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Common trends in global volatility ☆

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ABSTRACT

This paper investigates the long-term patterns in global foreign exchange, equity and bond markets in three different trading zones, namely, Japan, Europe and the United States. Recent advances in the measurement of volatility from high-frequency data are used together with the concepts of fractional integration and cointegration. The specific objective is to consider whether there are common trends that drive volatility in the global marketplace. This so-called commonality in volatility hypothesis is formulated using a cofractional model. The results confirm that volatility in all three financial asset markets, across all three trading zones share a single common trend which lends itself to interpretation as a global news stream.

1. Introduction

An important strand of research into the volatility of global financial markets has been the examination of the transmission of volatility across different international trading zones. The seminal paper is that of Engle et al. (1990) who examine international linkages in foreign exchange volatility. Using the framework of Ito (1987) and Ito and Roley (1987), Engle et al. (1990) partition each 24 hour period (calendar day) into four non-overlapping trading zones, namely, Asia, Japan, Europe and finally the United States. An important conclusion that emerges from this line of research is that periods of high

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volatility in the foreign exchange markets are expected to be followed by high volatility in the subsequent trading zone within the same calendar day (the so called meteor-shower pattern). Given that volatility is linked to news arrival (Andersen, 1996; Clark, 1973; Ederington and Lee, 1993; Tauchen and Pitts, 1983), one potential interpretation of this result is that news is a global phenomenon.[1] This paper seeks to provide a different perspective on this result.

The recent literature on financial asset volatility has used high frequency intra-day data on financial asset returns to construct a realized volatility measure, known as realized volatility (Andersen et al., 2003). The core idea of this paper derives from the observation of Andersen et al. (2001, 2003) that realized volatility is a long memory process or, more formally, that realized volatility has a fractional order of integration. The aim of this paper, therefore, is to investigate the long-term relationships between volatility in different international trading zones and hence examine whether or not there is evidence of common trends in these series. Put another way, evidence of common trends may be interpreted as the existence of global news. Common components in volatility will be identified by means of the presence of fractional cointegration (Baillie and Bollerslev, 1994; Shimotsu, 2012) between the series when their dynamics are given by a vector error correction representation. The longer-term fundamental determinants of volatility in different trading zones has not previously been investigated, and it is hoped that the proposed approach will allow a deeper understanding of the patterns of transmission of financial asset volatility in global markets and provide another perspective on whether or not volatility in financial markets is driven by a common global news stream.

In this paper a specially constructed data set comprising high-frequency foreign exchange, equity, and bond market data is used to explore the transmission of volatility and news between these markets and across international trading zones. The calendar structure used by Engle et al. (1990) is amended slightly so that three trading zones for Asia, Europe and the United States are established and high frequency returns are used to construct realized volatility estimates for each asset class in each zone for each calendar day. The behaviour of volatility is then examined from a number of perspectives, namely, transmission across asset classes in local markets, linkages between international trading zones for each asset class and finally the most general case of linkages between all asset classes in the global market.

The results obtained in the empirical sections of the paper can be summarized succinctly as follows. Although volatility linkages between different markets and across global trading zones are fairly complex, fractional cointegration exists between the volatility of financial assets in different trading zones. Evidence of commonality in volatility is found in the foreign exchange, equity and bond markets, suggesting that volatility in these markets is strongly interrelated in the long run. The results provide significant support for the conclusion that the volatility of financial assets in different markets and across global trading zones can be explained by a single stochastic trend. In addition, this common trend can be related to a global measure of news flow, obtained by collecting data on news items from the Thomson Reuters News Analytics database. Using the fact that volatilities in these markets are fractionally cointegrated provides economically significant information when formulating a strategy to trade on volatility. Specifically, a simple trading strategy based on positions in volatility taken on the basis of forecasts generated by a model in which the fractional cointegration restrictions are imposed generates smaller returns than positions taken on the basis of forecasts from a model in which these cofractional restrictions are ignored.

The rest of the paper proceeds as follows. Section 2 describes the construction of the global trading day, the high-frequency data set used in the paper and how jump-robust measures of realized volatility, which are used in the empirical analysis, are constructed. Section 3 discusses the presence of long memory in realized volatility. In Section 4 a fractionally cointegrated volatility model is proposed. Section 5 addresses the issue of the transmission of volatility between the foreign exchange, equity and bond markets of a single trading zone. This is the simplest case to address and uses cofractional vector autoregressive models to explore volatility linkages. Section 6 explores volatility patterns between

[1] This result is not found uniformly in all markets. For example, Fleming and Lopez (1999) and Savva et al. (2005) find that in bond and equity markets volatility is expected to be followed by high volatility in the same trading zone on the following calendar day (the heatwave hypothesis).

trading zones but within a given market. The analysis is now complicated by the calendar structure of the global trading day which allows intra-day influences between zones. Structural cofractional vector autoregressive models are used to account for the calendar restrictions required by the global trading day. Section 7 presents a general model that allows for unrestricted interaction between markets and global trading zones and confirms the presence of a common trend in global volatility. Sections 8 and 9, respectively, explore the interpretation of the common volatility trend by relating it to observed news flow and the significance of the information provided by the existence of fractional cointegration.

2. Data

The central purpose of this research is to explore volatility linkages between important financial markets and also between the main financial hubs of the global market, namely, Japan, Europe, and the United States. To achieve this, it is necessary to construct a comprehensive data set capturing the volatility of these asset markets and trading zones. Consequently, a data set was collected comprising high frequency (10 minute[2]) data for foreign exchange, equity and bond markets for each of three regions, Japan, Europe and the United States. The data were gathered from the Thomson Reuters Tick History database and covers the period from 4 January 1999 to 30 December 2015. Days where one market is closed are eliminated, as are public holidays or other occasions when trading is significantly curtailed. These high frequency data are then used to construct minimum realized volatility consisting of 3915 full trading days.

Before setting out the exact specification of the data that is collected it is necessary to define the global trading day which is integral to this research. Each calendar day is split into three trading zones, namely, Japan (JP), Europe (EU) and the United States (US). The Japan trading zone is defined as 12:00am to 7:00am, the European trading zone 7:00am to 12:30pm and the United States zone 12:30pm to 9:00pm, where all times are taken to be Greenwich Mean Time (GMT).[3] This setup may be illustrated as follows:

$$\underbrace{\overbrace{12:00\text{am}\cdots 7:00\text{am}}^{\text{JP}}\ \overbrace{7:00\text{am}\cdots 12:30\text{pm}}^{\text{EU}}\ \overbrace{12:30\text{pm}\cdots 9:00\text{pm}}^{\text{US}}}_{\text{One Trading Day}}$$

The foreign exchange rate data in each of the three trading zones consist of closing prices for 10 minute intervals on Yen–Dollar spot exchange contracts traded on the Chicago Mercantile Exchange. The bond market data consist of 10 minute prices for Japanese JGB, German Bund and United States Treasury note 10-year bond futures contracts. For equity markets, 10 minute prices were collected for TOPIX (JP), DAX (EU) and S&P500 indices.

The high-frequency returns data are now used to construct time series of realized volatility (Andersen et al., 2001, 2003). For the purposes of estimating volatility and its associated components, define a jump-diffusion process for the logarithm of price:

$$dp(t) = \vartheta(t)dt + \sigma(t)dW(t) + \zeta(t)dq(t) \tag{1}$$

in which $\vartheta(t)$ is a drift process, $\sigma(t)$ is a positive stochastic volatility process, $dW(t)$ is the increment of a Wiener process and $q(t)$ is a counting process with intensity $\lambda(t)$, $t = 1, \ldots, T$. $P[dq(t) = 1] = \lambda(t)$ and $\zeta(t)$ reflects the size of discrete price jumps. It is well known that realized variation (commonly known as realized volatility) is defined as:

[2] The 10-minute frequency was chosen using volatility signature plots of each of the assets, which are available on request.
[3] The period denoted as Asian trading (2 hours prior to Japan opening) by Engle et al. (1990) is excluded here.

$$RV_t(\Delta) \equiv \sum_{j=1}^{1/\Delta} r_{j,t}^2, \tag{2}$$

which is the sum of intraday squared returns and converges to the quadratic variation

$$QV_t = \int_{t-1}^{t} \sigma^2(s)ds + \sum_{t-1<s\leq t} \zeta_{j,t}^2. \tag{3}$$

The proxy for volatility in equation (3) includes contributions from both the continuous and jump components of prices. Andersen et al. (2007), however, demonstrate that information pertaining to future volatility is best captured by the persistent diffusive component of volatility. Using the diffusive component of realized volatility, therefore, is likely to provide more reliable estimates of volatility linkages. As these linkages are the primary focus of this research, a necessary prerequisite is a reliable method for estimating a continuous diffusive process.

