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Consistency and Some Other Requirements of a Formal Theory in the Context of Multiverse Models[1]

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Abstract:
The paper is devoted to the problem of describing reality in the language of mathematics and logic in connection with intellectual intuition. The question raised is how the basic requirements of mathematical theory and logic will change if some of the multiverse models of modern physics are taken as the basis. Mathematics is considered in the context of various historical approaches. It is shown that some of the well-known requirements of a formal theory (such as consistency) may begin to play a different role if the multiverse hypothesis is accepted. In the framework of theories based on the idea of multiple worlds, the logical consequence, the natural law of Duns Scotus, the law of excluded middle, and other well-known facts of classical logic which in some cases cause controversy due to their intuitive unacceptability are resolved. The paper discusses an approach based on paraconsistent logics: such logics can be considered the first to correspond to multiverse theories.
Keywords: consistency, contradiction, formal theory, multiverse theories, philosophy of science, intellectual intuition.

1. Introduction

A considerable number of works have been written on the philosophy of mathematics. Philosophy of mathematics experienced particularly significant development with the emergence of various programmes for establishing the foundations and development of mathematics in the first half of the twentieth century. It was as a result of a crisis, the prerequisites for which formed explicitly in the nineteenth century and even earlier in a latent form.

The main questions that the first philosophers of mathematics (who were themselves mathematicians) tried to answer were essentially what mathematics is, what exactly it studies, what the status of existence of what it studies is, what the criteria for the truth of mathematical knowledge are, what its place among other sciences is, and others. In the context of this study, mathematics is considered as the maximum abstraction of intellectual intuition in describing reality. The main goal of this study: to clarify the role and content of some mathematical concepts in the context of multiverse theories. This is not so much about the possible worlds of mathematical logic as about the models of multiple worlds that modern physics offers (for example, quantum multiverse, inflation, string theory, etc.).

Although multiverse theories are only hypotheses, they are all consequences of working mathematical structures describing the corresponding physical reality within the framework of specific physical theories, some of which rely on a strong experimental basis (quantum mechanics in the form of the standard model of particle physics and general relativity).[2] For this reason, they deserve studying, not necessarily in and of themselves (enough has already been written about this) but in connection with mathematical analysis and formal logic in relation to the cognition, perception, and description of reality. This is a relevant problem, and in this perspective, the classical foundational programmes of mathematics can suddenly sparkle with new colours and be further developed.

In this paper, a hypothesis will be formulated, according to which the requirements of a theory, such as consistency (as well as some accompanying requirements), may turn out to be non-obligatory (or even impossible) under conditions of theories that claim to describe all possible worlds as equal, and not just one particular world—our world, with which the theory is correlated in the experiment. It is substantiated that an increasing degree of abstraction in intellectual intuition naturally leads to such results. This allows us to take a fresh look at well-known logical principles, such as "anything follows from a contradiction", eliminating their intuitive unacceptability observed in a number of cases.

The structure of this article is as follows. The second section considers the nature of mathematics, while the third section explores the concept of intellectual intuition. The fourth section briefly examines the historical debate surrounding the essence of mathematics and logic. The fifth section discusses the connection between the logical principle of noncontradiction and intuition. In the sixth section, a possible solution is proposed using a paraconsistent logic and multi-world approach to address the issue. The seventh section explores the relationship between Platonism, contradictions, and intuition. In the eighth section, it will be demonstrated that a many-worlds interpretation of quantum mechanics provides a promising avenue to resolve the contradiction between intuition and some logical principles. Finally, in the ninth section, conclusions will be drawn and prospects for future research will be outlined.

2. Formulation of the Main Concept

In order to achieve the objectives of the study concerning the philosophical problems of mathematics in different aspects, it is necessary to understand the specifics of mathematics, its essential characteristics—or, speaking in the language of logic, to formulate the concept of mathematics.

There are many different approaches depending on the starting position as regards mathematics and what it does. Modern mathematics can be seen as the limit of abstraction, in the sense that it is an abstract description of reality;[3] there may already be serious objections here, but our position is close to that expressed in the article by Bangu (2020), where the explanatory role of mathematics is quite convincingly substantiated.

