A Survey on Business Cycles: History, Theory and Empirical Findings

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Abstract This work summarizes recent advances in modelling and econometrics for alternative directions in macroeconomics and cycle theories. Starting from the definition of a cycle and continuing with a historical overview, some basic nonlinear models of the business cycle are introduced. Furthermore, some dynamic stochastic models of general equilibrium (DSGE) and autoregressive models are considered. Advances are then provided in recent applications to economics such as recurrence quantification analysis and numerical tools borrowed from other scientific fields such as physics and engineering. The aim is to embolden interdisciplinary research in the direction of the study of business cycles and related control techniques to broaden the tools available to policymakers.

Keywords Business cycles · Nonlinearities in economics · DSGE models · RQA

JEL Classification C61 · E32 · E37

1 Introduction

The purpose of this paper is to embolden interdisciplinary research in the direction of the study of business cycles and related control techniques to broaden the tools available to policymakers. To do this we provide an overview of the evolution of complex dynamic theory in macroeconomics and then, to conclude, we present a concise treatment of advances in recent applications to economics such as recurrence quantification analysis and numerical tools borrowed from other scientific fields such as physics and engineering.

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With regard to the meaning of "dynamics", it is worthwhile to recall the different views on it. John Stuart Mill (1848), and later Hicks Hicks (1946), meant that "economic dynamics refers to that part of economic theory in which all quantities must be dated". Jevons (1879), followed by Wicksell (1898) and Keynes (1936), similarly to physical mechanics, by "statics" intended the relations of forces at equilibrium, versus the changes in movements towards equilibrium represented by the "dynamics". Those views were rejected by Kuznets (1930) who argued that "statics" concerns the conditions of equilibrium while "dynamics" relates to the changes leading towards equilibrium. An interesting account on the meaning of "statics" and "dynamics" in macroeconomics from a historical perspective is in Rivot and Trautwein (2020).

In the present work, by "economic dynamics" we refer to the definition given by Day (1994): dynamics in economics deals with the systematic study of changes in micro and macro-economic variables. Specifically, since we are focusing on business cycles, other aspects of economic dynamics are neglected.

The paper is organized as follows: Sect. 2 introduces the topic of nonlinear dynamics in economics which encompasses the definition of business cycles, a historical overview of the research, some well known models on business cycles such as the ones by Goodwin, Kalecky and Kaldor and, finally, a brief description of dynamic stochastic general equilibrium (DSGE) vector and autoregressions models. Section 3 describes Recurrence Quantification Analysis (RQA) which highlights the correlation structure of the observed phenomenon along with the Recurrence Plot (RP) and the RQE Correlation Index (RQCI). Section 4 describes an original setup of a Kaldor-Kalecki model on the business cycle displaying common features with real-world data. Section 5 concludes.

2 Background and Literature

Business Cycles

In the dynamics of the economic system, the alternation between recession and expansion is universally known as the business cycle. A recession consists of a decline in economic activity throughout the economy, lasting at least two quarters, and affecting employment, real GDP, real income, consumption, etc. A recession ends when the economy reaches its minimum and corresponds to the period between the minimum and the peak reached during the previous expansion. Such expansion is the norm and most recessions are short and were rare in recent periods.

When studying stock market crises in conjunction with credit and housing market, Claessens et al. (2021) adopted this classical definition and, employing Harding and Pagan (2002) algorithm, found that when "credit downturns coincide with equity price busts, their duration does not become significantly longer, but these downturns are more severe than others. If credit downturns are accompanied by financial crises, they are much longer, deeper, and more violent than other downturns (though these

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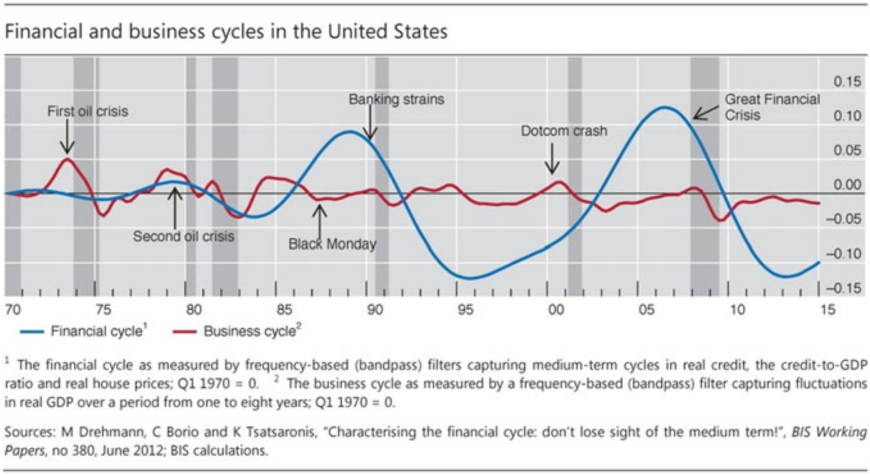


Fig. 1 BIS 85th annual report 2015

differences are not statistically significant)". Figure 1 shows how often economic and financial crises are not synchronized and that the latter is much stronger than the former.

Historical Overview

The study of the business cycle has always been at the core of classical and neo-classical inquiries in economics. However, in the past, economists did not employ mathematical formalizations to explain the ups and downs in economic activity (see, Sherman, 2014; Rosser, 2013). This implied that "logical inconsistencies could not always be avoided" (Lorenz, 1993).

Only after the Keynesian revolution Nicholas Kaldor, Michal Kalecki and Roy Harrod understood that Keynes's multiplier and Clark's (1917) acceleration principle were adequate tools to explain the business cycle. It was their mathematical approach to the business cycle that progressively made it possible to overcome the old theories.

However, it quickly became clear that their models were inadequate to describe the persistence of business cycles because they used linear differences or differential equations that were capable of generating only damped or undamped oscillations. Consequently, the main original purpose, which was the description of persistently oscillating behavior, could not be achieved.

In 1933, one of the first issues of Econometrica published a short note by the French mathematician Philip Le Corbeiller where he suggested the use of non-linear functions to describe cycles (Le Corbeiller, 1933). Referring to the van der Pol equation (e.g., see Ginoux & Letellier, 2012), Le Corbeiller hoped that economists

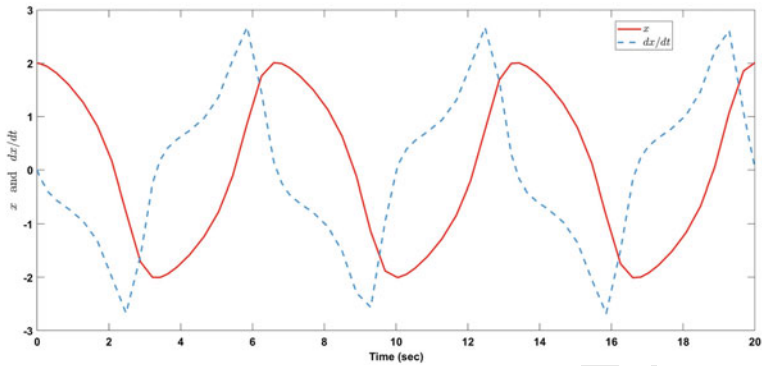


Fig. 2 Solution of the van der Pol equation, $\mu = 1$

would start using it in nonlinear models to describe business cycles. This because that equation produces cycles endogenously (see Fig. 2). However, neither Frisch nor Tinbergen and Schumpeter, the founders of the Econometric Society and its journal, gave credit to Le Corbeiller's arguments. This probably happened because Frisch, as argued by Slutzky (1937), was convinced that economic models should be stable, while cycles were generated and sustained by exogenous shocks.

Only at the beginning of the 1940s, thanks to the meeting with Le Corbeiller at Harvard University, did Richard Goodwin understand the great relevance and potential applications of nonlinear dynamics to Economics. In 1951, Goodwin published an article in Econometrica entitled "The nonlinear accelerator and persistence of business cycles" (Goodwin, 1951) showing that the interaction between accelerator and multiplier yielded a Lienard type equation (Liénard, 1928).

Since that equation can generate stable limit cycles, the persistence of oscillations seemed to well describe the fluctuations of the economic system. Although Goodwin's nonlinear accelerator model did not get much attention among contemporary scholars, it had the merit of opening Economics to the mathematical theory of dynamical systems (see, Orlando & Taglialatela, 2021b). Therefore, it represents a kind of watershed between the old and the new dynamic theory in economics.

In the 1970s, studies on deterministic chaos proliferated in pure and applied mathematics, especially after the paper by Li and Yorke (1975), where the complicated behaviour of iterated maps was investigated. In fact, in their work, Li and Yorke reconsidered a special case of the more general result previously obtained by Sharkovskij (1964), where the family of one-dimensional maps $x_{t+1} = \mathrm{F}(x_t)$ displays chaotic motions when the map has a period-3 cycle.