A number of methods exist to effect this calculation and provide volatility indicators that are robust to jumps, the earliest of which is the bi-power variation (Barndorff-Nielsen and Shephard, 2002, 2004), given by:

$$BV_t(\Delta) \equiv \frac{\pi}{2}\left(\frac{1}{1-\Delta}\right)\sum_{j=2}^{1/\Delta} |r_{j-1,t}||r_{j,t}|. \tag{4}$$

As this measure converges to integrated volatility, it is possible to extract from the total volatility the contribution from jumps:

$$RV_t(\Delta) - BV_t(\Delta) \to \sum_{t-1<s\leq t} \zeta_{j,t}^2. \tag{5}$$

An important result that follows from equations (4) and (5) is that by construction, the bi-power variation can be used as an estimator of quadratic variance robust to jumps. Ait-Sahalia et al. (2012) and Mancini (2009) propose two such estimators. These are truncated realized volatility, given by:

$$TRV_t(\Delta, u_n) \equiv \sum_{j=1}^{1/\Delta} r_{j,t}^2 \cdot 1_{\{|r_{j,t} \leq u_n\}}, \tag{6}$$

and truncated power variation:

$$TPV_t(\Delta, u_n, p) \equiv \sum_{j=1}^{1/\Delta} |r_{j,t}|^p \cdot 1_{\{|r_{j,t}| \leq u_n\}}, \tag{7}$$

in which $u_n = \iota\Delta\varpi$ is a suitable sequence going to 0, $\iota > 0$, ϖ is an arbitrary constant, and $p \geq 2$ is a positive integer.

In practice suitable choices for α and ϖ must be provided. Todorov et al. (2011) argue that $\iota = 3\sqrt{BV_{t+1}}$ and $\varpi \in (0, 1/2)$, and these conditions are intuitively reasonable. However, it is necessary to note that in choosing these parameters there is a risk of throwing away many Brownian increments, which makes it difficult to use this method in practice.

Andersen et al. (2012) introduce an alternative jump robust estimator known as minimum realized volatility:

$$\mathrm{MinRV}_t(\Delta) \equiv \frac{\pi}{\pi - 2}\left(\frac{1}{1-\Delta}\right)\sum_{j=2}^{1/\Delta} \min(|r_{j-1,t}|, |r_{j,t}|)^2. \tag{8}$$

Andersen et al. (2012) justify that minimum realized volatility measure provides better finite sample properties than bi-power variation. Due to this fact, and taking into account the arbitrary character of choosing the threshold ϖ in truncated power variation for volatility estimation, the MinRV measure of integrated volatility robust to jumps is used. The nine series for realized volatility, calculated using equation (8) applied to each asset class in each trading zone,[4] are plotted in Fig. 1.

To the naked eye it appears that the estimates of realized volatility in foreign exchange and equity markets have similar patterns across the trading zones. The volatility in the United States is perhaps a little more pronounced during the Global Financial Crises period of 2007–2009. However, the similarity across the three zones in not as pronounced in the bond markets. Fig. 1 indicates that while realized volatilities in Japan and the United States are very similar, realized volatility in the European zone appears to experience more volatility events (appears more spiked) than the other zones. In some instances, to enhance its appearance, the scale on the y-axes of Fig. 1 are different for some of the trading zones, making it difficult to compare the relative sizes of realized volatility. Table 1 reports summary statistics for the minimum realized volatility series multiplied by 1000.

Table 1 reveals that the mean level of volatility in the equity market is greater that in the FX market which in turn is greater that the mean volatility in the bond market. Volatility in all three markets is highest during trading in the United States whereas Japan experiences the lowest volatility. Engle et al. (1990) find volatility is substantially higher during the New York trading hours than during Tokyo or London trading hours. Their view is that much of this volatility seems to originate from macroeconomic announcements released during New York trading hours. Another conclusion that follows from Table 1 is that volatility series are skewed and leptokurtic. Guidolin and Timmermann (2008) point to importance of co-skewness and co-kurtosis which leads to a substantial increase in the investor's optimal holdings of stocks in the United States.[5] The issue of linkages between volatility across different trading zones will be considered in later sections.

3. Fractional integration in realized volatility

One manifestation of long memory in a volatility series is a slowly decaying autocorrelation function, a characteristic which is to be expected given that autocorrelation in the squares of financial returns is a well-documented phenomenon[6] (Pagan, 1996). The autocorrelations out to ten lags for each of the nine series of realized volatility are reported in Table 2, revealing strong, statistically significant persistence. It appears that volatility in the equity market is generally more persistent than volatility in the foreign exchange market which in turn is more persistent than the bond market. In the bond case, the autocorrelation coefficients appear to be smaller in Japanese volatility.

To explore whether or not there is a *prima facie* case for modelling commonality in realized volatility using fractional cointegration, the series must first be examined for the order of fractional integration. Bollerslev and Mikkelsen (1996) and Baillie et al. (1996) describe volatility as a long memory process with a fractional parameter d. Following Diebold and Inoue (2001) long memory can be defined by the rate of growth variances of partial sums as $\mathrm{var}(S_T) = O(T^{2d+1})$, in which $S_T = \sum_{t=1}^{T} y_t$, y_t is a financial series of interest, and T is a sample size. A commonly used test for existence of long memory is the R/S statistic discussed by Lo (1991) that is defined as:

$$Q_T = \frac{1}{\hat{\sigma}_T(q)}\left[\max_{1\le k\le T}\sum_{t=1}^{k}(y_t-\bar{y}) - \min_{1\le k\le T}\sum_{t=1}^{k}(y_t-\bar{y})\right], \tag{9}$$

in which $\bar{y}_t = (1/T)\sum_{t=1}^{T} y_t$, and $\hat{\sigma}_T(q)$ is the standard deviation of the Newey–West estimate of the long run variance with bandwidth q. Lo (1991) found that if there is short memory but no long memory in the series y_t, Q_T statistic will converge to the range of a Brownian bridge on the unit interval.

A semiparametric estimator of d was proposed by Geweke and Porter-Hudak (1983). This estimator implies, that for a long memory time series y_t, the spectral density $f(\omega)$ has a power-law decay $f(\omega) \sim c\omega^{-2d}$ when $\omega \to 0$. The GPH estimator $\hat{d}$ is represented by an ordinary least squares regression

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4 Each volatility series is divided by the square root of the trading time in each respective zone measured in hours.