Of course, mathematics itself—or rather, the way it was used—was not initially abstract: the mathematical descriptions of Babylon, Ancient Egypt, and Greek Antiquity were close to the natural language, which, in turn, was close to concrete things. However, the entities behind this description, such as geometric figures and numbers, are abstract in the sense that, for example, the square root or the operation of addition do not exist as observable natural phenomena. As science developed, mathematics became more and more abstract both in its content and in its language. Obviously, these were interconnected processes: the growth of abstraction contributed to the development of science, and vice versa.

One can also take the view that the whole of science, in almost all its fields, tends to move away from the description bound to usual observations of concrete phenomena to their abstract analogues.

For example, medical, geographical, mathematical, and physical representations of Antiquity and the Middle Ages were accompanied by images and metaphors (including mythological ones) of the cultural space of that time. However, science is currently experiencing a kind of evolution: it is gradually cutting off extraneous objects and moving towards the greatest level of abstraction in order to isolate the subject of research for the purpose of achieving greater accuracy and specificity, and, importantly, for the purpose of talking only about the object of the study and not about something else that has nothing to do with it.

Of course, this applies to the greatest extent to the natural sciences. The philosophy of science uses predominantly natural language, and the words of natural language and connections between them are inevitably ambiguous, metaphorical, and even mythological, although we can still observe an increase in the level of abstraction, which consists in moving away from colloquial concepts based on everyday experience of interaction with reality. At the same time, it cannot be said that philosophy in general has moved away from imaginary entities to which nothing corresponds in the observed reality: indeed, it actively operates with them. However, mathematics and even physics can be reproached for the same.[4]

For a number of scientists, this serves as an argument in favour of saying that there is no need for the philosophy of science (due to a lower level of abstraction compared to what it describes), and thus such description and interpretation is not progress in intellectual intuition but, on the contrary, a regression. From our point of view, even if this objection is to some extent true, the philosophy of science is necessary because it makes it possible to highlight and outline ways to solve the problems that science faces: in its baggage, it has the whole array of knowledge about the continuity and development of ideas in history of science.

Thus, mathematics is currently the height of abstract description.

3. Some Traditional Views That Need to Be Considered

The root of many problems in attempts to clarify the foundations of mathematics is the ambiguity of the essence and nature of intellectual intuition. There is no such category of intellectual intuition in modern neuroscience; it is a philosophical abstraction designed to denote a certain thinking ability. Of course, it is very likely that intuition is provided by certain neural connections in different parts of the brain, but our task here is not to engage in neurobiology. There is extensive literature in the philosophy of mathematics devoted to intellectual intuition—a recent monograph by Linnebo (2020) provides an overview of this and other problems based on modern research—but that is not exactly what is usually understood by intuition in philosophy.[5]

In philosophy, discussion of intellectual intuition most often refers to Descartes, as well as to the ideas of Spinoza, Hume, and Locke, and the more complex concepts of Kant, Fichte, Schelling, and others; though, in doing so, there is more often talk about “intellectual contemplation”, which can be equated with intuition. Intuition is declared to be characterized by such features as obviousness, clarity, distinctness, reliability, fundamentality, and others. In general, Descartes’s formulation in which he calls intuition the undoubting conception of an unclouded mind (Descartes, 1989, p. 84) is still relevant, though with some reservations (taking into account the vagueness of natural language definitions). For a better understanding of intellectual intuition, we should add that it is also a specific ability common to all mankind—the ability to think using some universal principles as the basis. The existence of such intuition is indicated, for example, by the general relative unanimity in acceptance of the basic rules of thought (the laws of classical logic, which are considered to be common to all), the ability to understand and accept mathematical proofs, and the existence of a consensus on this issue. In a recent article, Chudnoff (2020) proposes to distinguish three types of intuition: intuition obtained in the course of experience; improved intuition that contravenes what is based on common sense; and guided intuition, in which an expert teaches a novice). Van-Quynh (2019), adhering to the same position, shows an analogy between mathematical intuition and ordinary (perceptive) intuition based on phenomenology.

It is important for us that mathematical intuition is more abstract due to weaker association with the directly observable reality that forms the initial intuition. However, the differences are not so fundamental, in that we can talk about the development of intuition in general, and that it can be improved. Therefore, we will ignore the division of intuition into types, not distinguishing between ordinary and mathematical intuition, believing that the latter is the result of the growth of abstraction level in the former.

