3618
β_1	0.90	0.73	1.11	1.13
β_2	0.84	0.78	1.11	1.20
β_3	0.90	0.87	1.12	1.25
β_4	0.97	1.00	1.15	1.30
β_5	1.01	1.10	1.14	1.32
β_6	0.70	1.95	0.94	2.07
$\Delta\beta_1$	−0.20	1.22	−0.17	0.95
$\Delta\beta_2$	0.10	0.37	0.03	0.19
$\Delta\beta_3$	0.07	0.27	0.03	0.17
$\Delta\beta_1$: 1st	−2.29	0.13	−1.42	0.06
50th	−0.09	1.03	−0.11	0.79
99th	0.51	3.97	0.51	3.10

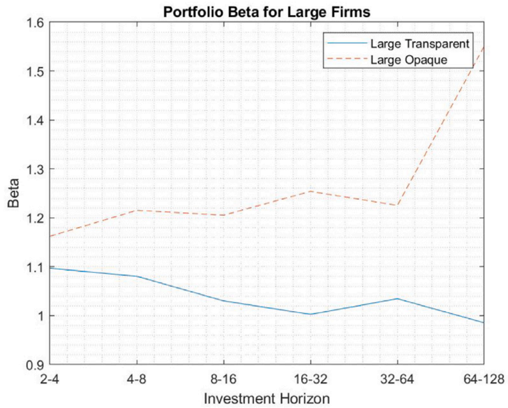
small and large opaque firms recorded positive whereas large transparent firms documented negative $\Delta\beta$. However, small transparent firms surprisingly reported a positive value for $\Delta\beta$ across all horizons instead of a negative value. Further, Table 3 also confirms the horizon-specific nature of opacity revealed by Table 2 where $\Delta\beta$ for all given portfolios changes with the varying frequency of returns.

Based on the results discussed above and findings of Gilbert et al. (2014), it can be said that the beta for transparent firms (opaque) will increase (decrease) as we transition from high to low frequency. Similarly, for large firms' the value of beta will rise and for small it will fall as we move from high to low frequency. In the case of large firms, the plots given in Fig. 2 are consistent with the findings of Gilbert et al. (2014) (i.e., beta is overall increasing for large opaque and decreasing for large transparent as we move from high to low frequency). However, surprisingly, for small firms, whether opaque or transparent, their beta follows the same increasing trend as we move from high to low frequency. Consistent with the expectations, if opacity was the driving force behind the frequency dependence of beta, then the beta of small transparent firms should have decreased from high to low frequency. Thus, these somewhat contradictory findings imply that in the case of small firms, opacity may not be responsible for the frequency dependence of beta.

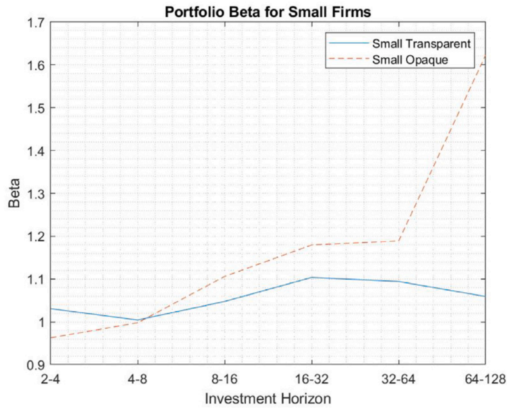
In addition, the given tables and plots offer compelling proof that, regardless of how transparent or opaque a firm may be, if it is small, its systematic risk will increase as we move from high to low frequency (i.e., it will have a lower high-frequency beta as compared to its low-frequency counterpart and in turn a negative delta beta). According to Gilbert et al. (2014) negative delta beta is associated with opaque firms and indicates a delay in adjustment of prices to new information which means small firms will diffuse

Table 3
Horizon specific portfolio β and $\Delta\beta$ based on wavelet transform ; This table shows horizon-specific portfolio β and the difference of β estimated with the help of a wavelet transform using daily portfolio returns of the years 1991 to 2023. Where β_1, β_2, β_3, β_4, β_5 and β_6 are the β estimated using scale one, scale two, scale three, scale four, scale five and scale six returns respectively, $\Delta\beta_1$ is a difference of scale six and scale one β, $\Delta\beta_2$ difference of scale five and scale one β, $\Delta\beta_3$ difference of scale four and scale one β, ST is a portfolio which comprised of firms that are both small and transparent, SOp is small Opaque, LT large Transparent and LOp large Opaque.