In 1976, the work by Li and Yorke was successfully publicized by Robert May in a paper published in Nature (May, 1976, 2004), where the Malthus hypothesis of exponential population growth was replaced by the Verhulst (1847) logistic equation $\lambda_{t+1} = \lambda_t(\alpha - \beta\lambda_t)$. . For a wide list of models using a one-dimensional map, see (Lorenz, 1993; Orlando et al., 2021a). For a specific example on the logistic map, see (Orlando & Taglialatela, 2021a; Orlando et al., 2021b).

532292_1_En_2_Chapter ☑ TYPESET ☐ DISK ☐ LE ☑ CP Disp.:3/4/2023 Pages: 34 Layout: T1-Standard

The emergence of chaos in a one-dimensional map had great success in economics (see, Yoshida, 2021). Through a difference equation like a logistic map, a plethora of contributions appeared in the field of overlapping generation models and optimal economic growth. While the emergence of chaos may seem relatively simple in discrete time models, in contrast, a chaotic movement is very difficult to detect when time is continuous. In this case, chaos appears only when the system is described by at least three nonlinear differential equations. This is because trajectories of two-dimensional systems cannot intersect themselves and therefore only a simple dynamic motion is possible (i.e., limit cycles).

During the first half of the 1980s, economic models only featured discrete-time dynamics. In 1991, a survey of chaotic dynamics and economics by Brock and Dechert (1991) in the volume "Handbook of Mathematical Economics" mentioned as continuous-time models only the Lorenz (1963) geometric butterfly object and the Mackey and Glass (1977) attractor. Both have nothing to do with economics (for example, the MacKey–Glass attractor investigates the hematologic disorder in leukemic patients).

Only in the second half of the 1980s and in the 1990s, models generating chaotic motions in continuous time emerged in economics. For example, Chiarella and Flaschel (1996) and Chiarella et al. (2013), studied macroeconomic models of monetary growth in the Tobin and Keynes-Wicksell tradition. Their purpose was to build a framework where the non-market-clearing approach to macroeconomics led to integrated models of disequilibrium growth. Further examples are the contribution by Goodwin (1990), which is an extension of his predator–prey model, where the Rössler (1977) Rössler (1976); Letellier and Rossler (2006) attractor (which originates in chemical kinetics) is applied to account for aperiodic cycles; the non-linear version of the Metzler (1941) inventory cycle model suggested by Lorenz (1992); the formalization of Harrod's dynamics by Sportelli (2000); Piscitelli and Sportelli (2004).

In summary, there is a long debate on chaos and non-linear dynamics in economics, and even the use of these concepts has been questioned. Although stochastic modeling has proven effective, the theoretical implication is that reality is made up of exogenous randomness. The opposite view of the chaos theory is that reality is deterministic and nonlinearities are endogenous.

To the criticism that chaos theory would explain little in terms of real economics, Orlando and Della Rossa (2019) carried out an empirical test on a chaotic model specification of the Harrod's open economy showing the agreement between theoretical predictions and actual data. Similarly, Araujo and Moreira (2021) tested a Goodwin's model with capacity utilization to the US economy. Furthermore, Orlando and Zimatore (2020a) proved that reality can be represented by a chaotic model as well as a stochastic model. can do. In the same work, it was shown that a chaotic model can reproduce an extreme event such as a black swan. Further evidence can be found in (Orlando & Bufalo, 2022; Orlando, 2022; Orlando et al., 2022; Lampart et al., 2022).

Some Basic Nonlinear Business Cycle Model

In recent decades, a growing number of economists agree with the non-linear approach to the business cycle, because it better describes the complexity of the real economy. Therefore, in this section, we present an overview of three seminal models, which still act as a reference for new and more advanced theoretical works.

Wage Share-Employment Dynamics (Goodwin Model)

A relevant contribution developed by Goodwin in the late 1960s (Goodwin, 1982) was intended to describe how the Marxian class struggle could cause persistent swings in the growth rate of the economic system. That work is an economic translation of the predator–prey model originally developed by Lotka (1925) and Volterra (1931) for the study of the antagonistic growth of two populations (Anisiu, 2014; Orlando & Sportelli, 2021).

Goodwin considered an economy consisting of workers and capitalists. Workers spend all their income on consumption, while capitalists save and invest all their profits. Given the labour productivity $Y/L = a_0 \exp^{\alpha t}$ ($0 < \alpha =$ constant), the labour supply N = N_0exp{βt} (0 < β = constant) and the capital/output ratio K/Y = σ (σ = constant), Goodwin set v = the employment rate and u = the labor income share and assumed that the real wage rate ($\dot{w}$/w $= -\gamma +$) changes according to a linear Phillips curve.

The logarithmic differentiation of v and u and the necessary rearrangement yields

$$\begin{cases} \frac{\dot{v}}{v} = \left[\frac{1}{\sigma} - (\alpha + \beta)\right] - \frac{1}{\sigma} u \\ \frac{\dot{u}}{u} = (\gamma - \alpha) - \rho v \end{cases} \tag{1}$$

By setting $1/\sigma > \alpha + \beta$ the system has two equilibrium points: the origin, which is a saddle point (every trajectory approaching the equilibrium is always pushed away from it) and (v∗, u∗), which is a center of infinitely many closed orbits. The specific closed orbit the system is in depends on the initial conditions.

In this model (which is a rare example of an integrable system of nonlinear differential equations) the employment rate v serves as the prey, while the wage bill share acts as the predator. When there is no employment, the wage bill tends to zero. When the wage bill tends to zero, the employment rate increases because there are no relevant labor costs (see Figs. 3 and 4).

As mentioned by Semmler (1986) this model explains cyclical growth and was applied by Goodwin to explain the Marxian idea of the industrial reserve labor army and its role in the capitalist economy. Goodwin has the merit of representing a growing economy, while most other non-linear oscillation models refer only to a stationary economy. Moreover, Goodwin's predator–prey model "does not really model business cycles but rather long cycles. On the other hand, for a theory of long cycles, the dynamical interaction of other important variables (such as waves

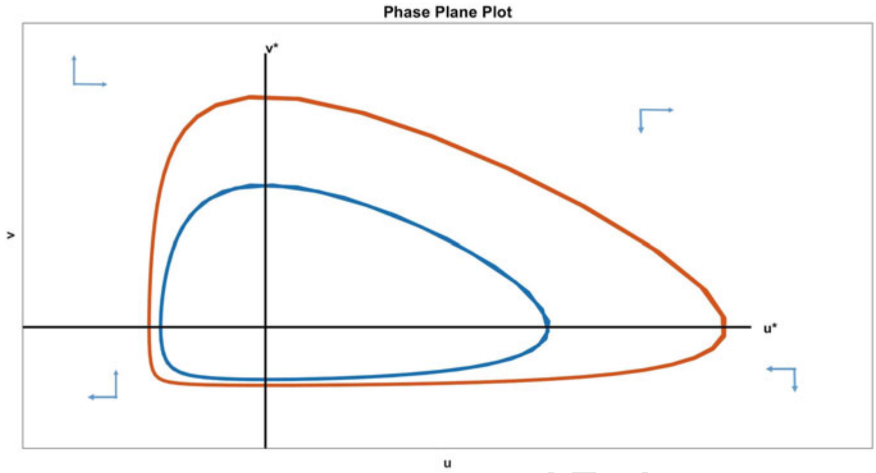


Fig. 3 Ordinate v, abscissa u. In the northwest region (low labor, high employment, share in production) the economy moves north-east (employment increases as well as the share of workers). Once the u∗ line is crossed, the dynamics start moving southwest

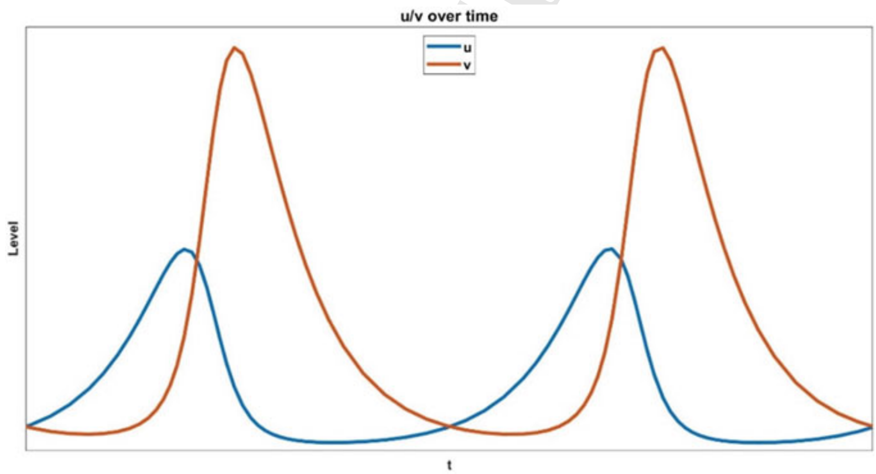


Fig. 4 Ordinate v and u, abscissa t (time). Oscillation of v and u over time

of innovations, changes of capital/output ratio, relative prices and interest rates) are unfortunately neglected" (Semmler, 1986).