5 Jondeau and Rockinger (2003) also highlight the importance of modelling the skewness and the kurtosis of volatility.

6 Similar findings for realized volatility were reported by Andersen et al. (2003) and Deo et al. (2006).

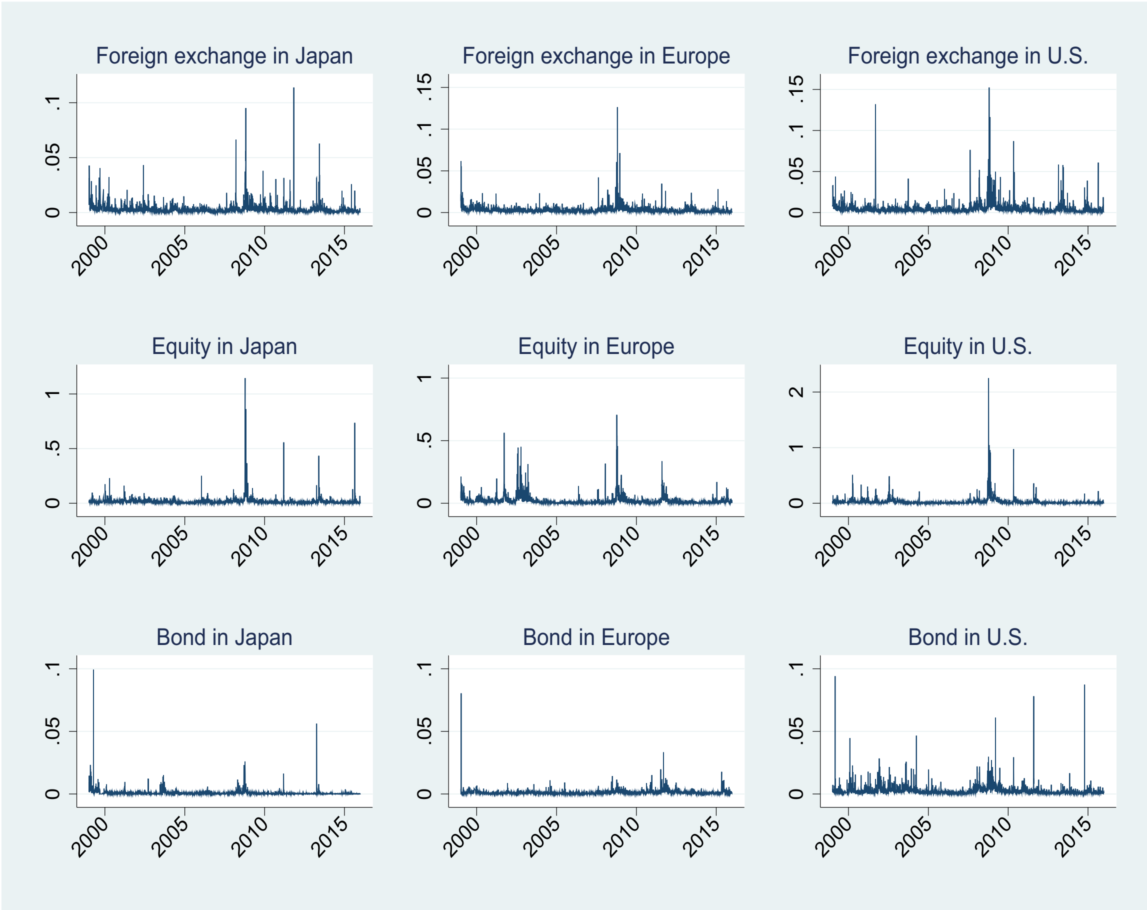

Fig. 1. Minimum realized volatility estimates for the foreign exchange, equity and bond markets in Japan, Europe and the United States, respectively. The daily estimate of realized volatility for the period 4 January 1999 to 30 December 2015 is computed using (8) and then scaled by 1000 before plotting.

Table 1
Descriptive statistics for daily estimates of realized volatility in the foreign exchange, equity and bond markets in Japan, Europe and United States for the period 4 January 1999 to 30 December 2015.

		Mean	St.dev.	Min.	Max.	Skew.	Kurt.
FX	Japan	0.0029	0.0047	0.0001	0.1140	9.6033	158.13
	Europe	0.0030	0.0041	0.0001	0.1263	11.604	257.58
	U.S.	0.0046	0.0069	0.0001	0.1525	8.6382	127.68
Equity	Japan	0.0174	0.0411	0.0001	1.1453	15.091	304.67
	Europe	0.0209	0.0379	0.0002	0.7068	7.0145	79.401
	U.S.	0.0272	0.0686	0.0002	2.2511	13.972	337.97
Bond	Japan	0.0008	0.0024	0.0001	0.0994	23.448	845.74
	Europe	0.0012	0.0019	0.0001	0.0804	20.250	748.45
	U.S.	0.0026	0.0041	0.0001	0.0941	10.121	174.37

Table 2
Sample autocorrelations of the realized volatility in the foreign exchange, equity, and bond markets of Japan, Europe, and United States, respectively.

	Foreign exchange			Equity			Bond		
Lag	JP	EU	US	JP	EU	US	JP	EU	US
1	0.4048	0.3890	0.4407	0.5523	0.5669	0.5866	0.2442	0.2562	0.3713
2	0.3520	0.3372	0.3506	0.5051	0.5533	0.5322	0.2111	0.2095	0.2417
3	0.2648	0.3074	0.2990	0.3589	0.5507	0.4893	0.2088	0.1958	0.2151
4	0.2446	0.2380	0.2949	0.2884	0.4875	0.4773	0.2113	0.1853	0.2121
5	0.1735	0.2523	0.3100	0.2572	0.4852	0.4381	0.1645	0.1730	0.2172
6	0.1608	0.2277	0.2729	0.2510	0.4737	0.4196	0.1783	0.2030	0.2016
7	0.1501	0.2071	0.2749	0.3002	0.4489	0.3884	0.1584	0.1751	0.1966
8	0.1269	0.1881	0.2665	0.2960	0.4597	0.4764	0.1502	0.1774	0.1948
9	0.1626	0.2008	0.2893	0.3715	0.4304	0.4865	0.1679	0.1784	0.1842
10	0.1673	0.1764	0.2989	0.3408	0.3795	0.3895	0.1527	0.1821	0.2026

of the log periodogram $\{\log(I(\omega_j))\}_{j=1}^{m}$ on $\{\log(\omega_j)\}_{j=1}^{m}$, where $\omega_j = 2\pi j/T,\ \ j = 0, \ldots, T-1$ are the Fourier frequencies:

$$I(\omega_j) = \frac{1}{2\pi n}\left|\sum_{t=1}^{T} y_t \exp(-i\omega_j t)\right|^2, \tag{10}$$

is the periodogram and m is a positive integer. Then the GPH estimator is given by –0.5 times the least-square slope estimate in the ordinary least squares regression. To specify m an automatic procedure of Hurvich and Deo (1999) or visual inspection of log–log periodogram plots can be used. When $|d| > 0.5$, y_t is non-stationary; if $0 < d < 0.5$, y_t is stationary and has long memory; and when $-0.5 < d < 0$, y_t is stationary and has short memory.

To check for the existence of long memory in the volatility series formally, the R/S test defined by equation (9) can be used. The test clearly rejects the short memory hypothesis for all time series. The estimates of the parameter d, using the GPH estimator from (10), are reported in Table 3. The United States volatility in the foreign exchange and bond markets are non-stationary, perhaps related to the strong influence of the global financial crisis on this zone. Japan is the most stable zone with all volatility series stationary. In the European zone both equity and foreign exchange market volatilities are stationary.

4. Fractional cointegration

Given that all volatility series exhibit long memory, it is natural to conjecture if these volatilities share a common stochastic trend(s) which can be formulated using the concept of cointegration. For a nonstationary n dimensional series the cointegrated VAR model is:

Table 3
The GPH estimates of fractional differencing parameter d from equation (10) with t-statistics for the foreign exchange, equity and bond markets in Japan, Europe, and the United States. The power $m = T^{0.5}$.

		$\hat{d}$	t-Statistic
FX	Japan	0.2612	2.8723
	Europe	0.4155	4.5681
	US	0.5108	5.6162
Equity	Japan	0.2143	2.3563
	Europe	0.4652	5.1149
	US	0.4190	4.6071
Bond	Japan	0.4845	5.3269
	Europe	0.5022	5.5212
	US	0.5167	5.6808

$$\Delta y_t = \alpha(\beta' y_{t-1} + \rho') + \sum_{j=1}^{k} \Gamma_j \Delta y_{t-j} + \varepsilon_t, \tag{11}$$

where $\Delta y_{t-j} = y_{t-j} - y_{t-j-1}$ and α, β, ρ, Γ are the parameters of the model. The long run dynamics are captured by the stationary combinations $\beta' y_t$, while the short-term dynamics are determined by the Γ_j matrices. Supposing that matrix Π is rank deficient and suppressing the constant term ρ for simplicity, the model (11) can be rewritten in a structural form as:

$$A\Delta y_t = \alpha^* \beta' y_{t-1} - \sum_{j=1}^{k} B_j \Delta y_{t-j} + \varepsilon_t, \tag{12}$$

in which A and B_j are matrices of structural coefficients and $\alpha^* = A\alpha$. The main task in the structural VECM in equation (12) is to estimate the coefficients contained in matrix A. Note that the structural models proposed in this paper are identified from the calendar structure of the trading day. However, different identification schemes can be easily used within the proposed framework.

