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Reviving stagnated debates in Group Decision Making environments with high number of alternatives

J. A. Morente-Molinera^{*a}, M. Barragán-Guzmán^a, J.R. Trillo^a, M. A. Martínez-Sánchez^b, F. J. Cabrerizo^a, E. Herrera-Viedma^a

^aAndalusian Research Institute in Data Science and Computational Intelligence, jamoren@decsai.ugr.es, University of Granada, Granada, Spain.

^bDepartment of Social Work and Social Services, University of Granada, 18001 Granada, Spain.

Abstract

In group decision-making, experts try to obtain a consensus to determine how to order a series of alternatives. A consensus is a decision that reflects the opinions of every group member. Consensus requires discussion and deliberation between the group members. During the process, it is normal that the discussion process to stagnate. In this article, we present a new method of group decision-making that solves the stagnation of the process by including new information. The timing of this is determined by a process stagnation analysis. Fuzzy Ontologies allow experts to work in environments with large numbers of alternatives. The method takes into account the experts' rankings of alternatives to determine which criteria the experts seem to like the most. When introducing new information into the debate, experts are given the possibility to explore alternatives that are very different from those already seen or to look for alternatives that meet the criteria that are preferred the most.

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1. Introduction

One of the most common problems dealt by humans is Group Decision Making (GDM). Group Decision-Making problems have been investigated by scientific literature since the 80's until nowadays. In fact, it is possible to find a lot of publications in the recent literature [1, 2, 3]. Recent GDM methods are about how to carry out decision processes on the Web and social media [4]. In these environments, there are a lot of information that experts have to deal with in order to carry out responsible and accurate decisions. Nevertheless, the number of elements that experts can deal with in a single moment is 7 [5]. Thanks to social media and Big Data, recent environments can have around 1000 or even 10000 or more different possibilities to discuss about. Therefore, there is a need of new intelligent and adaptable GDM methods that are scalable and help experts to deal with those amounts of information. For this purpose, new structures need to be developed. Fuzzy Ontologies (FOs) are used to deal with imprecise information providing the experts with a quantity of information that they can deal with with no problem.

^{*}Corresponding author.

E-mail address: jamoren@decsai.ugr.es.

The amount of information that such methods have to deal with in the future will only grow. Therefore, we need methods that are scalable and that are suitable for any number of available alternatives. FOs, in addition to being able to organise information in an appropriate manner, are capable of working with imprecise information, which is what is usually used in this type of process. Moreover, their structure allows us to represent large amounts of information, which makes them ideal for dealing with decision environments where the number of alternatives and criteria available is high.

It is normal, when conducting GDM processes, that there comes a point where the debate stagnates and no one is able to contribute anything new to the discussion. This is because the experts are not able to move the discussion forward or provide new ideas or points to discuss. To solve this, our method proposes a new decision support tool that helps the experts to continue the debate by providing new alternatives to discuss. In this way, we not only focus on the most promising alternatives but, when the debate has stalled, it is possible to remove the most unpopular ones and replace them with new alternatives that re-launch the debate.

GDM papers available in the literature, [6-8], solves this problem by making the experts to deal with groups of alternatives, that is, categories. Nevertheless, this approach makes the discussion very general since the experts cannot discuss the specific and concrete aspects of each of the alternatives. There are also available some methods that use FOs [9, 10]. These methods extract the alternatives that better fulfill a certain criteria and allow the experts to discuss about them. Nevertheless, these methods tend to oversimplify the process by not allowing the experts to discuss about most of the alternatives that are not available on the FO. Also, it is difficult for the experts to know from the start which are the criteria that they are looking for. Also, the alternatives that the experts discuss on these methods are very similar among them making the debate almost unnecessary. All these mentioned drawbacks of the already existing FO methods need to be overcome. The methods currently available in the literature can be improved if we allow experts to discuss alternatives that are very different from each other.

Our contribution to the solution of this problem consists in the development of a new GDM method that uses FOs and that, in turn, modifies the set of alternatives. It is designed to be used in environments with a large number of alternatives and criteria. Highlights of our methods are shown below:

- We present a fully scalable method that uses FOs to represent the alternatives and criteria that are part of the decision process. It is necessary to know this information a priori in order to build the FO properly.
- Experts debate all the time with a small number of alternatives. They can analyse different sets of alternatives at any given time, which makes it easier for them to carry out a thorough analysis and for the debate not to stagnate.
- In order to prevent the debate from stagnating, we have developed a new process that adds new information to the debate when this situation is detected. In this way, we carry out the following process. When the debate has stalled, the experts are given new alternatives to discuss. In order to carry out this process, we propose three different approaches, the last one being a combined approach of the first two. The exploration approach consists of finding alternatives that are very different from those already discussed. In this way, experts see other points of view. The exploitation approach is to find alternatives that are very similar to the most popular ones. In this way, experts can discuss alternatives that meet the criteria that best fit the problem. Finally, we propose a combined strategy where we first have the experts discuss very different alternatives and finally use the second approach to find the best alternative that meets the most popular criteria.
- When experts debate, they are allowed to conduct a thorough discussion of the set of alternatives selected at that time. The set of alternatives is only modified if the debate has stalled.
- Our GDM method works with criteria. It is possible to extract, from the discussions of several sets of alternatives, which criteria the experts are taking most into account. This helps to refine the alternatives presented to them in the discussion.

In section 2, we described the FO assisted GDM method in detail. In section 3, an application example is presented. Finally, some conclusions are pointed out.

2. A novel method for reaviving debates

The main goal of this section is to carry out a detailed description of the proposed method. In subsection 2.1, the structure and alternatives extraction process of the FO is described. In subsection 2.2, the stagnation process and GDM steps are detailed.

2.1. Fuzzy Ontology structure and reasoning process

The FOs are a key part of the functioning of the developed method. Thanks to them, we can store information on the criteria and alternatives that conform the decision environment. Within the FO, each of the alternatives is related to each of the criteria through the use of linguistic labels. Therefore, a linguistic set with a specific size is defined. For the examples, we will use a granularity of 7. Therefore, each alternative and criterion is related through the association of a label from the set $S^7 = \{s_1, \dots, s_7\}$. In turn, each label has an associated membership value which is located in the interval $[0,1]$. If the associated value is 0, then the alternative does not meet the criterion. Our method also allows a set of labels to be associated to each alternative-criterion relation.

For instance, it is possible to state that labels s_2 and s_3 have a membership value of 1 for alternative x_1 and criterion cr_2 . This can be defined by a hesitant fuzzy relation [10, 12] in the following way: $\{s_2, s_3\}$. Since labels s_1 and s_4 are close to the set $\{s_2, s_3\}$, they can also have a high similarity value. From now on, for exemplary purposes, it is assumed that only one label of S^7 is assigned to each relation. In this paper, the similarity values of the labels are defined as follows [10]. The same labels have a similarity value of 1. Closest labels have a similarity value of 0.7. Labels whose index distance is 2 have a similarity value of 0.3. Finally, distances larger than 2 have a similarity value of 0.

To calculate the reduced set of alternatives, we perform a search and obtain those alternatives that best meet a set of criteria. It is possible to assign different weights to the criteria values in order to establish their importance.

2.2. Group Decision Making process description

In this section, the presented method is described. Subsection 2.2.1 defines initial parameters that the method need at the beginning. Subsection 2.2.2 defines how the alternatives ranking is calculated. Finally, subsection 2.2.3, defines the stagnation process that allow the method to update the alternatives set.

2.2.1. Defining the initial parameters

The following parameters are important and need to be set before starting the GDM process [15]:

- **Stagnation threshold, β :** This value measures the number of changes experts make to their preferences. When these changes are low, then it is a good time to modify the set of alternatives. It is calculated as a value in the interval $[0,1]$.
- **Replacement of alternatives, ar :** This parameter defines the number of alternatives we replace within the set of alternatives that are part of the expert discussion. The lower the value chosen, the fewer alternatives are changed. An excessively low value of ar means that hardly any new alternatives can be discussed. Too high a value of ar may cause some interesting alternatives to be lost. The ar value should always be lower than the number of available alternatives in order not to lose the best solutions found.
- **Calculating the initial alternatives set, X_1 :** The calculation of the initial set of alternatives that the experts will use in their first round of discussion can have a great impact on the final outcome. We propose 3 different ways of doing this calculation: Random selection; promoting certain characteristics on the initial set of alternatives; Providing alternatives that are very different among them.

GDM process starts after this parameters have been defined.