As a demonstration of the long-lasting interest in the scientific debate opened by Goodwin, there is a number of recent generalizations and extensions of his model such as Fanti (2003), Yoshida and Asada (2007), Sportelli and De Cesare (2019), Haddad et al. (2020), etc. For a test to the USA economy, see (Araujo & Moreira, 2021) and for a review, see (Gonze & Ruoff, 2021).

532292_1_En_2_Chapter ☑ TYPESET ☐ DISK ☐ LE ☑ CP Disp.:3/4/2023 Pages: 34 Layout: T1-Standard

Profit-Investment Dynamics (Kalecki Model)

Kalecki (1971) describes cycles as based on the dynamic interaction of profits and accumulation of capital originally developed by Marx and McLellan (2008). Other comparable approaches can be found in Veblen (1904), Lowe (2017), etc.

The aforementioned dynamics of capital (K) and profit (Π) are described by

$$\begin{cases} \dot{K} = \alpha \Pi \\ \dot{\Pi} = -\beta \Pi - \gamma K \end{cases} \tag{2}$$

with $\dot{K} = (I - \delta K) \gtreqless 0$ where I and δK represent gross investment and depreciation, respectively.

In this model "the net increment of capital equipment per unit of time affects adversely the rate of investment decision, i.e., without the effect, the rate of investment decision would be higher" (Kalecki, 1971).

Thus, the second equation in (2) has a negative sign. The interaction between Π and $\dot{\Pi}$ implies that whilst profits derive from past investments (of profits), the accumulation of capital leads to $\dot{\Pi} < 0$ at some point. This model "depicts only a stationary economy where the capital stock remains constant in the long run. This and the fact that linear differential (or difference) equations cannot be used to produce limit cycles (i.e. economic cycle) are limitations of his early attempt to model the dynamic interaction of profits and capital accumulation" (Semmler, 1986). However, in the Kalecki (non-formalized) description of business cycles, denoted K^* as equilibrium value, past investment has positive effects on the current change of profits if $K < K^*$ and negative if $K > K^*$. These profit-investment dynamics allow the generation of turning points.

Income-Investment Dynamics (Kaldor Model)

The Kaldor model is based on the geometrical characteristics of the saving and investment function that, depending on their shape and relative positioning, generate endogenously cycles.

A hypothesis adopted by Kaldor is that the propensity to save of the capitalists (S^p) is higher than the propensity of wage earners (S^w). The dynamics of the economy are described by the following equations:

$$\begin{cases} \dot{Y}_t = \alpha (I_t - S_t) \\ \dot{K}_t = I_t - \delta K_t, \end{cases} \tag{3}$$

where the subscripts denote the macroeconomic variables income (Y), investment (I), saving (S) and capital (K) at time t. In the Eqs. (3), α is the rate at which the output responds to the excess investment I-S and δ represents the capital depreciation rate K.

532292_1_En_2_Chapter ✓ TYPESET DISK LE ✓ CP Disp.:3/4/2023 Pages: 34 Layout: T1-Standard

Furthermore, Kaldor assumes that

$$\begin{cases} I_Y > 0, \ I_K < 0, \\ S_Y > 0, \ S_K > 0. \end{cases} \tag{4}$$

A stable equilibrium is the only income level where savings and investment are equal. When S and I are linear, there is only one equilibrium and it is stable or unstable. In the first case the model shows greater stability than what appears to be present in reality (Fig. 5), in the second case the equilibrium is unstable and the resulting income is infinite or zero (Fig. 6).

To explain the dynamics of I and S, Kaldor assumed that $I = I(Y, K)$ and $S = S(Y, K)$ are nonlinear functions of income and capital.

Kaldor's inspiration was to conceive a structure in which nonlinear functions move dynamically. Figure 7 illustrates that the curves $I(Y)$ and $S(Y)$ cross at three

Fig. 5 Ordinate S and Y, abscissa t (time)

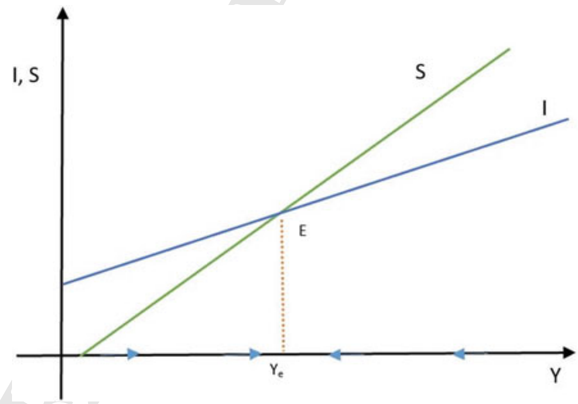
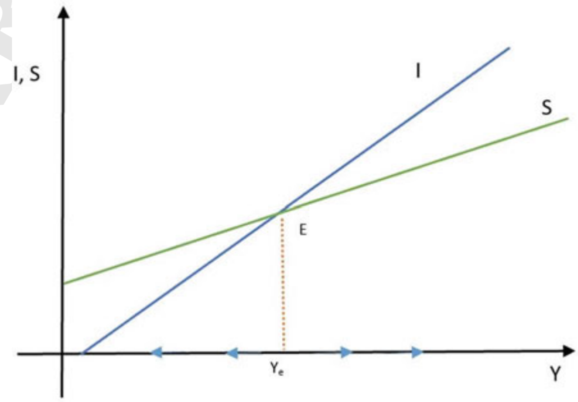


Fig. 6 Savings (green) and investment (blue) versus income (abscissa). Stable equilibrium



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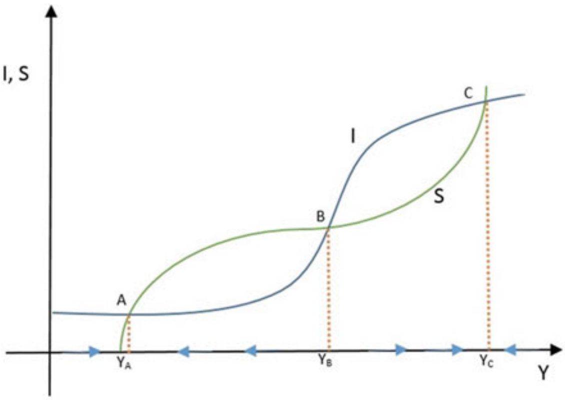

Fig. 7 Savings (green) and investment (blue) versus income (abscissa). Unstable equilibrium

points A, B and C. These points correspond to three different equilibria defined by the equality $I = S$.

The A equilibrium corresponds to a low level of Y_A production and overcapacity. Any increase in aggregate demand is absorbed and, consequently, in this situation, there is little or no investment. In the opposite case, when $Y = Y_C$, the production capacity is full and therefore rises the cost of a further unit of capital. However, the return on investment decreases as more profitable activities have already been funded. This motivates nonlinear investments.

Savings (green) and investment (blue) versus income (abscissa). Equilibrium is when $I = S$. If the income is greater than Y_B the savings are greater than the investments, so the total output decreases. Conversely, income is less than Y_B, investments are greater than savings, and the economy grows.

Equilibrium is when investment equals savings. To the right of Y_B, the high investment pushed the economy further. To the right of Y_B, on the other hand, savings are greater than investments and the economy gradually declines.

The equilibrium exists for the level of income corresponding to the investment equal to the savings as for the savings rates, it can be assumed that they are high both when production is low and when it is high. The cause is that for $Y = Y_A$, the income is almost completely used and families have presumably exhausted their finances. For this reason, in the event of an increase in income, savings are likely to be reinstated. On the other hand, when the income is high and corresponds to $Y = Y_C$, the consumption is already high and therefore the additional income is saved. Figure 8 shows the three equilibria (Y_A, Y_B and Y_C) between investment and savings corresponding to the different output level. Note that while Y_A and Y_C are stable, Y_B is not because on the left the savings exceed investment while on the right the opposite happens.

According to Kaldor, the business cycle is caused by the accumulation of capital. For example, suppose $Y = Y_C$ and I depend on K such that $\frac{dI}{dK} < 0$. This implies that on the one hand, the stock of capital increases and on the other hand the marginal productivity of capital decreases as does the investment curve. For high levels of

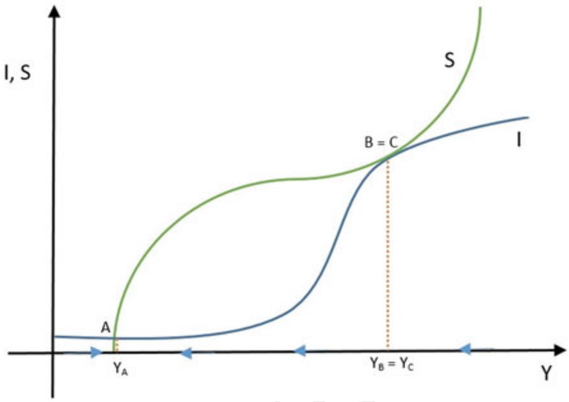

Fig. 8 Dynamic analysis of investments and savings

output, prices decrease with a positive effect on savings. This produces $\frac{dS}{dK} > 0$, which means that the savings curve shifts up.