Both of the models defined in equations (11) and (12) imply that data are I(1) processes. In order to work the VECM representation into a form suitable for use with fractionally integrated series, the idea of Hosking (1981) can be used. Given an estimate of the fractional differencing parameter d is available, define the lag operator L as:

$$(1-L)^d = \sum_{k=0}^{\infty} \delta_k(d) L^k, \tag{13}$$

where $\delta_k(d) = \delta_{k-1}(d)(k-1-d)/k$, $\delta_0(d) = 1$ and following Künsch (1986) infinite process in (13) is truncated at $\sqrt{T}$. In this case, the VECM defined in (11) can be rewritten (suppressing the constant term) to allow y_t to be fractional of order d (Johansen, 2008; Johansen and Nielsen, 2010) as:

$$\Delta^d y_t = \Delta^{d-b} L_b \alpha \beta' y_t + \sum_{j=1}^{k} \Gamma_j \Delta^d L_b^j y_t + \varepsilon_t, \tag{14}$$

where $L_b = 1 - \Delta b$, $\Delta^b = (1-L)^b$, ε_t is an n-dimensional i.i.d process with a positive definite covariance matrix V, and α and β are $n \times r$ parameter matrices. The model $VAR_{d,b}(k)$ defined in (14) accommodates classical VARFIMA if $b = 1$. The special case of the model with $d = b$ is:

$$\Delta^d y_t = \alpha L_d \beta' y_t + \sum_{j=1}^{k} \Gamma_j \Delta^d L_d^j y_t + \varepsilon_t. \tag{15}$$

The structural analogue of model (15) can be presented as:

$$A\Delta^d y_t = \alpha^* L_d \beta' y_t - \sum_{j=1}^{k} B_j \Delta^d L_d^j y_t + \varepsilon_t. \tag{16}$$

Note that model (16) is a special case of system (14) with the additional calendar structure represented by matrix A. In the empirical analysis, system (16) is always identified. The time series y_t from equation (14) is cofractional of order $d-b$ when there exist vectors β for which $\beta' y_t$ has a unique fractional order $d-b$.

The coefficients in model in (14) can be estimated by maximum likelihood (Johansen and Nielsen, 2012). The log-likelihood function is given by:

$$\log L_T(\theta) = -\frac{T}{2}\log\left(\det\left(T^{-1}\sum_{t=1}^{T}\varepsilon_t\varepsilon_t'\right)\right), \tag{17}$$

in which θ represents the model parameters and:

$$\varepsilon_t = \Delta^d y_t - L_d \alpha\beta' y_t - \sum_{j=1}^{k}\Gamma_j \Delta^d L_d^j y_t, \tag{18}$$

are residuals from model (15). Johansen and Nielsen (2012) showed that the parameters α, β, Γ can be concentrated out of the likelihood function, and numerical optimization is only required for the fractional parameter d. Moreover, under i.i.d. errors the maximum likelihood parameter estimates $\left(\hat{d}, \hat{\alpha}, \hat{\Gamma}_j\right)$ are asymptotically Gaussian, while β is asymptotically mixed normal. These results allow the use of standard inference for all parameters of the model.

Since the parameters of the reduced model are estimated the residuals $\hat{\varepsilon}_t$ can be used to obtain parameter estimates of structural model (16). Implying normality of the residuals ε_t the full information maximum likelihood estimates of A and the structural covariance matrix D are obtained by maximizing:

$$\log l_T(\theta^s) = -\frac{n}{2}\log 2\pi - \frac{1}{2}\log\det(V) - \frac{1}{2(T-p)}\sum_{t=p+1}^{T}\varepsilon_t' V^{-1}\varepsilon_t, \quad \theta^s = (A, D), \tag{19}$$

in which p is a number of initial observations, $V = SS'$ and $S = A^{-1}D^{1/2}$. This maximization problem is equivalent to solving the nonlinear system of equations $\hat{V} = SS'$. Since A is estimated, the structural parameters B_j can be obtained from $B_j = \hat{A}\hat{\Gamma}_j$, where the $\hat{\Gamma}_j$ are computed from the reduced form model.

A key ingredient for investigating commonality in volatility is a cointegration test for rank r. When $0 < r < n$ (Johansen and Nielsen, 2012), y_t is fractional of order d and $\beta' y_t$ is fractional of order $d-b$. More specifically, the hypothesis of interest is $rank(\Pi) \le r$ against $rank(\Pi) \le n$. In this situation, the (S)VECM model can be used to capture both the long and the short run dynamics of volatility. If, on the other hand, the volatility series are not cofractional and $r = 0$, the commonality hypothesis is rejected in favour of the alternative hypothesis of dissimilarity. In this case the dynamics of volatility can be modelled using a simple fractional (S)VAR.

To test these hypotheses the profile likelihood function (17) is maximized to give the values $L\left(\hat{d}_n, \hat{b}_n, n\right)$ and $L\left(\hat{d}_r, \hat{b}_r, r\right)$ under the null and alternative hypothesis. The likelihood ratio test statistic for model (14) is then $LR(n-r) = 2\log L\left(d_r, b_r, r\right)/L\left(d_n, b_n, n\right)$. Theorem 11 from Johansen and Nielsen (2012) presents the limit distributions for the likelihood ratio test. In the case, of *weak cointegration*, that is, $0 < b < 0.5$, the likelihood ratio (LR) statistic has a standard asymptotic distribution $\chi^2(n-r)^2$. When it comes to *strong cointegration*, which means that $0.5 < b \le d$, asymptotic theory is nonstandard and the LR statistic is:

$$LR(n-r)\xrightarrow{D} tr\left(\int_0^1 (dW)W_{b-1}'\left(\int_0^1 W_{b-1}W_{b-1}'du\right)^{-1}\int_0^1 W_{b-1}(dW)'\right), \tag{20}$$

where dW is the vector of increments of standard Brownian motion and W_{b-1} is the correspondent vector fractional Brownian motion. The distribution (20) is continuous in b and can be computed using the numerical algorithm of MacKinnon and Nielsen (2014).

5. Common trends in volatility between financial markets

This section analyses volatility interaction between the foreign exchange, equity, and bond markets within each of the three trading zones. For the moment, the assumption is that each of the trading zones is unaffected by the others, which is a rather strict assumption which will be subsequently be relaxed, after these simple benchmark models have been examined. The econometric model to be estimated here is the fractional VECM (15) in each of the three global trading zones. In this case, y_t is the vector of minimum realized volatilities for the foreign exchange, equity and bond markets for a given trading zone.

The first practical issue is the correct choice of optimal lag length, k in equation (15). Given the rapid dissemination of news in financial markets, intuition would suggest that one week (5 trading days) would be enough to capture all the relevant information in lagged values of realized volatility. Moreover, as reported by Andersen et al. (2003), long-memory VAR models of volatility requires less lags than classical VARs. Formally, the choice may be guided by relying on well known information criteria (AIC and BIC) for lag length selection and checking if the included Γ_k, the coefficient matrices on the lag differences of the dependent variables, are statistically significant. As expected, the BIC favours a more parsimonious lag structure than the AIC. Overall, two lags were chosen as being sufficient for all models.

Table 4 presents the estimates of the cofractional rank r for the VECM estimated for each of the regions. The estimates of r were obtained by applying the likelihood ratio test outlined in the previous section to each of the three models. It is interesting to note that every zonal model contains two cofractional equations, which implies that volatility in the two markets can be expressed as a function of volatility in the third market, or equivalently that there is one driving factor for volatility in each of these regions.

The estimates of the system (15) for each of the trading zones are reported in Table 5. The estimates are obtained by imposing the triangular restrictions on the model which require that the top block of β be the identity matrix (Phillips, 1991). As the primary interest of this section are the linkages between markets, only the estimates of α, β and d are reported.

Turning first to the results for the Japanese region, the coefficient $\hat{d}$ is significant, indicating that Japanese volatility is stationary. The matrix $\hat{\alpha}^{JP}$ captures the long-run dynamic volatility feedback effect implied by the model. The insignificant parameters α_{22} and α_{23} implies the weaker connection between the bond and equity markets in Japan. It is possible to conjecture that the cointegrating vector $\hat{\beta}_1^{JP}=[1,0,-4.6044]'$ is associated with the volatilities in the foreign exchange and bond markets, which shows that the hypothesis of the absence of fractional cointegration, $\beta_{32}=0$, is rejected. The long-run ties represented by the $\hat{\beta}_2^{JP}$ are also significant. The general pattern in the United States volatility reported in Table 5 is similar to the Japanese market and is also stationary, which is confirmed

Table 4
Optimal choice of lag length k and cointegration rank r for equation (15) for Japan, Europe, and the United States.