What is the nature of this intuition? Is it due solely to cultural factors, the environment, or are there genetic factors that set some general principles for the formation of the human brain? Both are likely to be the case; recent brain-mapping projects (Filler, 2009) provide indirect evidence that some of the sociocultural factors that have traditionally been considered a product of social life may be of biological origin and arise in the course of evolution (requiring, nevertheless, development in the social environment): take, for instance, the capacity for abstract thinking, which is closely related to the assimilation of grammatical structures.

Leaving this complex issue aside, let us confine ourselves to some general remarks. First, intuition evolves.[6] This is easy to prove in the example of the development of mathematics. Such concepts as negative numbers, irrational numbers, or complex numbers seemed alien and incomprehensible even to medieval mathematicians in the sense that these were counterintuitive constructions. Today, it is difficult to surprise anyone with negative numbers, except for those tribes that are relatively isolated from civilization and lead a primitive way of life, where mathematics has not received any development and, for example, where there is practically no counting system (Everett, 2007). To us, negative numbers seem quite intuitive because we master the basics of mathematics at an early age. The same applies, for example, to the principles of general relativity: to scientists of the Newtonian era, they would have seemed unimaginable and counterintuitive, while to modern physicists, they seem to be completely obvious in terms of intuition (and, indeed, representable: Einstein formulated them based on geometric representability). A similar situation can probably be observed as regards quantum mechanics: a more than a century-old problem of interpreting quantum mechanics ceases to be a problem due to the gradual assimilation of its provisions as fundamental, as part of everyday (scientific) experience (for example, the adoption of Everett's interpretation of decoherence makes quantum mechanics intuitively acceptable[7]).

Thus, intuition is apparently developing, gradually embracing what was previously unimaginable and incomprehensible in the scientific worldview system. Presumably, this is a natural process.

Of course, not everything is as obvious with quantum mechanics as with general relativity. That is explainable (moreover, the explanation itself points to the fact that intuition is at least conditioned by evolution, if not fully formed by it). Indeed, in the course of evolution, man, like any other organism, adapted to environmental conditions. However, perception of macroworld objects is essential for survival: it is they (and not electrons, photons, or quarks) that influence the effectiveness of everyday decision-making and participation in natural selection. In other words, knowledge about the microworld, about its properties, and about the behaviour of elementary particles is useless from the point of view of survival in nature. The sensory organs and the corresponding brain structures of organisms are hardwired for orientation and action in the macroworld and are not intended (or are intended, but to a very small degree) for interaction with objects in the microworld.

It is logical to assume that intuition is conditioned by the same evolutionary factors. Thus, it becomes clear why quantum mechanics is usually considered counterintuitive: in fact, many of its provisions contradict classical intuition and the classical physics based thereon. But as it becomes part of experience—and quite tangible, at that: the operation of modern electronics is based on the laws of quantum field theory—it becomes intuitively acceptable.

Of course, the statement about the universal character of human intuition is debatable, which can be illustrated by the axiom of choice, the law of excluded middle, double negation elimination, infinite sets, and so on. There is no complete consensus yet concerning these constructions, and one cannot exclude the possibility that one will not be reached at all.

It is clear that there can be different intuitions—but within certain limits: there is hardly an intuition for which the statement 2+2=4 turns out to be counterintuitive, although there are objections here as well.

4. The Schools

Here we will consider some of the traditional positions in the philosophy of mathematics and try to formulate an answer to them that is consistent with our position.

The crisis in the foundations of mathematics, which arose in the context of the interpretation of Cantor's set theory, led to the emergence of various programmes establishing the foundations of mathematics. It is worth remembering that some paradoxes arose in set theory which actually indicated its inconsistency—for example, Russell's paradox, Cantor's paradox, Richard's paradox, Burali-Forti's paradox—as well as a number of other problems, such as the problem of doubling the ball as a result of applying the axiom of choice. For example, Cantor's paradox states that if the set of all sets V exists, then the cardinality of no set can exceed its cardinality. However, it turns out that the set of all subsets has a greater cardinality than the set of all sets; hence the latter cannot exist.[8] In other words, there is no greatest cardinal number. In philosophical terms, this paradox is close to the omnipotence paradox.