This implies that on the one hand, the stock of capital increases and on the other hand the marginal productivity of capital decreases as does the investment curve. For high levels of output, prices decrease with a positive effect on savings.

The effect of this process is to move Y_C down and Y_B up (see Fig. 8), until the curves meet at the tangent point. On the left, the next equilibrium point is for $Y = Y_A$ which represents a severe economic downturn.

As regards the equilibrium point $Y_B = Y_C$ it can be observed that it is stable since, on the left, when $S < I$, the economy increases and on the right, when $S > I$, the output shrinks.

Due to the decline in productivity, the investment shifts downwards and the consequent reduction in price shifts savings upwards.

The special characteristics of the cyclical process just described are self-generation and dynamic adjustments of macroeconomic variables. In case the income is high, opposing dynamics keep it under control, producing a downward movement. The opposite thing happens when the income is low. In particular, the dynamics that elastically bring income down or up correspond to the shift of the two investment and savings curves and accumulation towards the reduction of capital. These events occur during the cycle and are embedded in the dynamics of the model. In terms of fiscal policy, the implication of Kaldor's model concerns the observation that the different distribution of income between capitalists and workers has effects on investment and saving. Income distribution can serve to bring the economy back into equilibrium. This aspect differentiates Kaldor's thinking from that of other contemporary economists dealing with cycle theory such as Harrod. While for Kaldor the system dynamically self-regulates and the distribution mechanism can help achieve a higher equilibrium, for Harrod a change in the investment curve triggers a cumulative process of decline (or growth) in income and production without counterweight. Finally, inflation in the Kaldor model plays an important role. In fact, when there is greater use of factors, investments generally grow and are greater than savings.

532292_1_En_2_Chapter ☑ TYPESET ☐ DISK ☐ LE ☑ CP Disp.:3/4/2023 Pages: 34 Layout: T1-Standard

This increase in investment, accompanied by induced growth in demand, leads to higher prices than wages in the presence of full employment. This changes the share of total income in favor of the capitalists and reduces that in favor of the workers. Since capitalists have a greater propensity to save, the saving will increase more than investment, to the point of re-establishing equality between saving and investment. Furthermore, as investment and consequently demand fall, wage prices will tend to fall. This means that the new balance between saving and investing will be restored for a lower level of income. This process is usually called the "Kaldor Effect".

Dynamic Stochastic General Equilibrium (DSGE) Vector and Autoregressions Models

As a tool for analyzing how in general the entire economy evolves, stochastically and in equilibrium, dynamic stochastic general equilibrium (DSGE) models are used. Their linearized version can be expressed in form of linear vector autoregressions (VARs).

DSGE models stem from the idea of providing microeconomics foundations to econometric models. The process starts with the equilibrium conditions of a nonlinear DSGE model and it is followed by a linearization around the non-stochastic steady state. Then, the log-linearized state transition equation is found in terms of a vector of observable variables represented by a VAR whose parameters are suitably calibrated. DSGE models have been adopted by many central banks for policy analysis and forecasting: the IMF (GEM), Norges Bank (NEMO), Bank of Canada (ToTEM), the European Commission (QUEST III), European CentralBank (NAWM), Sveriges Riksbank (RAMSES), Bank of England (BEQM), the US Federal Reserve (SIGMA).

While the whole framework has provided new insights and helped in identifying the consequence of the change of a given variable (the so-called impulse response analysis), several issues need to be addressed: (a) The mapping from the DSGE to the VAR model (Giacomini, 2013), (b) The wrong microfoundations (Stiglitz, 2018), (c) The lack of regime dependent VAR specification (Mittnik & Semmler, 2012).

On the latter, Mittnik and Semmler (2012) show that a fiscal multiplier that varies according to the state of the business cycle can be modeled with a two-regime VAR. In particular, for the U.S.A. the "expansion multiplier is much higher in a regime of a low economic activity than in a regime of high activity" Mittnik and Semmler (2012). Moreover, they prove that it is size-dependent. So, multi-regime models can capture different states of business cycles and are policy-relevant. Figure 9 provides an example of a DSGE-VAR model.

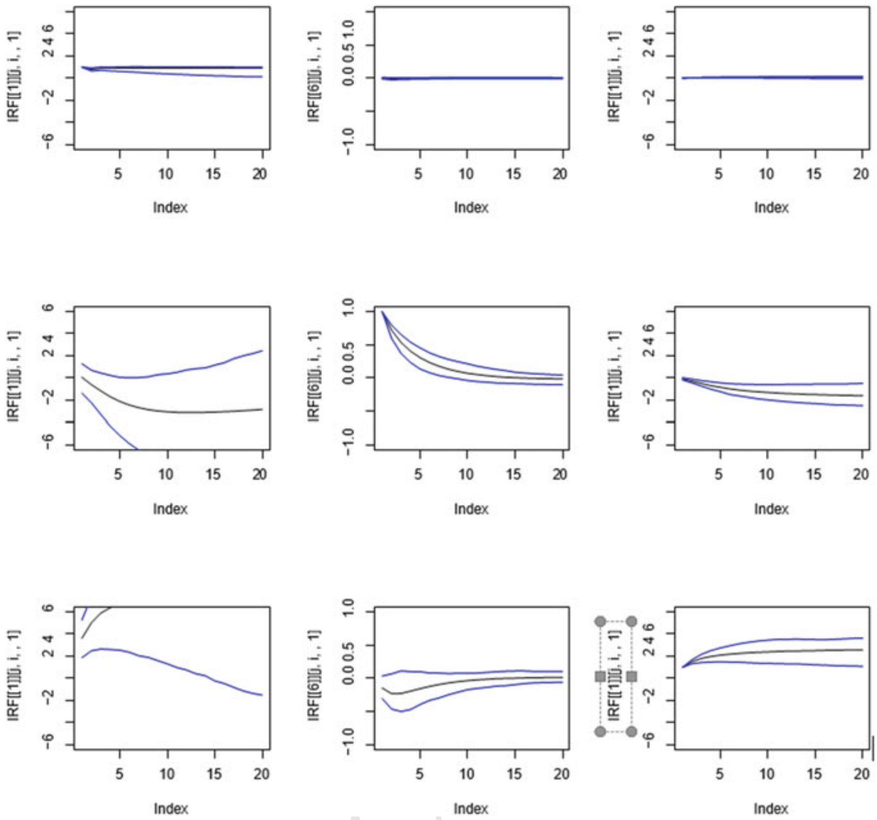

Fig. 9 Impact of a rate cut (see Chen & Semmler, 2021)

3 Recurrence Quantification Analysis (RQA)

Recurrence Quantitation Analysis (RQA) is based on the change in the correlation structure of the observed phenomenon and therefore is used to predict catastrophic changes in various systems: from geophysics (Zimatore et al., 2017) and physiology (Zimatore et al., 2011) to economy (Crowley, 2008; Orlando & Zimatore, 2021). For a brief overview, see (Orlando et al., 2021e).

Among the first applications to economics we can mention the study by Gorban et al. (2010) which demonstrates how, in the UK stock market, correlation (i.e. determinism) increases during a crisis and decreases when the market recovers. More recently, Orlando and Zimatore (2017, 2021) found that RQA and statistical techniques applied to real-world time series highlight potential indicators of structural changes in economic time series that are harbingers of downfall.

Recurrence Plot (RP)

The Recurrence Plot (RP) is also called the Distance Matrix (DM) as it is denoted as R^u_{ij} the distance between the vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ based on Phase Space Reconstruction as defined by Eckmann et al. (1987).

For example, Fig. 10 at the top shows the historical series of US GDP % and at the bottom the relative RP. In correspondence with the grey areas that denote periods of economic recession in the USA economy, it is possible to observe the anticipations of the transitions in turbulent phases represented by vertical lines.

Recurrence Quantification Epoch (RQE)

When RQA is performed on windows/sub-intervals, rather than not on the whole time series, it is called Recurrence Quantification Epoch (RQE) analysis. Determinism (DET) and laminarity (LAM) are among the most important pieces of information provided by the RQA, in fact, A Bastos and Caiado (2011) found a reduction in DET and LAM during the sub-prime mortgage crisis. Fabretti and Ausloos (2005) and Kousik et al. (2010) reported the highest value of DET and LAM during the bullish period. Figure 11 shows an example of Recurrence Quantification Epoch (RQE) applied to USA GDP %.

RQE Correlation Index (RQCI)

In this section, we first introduce a newly built RQE Correlation Index (RQCI) based on RQA measures and, then, we show how the RQCI performs in detecting structural changes (such as mean and volatility) in both simulations and real data.

