AFFINE CONES AS IMAGES OF AFFINE SPACES

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To Yuri Prokhorov on the occasion of his 60th birthday

Abstract. We prove that an affine cone X admits a surjective morphism from an affine space if and only if X is unirational.

1. Introduction

We work over an algebraically closed field $\mathbb{K}$ of characteristic zero. Recall that an irreducible algebraic variety X is *rational* if the field of rational functions $\mathbb{K}(X)$ is isomorphic to the field of rational fractions $\mathbb{K}(x_1, \ldots, x_n)$, and X is *unirational* if the field $\mathbb{K}(X)$ is a subfield the field of rational fractions $\mathbb{K}(y_1, \ldots, y_s)$ for some positive integer s; see [15, Chapter III]. Geometrically speaking, rationality means that there is a birational map $\mathbb{A}^n \dashrightarrow X$ and unirationality means that there is a dominant rational map $\mathbb{A}^s \dashrightarrow X$.

Assume that there is a surjective morphism $\mathbb{A}^m \to X$ for some positive integer m. Then the variety X is irreducible, unirational, and $\mathbb{K}[X]^\times = \mathbb{K}^\times$. One may expect that the converse implication also holds.

Conjecture 1. *Let X be a unirational algebraic variety with $\mathbb{K}[X]^\times = \mathbb{K}^\times$. Then there is a surjective morphism $\mathbb{A}^m \to X$ for some positive integer m. Moreover, one may take $m = \dim X + 1$ or even $m = \dim X$.*

In [1], several results confirming this conjecture are obtained. Namely, it is shown that every non-degenerate toric variety, every homogeneous space of a connected linear algebraic group without non-constant invertible regular functions, and every variety covered by affine spaces admits a surjective morphism from an affine space. Further, it is proved in [4, Theorem 1.7] that Conjecture 1 holds for any complete variety X, and it follows from a result of Kusakabe [13] that the number m can be taken $n+1$, where $n = \dim X$. Moreover, if $\mathbb{K} = \mathbb{C}$, then by a result of Forstnerič there is a surjective morphism $\mathbb{A}^n \to X$; see [7, Theorem 1.6].

The proof of [4, Theorem 1.7] is based on the concept of an elliptic algebraic variety in the sense of Gromov [8]; see [7, 4, 10, 11, 13] for more information on elliptic varieties. More precisely, let us recall that an algebraic variety X is *uniformly rational* if for any point $x \in X$ there is an open neighborhood X_0 of x in X isomorphic to an open subset of $\mathbb{A}^n$. Clearly, any uniformly rational variety is smooth and rational. We prove in [4, Theorem 3.3] that any complete uniformly rational variety is elliptic. It follows from Chow's Lemma and Hironaka's Theorem on elimination of indeterminacy that for any complete unirational variety X there is a surjective morphism $\widetilde{X} \to X$ from a uniformly rational complete

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variety $\widetilde{X}$. Kusakabe [13] proved that any elliptic variety $\widetilde{X}$ admits a surjective morphism $\mathbb{A}^m \to \widetilde{X}$ with $m = \dim \widetilde{X} + 1$, and [4, Theorem 1.7] follows.

Moreover, [4, Theorem 3.3] claims that the punctured cone $\widehat{Y}$ over a uniformly rational subvariety $X \subseteq \mathbb{P}^r$ is elliptic, where the punctured cone $\widehat{Y}$ is the affine cone Y over X with the vertex removed. In particular, there is a surjective morphism $\mathbb{A}^{n+2} \to \widehat{Y}$, where $n = \dim X$. Note that $\widehat{Y}$ is a smooth quasi-affine, but not affine variety.

The aim of this note is to prove Conjecture 1 for a class of affine varieties. We say that a closed irreducible subvariety $Y \subseteq \mathbb{A}^k$ is an *affine cone* if Y is stable under scalar multiplications. Equivalenlty, Y is the affine cone over an irreducible closed subvariety $X \subseteq \mathbb{P}^{k-1}$.

Theorem 1. *An affine cone $Y \subseteq \mathbb{A}^k$ admits a surjective morphism $\mathbb{A}^m \to Y$ for some positive integer m if and only if Y is unirational or, equivalently, its projectivization X is unirational. Moreover, one may take $m = \dim Y + 1$.*

Let us come to an algebraic version of the results discussed above. Given a commutative associative algebra A, one may ask whether elements of A can be expressed as polynomials in finitely many algebraically independent variables. Or, equivalently, can the algebra A be realized as a subalgebra of the polynomial algebra $\mathbb{K}[x_1, \ldots, x_m]$ for some positive integer m. Necessary conditions for such a realization are absence of zero divisors and absence of non-constant invertible elements. A more delicate necessary condition is that A is embeddable into the field of rational fractions $\mathbb{K}(y_1, \ldots, y_s)$ for some positive integer s. When A is finitely generated, this condition means that the affine variety $Y := \operatorname{Spec} A$ is unirational.