The three programmes designed to build mathematics on a solid foundation free from paradoxes have become known as formalism, intuitionism, and logicism (for a detailed analysis of the approaches with authentic texts of their main representatives, see van Heijenoort, 2002; Snapper, 1979).

In short, the task of the formalist movement, led by David Hilbert, was to try to reduce the foundations of mathematics to the study of formal systems. That is, he believed that for any mathematical theory it is possible to build a system of axioms and rules and set the rules of inference on the basis of which all possible theorems of this theory are derived; thus it is possible to prove its consistency and completeness. An example of such a system is Gentzen's sequent calculus. It is clear that the creation of such systems is based on the principles of logic.

Intuitionism, led by Luitzen Brouwer, considers it necessary to build mathematics on obvious intuitive foundations, as structures that are completely generated by our mind and do not exist independently. Thus, the axiom of choice was declared counterintuitive, since it does not allow constructing objects: it only says that they exist, and, according to intuitionists, mathematics must propose a way to construct the objects under study. In intuitionistic logic, double negation elimination and the law of excluded middle are questioned, and hence also proof by contradiction. Proving consistency of a theory is declared superfluous, since intuition itself does not contain contradictions; actual infinity is denied, as intuition deals only with finite sets, and so on.

Logicism (Frege, Russell, Whitehead) insists on the possibility of deriving all mathematics from logical foundations. The idea in general terms is that it is possible to build a logical system that will serve as the basis for any mathematical constructions. This can be interpreted as stating that there is some basic logical intuition (laws of thought inherent in human consciousness), and it is primary in world cognition. Here, too, there were some paradoxes: Russell's paradox shows that Cantor's set theory and Gottlob Frege's attempt to formalize it are contradictory. Russell, trying to get around the problem, introduces in particular the axiom of infinity, which, strictly speaking, is not logically justified or intuitively acceptable (at least not for everyone—it requires the existence of an infinite set).

As for the formalist programme, the hope of proving the completeness and consistency of any correct system based on formal arithmetic turned out to be unrealizable, as is well known. Completeness of a theory requires that for every statement that is provable in this theory, either A or the negation of A can be derived. Consistency requires that statements A and *not A* should not be simultaneously derived in the theory. Consistency is so important because derivability in the theory of contradictions leads to the fact that any statement in this theory becomes true. (In classical logic, anything follows from a contradiction: if the antecedent is false, then the statement is true for any consequent, and the contradiction is always false—that is statements *A* and *not A* are an always-false formula.)

Kurt Gödel showed in 1930 (see new edition; Feferman, 2001), firstly, that if formal arithmetic does not contain contradictions, then it has a formula that is underivable and true (irrefutable). This actually indicates the incompleteness and insolvability (absence of an algorithm for proving any true theorem in the system) of a consistent theory. Secondly, he showed that if formal arithmetic is consistent, then it is impossible to derive a formula asserting its consistency. So, consistency cannot be proved, and formal arithmetic is the basis of any mathematical theory (where natural numbers, addition, and multiplication must be defined); thus, this is a problem of all mathematics, and Hilbert's programme—the second problem in Hilbert's list—turns out to be impossible (Hilbert, 1902). Gerhard Gentzen's proof of the consistency of arithmetic using primitive recursive arithmetic requires an additional axiom for transfinite induction and does not solve the problem of completeness (Kleene, 2012).

The intuitionistic programme on the whole turned out to be very difficult in terms of proofs due to the restrictions it introduced. It places severe restrictions on the area of the unprovable (including what has already been proved) and has drawn criticism due to its counter-intuitiveness and its disputes over what is to be considered intuitive and what is not.

In general, though, such statements as the law of excluded middle may indeed seem counterintuitive: it states that either *A* or *not A* is true, regardless of what *A* is—i.e. regardless of whether we can define and construct it. If we cannot, then, from the point of view of intuitionism, it is neither true nor false until it is proven true. However, for most logicians and mathematicians, any statement is still true or false regardless of our knowledge of it. Generally speaking, this leads us to mathematical Platonism (see Tieszen, 2011, which discusses the emergence of mathematical Platonism and its roots in the philosophy of Plato, Leibniz and Husserl, and Gödel's attitude to Kant's position), which is not acceptable for intuitionism.