Horizon specific β and $\delta\beta$ for double sorted portfolios									
	β_1	β_2	β_3	β_4	β_5	β_6	$\Delta\beta_1$	$\Delta\beta_2$	$\Delta\beta_3$
ST	1.03	1.00	1.05	1.10	1.09	1.06	0.03	0.06	0.07
Sop	0.96	1.00	1.11	1.18	1.19	1.62	0.66	0.23	0.22
LT	1.10	1.08	1.03	1.00	1.03	0.99	−0.11	−0.06	−0.09
Lop	1.16	1.22	1.21	1.25	1.23	1.55	0.39	0.06	0.09



(a) Wavelet β for Large Transparent and Opaque firms



(b) Wavelet β for Small Transparent and Opaque firms

Fig. 2. Wavelet Portfolio Beta: This figure reports the average Wavelet betas for portfolios that are sorted based on size and $\Delta\beta$ as small transparent, small opaque, large transparent, and large opaque from$t = 1991$ to 2023. Firms are first sorted into four portfolios based on both size and $\Delta\beta$ and portfolio β is then estimated across different horizons using wavelet transform.

information at a much slower speed as compared to a large firm. These findings confirm the results of Hong and Stein (1999), who argue that private information diffuses slowly for small firms; this gradual information diffusion is believed to be the root cause of under-reaction. In other words, this shows opacity is not relevant for small firms in the context of frequency dependence of beta. The frequency dependence of beta in small firms is probably because of its small size and not because of opacity.

4.1. Panel regression

To deepen the knowledge of the frequency dynamics of opacity and size-based portfolios and to test the effect of opacity on beta measured at different frequencies this section now turns to panel regression analysis. Based on median size (log Mcap) and median opacity (calculated as $\Delta\beta$), the entire sample is first sorted into four portfolios: Small Transparent, Small Opaque, Large Transparent, and Large Opaque. Where, a small(large) transparent portfolio consists of firms that are both small (large) and transparent, a small(large) opaque portfolio is made up of firms that are both small(large) and opaque at the same time.

After grouping firms into the aforementioned portfolios, panel regression analysis is performed for each portfolio with the help of Eq. (5) using three different dependent variables estimated based on Wavelet Transform (i.e., scale-six minus scale-one beta; scale-five minus scale-one beta; and scale-four minus scale-one beta, henceforth $\Delta\beta_1$, $\Delta\beta_2$, $\Delta\beta_3$). In addition, to account for time-invariant unobserved heterogeneity, both time and firm fixed effects were used.

The independent variable is opacity, measured as the variance of abnormal accruals with the help of Eq. (4). Following Gilbert et al. (2014), residuals are first obtained from the estimation of the expected accrual model of Jones (1991) given in Eq. (4). The five-year rolling variance of these residuals is then estimated and used as a proxy for opacity. It is expected that the higher the variance of abnormal accruals, the more opaque the firm will be. Moreover, to investigate the effects of decimalization on the relationship between opacity and frequency dependence of systematic risk, a dummy variable is used. The dummy variable takes a value of 1 if the year is less than 2001 and 0 otherwise. The primary coefficient of interest in terms of the effects of decimalization on the relationship of opacity and frequency dependence of beta is γ_2. In addition, a set of controls is also used, which is explained in detail in the methodology section. All variables are winsorized by 1% to minimize the influence of outliers.

The findings reported in Table 4 based on regression Eq. (5), depict that in the case of small transparent and opaque firms, opacity is positively related to the frequency dependence of beta only at scale six (64 to 128 days). The relationship is insignificant on both scale five and scale four. Moreover, ILLQ is the only significant (negative) variable for small transparent firms at scales five