Let us say that a subalgebra $A \subseteq B$ is *proper* if any maximal ideal in A is contained in a maximal ideal of B. If A and B are finitely generated, this condition means that the corresponding morphism of affine varieties $\operatorname{Spec} B \to \operatorname{Spec} A$ is surjective. Denote by $\operatorname{tr.deg} A$ the transcendence degree of an algebra A.

Conjecture 1 for affine varieties can be reformulated as follows.

Conjecture 2. *Let A be a finitely generated subalgebra without non-constant invertible elements in the field of rational fractions $\mathbb{K}(y_1, \ldots, y_s)$ for some positive integer s. Then A can be properly embedded into the polynomial algebra $\mathbb{K}[x_1, \ldots, x_m]$. Moreover, the number m can be taken* $\operatorname{tr.deg} A + 1$ *or even* $\operatorname{tr.deg} A$.

Clearly, an affine variety Y can be realized as an affine cone in some affine space $\mathbb{A}^k$ if and only if the algebra $A := \mathbb{K}[Y]$ admits a $\mathbb{Z}_{\geqslant 0}$-grading such that A is generated by elements of degree 1.

We come to the following algebraic reformulation of Theorem 1.

Theorem 2. *Let A be a finitely generated subalgebra in the field $\mathbb{K}(y_1, \ldots, y_s)$ for some positive integer s. Assume that A admits a $\mathbb{Z}_{\geqslant 0}$-grading such that A is generated by elements of degree 1. Then A can be properly embedded into the polynomial algebra $\mathbb{K}[x_1, \ldots, x_m]$ with $m = \operatorname{tr.deg} A + 1$.*

It is interesting to find out whether Theorem 2 holds for not finitely generated subalgebras, over non-closed fields and in positive characteristic.

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2. Proof of Theorem 1

Since the "only if" part of Theorem 1 is clear, we have to prove the "if" part. Let $Y \subseteq \mathbb{A}^k$ be a unirational affine cone and $X \subseteq \mathbb{P}^{k-1}$ be its projectivization. Consider the quotient morphism $\pi\colon \mathbb{A}^k \setminus \{0\} \to \mathbb{P}^{k-1}$ by the one-dimensional torus T of scalar matrices and its restriction $\pi\colon Y \setminus \{0\} \to X$.

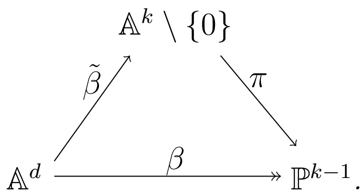
By Hilbert's Theorem 90, the variety Y contains an open subset U isomorphic to $T \times V$, where V is an open subset of X; see [14, Section 2.6]. So we have

$$\mathbb{K}(X) = \mathbb{K}(V) \subseteq \mathbb{K}(T \times V) = \mathbb{K}(U) = \mathbb{K}(Y) \subseteq \mathbb{K}(y_1, \ldots, y_s),$$

and the variety X is unirational.

Since X is complete, we deduce from [4, Theorem 1.7] that there is a surjective morphism $\beta\colon \mathbb{A}^d \to X$, where $d = \dim X + 1$. This morphism can be considered as a morphism $\beta\colon \mathbb{A}^d \to \mathbb{P}^{k-1}$, whose image is X.

Lemma 1. *For any morphism $\beta\colon \mathbb{A}^d \to \mathbb{P}^{k-1}$ there is a morphism $\tilde{\beta}\colon \mathbb{A}^d \to \mathbb{A}^k \setminus \{0\}$ such that the following diagram is commutative:*



Proof. Let $[z_1 : \ldots : z_k]$ be homogeneous coordinates on $\mathbb{P}^{k-1}$ and let $\mathbb{P}^{k-1} = \bigcup_{i=1}^k U_i$ with $U_i = \{z_i \neq 0\}$ be the standard affine covering. We may assume that $\beta(\mathbb{A}^d) \cap U_1 \neq \emptyset$. Then $f_i =: \beta^*(z_i/z_1)$ is a rational function on $\mathbb{A}^d$. Multiplying the presentation $[1 : f_2 : \ldots : f_k]$ by the denominators of f_i we come to the presentation $[h_1 : h_2 : \ldots : h_k]$. We may assume that the polynomials $h_1, h_2, \ldots, h_k$ are coprime.