Japan		Europe		United States	
Lag length k	Rank r	Lag length k	Rank r	Lag length k	Rank r
2	2	2	2	1	2

Table 5
Coefficient estimates of the VAR in (15) estimated using realized volatility in Japan, Europe and the United States for each of the three markets. Coefficients that are significant at the 5% level are marked (*).

Parameters	Estimates
$\hat{d}^{JP} =$	0.3723*
$\hat{\beta}^{JP} =$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -4.6044^* & -27.662^* \end{bmatrix}$
$\hat{\alpha}^{JP} =$	$\begin{bmatrix} -0.2967^* & 0.0265^* \\ 0.8714 & -0.1909^* \\ 0.0306 & -0.0096^* \end{bmatrix}$
$\hat{d}^{EU} =$	0.5030*
$\hat{\beta}^{EU} =$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1.5279^* & -6.5171^* \end{bmatrix}$
$\hat{\alpha}^{EU} =$	$\begin{bmatrix} -0.1035^* & 0.0099^* \\ -0.4996 & 0.0278 \\ 0.0068 & 0.0008 \end{bmatrix}$
$\hat{d}^{US} =$	0.4024*
$\hat{\beta}^{US} =$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1.2281^* & -1.1100^* \end{bmatrix}$
$\hat{\alpha}^{US} =$	$\begin{bmatrix} -0.0730^* & 0.0146^* \\ 0.3295 & 0.0167 \\ 0.0619^* & 0.0035 \end{bmatrix}$

by the significant $\hat{d}^{US} = 0.4024$. Both Japanese and the United States markets in this case have a single driving factor behind the $I(d)$ features of the volatility series.

The model estimates in the European market, however, are slightly different. First of all the volatility series are not stationary (albeit marginally so), with $\hat{d}^{EU} = 0.5030$. This result is surprising given the results from Table 3 where two out of three volatility series in Europe are stationary. The volatility linkages in the European market appear to be weak. Most of the coefficients in $\hat{\alpha}^{EU}$ are insignificant at the 5% level which means a weaker long-run relation within this market. This result is quite surprising as the European market is normally connected to other markets (e.g., Connolly and Wang, 2003). However, the LR test rejects the hypothesis $H_0 : \alpha_{ij} = 0, \forall i, j$ relating to the absence of fractional cointegration in the model. The results of this section establish weaker long-run ties between volatilities in Europe, while the patterns are similar in Japan and the United States. A fairly strong *prima facie* case for the 'commonality in volatility' hypothesis in all three zones is confirmed.

6. Common trends in volatility across trading zones

This section considers volatility patterns between the three global trading zones, Japan, Europe, and the United States in each of the three financial markets. In this case, y_t is defined as the vector of realized volatilities for a particular asset in each of the three trading zones. The question of interest now is whether or not the volatilities of returns to this particular asset class are linked across trading zones. The first problem to overcome is that, unlike the analysis of Section 5, there is scope for intraday interaction between the volatility of a given financial asset across the trading zones. For example, events in the foreign exchange market in Japan can influence the volatility of the foreign exchange market in both Europe and the United States on the same trading day. There is a natural ordering in each calendar day t that imposes the structure $\mathrm{JP}_t \rightarrow \mathrm{EU}_t \rightarrow \mathrm{US}_t$. Consequently, the VECM methodology must be augmented slightly and a structural VECM (SVECM) must be estimated, in which the calendar structure of the trading day imposes a recursive set of short-run restrictions on the intra-day interactions of the variables.

The SVECM model for the realized volatility of a particular asset in each of the trading zones is now defined by the system of equations (16) in which A is a (3×3) matrix representing the interaction between the variables that takes place on the same trading day. All the other parameters have the same meaning as in the previous sections. As already discussed, the model has a natural recursive structure, with the intra-day matrix, A, restricted to be the lower triangular matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -\eta_{21} & 1 & 0 \\ -\eta_{31} & -\eta_{32} & 1 \end{bmatrix}, \tag{21}$$

in which η_{21} captures the influence of the Japanese zone on the European zone, and η_{31} and η_{32} model, respectively, the effects of Japan and Europe on the United States.

Once again, the absence of any common trend in the realized volatilities will imply that the cointegrating rank of the system (16) is zero and the volatility dynamics can be estimated by a simple fractional SVAR without an error correction term. On the other hand, a single underlying common trend which may be interpreted as the common global news flow will be represented by a non-zero cointegration rank given by $r = n$–1. In this case, the impact of volatility is independent of the trading zone, with one process, namely, global news, describing the evolution of volatility in all zones.

The results of estimating model (15) with structural ordering (21) for the foreign exchange, equity and bond markets are Tables 6, 7 and 8 respectively. In particular, the lagged parameters α, β and Γ_j are estimated from equation (15), while the matrix of intra-day effects, A, is obtained from structural model (16). Similar to the previous section, the cointegrating rank, r, is estimated using the LR test.

Complex dynamics are observed in the foreign exchange market, with most of the coefficients of the SVECM significant at the 5% level. The intra-day linkages, represented by the coefficients η_{21} for Europe and η_{32} for the United States, are significant, while the effect from Japan to the United States, captured by the coefficient η_{31}, is not significant. A robust conclusion seems to be that intra-day effects matter in the foreign exchange market. The size of these effects do differ, however, implying the largest

Table 6
Coefficient estimates of the VAR in (16) with the relevant calendar structure represented in A from (21) estimated using realized volatility in the foreign exchange market for each of the three trading zones. Coefficients that are significant at the 5% level are marked (*).

Parameters	Estimates
$\hat{r} =$	2
$d =$	0.4641*
$A =$	$\begin{bmatrix} 1 & 0 & 0 \\ -0.2526^* & 1 & 0 \\ -0.0650 & -0.5960^* & 1 \end{bmatrix}$
$\alpha =$	$\begin{bmatrix} -0.5640^* & 0.4677^* \\ 0.0846 & -0.3354^* \\ 0.1582 & 0.1660 \end{bmatrix}$
$\beta =$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -0.5321^* & -0.5867^* \end{bmatrix}$
$\Gamma_1 =$	$\begin{bmatrix} -0.2026^* & -0.1597^* & 0.2958^* \\ -0.1042^* & -0.3680^* & 0.1394^* \\ -0.0856 & 0.0702 & -0.3221^* \end{bmatrix}$
$\Gamma_2 =$	$\begin{bmatrix} -0.1225 & 0.7635^* & 0.2549^* \\ 0.0351 & -0.2583^* & 0.0733^* \\ -0.1122 & 0.3032 & -0.4494^* \end{bmatrix}$

Table 7
Coefficient estimates of the VAR in (16) with the relevant calendar structure represented in A from (21) estimated using realized volatility in the equity market for each of the three trading zones. A rank of the system is estimated at the 10% level. Coefficients that are significant at the 5% level are marked (*).

Parameters	Estimates
$\hat{r} =$	2
$d =$	0.5139*
$A =$	$\begin{bmatrix} 1 & 0 & 0 \\ -0.2106^{*} & 1 & 0 \\ -0.3245 & -0.6892^{*} & 1 \end{bmatrix}$
$\alpha =$	$\begin{bmatrix} -0.4361^{*} & 0.0494^{*} \\ -0.1961^{*} & -0.0155 \\ 0.0183 & -0.0270 \end{bmatrix}$
$\beta =$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -0.5276^{*} & -0.7913^{*} \end{bmatrix}$
$\Gamma_1 =$	$\begin{bmatrix} -0.0967 & 0.0274 & 0.3162^{*} \\ 0.0867^{*} & -0.6314^{*} & -0.0043 \\ 0.1825^{*} & -0.0675 & -0.4140^{*} \end{bmatrix}$
$\Gamma_2 =$	$\begin{bmatrix} 0.5106^{*} & -0.2790^{*} & -0.1073^{*} \\ 0.2430^{*} & -0.3031^{*} & 0.2807^{*} \\ -0.4367^{*} & 0.7097^{*} & -0.2782^{*} \end{bmatrix}$

impact from Europe to the United States. Another interesting result is that in the long run volatility linkages between Europe, Japan and the United States are represented by a single common trend. Such commonality is also supported by the significant coefficients β_{32} and β_{31}. The only zone which has a set of lagged coefficients γ_{i3} that are all statistically significant for both Γ_1 and Γ_2 is the United States. In the short run, lagged volatility from the United States is important for explaining volatility in all zones. This result supports the conjecture of Engle et al. (1990) that, if volatility spillovers do occur, they probably flow from New York to the overseas trading centres.