RQE Correlation Index on Test Data

As explained in (Orlando & Zimatore, 2017, 2018), it is possible to define the so-called *RQE correlation index* (RQECI) composed of the correlations of the recurrence quantification measures of the recurrence such as the aforementioned DET and LAM obtained by performing the RQE several times over a given time series.

To test if the RQECI can detect changes in a time series, we take $\varepsilon \sim \mathcal{N}(\mu, \sigma^2)$ normally distributed and simulate two signals, one not perturbed and the other perturbed (both in average and in variance) as shown in Fig. 12.

Although the RQECI on the original time series shows nothing of note (see Fig. 13), the RQECI on the perturbed time series detects 9 out of 10 changes in mean and variance (see Fig. 14).

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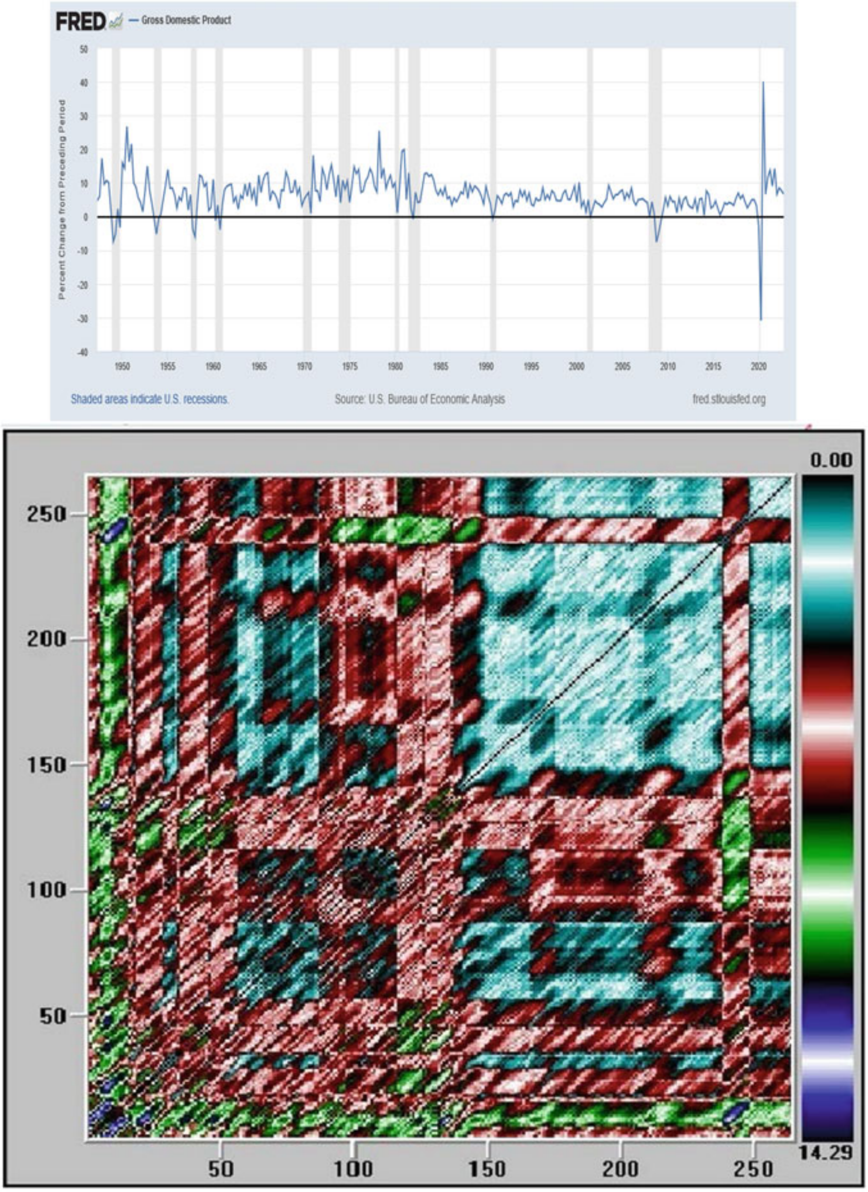


Fig. 10 Percent change of USA GDP-A191RP1Q027SBEA (above) versus the its RP (below). *Source* St. Louis Fed, Orlando and Zimatore (2017, 2021)

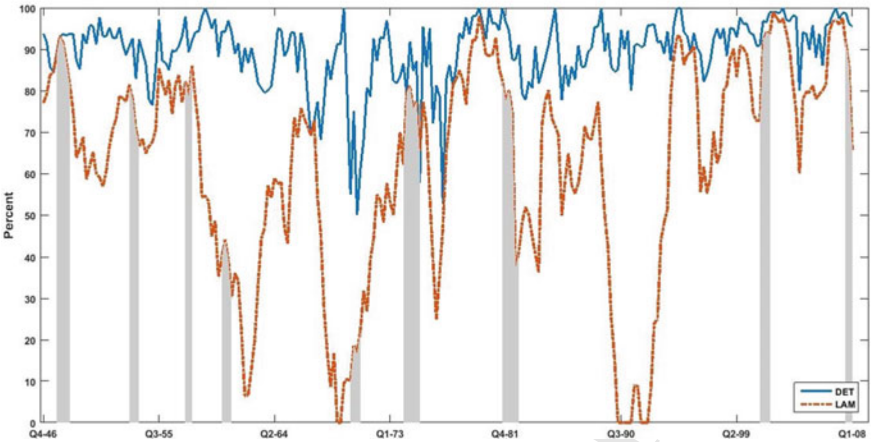


Fig. 11 RQE (i.e., dynamic RQA) with respect to laminarity (LAM) and determinism (DET) applied to the same time series as Fig. 10

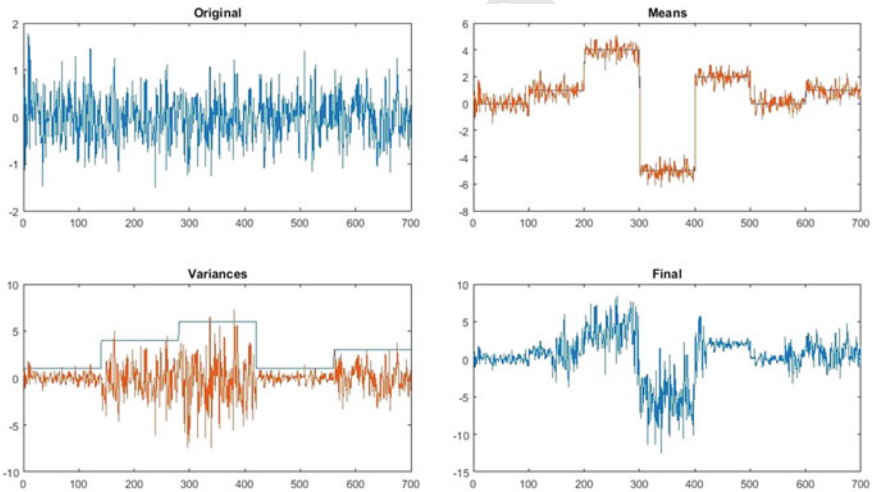


Fig. 12 Clockwise from top left: original non perturbed signal, shifts in mean, changes in variance and resulting final perturbed signal

RQE Correlation Index on Real Data

As shown in the previous paragraph, the RQECI can detect regimes' changes that are difficult to see at a glance. Therefore, the additional potential use of the index is as an early indicator in economics for recessions and market crashes, in seismology for earthquakes, etc. To show an application to economics, we have retrieved from the OECD database the USA GDP OECD (2016) and then we have run an RQE

532292_1_En_2_Chapter ☑ TYPESET ☐ DISK ☐ LE ☑ CP Disp.:3/4/2023 Pages: 34 Layout: T1-Standard

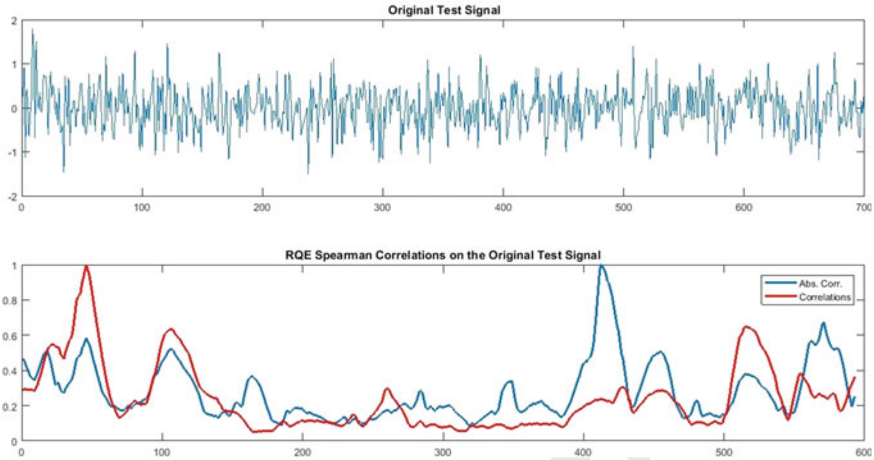


Fig. 13 Original test signal (above) and RQCI correlations (below)