Nevertheless, it would be wrong to dismiss intuitionism as an incident . The intuitionistic logic built on its basis turned out to be quite efficient, and the requirement that the objects in question be constructible is welcomed in some important sections of modern mathematics.

One of the problems that gave rise to the abovementioned approaches was the problem of consistency. By itself, the requirement that a theory should not produce theorems that contradict each other seems quite intuitively acceptable. But, as already noted, in classical propositional logic, there is the law that anything follows from a contradiction. Thus, if there is a contradiction in a theory, any formula in it is a theorem (any one that can be constructed according to the definition of a formula in this theory). This position no longer seems intuitively obvious: why should anything follow from a contradiction from a substantive point of view? (It is, though, from a formal point of view, easy to prove in classical logic; as already noted, with a false antecedent, the whole formula will be a theorem).

5. Logic and Intuition

The question arises: how does classical logic correlate with physical reality? And can there be situations (worlds) that allow contradictions?

There is an infinite number of contradictory formal systems, which are often referred to as paraconsistent logic, the prefix *para-*, of course, not implying any mystical content in this case. Szmuc et al.'s (2018) collection of articles *Contradictions, from consistency to inconsistency*, devoted to the problems of contradiction and consistency, gives an overview of the nature of such systems. These are logical systems in which the law of non-contradiction is not a law. Therefore, nothing follows from a contradiction in them. Obviously, such logics describe worlds in which contradictions are admissible. Certain facts and negations of those facts can occur simultaneously, but that does not lead to triviality.[9]

Interestingly enough, one can obtain paraconsistent logic from any multi-valued logic:

> The simplest and most illustrative way to construct paraconsistency is to add a third truth value *S*, interpreted by different authors as "antinomical", "paradoxical", or "contradictory", to the two classical truth values 1 (true) and 0 (false). Taking 1 and *S* as designated truth values, and taking *S* as a fixed point, which allows us to define negation as $\neg(S) = S$, we have a case where $(p \ \& \ \neg p)$ takes a designated value. Obviously, these logics, like the classical one, are truth-functional. It is not difficult to construct such three-valued logical matrices in which the *modus ponens* rule works, but Duns Scotus's law does not (Karpenko, 2001, web).

The same applies to relevant and modal logics: it is easy (and natural) to construct paraconsistent logics in them (see Jaśkowski's discursive logic, which some consider the first consistent logic historically; Jaśkowski, 1969).

In all these systems, one can consider situations (choosing appropriate interpretations) when *A* and *not A* occur, but, say, *B* does not, and thus there is a contradiction (*A and not A* is true), but one cannot say that anything follows from it.

A separate question is how negation should be interpreted. This problem arises in intuitionism in connection with double negation elimination: what exactly is meant by negation and can negations be semantically different? For example, the statement "It is not true that it is not raining" in classical propositional logic is equivalent to the statement "It is raining". Intuitionists do not agree: maybe there is some third option. From here, it is one step to multi-valued logic and the transition to paraconsistency.

However, here we will concentrate on another problem, closer to our objective. The question we are primarily interested in is what world (worlds) in the physical sense can be described by this or that logical theory and the corresponding mathematical structure.

6. Possible Solution

The approach to solving the problem that we propose is based on multiverse physical concepts. In this case, it does not really matter on which ones exactly; for our purposes, we can use Everett's many-worlds interpretation (Everett, 2015), the chaotic inflation scenario (Linde, 1982, 1983), the string landscape model (Susskind, 2003), the brane world scenario (Yau & Nadis, 2010), and others. Though all of these multiverse models are certainly hypothetical, and no experimental verification is expected to confirm any of them, this does not matter so much here, since we are discussing the problem in the context of logical and mathematical approaches (and the corresponding intellectual intuition), and all these hypotheses are consequences of working mathematical theories, some of which do not raise any doubts about their adequacy and effectiveness due to their compliance with the experimental criterion.