Results for equity market in Table 7 show that two coefficients of the matrix of intra-day effects, A, are significant at the level 5%. These significant effects are intra-day volatility spillovers in the equity market from Japan to Europe and from Europe to the United States.[7] This result suggests that the effect from Japan to the United States operates via Europe. The most striking result is the presence of two cointegrating vectors, a result that suggests one underlying common trend and hence an acceptance of the world-wide news hypothesis. All zones are interrelated in the long term, meaning that all coefficients in β are significant. In contrast to the foreign exchange market, volatility in the equity market is a non-stationary process. The overall pattern in the equity market can be summarized as one of significant interactions between zones driven by a single factor.

The results for the bond market, Table 8, indicate that although the patterns are generally similar to those in the foreign exchange market, there are a number of important distinctions that should be noted. Surprisingly, the intra-day effects from Japan to Europe and from Japan to the United States represented by the coefficients η_{21} and η_{31} are not significant. In the long run, however, Japan and the United States are closely related, a conclusion that is supported by the significant coefficient $\beta_{31} = -0.1500$.

Overall, these results lead to an important conclusion that it is not possible to regard each trading zone as being completely independent of all the others. A second interesting conclusion to emerge from this analysis is that there are strong linkages between realized volatility in all of the three trading

[7] The finding that volatility spillovers from Japan to the United States on the same day are not significant is also confirmed by Becker et al. (1990).

Table 8
Coefficient estimates of the VAR in (16) with the relevant calendar structure represented in A from (21) estimated using realized volatility in the bond market for each of the three trading zones. Coefficients that are significant at the 5% level are marked (*).

Parameters	Estimates
$\hat{r} =$	2
$d =$	0.4232*
$A =$	$\begin{bmatrix} 1 & 0 & 0 \\ -0.0038 & 1 & 0 \\ -0.0391 & -0.2801^* & 1 \end{bmatrix}$
$\alpha =$	$\begin{bmatrix} -0.0173 & -0.0052 \\ 0.0075 & 0.0495^* \\ 0.3490^* & -0.0075 \end{bmatrix}$
$\beta =$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -0.1500^* & -0.0060 \end{bmatrix}$
$\Gamma_1 =$	$\begin{bmatrix} -0.7337^* & 0.0851 & 0.0277 \\ 0.0010 & -0.5086^* & 0.0702^* \\ -0.3569^* & 0.4914^* & -0.3627^* \end{bmatrix}$
$\Gamma_2 =$	$\begin{bmatrix} -0.5274^* & 0.1911 & 0.0644 \\ 0.0559 & -0.3720^* & 0.0598^* \\ -0.4240^* & 0.3715 & -0.6572^* \end{bmatrix}$

zones in all of the markets in the long run. The existence of a single global news stream driving volatility is accepted for the foreign exchange, equity and bond markets. Consequently, there is a strong motivation to pursue this avenue of inquiry in a general model that encompasses the models addressed in the previous two sections.

7. A general model of volatility interaction

A model capable of analysing volatility patterns across both international trading zones and between financial markets simultaneously is now proposed. Essentially, the fractional VECM of Section 5 and the structural VECM of Section 6 are combined in an unrestricted model. This general model is given by:

$$\mathbb{A}\Delta^d Y_t = \alpha^* L_d \beta' Y_t - \sum_{j=1}^{k} \mathbb{B}_j \Delta^d L_d^j Y_t + \varepsilon_t, \qquad \varepsilon_t \sim \text{iid}\, N(0, \mathbb{D}), \tag{22}$$

in which

$$\mathbb{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\eta_{21} & 1 & 0 & -\eta_{24} & 0 & 0 & -\eta_{27} & 0 & 0 \\ -\eta_{31} & -\eta_{32} & 1 & -\eta_{34} & -\eta_{35} & 0 & -\eta_{37} & -\eta_{38} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -\eta_{51} & 0 & 0 & -\eta_{54} & 1 & 0 & -\eta_{57} & 0 & 0 \\ -\eta_{61} & \eta_{62} & 0 & -\eta_{64} & -\eta_{65} & 1 & -\eta_{67} & -\eta_{68} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -\eta_{81} & 0 & 0 & -\eta_{84} & 0 & 0 & -\eta_{87} & 1 & 0 \\ -\eta_{91} & -\eta_{92} & 0 & -\eta_{94} & -\eta_{95} & 0 & -\eta_{97} & -\eta_{98} & 1 \end{bmatrix}, \quad Y_t = \begin{bmatrix} \mathrm{jp}_t^{\mathrm{fx}} \\ \mathrm{eu}_t^{\mathrm{fx}} \\ \mathrm{us}_t^{\mathrm{fx}} \\ \mathrm{jp}_t^{\mathrm{eq}} \\ \mathrm{eu}_t^{\mathrm{eq}} \\ \mathrm{us}_t^{\mathrm{eq}} \\ \mathrm{jp}_t^{\mathrm{bd}} \\ \mathrm{eu}_t^{\mathrm{bd}} \\ \mathrm{us}_t^{\mathrm{bd}} \end{bmatrix},$$

the matrices $\mathbb{B}_j$, $j \geqslant 1$ are parameter matrices for lag j, $\alpha^* = \mathbb{A}\alpha$ and ε_t is a vector of non-correlated disturbances with covariance matrix $\mathbb{D}$.

The upper left, middle and lower right shaded blocks of the matrix highlight coefficients describing the behaviour of intra-day volatility interaction within the foreign exchange, equity and bond markets. As in the previous section, the structure of these matrices incorporate the calendar restrictions imposed by the definition of the global trading day. Each of these matrices corresponds to the matrix of intra-day effects estimated in Section 6 as separate entities for each market. The main innovation in this general model is in the off-diagonal coefficient blocks which now describe the intra-day effects from all of the other asset markets in all the trading zones, which the single-market analysis of Section 6 ignored. For example, the coefficient η_{51} measures the intra-day influence of the Japanese foreign exchange market on the European equity market. Similarly, η_{62} measures the intra day effect from the European foreign exchange market on the United States equity market. It is important to note that events in the United States and Europe can only affect Japan at the opening of the following global trading day.

The cointegration rank, estimated using the LR test discussed earlier, is found to be $r = 8$, a very significant result because it confirms that in this general model there is a single common trend, global news, which underlies realized volatility in all these markets and across all zones. The parameters $\theta^s = \{\mathbb{A}, d, \alpha, \beta, \mathbb{B}_1, \mathbb{B}_2\}$ of system of equations (22) are estimated by maximum likelihood for $k = 2$ lags using the same procedure, as outlined in Section 4. Results are only reported for and d for the sake brevity. Estimates of the coefficients in matrix $\mathbb{A}$, with asterisks indicating the significance of individual coefficients at the 5% level, are:

$$\mathbb{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.2556^* & 1 & 0 & 0.0038 & 0 & 0 & -0.0737 & 0 & 0 \\ -0.0719 & -0.5577^* & 1 & 0.0062 & -0.0092^* & 0 & -0.0563 & -0.0299^* & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -0.5387 & 0 & 0 & -0.1758^* & 1 & 0 & -0.7806 & 0 & 0 \\ -0.1508 & -1.5490 & 0 & -0.2916 & -0.6116^* & 1 & -1.4850 & -1.0855 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -0.0036 & 0 & 0 & -0.0015 & 0 & 0 & -0.0018 & 1 & 0 \\ -0.0012 & -0.0484^* & 0 & -0.0008 & -0.0155^* & 0 & -0.0113 & -0.1997^* & 1 \end{bmatrix}.$$

The first thing to note is that the intra-day volatility patterns between the zones for a particular market for the general model (shaded areas) are similar to the results presented in Section 6 (matrices $\mathbb{A}$ of Tables 6, 7 and 8 for the foreign exchange, equity, and bond markets respectively). An insignificant intra-day effect from the Japanese equity market to the United States equity market is also confirmed by Hamao et al. (1990). The coefficients in the highlighted block related to the effect from Japan to the United States are not significant, which provides further evidence of how this zone is separated from the others.