Let us check that the polynomials $h_1, h_2, \ldots, h_k$ have no common zero. Assume that

$$h_1(c) = h_2(c) = \ldots = h_k(c) = 0$$

for some $c \in \mathbb{A}^d$. Let $\beta(c) \in U_i$ for some $1 \leqslant i \leqslant k$ and $p(x)$ be an irreducible divisor of $h_i(x)$ such that $p(c) = 0$. Then there is $j \neq i$ such that $p(x)$ does not divide $h_j(x)$. Then the function $\beta^*(z_j/z_i) = h_j(x)/h_i(x)$ is not regular at c, a contradiction. This proves that the polynomials $h_1, h_2, \ldots, h_k$ define the desired morphism $\tilde{\beta}\colon \mathbb{A}^d \to \mathbb{A}^k \setminus \{0\}$. □

Let us return to the original morphism $\beta\colon \mathbb{A}^d \to X$ and let the morphism $\tilde{\beta}\colon \mathbb{A}^d \to \mathbb{A}^k \setminus \{0\}$ be given by polynomials $h_1, \ldots, h_k \in \mathbb{K}[\mathbb{A}^d]$, which have no common zero. We consider the morphism

$$\gamma\colon \mathbb{A}^{d+1} = \mathbb{A}^d \times \mathbb{A}^1 \to \mathbb{A}^k, \quad (x, z) \mapsto (h_1(x)z, \ldots, h_k(x)z).$$

Then the image $\gamma(\{a\} \times \mathbb{A}^1)$ is a line in $\mathbb{A}^k$ corresponding to the point $\beta(a) \in X \subseteq \mathbb{P}^{k-1}$ for any $a \in \mathbb{A}^d$. We conclude that the image of γ is Y. So we obtain a surjective morphism $\gamma\colon \mathbb{A}^m \to Y$ with

$$m = d + 1 = \dim X + 2 = \dim Y + 1.$$

This completes the proof of Theorem 1.

Remark 1. It follows from the proof given above that if $X \subseteq \mathbb{P}^{k-1}$ is a locally closed subset that admits a surjective morphism from an affine space, then the same holds for the cone Y over X in $\mathbb{A}^k$, where Y is considered as a quasi-affine variety.

Remark 2. The referee observed that Lemma 1 can be proved in another way. Namely, the morphism $\pi\colon \mathbb{A}^k \setminus \{0\} \to \mathbb{P}^{k-1}$ is a fiber bundle with fiber $\mathbb{A}^1 \setminus \{0\}$. It can be obtained from the tautological line bundle $\mathcal{O}_{\mathbb{P}^{k-1}}(-1)$ over $\mathbb{P}^{k-1}$ by deleting the zero section. The line bundle $\beta^*(\mathcal{O}_{\mathbb{P}^{k-1}}(-1))$ over $\mathbb{A}^d$ induced by the morphism $\beta\colon \mathbb{A}^d \to \mathbb{P}^k$ is trivial, since $\mathrm{Pic}(\mathbb{A}^d) = 0$. Hence, the latter line bundle admits a non-vanishing section. This yields the desired lift $\widetilde{\beta}$ of the morphism β.

Remark 3. By [4, Theorem 3.3], the punctured cone $\widehat{Y}$ over a uniformly rational subvariety $X \subseteq \mathbb{P}^r$ is elliptic. This implies that there is a surjective morphism $\alpha\colon \mathbb{A}^{n+2} \to \widehat{Y}$, where $n = \dim X$. Assume that α is given by polynomials $h_0, \ldots, h_r$. Consider the morphism

$$\alpha'\colon \mathbb{A}^{n+3} = \mathbb{A}^{n+2} \times \mathbb{A}^1 \to \mathbb{A}^{r+1}, \quad (x, z) \mapsto (h_0(x)z, \ldots, h_r(x)z).$$

Then the image of α' is the affine cone Y over the subvariety $X \subseteq \mathbb{P}^r$. Hence [4, Theorem 3.3] implies a weaker version of Theorem 1.

Remark 4. One may try to generalize Theorem 1 to affine varieties Y that are closures in $\mathbb{A}^k$ of preimages of closed unirational subvarieties X in a weighted projective space $\mathbb{P}(d_1, \ldots, d_k)$ under the quotient morphism $\pi\colon \mathbb{A}^k \setminus \{0\} \to \mathbb{P}(d_1, \ldots, d_k)$ by the one-dimensional diagonal torus $(t^{d_1}, \ldots, t^{d_k})$. But the problem is that Lemma 1 does not hold in this case.