Table 4

$\Delta\beta$ panel regressions: This table reports results of a panel regression (given below) of annual, firm-level $\Delta\beta_1$ $\Delta\beta_2$ $\Delta\beta_3$ (i.e.the difference between β_6 and β_1, β_5 and β_1, β_4 and β_1) onto a measure of opacity and lagged controls such as size, volume turnover (TO). where $\beta_6(\beta_5,\ \beta_4,\ \beta_1)$ is β estimated at the end of every year (t) using Scale6 (Scale5, Scale4, Scale1) returns over the previous 60 months (i.e. years t_4 to t). Ret is the average return estimated over scale 2. Where scale 2 represents a horizon of 4 to 8 days. AAV stands for Abnormal accrual variance which is the five-year rolling variance (t_5 to t_1) of the residual from estimating the expected accrual model by Jones (1991). D is a dummy variable that captures the effect of decimalization on the relationship between opacity and frequency dependence of beta. It takes a value of 1 if year <2001 and 0 otherwise. ILLQ is the measure of illiquidity from Amihud (2002) and is calculated based on the previous year (t_1). To be included, a stock is required to trade at least 75% of trading days in a year. The sample period is 1993–2024. T-stats clustered on firm and year are reported in parentheses. Panel (A) reports results for the Small Transparent portfolio and Panel(B), (C), and (D) for Small Opaque, Large Transparent, and Large Opaque respectively. $\Delta\beta_{it} = \alpha + \gamma_1 Opacity_{i,t-1} + \gamma_2 D \times Opacity_{i,t-1} + \gamma_3 Controls_{i,t-1} + \mu_t + \mu_j + \epsilon_{i,t}$.

	Panel A: Small transparent			Panel B: Small opaque			Panel C: Large transparent			Panel D: Large opaque		
	$\Delta\beta_1$	$\Delta\beta_2$	$\Delta\beta_3$	$\Delta\beta_1$	$\Delta\beta_2$	$\Delta\beta_3$	$\Delta\beta_1$	$\Delta\beta_2$	$\Delta\beta_3$	$\Delta\beta_1$	$\Delta\beta_2$	$\Delta\beta_3$
AAV	0.042*	−0.014	−0.008	0.088**	0.017	−0.004	0.092*	0.079*	0.017	0.084*	0.082**	−0.000
	(1.65)	(−0.65)	(−0.57)	(2.05)	(0.46)	(−0.18)	(1.82)	(1.76)	(0.54)	(1.76)	(2.11)	(−0.00)
D ×AAV	−0.014	0.029	−0.012	−0.082	−0.020	0.016	−0.081	0.095	−0.107	−0.029	0.019	0.020
	(−0.19)	(0.52)	(−0.30)	(−0.68)	(−0.26)	(0.28)	(−0.51)	(0.72)	(−1.21)	(−0.24)	(0.19)	(0.29)
Size	−0.060*	−0.022	−0.037*	−0.026	−0.044*	−0.014	−0.028	−0.022	−0.004	−0.027	−0.003	−0.024
	(−1.74)	(−0.75)	(−1.87)	(−0.92)	(−1.87)	(−0.94)	(−1.17)	(−1.05)	(−0.29)	(−0.79)	(−0.13)	(−1.32)
TO	0.171	0.064	−0.019	0.139**	0.076	0.030	0.219	0.263**	0.162**	0.439***	0.210**	0.171***
	(1.40)	(0.65)	(−0.28)	(2.31)	(1.45)	(1.08)	(1.48)	(2.24)	(2.02)	(3.43)	(2.12)	(2.70)
ILLQ	−0.007	−0.008**	−0.006***	0.001	0.001	−0.002	−0.186	0.577**	0.435**	0.232	0.514*	0.047
	(−1.58)	(−2.22)	(−2.64)	(0.40)	(0.43)	(−1.46)	(−0.55)	(2.02)	(2.42)	(0.70)	(1.89)	(0.25)
Ret	−1.033	−1.842	−1.579	2.123	−1.689	0.237	0.154	−5.793**	0.125	−6.134	−2.437	−0.081
	(−0.27)	(−0.61)	(−0.77)	(0.73)	(−0.73)	(0.16)	(0.05)	(−2.09)	(0.07)	(−1.58)	(−0.87)	(−0.04)
ΔR_1	0.027			1.143			−0.406			0.443		
	(0.02)			(1.29)			(−0.47)			(0.47)		
ΔR_2		−0.485			−0.056			−0.346			0.539	
		(−0.44)			(−0.06)			(−0.37)			(0.49)	
ΔR_3			0.291			1.167			0.760			2.961***
			(0.27)			(1.51)			(0.85)			(3.08)
Fixed effects	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
N	8645	8645	8645	9547	9547	9547	9808	9808	9808	8871	8871	8871
adjR^2	0.245	0.309	0.276	0.237	0.260	0.260	0.242	0.261	0.282	0.290	0.310	0.280