Most of the coefficients in the non-shaded panels of the matrix $\mathbb{A}$ are insignificant, which means that intra-day effects between the respective markets are not strong. However, there are four significant coefficients, which are:

1. η_{35}, the effect of the European equity market on the United States foreign exchange market;
2. η_{38}, the effect of the European bond market on the United States foreign exchange market;
3. η_{92}, the effect of the European foreign exchange market on the United States bond market; and
4. η_{95}, the effect of the European equity market on the United States bond market.

Taken together with the previous results for the shaded blocks that show a strong and significant pattern of influence from Europe to the United States in each of the foreign exchange, equity and bond markets (η_{32}, η_{65} and η_{98}, respectively), the overall pattern that seems to emerge is one in which the developments in European markets have a significant influence on what happens in the United States markets later on the same day. Moreover, the inter-market effects from Japan are not significant, which is also confirmed by Hamao et al. (1990).

The long-term volatility impact is given by the coefficients of the matrix β. Parameter estimates for β, with stars indicating significance at the 5% level, are as follows:

$$\beta = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -0.943^* & -1.286 & -2.552 & -11.19^* & -2.297^* & -26.27 & 0.072 & -0.394^* \end{bmatrix}.$$

The columns of the matrix β represent cointegration vectors of the general model. Only four values are significant, which might be interpreted as following. Each of the foreign exchange and the bond markets has a single cointegrating relation, while the equity market is driven by a common stochastic trend. Such a result leads to the important conclusion that the global market can be explained by the worldwide meteor shower hypothesis. The matrix of the coefficients α capturing the speed of convergence in the long run:

$$\alpha = \begin{bmatrix} -0.714^* & 0.346^* & 0.109^* & 0.008^* & 0.011^* & -0.007 & 0.273^* & 0.177^* \\ 0.008 & -0.502^* & 0.116^* & -0.008^* & 0.002 & 0.006^* & 0.318^* & 0.171^* \\ 0.023 & 0.114 & -0.378^* & 0.045^* & -0.010 & 0.007 & 0.338^* & 0.150 \\ -0.393 & 1.988^* & -0.131 & -0.524^* & 0.021 & 0.162^* & 0.463 & 0.274 \\ 1.979^* & -2.349^* & 0.032 & -0.269^* & -0.038^* & 0.132^* & 0.418 & 0.797^* \\ 0.190 & -3.702^* & 1.813^* & 0.040^* & -0.009 & 0.003 & 2.666^* & 1.687 \\ -0.029 & 0.058 & 0.019 & -0.004 & 0.001 & -0.004^* & -0.167^* & -0.040 \\ 0.043 & -0.034^* & 0.005 & -0.003 & 0.001 & 0.001 & -0.001 & -0.003 \\ -0.089 & 0.205^* & 0.018 & -0.010 & -0.011^* & 0.006 & 0.213^* & 0.007 \end{bmatrix}.$$

An interesting pattern is represented by the block of significant coefficients in the fourth column. All these coefficients are related to the foreign exchange and equity markets. Finally, the significant fractional parameter $d = 0.4861$ reveals stationarity of the global market.

There are two broad conclusions concerning intra-day interaction between markets and trading zones to be drawn. First, there is compelling evidence for a commonality in volatility in the short run for the equity and foreign exchange markets in which volatility in one trading zone is driven by events from other zones. In the bond market only European volatility plays an important role in influencing events in other markets. Second, there are enough significant coefficients outside of the diagonal blocks to suggest that the volatility spillovers also occur between markets for different assets. Once again, this emphasizes that volatility linkages are particularly complex and one simple explanation is not available.

The methodology of Hasbrouck (1995) can be used to summarize the effects from different zones across different markets. Following the idea of Hasbrouck (1995), the information share of a market is calculated from the covariance matrix, $\mathbb{D}$, estimated from equation (22). This measure is defined as the proportion of the contribution of a specific market relative to the total variance with a normalization ensuring that the information shares sum to unity. The respective information shares for the foreign exchange, equity and bond markets in Japan, Europe and the United States are presented in Table 9.

Table 9
The information share estimated from equation (22) for the period 4 January 1999 to 30 December 2015 (Total), 4 January 1999 to 14 September 2008 (pre-GFC), 15 September 2008 to 30 March 2010 (GFC) and 1 April 2010 to 30 December 2015 (post-GFC – European bond market crisis). The estimates are normalized to sum to unity.

	JP-FX	EU-FX	US-FX	JP-EQ	EU-EQ	US-EQ	JP-BD	EU-BD	US-BD
Total	0.0038	0.0028	0.0080	0.2276	0.1912	0.5610	0.0013	0.0005	0.0036
Pre-GFC	0.0049	0.0031	0.0122	0.1182	0.4106	0.4396	0.0035	0.0005	0.0071
GFC	0.0026	0.0056	0.0099	0.1909	0.0961	0.6924	0.0001	0.0002	0.0021
Post-GFC	0.0064	0.0018	0.0108	0.3625	0.1032	0.5038	0.0011	0.0120	0.0072

The results indicate that the main source of volatility for the whole sample (Total) appears to be concentrated in the equity markets, where the information share is 97.9%. Only 2.1% of the information share is generated by the foreign exchange and bond markets.

To explore any time variation in this pattern, the sample period in Table 9 is also divided into three sub-periods: the pre-GFC period, before 2008; the GFC between 2008 and 2010; and the period characterized by instability in the European (Greek) bond markets from 2010 to 2015. During the pre-GFC period the information share is largest for the United States and European equity markets. The main source of volatility during the GFC stems from the United States equity market, with 69.2% of the information share, a result that is unsurprising given the unprecedented volatility in the United States at that time. A gratifying result is that the influence of the bond market appears to be more significant (increased information shares) in the post GFC period characterized by turbulence in European bond markets. Moreover, the largest information share for the bond market is observed in Europe. Overall, the information shares from the estimated volatility model are consistent with prior expectations about the importance of equity markets, but the time variation in information shares is also consistent with major economic events during the sample period.

8. News and the common trend

The fractionally integrated VECM models, of the general form shown in (14), estimated on foreign exchange, equity and bond realized volatility data, all suggest that there exists at least one fractional cointegrating vector, that is, $r > 0$. In this situation, the volatility series y_t can be decomposed into permanent (fractionally integrated) and transitory (stationary) components using the orthogonal decomposition proposed by Johansen and Nielsen (2010) given by:

$$y_t = \beta(\beta'\beta)^{-1}\beta' y_t + \beta_\perp(\beta_\perp'\beta_\perp)^{-1}\beta_\perp' y_t. \tag{23}$$

In this representation the first summand defines the stationary (fractionally cointegrated) component while the second summand defines the permanent (fractionally integrated) component, where the $\beta_\perp$ are normalized weights that represent the loadings of each volatility series on potentially $n - r$ common stochastic trends.

The results from Section 6 show that, for the all these asset markets, the cointegration rank r of model (15) is exactly equal to $n - 1$. In this important special case there is only one common stochastic trend of fractional order $I(d)$. The conjecture here is that this single fractionally integrated component has the natural interpretation of being related to the global news stream. To identify a global news stream, news items relating to individual firms, which are constituents of the relevant index in each trading zone, are collected from the Thomson Reuters News Analytics database. The data represent all news headlines for the stocks included in the S&P 500 (United States), TOPIX 100 (Japan) and DAX 30 (Europe) from 4 January 2005 to 28 September 2012.[8] Each news item is associated with a

[8] The news data are collected on the same days as the index data discussed in Section 2. The sample period is consistent with availability of the news data.

Table 10
Coefficient estimates and robust t-statistics of equation (24) for the period 4 January 2005 to 28 September 2012 (total), 4 January 2005 to 14 September 2008 (pre-GFC), 15 September 2008 to 30 March 2010 (GFC), 1 April 2010 to 28 September 2012 (post-GFC). Coefficients that are significant at the 5% level are marked (*).