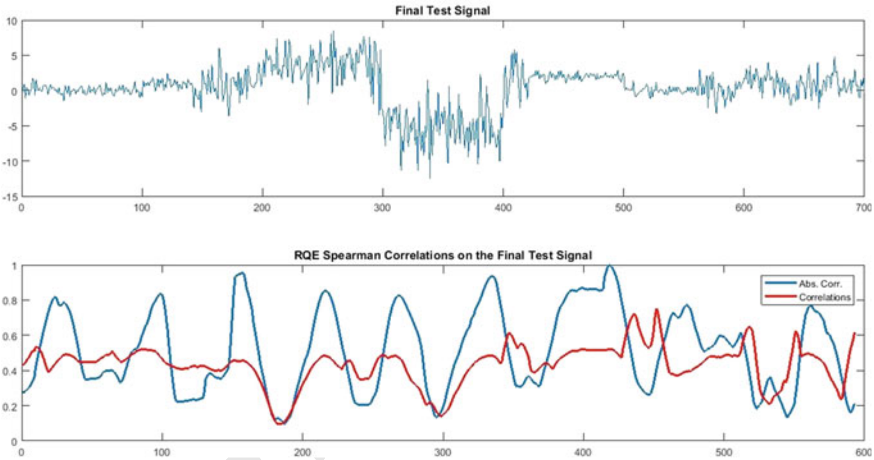


Fig. 14 Perturbed test signal (above) and RQCI correlations (below)

on the data. In the following graphs, we show first the set of RQE indicators on USA (quarterly) GDP changes as taken from OECD database Fig. 15 and second the Spearman correlation indices next to the USA GDP changes Fig. 16. Finally, by considering the correlation among RQE measures (see Fig. 11), business cycles of 5–7 years were found (which is consistent with existing literature, e.g. Prescott (1986) defines business cycles as 12–32 quarter cycles).

RQCI is performed either by considering the absolute values of the correlations (blue) or the simple Spearman correlation (red). The difference in abscissa between the top and bottom graphs is due to the windowing mechanism.

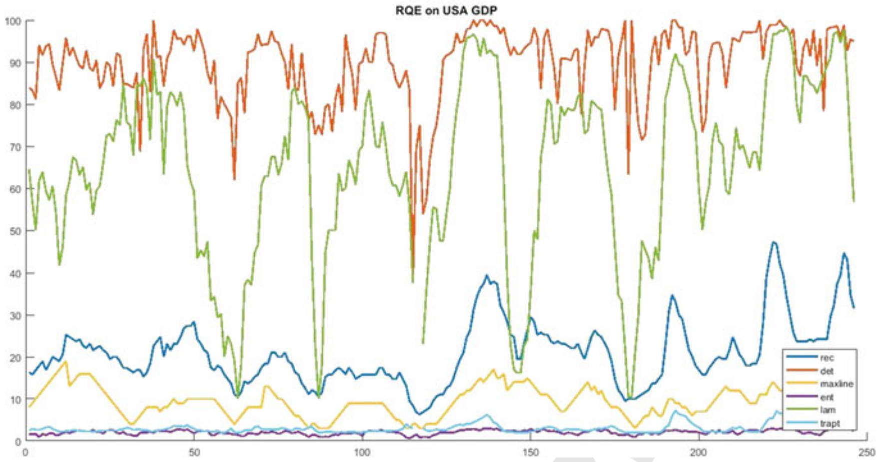


Fig. 15 RQE indicators on USA (quarterly) GDP as retrieved from OECD database ("USA QGDP TOT PC_CHGPP Q")

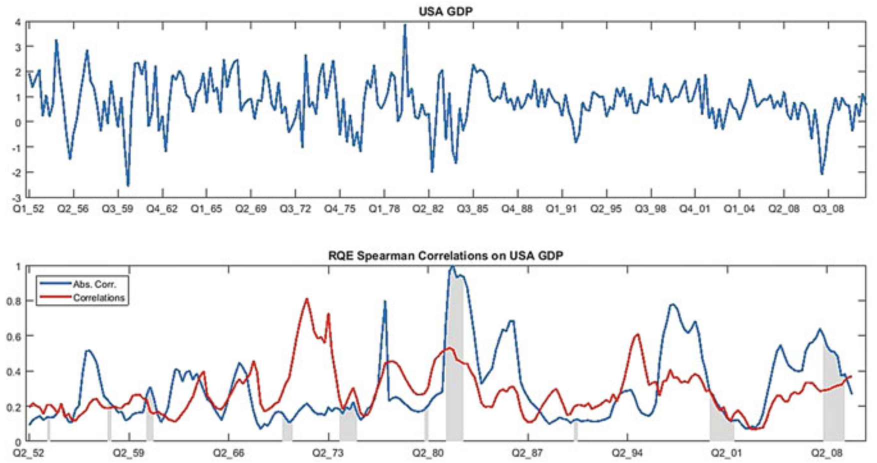


Fig. 16 Top, time series reported in Fig. 15

RQCI is performed either by considering the absolute values of the correlations (blue) or the simple Spearman correlation (red). The difference in abscissa between the top and bottom graphs is due to the windowing mechanism.

Bottom, RQECI versus recession periods (in grey). RQCI is performed either by considering the absolute values of the correlations (blue) or the simple Spearman correlation (red). A change in the RQECI is often linked to a recession. The difference in abscissa between the top and bottom graphs is due to the windowing mechanism.

532292_1_En_2_Chapter ✓ TYPESET DISK LE ✓ CP Disp.:3/4/2023 Pages: 34 Layout: T1-Standard

4 An Original Set-Up of a Kaldor-Kalecki Model

This section describes an original setup of a Kaldor-Kalecki model on the business cycle that not only adheres to theoretical specifications but, also, displays common features with real-world data.

Forewords

As mentioned, the Kaldor business cycle model was a major departure from the traditional Keynesian concept of a multiplier accelerator to explain the main reasons for cycles in the economy.

Below, we propose a sketch of a Kaldorian-type model exhibiting chaotic dynamics by adding a delay mechanism à la Kalecki. Following Kaldor, investment and savings functions are set to be nonlinear, regular and not decreasing when income and capital grow.

One of the main objectives of the model proposed by Orlando (2016, 2018) is to study the chaotic dynamics generated not by the use of the usual arcotagent function but by a variant of the hyperbolic function.

The discretized Kaldor model is

$$\begin{cases} Y_{t+1} - Y_t = \alpha(I_t - S_t) = \alpha[I_t - (Y_t - C_t)] \\ K_{t+1} - K_t = I_t - \delta K_t \end{cases} \tag{5}$$

where $I = I(Y, K)$ and $S = S(Y, K)$ are non-linear functions of income and capital and α, δ are some parameters describing the speed of adjustment between investment and saving and the depreciation of capital, respectively (for further details, see Orlando, 2016, 2018).

Graphs: Simulations and Strange Attractor

Figures 17 and 18 show the model in Eq. (5) behavior where the only difference is a perturbation on the initial condition. These simulations show the irregular behavior of the variables over time and the sensitive dependence on the initial conditions.

Figure 19 displays a strange attractor for the considered model in Eq. (5). Note that a strange attractor is defined as an attracting set that has a fractal dimension and zero measure in the embedding phase space (see, Adachi, 1993; Orlando et al., 2021d).