Indeed, let us consider the weighted projective plane $\mathbb{P}(1, 1, 2)$ and its open affine chart $U_3 = \{z_3 \neq 0\}$ with coordinates $(z_1^2/z_3, z_1z_2/z_3, z_2^2/z_3)$. Note that U_3 is a quadratic cone given by the equation $b^2 = ac$. Take the morphism

$$\beta\colon \mathbb{A}^3 \to U_3 \subseteq \mathbb{P}(1, 1, 2)$$

given by

$$(x_1, x_2, x_3) \mapsto (x_1^2x_3, x_1x_2x_3, x_2^2x_3).$$

The preimage $\pi^{-1}(U_3)$ is the principal open subset W_3 in $\mathbb{A}^3 \setminus \{0\}$ given by $z_3 \neq 0$. Assume that there is a desired morphism $\tilde{\beta}\colon \mathbb{A}^3 \to \mathbb{A}^3 \setminus \{0\}$. This is in fact a morphism from $\mathbb{A}^3$ to W_3. Since the function z_3 is invertible on W_3, its image $\tilde{\beta}^*(z_3)$ is a nonzero constant λ on $\mathbb{A}^3$. So the morphism $\tilde{\beta}$ is given as

$$(x_1, x_2, x_3) \mapsto (h_1(x_1, x_2, x_3), h_2(x_1, x_2, x_3), \lambda),$$

with some polynomials h_1 and h_2. On the first coordinate of the equality $\pi(\tilde{\beta}(x)) = \beta(x)$ we have $h_1(x_1, x_2, x_3)^2/\lambda = x_1^2x_3$, a contradiction.

3. Examples and applications

We begin with some examples of unirational affine cones.

Example 1. Consider a hypersurface Y in $\mathbb{A}^{2n}$ given by the equation

$$x_1^{k_1} + \ldots + x_n^{k_n} + x_1y_1 + \ldots + x_ny_n = 0$$

for some positive integers $k_1, \ldots, k_n$. The automorphism

$$(x_1, \ldots, x_n, y_1 - x_1^{k_1-1}, \ldots, y_n - x_n^{k_n-1})$$

of $\mathbb{A}^{2n}$ sends Y to a quadratic cone, so the variety Y is rational. By Theorem 1, there is a surjective morphism $\mathbb{A}^{2n} \to Y$.

Example 2. Let us consider the class of so-called trinomial hypersurfaces. Fix a partition $k = n_0 + n_1 + n_2$ with $n_0, n_1, n_2 \in \mathbb{Z}_{>0}$. For every $i = 0, 1, 2$ let $l_i := (l_{i1}, \ldots, l_{in_i}) \in \mathbb{Z}^{n_i}_{>0}$ and define a monomial

$$\mathbf{x}_i^{l_i} \ := \ x_{i1}^{l_{i1}} \cdots x_{in_i}^{l_{in_i}}.$$

The hypersurface

$$\mathbf{x}_0^{l_0} + \mathbf{x}_1^{l_1} + \mathbf{x}_2^{l_2} = 0$$

in $\mathbb{A}^k$ is called a *trinomial hypersurface*. Let $\mathfrak{l}_i := \gcd(l_{i1}, \ldots, l_{in_i})$. The following result characterizes rational trinomial hypersurfaces.

Proposition 1. [2, Proposition 5.5] *A trinomial hypersurface Y is rational if and only if one of the following conditions holds:*

(1) *there are pairwise coprime positive integers c_0, c_1, c_2 and a positive integer s such that, after suitable renumbering, one has*

$$\gcd(c_2, s) = 1, \quad \mathfrak{l}_0 = sc_0, \quad \mathfrak{l}_1 = sc_1, \quad \mathfrak{l}_2 = c_2;$$

(2) *there are pairwise coprime positive integers c_0, c_1, c_2 such that*

$$\mathfrak{l}_0 = 2c_0, \quad \mathfrak{l}_1 = 2c_1, \quad \mathfrak{l}_2 = 2c_2.$$

Any trinomial hypersurface Y carries an effective action of a torus T of dimension $k-2$; see [2, Section 2]. By Hilbert's Theorem 90, the variety Y contains an open subset isomorphic to $T \times C$, where C is a curve. Since C is unirational if and only if C is rational, we conclude that Y is unirational if and only if Y is rational.