*** Indicate significance at the 1% level.

** Indicate significance at the 5% level.

* Indicate significance at the 10% level.

and four. Conversely, for large transparent and opaque firms, the coefficient for AAV is positive and significant at scale six and scale five (32 to 64 days) and insignificant at scale four (16 to 32 days). Furthermore, it is clear from Table 4 that all the coefficients for the interaction term are insignificant. This means decimalization does not affect the relationship between opacity and frequency dependence of beta. These results show that, in general, for large firms, opacity is responsible for the frequency dependence of beta at both low and high frequencies. Whereas for small firms, while opacity leads to the frequency dependence of beta at low frequencies, at high frequencies, the observed frequency dependence of beta is not caused by opacity but rather stems from liquidity.

A possible explanation for these contradictory results may be that small transparent firms are transparent and small at the same time. As they are transparent, there ought to be no information asymmetry for these firms, thus, they should be liquid Jiang et al. (2021). However, their small size makes them less desirable and less frequently traded. Therefore, even though they are transparent by nature, the information diffuses at a slower speed because of lower volume and lower liquidity. Hence, in small firms, low volume and size are generally responsible for the frequency dependence of beta, not the opacity.

5. Conclusion

Opacity has gained increasing attention from academics and practitioners over the last two decades to understand the relationship of opacity to investors' behavior, asset prices, and welfare Sato (2014). This study examines the relationship of opacity to the frequency dependence of beta. The findings suggest that depending on how opaque or transparent a firm is, the value of its market risk can increase or decrease across different investment horizons. Contrary to prior literature Gilbert et al. (2014) we provide evidence that for small firms the relationship between opacity and frequency dependence of beta is insignificant.

We also confirm the positive relationship between liquidity and frequency dependence of beta. A positive relationship means an increase in the liquidity of a firm leads to an increase in the value of delta beta, where a higher value of delta beta, in turn, indicates opacity. This positive relationship between liquidity and opacity observed and documented in this study provides empirical evidence for the theoretical explanation of Dang et al. (2010). Theoretically, a perfectly opaque asset could be very liquid as there are no informed traders and in turn, there is no adverse selection problem. In addition, a possible explanation for these results might be that opacity leads to higher trading volume as because of its opaque nature, investors will have conflicting views regarding the true value of the security and will tend to trade more frequently. Therefore, trading volume will have a positive relationship with opacity Flannery et al. (2013).

Practical implications of our findings suggest that while assessing security or selecting assets for a portfolio, one should consider the frequency dimension of risk. In the spirit of a CAPM model augmented with a factor based on opacity by Gilbert et al. (2014) we advocate for an investment horizon being important factor in the CAPM. Thus, ignoring this investment horizon factor creates a misspecified model leading to biased estimates of systematic risk and in turn biased estimates for cost of capital. The frequency-dependent CAPM capturing investment horizon, advocated in this study, is a step toward measuring systematic risk more accurately.

CRediT authorship contribution statement

Sana Ejaz: Writing – original draft, Visualization, Validation, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation. **Vladimir Volkov:** Writing – review & editing, Supervision, Methodology, Conceptualization.

Data availability

The authors do not have permission to share data.

Appendix. Wavelet transform

Haar Wavelet is a filter of length $L = 2$ which consists of wavelet (high-pass) filter coefficients $h_0 = 1/\sqrt{2}$ and $h_1 = -1/\sqrt{2}$ and scaling (low-pass) filter coefficients $g_0 = 1/\sqrt{2}$ $g_1 = 1/\sqrt{2}$. Applying wavelet and scaling filters to a data series allows us to separate the low-frequency components of the data series from its high-frequency (rapidly changing) components. After applying the Haar Wavelet filter coefficients h_0 and h_l to a return series, the following wavelet coefficients are obtained.