γ_{total}	γ_{preGFC}	γ_{GFC}	γ_{Greek}
0.5287*	0.7271*	0.2538*	0.3562*
(2.7603)	(2.6484)	(3.2791)	(5.2529)

specific firm and is scored with a relevance score in the [0,1] interval, with 1 being most relevant to the firm, and a sentiment score taking the values of +1, 0, or −1 for a positive, neutral and negative tone. News items with a relevance of 1 from each firm are aggregated to construct a daily time series representing the volume of news flow related to each of the equity markets in Japan, Europe and the United States.[9]

The Thomsom Reuters News Analytics database applies only to equities. As a consequence, the estimates from Table 7, which report the results for the equity market, are used to obtain the common trend for the equity market by applying decomposition (23). Consider now a simple linear regression of innovations in the common trend, CT_t, and innovations in the global news, GN_t, given by:

$$\Delta^{0.77}\log CT_t = \gamma\Delta^{0.36}\log GN_t + \varepsilon_t, \tag{24}$$

in which the fractional difference parameters are obtained by using the GPH procedure. The coefficient γ captures the contribution of news, GN_t, to the innovations in trend CT_t.

Estimates of γ for the same sub-periods as used in Table 9 are presented in Table 10. As to be expected, both the fractionally integrated common trend and news exhibit strong persistence, but the really interesting result is that for the whole sample the coefficient γ capturing the contemporaneous news effect is significant at the 5% level and, although not equal to unity, the coefficient estimate of 0.5287 for the entire sample period is indicative that news is a primary driver of innovations in trend. Moreover, during the period of relative calm in the equity market pre-GFC, the estimate rises to 0.7271. Not surprisingly, during the period of immense turmoil of the GFC, there is a decoupling of news and the permanent component of volatility during the crisis period, but there is evidence that the importance of news increases during the period of bond market crisis. Overall, the common trend and news flow of the equity market are strongly interrelated, which in turn confirms that commonality in volatility can be linked to the existence of global news. These findings confirm the important role of news in propagating the volatility around the global market, which was documented by Engle and Ng (1993) and Ederington and Lee (1993).

9. Trading volatility

In order to assess the economic significance of fractional cointegration structure underlying global volatility in the foreign exchange, equity and bond markets, a simple volatility trading strategy is proposed which is based on the VIX index which represents S&P 500 index option implied volatility and is one of the world's most widely quoted measures of volatility. The strategy comprises two actions:

1. Take a **short** position in VIX futures when global volatility is forecast to decline, that is, when forecast volatility is lower than its current value.
2. Take a **long** position in VIX futures when global volatility is forecast to increase, that is, when forecast volatility is higher than its current value.

[9] Such filtering of data was used by Groß-Klußmann and Hautsch (2011), who found that the articles with relevance 1 were the most frequent in the data.

Table 11
Total and average log-returns with respective standard deviations and Sharpe ratios are generated from trading on VIX futures for the period 5 January 2015 to 30 December 2015.

	FIVAR			VECM		
	1 day	5 days	10 days	1 day	5 days	10 days
Total returns	1.4625	1.5733	1.1839	0.9457	1.5838	1.4797
Average returns	0.0067	0.0072	0.0054	0.0043	0.0072	0.0068
Standard deviation	0.0607	0.0606	0.0608	0.0609	0.0606	0.0607
Sharpe ratio	0.1101	0.1185	0.0889	0.0709	0.1193	0.1114

Transactions costs are assumed to be zero for this simple exercise.

To implement this trading strategy forecasts of volatility are required. Two sets of forecasts will be used; one forecast will be based on the VECM model in which the fractional cointegration structure is imposed, while the other forecast will be generated from a FIVAR model in which the fractional cointegration structure is ignored. To generate forecasts from the VECM the method proposed by Dolatabadi et al. (2015) is used. Given the estimated parameters of the model $\left(\hat{d}, \hat{\alpha}, \hat{\beta}, \hat{\Gamma}_1, \ldots, \hat{\Gamma}_k\right)$, and using data available up to time t, the multi-period h-step ahead forecasts can be generated from:

$$\hat{Y}_{t+j|t} = L_{\hat{d}}\hat{Y}_{t+j|t} + \hat{\alpha}\hat{\beta}' L_{\hat{d}}\hat{Y}_{t+j|t} + \sum_{i=1}^{k}\hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^{i}\hat{Y}_{t+j|t}, \tag{25}$$

where $\hat{Y}_{s|t} = Y_s$ for $s \le t$. Forecasts $\hat{Y}_{t+h|t}$ can be constructed recursively from (25) for $j = 1, \ldots, h$. Note that the second term on the right hand side of (25) represents the contribution of the common trend to the forecast. If this term is equal to zero, then equation (25) defines the best linear predictor from a FIVAR (or VARFIMA) model.

To implement the proposed strategy, model (22) is estimated over the period 4 of January 1999 to 31 December of 2014, and the parameter estimates are used to generate forecasts from the period 5 January 2015 to 30 December 2015 using equation (25). A comparison of the performance of the strategy based on the FIVAR and VECM forecasts will highlight the economic importance of the common trend (or in other words, the cost of ignoring the common trend).

Based on (non-overlapping) forecast horizons of 1, 5 and 10 days, the performance of the proposed strategy using both FIVAR and VECM forecasts is summarized in Table 11. Total and average returns, along with the standard deviation of returns and the associated Sharpe ratios are reported. The first major conclusion to note is that a trading strategy that uses either of the forecasting schemes provides superior results when compared with a simple long position in the VIX. The latter strategy yields, respectively, a standard deviation of 0.0610 and a Sharpe ratio of –0.0002, a heartening result as it suggests that using a model is better than not using a model at all.

More interesting from the point of view of the current research is the comparison of returns to the trading rule generated by the VECM and FIVAR forecasts. As the forecast horizon increases, the VECM becomes the preferred forecast. At the 1 day forecast horizon the FIVAR model (which uses only short run-dynamic information) produces forecasts that lead to better outcomes in terms of Sharpe ratios. However, the theoretical advantage of the VECM model is that the longer-memory fractional cointegration component is embedded in the forecasts and thus it is no surprise that using the VECM leads to superior performance for horizons of longer than 1 day. Despite the simplicity of this trading strategy, it confirms that using the information encapsulated in the common volatility trend contains enough meaningful information to support its use in longer-term decision making.

10. Conclusion

An enormous amount of research has focused on the issue of short-run volatility transmission through time within a specific country or asset class. This paper has considered volatility transmission in global foreign exchange, equity and bond markets, while simultaneously paying attention to the long memory characteristics of volatility. Realized volatility estimates were constructed using high frequency data for three asset (foreign exchange, equity and bond) markets and three trading zones (Japan, Europe and the United States), which constitute the global trading day. The use of realized volatility marks a significant departure in the literature on volatility transmission as it allowed traditional time-series techniques, such as VARs, structural VARs and VECMs, to be used to test hypotheses about volatility linkages.

Volatility linkages between global financial markets, which were based on the long-memory characteristics of volatility, were formulated in terms of fractional cointegration. Structural fractional VECM models, with the calendar restrictions implied by the global trading day carefully imposed, were proposed and estimated. In this way the models were able to capture both short-run dynamics and common long-run dynamics in volatility.

The conclusion that emerged from this work is that volatility transmission is governed by a series of complex short- and long-run relationships linking the different asset markets and trading zones. In the short-run, there exists a meteor shower effect in which volatility in a particular zone experiences significant intra-day influences from the trading zone which immediately precedes it in the global trading day. In the longer term, the dynamics of volatility invariably exhibit a single common fractional component which is interpreted as a common volatility factor.

The existence of this single fractional trend in volatility is a remarkably robust result, which was obtained in all of the estimated models. To deepen our understanding of the source of the common volatility factor, at least in equity markets, it has been shown to be related to an observed measure of global news, which was collected using data from the Thomson Reuters Analytics database.

Finally, it was shown that forecasts of global volatility can be harnessed to inform a volatility trading strategy. At longer horizons, forecasts taking into account the common trend in volatility produce superior trading outcomes relative to those that ignore a possible common trend. These results, coupled with those relating to news flow, show that the common volatility trend has an economic interpretation and that its use in forecasting volatility has significant implications in the medium to longer term.

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