Multiverse theories assume the existence of a set of worlds (it can be finite or infinite, as, for example, in the chaotic universe model) which realize all statistically possible states of reality (for example, in the many-worlds interpretation). These states are a reflection of the possible outcomes of events and ways of realization of the known laws of physics, and, more broadly, ways of realization of the laws of nature—i.e. the fundamental principles that underlie any possible universe. Thus, if we take our universe as the starting point, then among the many worlds we can find duplicates of our universe, worlds that differ to a certain extent, and fundamentally different worlds with other fundamental physical principles.

Models that claim to be extremely broad descriptions of reality (superstring theory, M-theory, chaotic inflation theory, and the string landscape model combining these approaches) describe sets of possible worlds and do not have tools in their arsenal for binding the theory specifically to our world (Karpenko, 2017). They speak about the multiple possible universes as equal in their rights, not describing only one kind of universe: therefore, the characteristics of our universe in particular (for example, the properties of the observed elementary particles) are not deduced in those theories; they have to be substituted into the equations manually within certain models (e.g. in the standard model of particle physics). An experiment does not play the traditional role in such a theory because in different worlds, depending on different initial physical conditions, it will give different results.[10]

In the context of this study, it is important that among the many worlds, there may be such worlds in which conflicting events occur. For example, the statement "Intelligent life exists" is true in world *N*, but it is false in world *M*. Suppose there is some theory *T* of a set of worlds which is not bound to the description of any one of the worlds (as, for example, Newton's mechanics is bounds to ours), and it describes a variety of worlds, including *N* and *M*. In that theory, a conjunction of these statements ("Intelligent life exists" and "Intelligent life does not exist") will seem, of course, a contradiction (*A* and *not A*). However, in the theory *T*, it will be true.[11] Does anything follow from it? Obviously not.

7. Platonism, Contradiction, and Intuition

Simplifying somewhat, Plato's theory of ideas (developed in his dialogues, the *Timaeus*, the *Parmenides*, and the *State*) can be expressed as follows: there are ideal prototypes of things in the observed world (*eidos*), and it is they that really exist, not the observed world. The latter is rather a shadow, a reflection of the world of ideas (although this is not entirely correct, because in some places it is argued that there is no connection between the world of ideas and the world of shadows, and therefore knowledge should be directed not to the study of the properties of the observed world but to the ideal world). It is this notion—that cognition should be directed to the world of ideas, bypassing the sensorially cognizable—that appeals so much to many mathematicians. Modern adherents to mathematical Platonism regard only mathematical objects as ideas, and it is their combinations into various structures that give rise to all the diversity of what we perceive sensorially. Extensive literature is devoted to the issue of mathematical Platonism. We proceed from the formulations set out in the classic work by Balaguer (1998), where anti-Platonic positions are also considered, as well as from a more modern and detailed monograph (Panza & Sereni, 2013): mathematical Platonism can be considered precisely as the maximum level of abstraction, moving from observed phenomena to their mathematical prototypes. In his important article, Côté (2013), considering the counterintuitive properties of such an abstraction as infinity, proves that they lead to mathematical Platonism. He also shows how the presence of abstract infinity explains why there is something in the universe instead of nothing, thus solving the old Leibnizian problem in an original way.

Plato, though, has an intermediary between the ideal world and the sensible world—the space, which is necessary for practising geometry. Modern mathematical Platonists do not need such an intermediary; for them, mathematical structures are the reality that stands behind the visible reality, and these structures are abstractions that are not connected with anything sensorially perceived: they do not look like anything; they are completely abstract ideas. Obviously, for the mathematical Platonist there is no problem of cognizability of things in themselves, because noumena are comprehended by mathematical intuition.

The mathematical universe hypothesis, proposed by Mac Tegmark (1988) as the basis for a "theory of everything", adheres to this very position—to the reality of the variety of mathematical structures that can be considered as bits of information describing this or that virtual reality (everything observed, sensorially perceived, turns out to be virtual in this sense). Our mind in this case is able to perceive pure ideas (mathematics) but for some reason transforms them into what we call the sensorially perceived.

Tegmark (1988) proposes a model in which the multiverse is realized as a set of different mathematical structures. There are many such structures, probably an infinite number, since each mathematical structure can be considered a kind of universe.