$$\sqrt{2}w_{1\tau} = h_0 r_t + h_1 r_{t-1} \tag{A.1}$$

When $t = 0, 1, \dots, T-1$

Where wavelet coefficient $w_{1\tau}$ are the coefficients at level one and time τ and are the weighted difference between consecutive blocks of returns. Whereas $\sqrt{2}$ is necessary to preserve the variance of the original data. Moreover, the time in Wavelet Transforms differs from the time in the time domain. It represents the shifting of wavelet function in time and is separated by multiples of 2^j(i.e., $\tau = k \times 2^j$ where k = 1,2,3. . . .). Furthermore, applying Haar scaling filter coefficients g_0 and g_1 to return series produces the scaling coefficients.

$$\sqrt{2}v_{1\tau} = g_0 r_t + g_1 r_{t-1} \tag{A.2}$$

where $t = 0, 1, \dots, T-1$

One of the crucial parameters in wavelet analysis is "Scale". Scale is represented by j, which can take values of 1,2. . . . . . J. High scale (low frequency) corresponds to non-detailed information of a data series that usually spans the entire data series. In contrast, low scale (high frequency) is related to detailed information about a hidden pattern in the time series that usually lasts for a relatively short time.

In practice, the Haar Wavelet Transform is implemented using a pyramid algorithm. After applying Haar Wavelet and Haar Scaling filter coefficients to the original data series, a series of high-frequency and a series of low-frequency components can be obtained. The idea of the pyramid algorithm is to further decompose the scaling coefficients $v_{1\tau}$ (low-frequency components) into high and low-frequency components to obtain $w_{2\tau}$ and $v_{2\tau}$. For instance, in order to obtain wavelet coefficients $w_{2\tau}$ based on frequencies 1/8< f ≤ 1/4 and scaling coefficient $v_{2\tau}$ based on frequencies 0 ≤ f ≤ 1/8, the Haar wavelet and Haar scaling filters from Eq. (A.1) and Eq. (A.2) can be applied to scaling coefficient $v_{1\tau}$ (instead of returns). The same procedure can be repeated for each subsequent scaling coefficient until level J, where J is the largest number of scales or levels for a given wavelet transform. Thus, the final collection of wavelet and scaling coefficient can be written as W = (w_1, w_2, ...$w_J v_J$). The frequency interval associated with wavelet coefficients from level $j = 1, 2.....J$ can be written as $1/2^{j+1}$ < f ≤ $1/2^j$. Whereas, the frequency interval related to the scaling coefficients from level $j = 1, 2.....J$ is denoted by 0 ≤ f ≤ $1/2^{j+1}$.

References

Bandi, F.M., Chaudhuri, S.E., Lo, A.W., Tamoni, A., 2021. Spectral factor models. J. Financ. Econ. 142 (1), 214–238.
Barron, O.E., Qu, H., 2014. Information asymmetry and the ex-ante impact of public disclosure quality on price efficiency and the cost of capital: Evidence from a laboratory market. Account. Rev. 89, 1269–1297.
Barry, C.B., Brown, S.J., 1985. Differential information and security market equilibrium. J. Financ. Quant. Anal. 20, 407–422.
Barth, M.E., Konchitchki, Y., Landsman, W.R., 2013. Cost of capital and earnings transparency. J. Account. Econ. 55, 206–224.
Bloomfield, R., O'Hara, M., 2000. Can transparent markets survive? J. Financial Econ. 55 (3), 425–459.
Blume, M.E., Stambaugh, R.F., 1983. Biases in computed returns: An application to the size effect. J. Financ. Econ. 12, 387–404.
Cheynel, E., 2013. A theory of voluntary disclosure and cost of capital. Rev. Account. Stud. 18, 987–1020.
Christensen, P.O., de la Rosa, L.E., Feltham, G.A., 2010. Information and the cost of capital: An ex-ante perspective. Account. Rev. 85, 817–848.
Coles, J.L., Loewenstein, U., Suay, J., 1995. On equilibrium pricing under parameter uncertainty. J. Financ. Quant. Anal. 30, 347–364.
Dang, T.V., Gorton, G., Holmstrom, B., 2010. Financial crises and the optimality of debt for liquidity provision. Yale School of Management Working Paper.
Dechow, P.M., Sloan, R.G., Sweeney, A.P., 1995. Detecting earnings management. Account. Rev. 193–225.

Dimson, E., 1979. Risk measurement when shares are subject to infrequent trading. J. Financ. Econ. 7, 197–226. http://dx.doi.org/10.1016/0304-405x(79)90013-8.