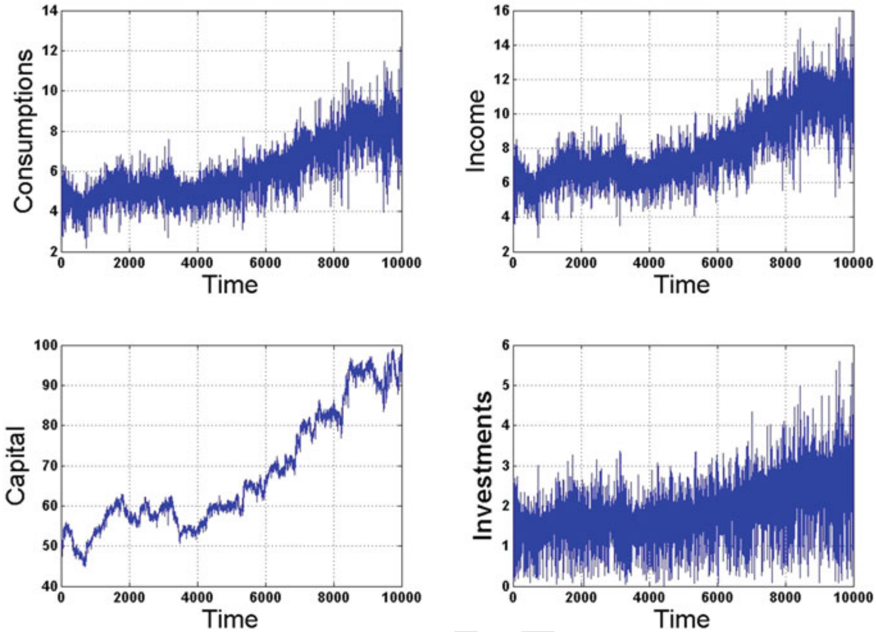


Fig. 17 Simulation of a growing economy

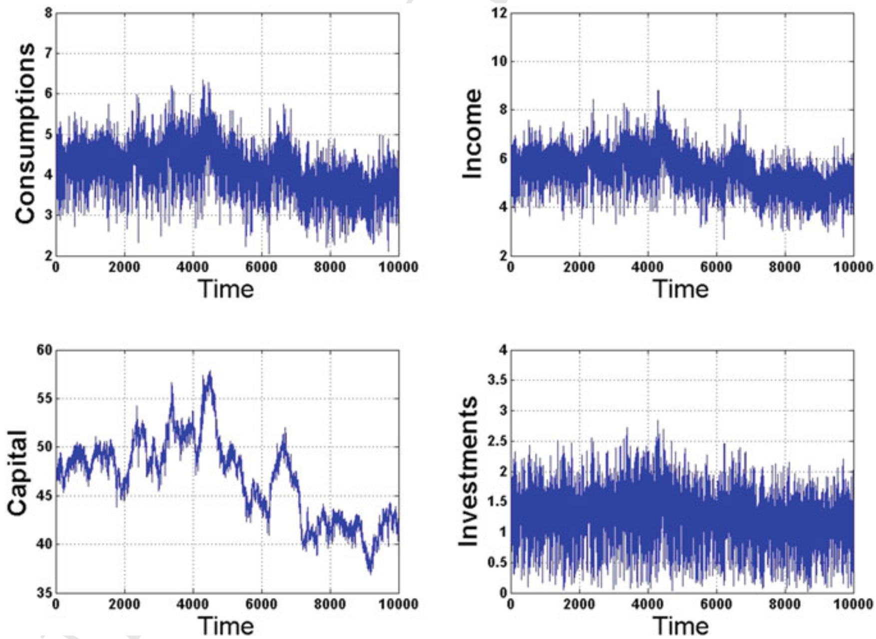


Fig. 18 Simulation of a declining economy

532292_1_En_2_Chapter ✓ TYPESET DISK LE ✓ CP Disp.:3/4/2023 Pages: 34 Layout: T1-Standard

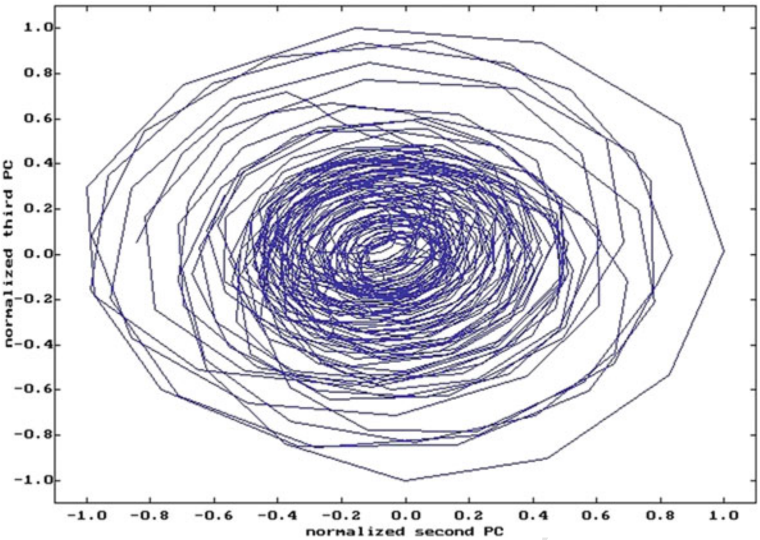


Fig. 19 Strange attractor for the system (3) obtained with RRChaos

Numerical Analysis

To better understand the nature of the system, we use some additional tools such as Lyapunov exponents, Kolmogorov entropy, spectral analysis, correlation integral and embedding dimension.

Lyapunov Exponents

Lyapunov exponents indicate the speed with which the neighbouring trajectories of a dynamic system diverge. Since dissipativity is one of the characteristics of a chaotic system, the system is said to be chaotic if its maximum Lyapunov exponent is positive (see, Orlando et al., 2021c). For the system in Eq. 3 the calculated Lyapunov exponents range from a minimum of 5.511 to a maximum of 19.64 across the four macroeconomic variables.

Correlation Integral

In Table 1 it can be observed that the correlation integral does not increase with the embedding dimension confirming that the system behaves in a stochastic way even if we know that it is deterministic by construction.

Table 1 Correlation integral versus embedding dimension

Embedding dimension (average)							
Variable	2	3	4	6	6	7	8
C	0.302	0.260	0.231	0.211	0.197	0.186	0.178

Correlation integral and embedding dimension of *C*. As shown the correlation integral is quite stable for $m = 2,\ldots, 8$. Similar results are obtained for *K, I* and *Y* (see, Orlando et al., 2021a).

Spectral Analysis

Spectral analysis highlights the most important frequencies of a signal with peaks (for an advanced introduction, see Della Rossa et al., 2021). For example, in the case of the sine function the peak is unique (see Fig. 20) while in the case of a random signal there are several peaks (see Fig. 21) indicating that there is no main frequency. Thus, we can observe that the presence of different frequency peaks suggests that irregular orbits (chaos) can be identified in the model.

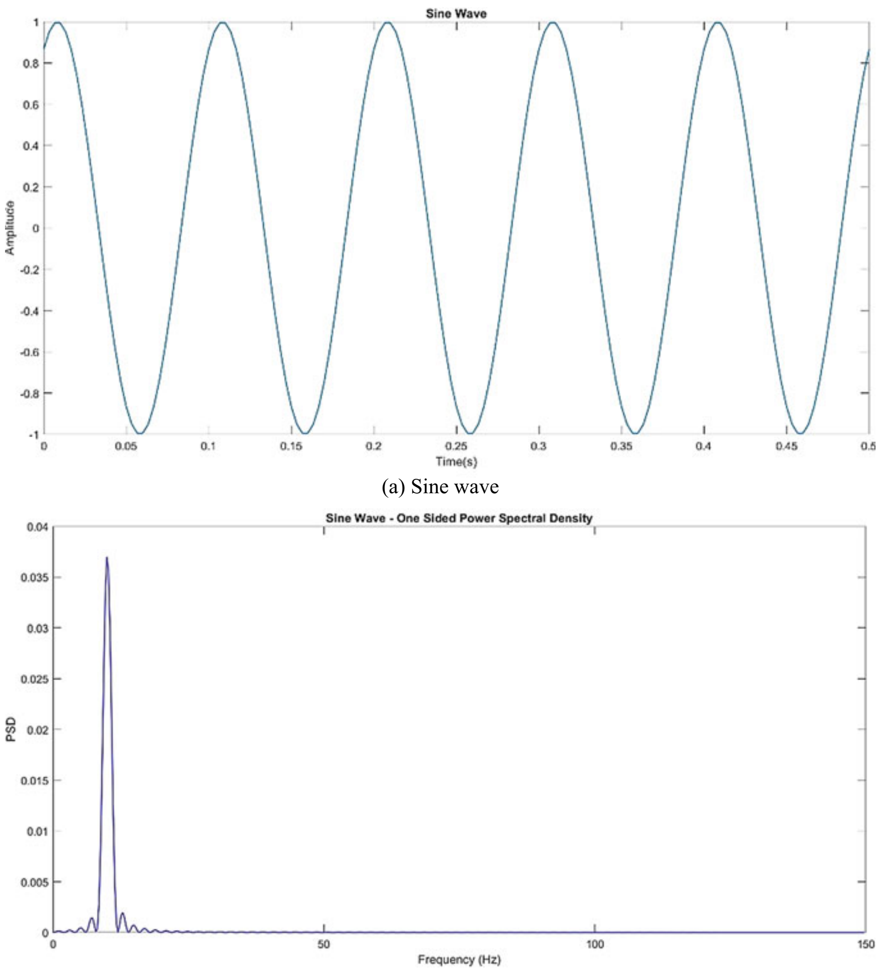
By applying the spectral analysis to the time series generated by the proposed model, similarly to the random signal, we can observe that the presence of multiple frequency peaks suggests the presence of chaos (see Fig. 22).

RP on Both Simulated and Real-World Data

The RP is particularly interesting also because it shows patterns that are not evident to other means of analysis. In particular, simulations can produce a time series of any length. This is especially useful to show fractal structures that (if existing in real data) are difficult to assess with certainty. Figure 23 shows the RP for both a simulation (for which fractal recurrence can be seen) of K and real capital stock. While real data span for a limited time, simulations are virtually unlimited.