In this context, the question arises: is mathematics as a means of describing reality universal or not? If the answer is positive—i.e. there is universal mathematics, as well as universal intellectual intuition, then the entire multiverse is cognizable within the framework of universal intuition. However, this has nothing to do with the requirement for consistency. That is, different mathematical structures in the multiverse can contradict each other and be incompatible within the same universe.

In other words, the consistency criterion can be considered as a requirement that implies being bound to describing the directly observed reality, which gives consistent results in an experiment. From the point of view of the multiverse, however, the requirement for consistency is unreasonable—just a convenient limitation for practical purposes.

Let us take an example from formal arithmetic: 2+2=4. Contradictory to it will be the statement "It is not true that 2+2=4", which does not mean at all that 2+2 must not be equal to only 4 and nothing else. It only says that 2+2 does not equal 4, but this does not mean that the sum cannot be equal to something else—for example, 5. That is, the statement "2+2=5" is not contradictory to "2+2=4." What is it contradictory to? Only to the statement "It is not true that 2+2=5".

These are very important nuances for understanding the essence of contradiction in general. The statement "This person is alive and dead" does not contain any contradiction, only an opposition of states, while the statement "This person is alive and this person is not alive" does contain a contradiction; it is false in classical propositional logic. Furthermore, one can argue whether "dead if and only if not alive" is a tautology, which is debatable, because different interpretations are possible. That is, the question is, how do truth functions correlate with various states? Quantum logic deals with states and not with truth functions—that is, in our example, with the states "alive" and "dead", as well as with superpositions ("alive and dead", "not alive and not dead"), which are between these extreme states, but not with questions like: "Is the statement 'an object is alive and not alive' true?" However, in quantum mechanics, the answer to such a question is obvious: "Yes, it is true" (that is a superposition), which in the many-worlds interpretation corresponds to intuition.

8. Many-Worlds Interpretation

If we adhere to the Copenhagen interpretation of quantum mechanics, with the collapse of the wave function, then the usual connection of the observed facts to a specific universe—the one in which the experiment is being set—is preserved. That is, the experiment shows the correspondence of the theory to the observed reality: in this case, the theory must predict the results of the observation. There is one physical reality—our world, with its set of laws of nature—and the theory describes it. The collapse of the wave function is the result of observing the world. The question remains: why do the objects of the microworld, when the wave function collapses, choose one single location out of many possible ones, although, according to the Schrödinger equation, before the act of measurement they were in a superposition—that is, in all possible places at once? This interpretation lacks an explanation, an answer to the question of why. The answer for the Copenhagen interpretation is "for no reason". This is how reality behaves, this is its property, and our attempts to explain it do not point to limitations of the theory but to limitations (underdevelopment) of our intellectual intuition, which is looking for classical explanations where they do not exist. In other words, you do not need to ask questions that do not make sense—you just need to describe the results of the experiment.

However, it seems to us that an explanation is still possible. The introduction of the wave function of the universe, which is proposed in Everett's many-worlds interpretation in 1957, provides such an explanation. All outcomes occur and there is no collapse of the wave function;

what we observe as a collapse is the realization of a specific outcome from a set of outcomes in parallel worlds. If we add decoherence—the interaction of the quantum mechanical system with the environment leading to violation of coherence, as a result of which it acquires the features of a classical (macroscopic) system, first described by Zeh (1970)—then the problem of contradiction between the behaviour of the microworld and the macroworld is also solved.

What does this mean in the context of the problem we are considering? The Schrödinger equation describes the evolution of the system in time; the set of its states is a superposition of all possible outcomes. Quantum mechanical probability does not coincide with classical probability: it allows for a kind of "mixing" of states—that is, not "the electron is either here or there", but "the electron is both here and there".

Let us consider the requirement for consistency in this context. Contradictory to the statement "The electron is at point *A*" will be the statement "It is not true that the electron is at point *A*". Actually, a conjunction of these statements will not be true in any specific universe but will be true—recall that a conjunction is only true when both of its terms are true—in the multiverse of the many-world interpretation. In a universe *X*, which differs from *Y* by nothing else but the location of the electron *N*, electron *N* is found at point *A*, while in universe *Y*, it is not found at point *A*. Does any statement follow from this conjunction? (We recall that in classical logic anything follows from a contradiction.) Obviously not. But some true statement may follow from it.