Easley, D., Hvidkjaer, S., O'Hara, M., 2002. Is information risk a determinant of asset returns? J. Finance 57, 2185–2221.

Flannery, M.J., Kwan, S.H., Nimalendran, M., 2013. The 2007–2009 financial crisis and bank opaqueness. J. Financial Intermediat. 22, 55–84.

Francis, J., LaFond, R., Olsson, P., Schipper, K., 2005. The market pricing of accruals quality. J. Account. Econ. 39, 295–327.

Gencay, R., Selcuk, F., Whitcher, B., 2003. Systematic risk and timescales. Quant. Finance 3, 108–116. http://dx.doi.org/10.1088/1469-7688/3/2/305.

Gilbert, T., Hrdlicka, C., Kalodimos, J., Siegel, S., 2014. Daily data is bad for beta: Opacity and frequency-dependent betas. Rev. Asset Pricing Stud. 4, 78–117.

Gray, P., Koh, P.-S., Tong, Y.H., 2009. Accruals quality, information risk, and cost of capital: Evidence from Australia. J. Bus. Finance Account. 36, 51–72.

Hambrick, D.C., Abrahamson, E., 1995. Assessing managerial discretion across industries: A multimethod approach. Acad. Manag. J. 38 (5), 1427–1441.

Handa, P., Kothari, S.P., Wasley, C., 1989. The relation between the return interval and betas. J. Financ. Econ. 23, 79–100. http://dx.doi.org/10.1016/0304-405x(89)90006-8.

Hawawini, G., 1983. Why beta shifts as the return interval changes. Financ. Anal. J. 39, 73–77.

Hong, H., Stein, J.C., 1999. A unified theory of underreaction, momentum trading, and overreaction in asset markets. J. Finance 54, 2143–2184. http://dx.doi.org/10.1111/0022-1082.00184.

Hughes, J., Liu, J., Liu, J., 2009. On the relation between expected returns and implied cost of capital. Rev. Account. Stud. 14, 246–259.

Jiang, C., John, K., Larsen, D., 2021. R&D investment intensity and jump volatility of stock price. Rev. Quant. Financ. Account. 57, 235–277. http://dx.doi.org/10.1007/s11156-020-00944-3.

Jones, J.J., 1991. Earnings management during import relief investigations. J. Account. Res. 29, 193–228.

Lambert, R., Leuz, C., Verrecchia, R.E., 2007. Accounting information, disclosure, and the cost of capital. J. Account. Res. 45, 385–420.

Lang, M., Lins, K.V., Maffett, M., 2012. Transparency, liquidity, and valuation: International evidence on when transparency matters most. J. Account. Res. 50, 729–774.

Lang, M.H., Lundholm, R.J., 1996. Corporate disclosure policy and analyst behavior. Account. Rev. 467–492.

Levhari, D., Levy, H., 1977. The capital asset pricing model and the investment horizon. Rev. Econ. Stat. 92–104.

Lo, A.W., MacKinlay, A.C., 1990. An econometric analysis of nonsynchronous trading. J. Econometrics 45, 181–211.

Longstaff, F.A., 1989. Temporal aggregation and the continuous-time capital asset pricing model. J. Finance 44, 871–887. http://dx.doi.org/10.1111/j.1540-6261.1989.tb02628.x.

Riedl, E.J., Serafeim, G., 2011. Information risk and fair values: An examination of equity betas. J. Account. Res. 49, 1083–1122.

Roll, R., 1981. A possible explanation of the small firm effect. J. Finance 36, 879–888. http://dx.doi.org/10.1111/j.1540-6261.1981.tb04890.x.

Roll, R., 1984. A simple implicit measure of the effective bid-ask spread in an efficient market. J. Finance 39, 1127–1139. http://dx.doi.org/10.1111/j.1540-6261.1984.tb03897.x.

Sato, Y., 2014. Opacity in financial markets. Rev. Financ. Stud. 27, 3502–3546.

Scholes, M., Williams, J., 1977. Estimating betas from nonsynchronous data. J. Financ. Econ. 5, 309–327.

Teoh, S.H., Wong, T.J., 1993. Perceived auditor quality and the earnings response coefficient. Account. Rev. 346–366.