2.2.2. Calculating the alternatives ranking

Once the experts have conducted the discussion round, their preferences are collected. Using a more formal description, we can identify this process as one in which the expert e_k provides the system, in decision round o , with the values p_{ij}^{ko} . These values indicate how preferred alternative x_i is over x_j . To help the experts provide the information in a convenient way, we allow them to use linguistic labels. Once all experts have provided the system

with their preferences, we then compute the collective preference matrix, C^o . This matrix contains the aggregation of all the preferences that the experts have provided to the system. For round o , we can calculate the C^o matrix as follows [18]:

$$C^o = \frac{\sum_{k=1}^n \text{index}(p_{ij}^{ko})}{n} \quad (1)$$

where index is a function that returns the index of the label. Once the C^o matrix is obtained, we can calculate the ranking of alternatives using the GDD operator [10]. Each alternative is assigned a GDD value according to the following expression:

$$GDD_i^o = \phi(c_{i1}^o, c_{i2}^o, \dots, c_{i(i-1)}^o, c_{i(i+1)}^o, \dots, c_{in}^o) \quad (2)$$

where ϕ is the mean operator and c_{ij}^o the collective value for alternatives x_i and x_j . The ranking R^o is calculated by sorting the alternatives according to the GDD value obtained.

2.2.3. Stagnation Analysis

In environments where the number of alternatives is very high, it is very important to provide tools that allow experts to carry out GDM processes that are accurate in the analysis of alternatives within the debate. In addition, it is important that experts discuss a large number of alternatives. In particular, it is interesting that they discuss those for which the criteria used are the most appropriate. However, it is very rarely the case that experts know in advance which criteria are the most appropriate without carrying out a round of discussion beforehand. To solve and provide an organised decision scheme in this type of environment, we propose the use of stagnation measures. In particular, we propose the analysis of the last 3 rounds of GDM. In the case that the variability in preferences is low, this means that the experts are clear about their ideas and that they have already thoroughly analysed the available alternatives. Therefore, it is necessary to include new alternatives to the set. On the contrary, if the variability is high, then the experts are still putting their ideas in order and need more time for discussion.

To calculate the degree of stagnation of a given expert, e_i in round o , we can use the following expression:

$$Stag_{e_i} = \phi(\text{sim}(P^{i(o-2)}, P^{i(o-1)}), \text{sim}(P^{i(o-1)}, P^{i(o)})) \quad (3)$$

where P^{io} is the preference matrix provided by e_i in the round o . To calculate the overall degree of stagnation, we aggregate the values of all experts as follows:

$$GStag = \phi(Stag_{e_i}), i = 1, \dots, n \quad (4)$$

The GStag value is what we use to determine whether the debate has stalled or not. If we detect that there is no stalemate, then the experts debate one more round. On the contrary, if there is a stalemate, we move on to the phase of modifying the set of alternatives. To do this, we can follow the following steps: - We select the ar worst alternatives: In order to always keep in the set those alternatives that are at the top of the ranking, we always eliminate the ar worst alternatives. It is very important to always keep at least the most voted alternative, otherwise it would be lost and this alternative could very well be the solution that best meets the needs of the experts.

1. **We select the ar worst alternatives:** In order to always keep in the set those alternatives that are at the top of the ranking, we always eliminate the ar worst alternatives. It is very important to always keep at least the most voted alternative, otherwise it would be lost and this alternative could very well be the solution that best meets the needs of the experts.

In this article we propose several ways to replace the least desirable alternatives. In doing so, we always rely on the criteria associated with the alternatives as these give us information about the general characteristics of the alternatives. By studying the alternatives that are at the top of the ranking, it is possible to guess which criteria are preferred by the experts. We represent the criteria as follows:

$$Cri(x_j) = (cr_1(x_j), \dots, cr_i(x_j), \dots, cr_p(x_j)) \quad (5)$$

where $cr_i(x_j)$ is the index of the linguistic label that defines, within the FO, the relationship between the alternative x_j and the criterion cr_i . Taking these criteria into account, we propose three different ways to carry out the replacement of the alternatives. The underlying idea of each of them is different and they can be combined to generate a hybrid approach. In the following, we define each of them in more detail:

- **Exploitation strategy:** With this approach, we fill the set of alternatives with those alternatives that most closely resemble the alternatives that are located at the top of the ranking. That is, we use the characteristics of the most popular alternatives, to extract from the FO those that most resemble them. In this way, experts can focus on discussing the most promising alternatives and decide which of them is the best. The idea of this approach comes from the exploitation approach optimisation algorithms. Formally, we define the search we perform on the FO using the following expression:

$$Q(\text{round}(\phi(\text{Cri}(R_1^o) + \text{Cri}(R_2^o)))) \quad (6)$$

where Cri is the set of values indicating how each alternative fulfil each criteria. The parameters of this expression are the following: R_1^o and R_2^o are respectively the 2 best alternatives in the set. *round* refers to the rounding operator. After applying this expression, we will obtain those alternatives whose criteria are more similar to those of the criteria of the two most voted alternatives. Although in this paper we propose the use of the first two alternatives, it is possible to use more when generating the search. In this paper we propose the use of the first two alternatives as we will not put the experts to debate more alternatives than they can manage at the same time.