The statement "It is not true that the electron is at point *A*" can be reformulated as "The electron is not at point *A*"; thus the conjunction becomes "The electron is at point *A* and the electron is not at point *A*". This contradiction is perhaps closer to quantum superposition and is also true.

9. Conclusion

In classical propositional logic, "Intelligent life exists and intelligent life does not exist; therefore, a round square exists" is true; however, one cannot apparently speak of the intuitive acceptability of such truths, at least because there is no connection between the premise and the conclusion. Relevant logic tries to get around this by reasonably demanding a connection between the antecedent and the consequent, which under certain conditions lead to a paraconsistent system which may seem intuitively acceptable only given the multiverse. In a set of possible worlds, the statement "Intelligent life exists and intelligent life does not exist" is true, and no unrelated statement must follow from it, especially because, if it is false, then the result will be false (since there is no reason to doubt the falsity of the formula with its true antecedent and false consequent). In multiverse theories, the statement "Intelligent life exists and intelligent life does not exist; therefore, a round square exists" is false, which, in our opinion, is more intuitively acceptable.

With the acceptance of multiverse concepts as they are presented in modern mathematics and physics, the requirement for consistency begins to look like a requirement for proximity to the description of the observable universe, since the law of consistency must apparently be observed within a single universe, although it is not clear whether that is obligatory for any possible universe. Rejection of the need for such a correspondence to the observed reality is the development of intellectual intuition, in the direction of increasing the level of abstraction, since multiverse theories are basically mathematical abstractions.

Logic based on quantum superpositions can be regarded as one contender for the description of such a reality: it is somewhat close to intuitionism—there are no laws of excluded middle in it, and the requirement for consistency is abandoned as restrictive.

Whether it is possible to build such a logic, which would form the basis for a "theory of everything" describing all possible worlds, is not yet clear. Studies in the field of quantum logic touch upon only some of the aspects of the problem that are discussed here.[12] This paper has aimed to show that consistency is not an unconditional intuitive condition reflecting the laws of nature. Rather, it is due to the applied functional nature of the theories focused on the description of the observable universe and the limitations of the corresponding intellectual intuition.

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Notes

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2. Extensive literature is devoted to the issue of multiplicity of worlds. The history of the issue from Antiquity to modern times is considered, for example, in Vizgin (2007), as well as in the classic monographs (Lewis, 2001; Pruss, 2011), where the problem is considered more generally. Talking about more modern approaches not considered in the indicated sources, we will refer to current studies in the field of philosophy, physics, and mathematics.

3. A recent collection of works edited by Lektorskii (2017) and devoted to the problem of realism in philosophy examines approaches from the standpoints of philosophy of science, epistemology, and philosophy of consciousness.

4. All mathematics can be regarded as imaginary if one does not adhere to mathematical Platonism, as well as a significant part of physics; for example, the key concept of quantum mechanics, the wave function, is a mathematical abstraction. For quantum realism, see Terekhovich (2019).

5. However, the article (Tieszen, 2015) shows the connection between mathematics and philosophy in the matter of intuition: the mathematical intuition of Kurt Gödel turns out to be derivative of the philosophy of Husserl and a number of other philosophers. See also Weinberg et al. (2001).

6. Porus (2021) essentially says this in his article on historical epistemology and the need for reform of the philosophic theory of knowledge, but in other terms, citing the evolution of intellectual intuition as the condition.

7. Though, of course, not for everyone.

8. A possible solution is to abandon the principle of abstraction, which states that for any property P there is a set that consists of those and only those objects that have this property P.

9. Triviality is observed when the set of formulas and theorems of the theory coincides (this is exactly what will happen with theories built in classical logics if contradictory theorems are derived in them).

10. But the experiment is important for finding out the parameters of the particles of our universe and substituting them into the theory. See also Pronskikh (2021) on the problem of reproducibility in a scientific experiment.

11. Since the theory describes all possible worlds and the totality of facts about them. Of course, within one particular world, such a statement will be false.

12. The first attempts were made quite a long time ago (see, for example, Birkhoff and von Neumann, 1936); a more recent work is the monograph by Vladimir Vasyukov (2005).