Notice that the different scale between the two sub-figures displayed in Fig. 23 is because the simulation has 10,000 points versus 62 of the real-world time series. For this reason, Fig. 23b can be seen as a zoomed frame of Fig. 23a.

The Kaldor-Kalesky business cycle model described by the system (3) has been further investigated by Orlando and Zimatore (2020a, 2020b). In there, simulated data are further analyzed with nonlinear techniques such as recurrence quantification analysis, the Poincaré graph and related quantifiers. Through a comparison with real-world data, the analysis conducted provides evidence on the fractal dimension and entropy for both real data and the ones produced by the simulations. This demonstrates that the dynamics of the real and simulated economic cycle have similar characteristics and that the model can be a useful tool for simulating reality.



(a) Sine wave

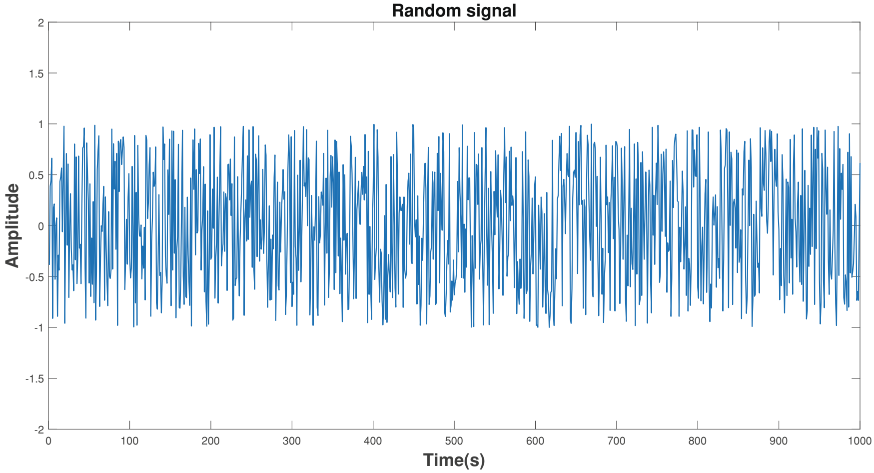
(b) Power spectrum of sine wave

Fig. 20 A sine wave signal and its power spectrum

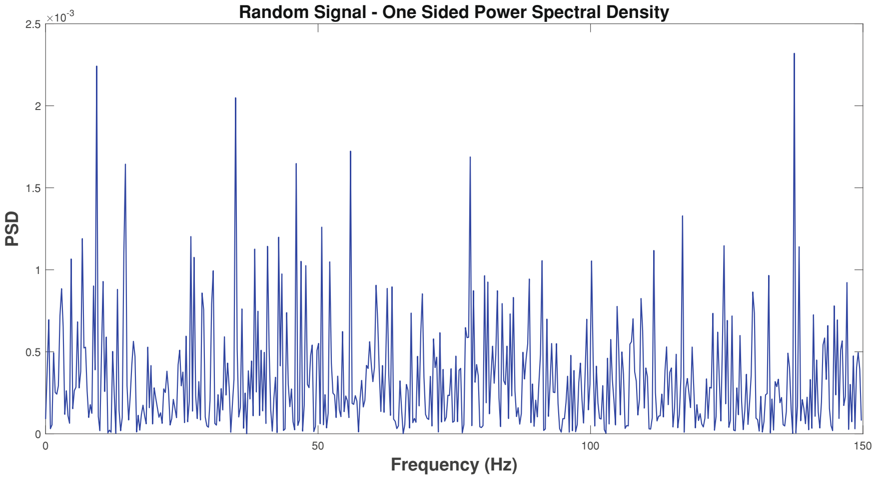
5 Conclusion and Future Research

As mentioned by Goodwin "economists will be led, as natural scientists have been led, to seek in nonlinearities an explanation of the maintenance of oscillation". For this reason, we have studied business cycles as generated endogenously by nonlinear modeling and we have shown its chaotic behavior.

The contribution of the present work is to provide a synthesis of recent advances in modeling and econometrics for alternative directions in macroeconomics and cycle theories. This was achieved by introducing business cycles and continuing with a historical overview. Subsequently, some basic non-linear models of the business



(a) Random signal



(b) Power spectrum of random signal

Fig. 21 A random signal and its power spectrum

cycle were introduced, as well as dynamic stochastic models of general equilibrium (DSGE) and autoregression models. Interdisciplinary advances such as the analysis of the quantification of recurrences and numerical tools borrowed from physics and engineering were provided along with their implementation in economics. These techniques were applied to a theoretical model to show how they can in practice help highlight common features between simulations and the real world that could be exploited by policymakers. In this regard, in terms of future research, we highlight the need to address the policy implications that a regulator or the government might set to achieve their goals. Interdisciplinary research applying chaos control theory

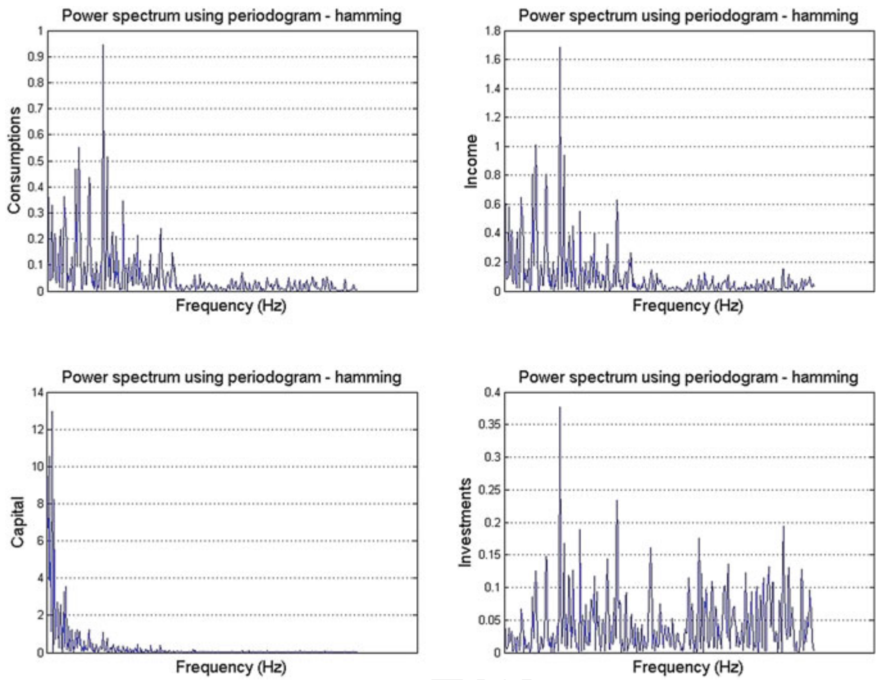
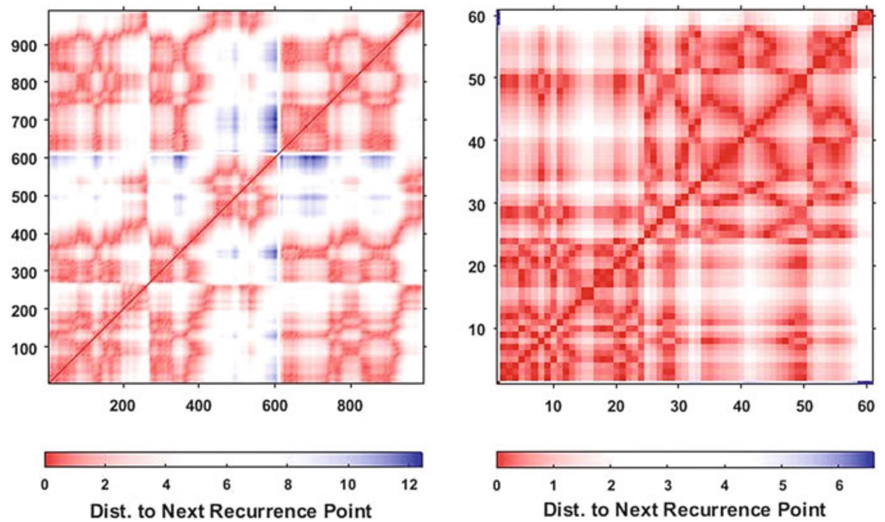


Fig. 22 Power spectrum with Hamming window

in economics (e.g., Stoop (2021)) can be of some help. Among the limitations of the present work is the lack of discussion of recent advances in weather forecasting using techniques such as ensembles (see, for example, (Taillardat et al., 2016; Buizza, 2018; Jouan et al., 2022). In fact, the aforementioned techniques could be usefully adopted to calibrate and predict nonlinear economic systems (e.g., Orlando et al. 2022).



(a) RP on simulated capital (b) RP on USA capital

Fig. 23 Simulated capital (left) versus capital stock (right) at constant national prices for United States—Series ID: RKNANPUSA666NRUG. Date Range: 1950-01-01 to 2011-01-01. Fractal structures in simulations look similar to RP on real data. *Source* University of Groningen, University of California, Davis

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