Clearly, Y is an affine cone in $\mathbb{A}^k$ if and only if

$$\sum_{j=1}^{n_0} l_{0j} = \sum_{j=1}^{n_1} l_{1j} = \sum_{j=1}^{n_2} l_{2j}.$$

By Theorem 1, in this case Proposition 1 gives necessary and sufficient conditions for Y to admit a surjective morphism from an affine space.

For a more general class of trinomial affine varieties (see [2, Construction 1.1] for a definition), a similar criterion of rationality is given in [2, Corollary 5.8]. All the arguments used above work in this case as well.

Example 3. Nowadays many examples of unirational but not rational varieties are known. Among the first examples were hypersurfaces built in the work of Iskovskikh and Manin [9]. It follows from this work that the affine cone Y in $\mathbb{A}^5$ given by

$$x_1^4 + x_2^4 + x_3^4 + x_4^4 + x_1x_5^3 + x_4^3x_5 - 6x_2^2x_3^2 = 0$$

is unirational but not rational. We conclude that there is a surjective morhism $\mathbb{A}^5 \to Y$.

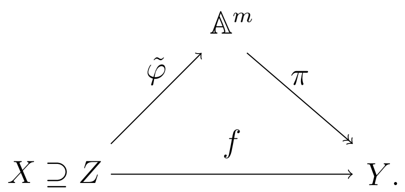
Let us mention some consequences of the fact that a variety admits a surjective morphism from an affine space.

Let X be an algebraic variety. We say that the monoid of endomorphisms $\mathrm{End}(X)$ acts on X *infinitely transitively*, if for any finite subset $Z \subseteq X$ and any map $f\colon Z \to X$ there is an endomorphism $\varphi \in \mathrm{End}(X)$ such that $\varphi|_Z = f$. This property is a version of the infinite transitivity property for the special automorphism group, see e.g. [3].

If Y is an affine variety that admits a surjective morphism from an affine space, then the monoid $\mathrm{End}(Y)$ acts on Y infinitely transitively; see [13, Corolalry 1.5] or [11, Proposition 5.1], where a more general result on infinite transitivity on zero-dimensional subschemes is proved. Let us give the following elementary lemma, which implies that the monoid $\mathrm{End}(Y)$ is infinitely transitive on Y. We learned this lemma from Mikhail Zaidenberg.

Lemma 2. *Let Z be a finite subset of a quasi-affine variety X and Y be an algebraic variety that admits a surjective morphism from an affine space. Then for any map $f\colon Z \to Y$ there is a morphism $\varphi\colon X \to Y$ such that $\varphi|_Z = f$.*

Proof. Let $Z = \{z_1, \ldots, z_k\}$ and $\pi\colon \mathbb{A}^m \to Y$ be a surjective morphism. Given $f\colon Z \to Y$, fix $a_1, \ldots, a_k \in \mathbb{A}^m$ with $\pi(a_i) = f(z_i)$:



Consider $h_j \in \mathbb{K}[Z]$, $j = 1, \ldots, m$, where $h_j(z_i)$ is the jth coordinate of a_i. There are $f_j \in \mathbb{K}[X]$ with $f_j|_Z = h_j$, and the functions $f_1, \ldots, f_m$ define a morphism $\tilde{\varphi}\colon X \to \mathbb{A}^m$ with $\tilde{\varphi}(z_i) = a_i$. Then with $\varphi := \pi \circ \tilde{\varphi}\colon X \to Y$ we have $\varphi|_Z = f$. □

Summarizing this discussion, we come to the following result.

Proposition 2. *Let Y be a unirational affine cone. Then the monoid* $\mathrm{End}(Y)$ *acts on Y infinitely transitively.*

It is observed in [5] that by the Noether Normalization Lemma [6, Theorem 13.3] any affine variety X of dimension n admits a surjective morphism $X \to \mathbb{A}^n$. This implies that there is a surjective morphism $X \to \mathbb{A}^s$ for every $s \leqslant \dim X$. So we come to the next result, which follows from Theorem 1 and [5, Proposition 1.6]; compare also with [7, Theorem 1.6].

Proposition 3. *Let X be an affine variety and Y be a unirational affine cone. If $\dim X > \dim Y$ then there is a surjective morphism $X \to Y$.*

We finish with the following observation. It is clear that an affine cone Y is smooth if and only if Y is isomorphic to an affine space. On the other hand, an affine homogeneous space G/H of a semisimple group G is smooth and is not isomorphic to an affine space by [12, Corollary 5.1]. So G/H can not be realized as an affine cone. At the same time, [1, Theorem C] implies that there is a surjective morphism $\mathbb{A}^m \to G/H$.

We note that Conjecture 1 remains open for arbitrary affine varieties.

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