- **Exploration strategy:** This approach is complementary to the previous one. Its main objective is to have all experts discuss as many different alternatives as possible. By applying this method we will choose alternatives that are very different from each other. This will make the experts come up with different approaches, which will make the number of possibilities analysed as large as possible. The approach of this strategy is based on the idea of exploration that arises in the optimisation algorithms [21, 22, 23]. Therefore, this approach chooses alternatives that are very different from the most voted alternatives. For this purpose, we use the following expression:

$$Q(\text{round}(\phi(\text{Cri}(R_1^o) + \text{Cri}(R_2^o))))^C \quad (7)$$

In this case, C represents the complementary operation on the linguistic set used. The idea is to choose alternatives that meet other different criteria. In this way, experts can discuss alternatives that are totally different from each other, thus maximising the analysis of all possible criteria available. In other words, the aim of this approach is to introduce new solutions and points of view to the debate. The previous approach was to discuss alternatives similar to the most popular ones in order to find the best one.

- **Hybrid strategy:** Most optimisation algorithms use, in their first iterations, exploration strategies. When the exploration is sufficiently extensive, or when the time interval established for it has passed, then exploitation strategies are applied that allow the best solution found so far to be optimised. The hybrid strategy combines the two approaches seen in this scheme. Thus, we apply, during a series of rounds, the exploration approach and, in the last rounds, we apply the exploitation approach. In this approach we define the parameter *extime* which indicates the number of rounds in which we are going to carry out the exploration process. Once this number of rounds has passed, we apply the exploitation process previously discussed. In this way, we allow experts in the first rounds to discuss alternatives that are very different from each other and therefore provide different approaches. In this first part, the experts implicitly define what an alternative must fulfil in order to adequately solve the problem posed. Finally, the experts discuss alternatives similar to the most voted alternatives in order to find, among all the alternatives that meet the desired criteria, the most optimal one.

All proposed strategies are equally valid and interesting. The choice depends on the problem at hand and the experts. Once we have done the search and extracted the alternatives, they replace the less popular ones and a new round of debate begins. In case that the alternatives have been replaced, stagnation analysis will not be performed again until three new rounds have passed.

3. Example

In order to improve the understanding of the proposed method, in this section we present an example of application of the measures and processes seen. Let us imagine that there is a company run by four managers,

Criteria	Description	Criteria	Description
cr_1	CPU speed	cr_7	OS support
cr_2	Size	cr_8	Storage
cr_3	Weight	cr_9	Security
cr_4	Connectivity	cr_{10}	Screen refresh
cr_5	Memory	cr_{11}	Price
cr_6	Camera	cr_{12}	Battery life

Table 1. Criteria description.

$E = \{e_1, e_2, e_3, e_4\}$, who have to select which enterprise mobile device they are going to purchase to give to their employees. The number of mobile devices is enormous in today's market. The managers investigate and find 150 different possibilities: $X = \{x_1, \dots, x_{150}\}$. The characteristics of the devices are known and they focus on evaluating 12 different criteria: $CR = \{cr_1, \dots, cr_{12}\}$. In the table 1, we show a list of the criteria and which feature of the phone each one refers to. As parameters, we set the stagnation parameter to 2. Also, the granularity of the alternatives set is set to 5. Finally, $ar = 3$. This means that the 2 best alternatives will always remain in the set of alternatives while, when the debate stalls, the 3 worst alternatives will be modified.

The first step is to search the FO to obtain the initial set of alternatives. As there is no preference specified with respect to the criteria, we select 5 alternatives that have very different criteria values from each other. In this way, we will promote the process of exploring the set. To obtain these alternatives, we perform the 5 searches detailed below:

$$Q_1 = \{7, 7, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$$

$$Q_2 = \{0, 0, 0, 0, 0, 0, 0, 0, 0, 7, 7, 0\}$$

$$Q_3 = \{0, 0, 7, 0, 0, 0, 0, 7, 0, 0, 0, 7\}$$

$$Q_4 = \{0, 0, 0, 7, 7, 0, 0, 0, 0, 0, 0, 0\}$$

$$Q_5 = \{0, 0, 0, 0, 0, 7, 7, 0, 0, 0, 0, 0\}$$

As can be seen, the search positions marked as 0 indicate the criteria that are not used in the process. The remaining values, [1,7] indicate the index of the linguistic label that determines the desired level of compliance of the alternative. Since we have considerably more criteria than alternatives, it is not possible to obtain alternatives that fulfil all criteria separately. Therefore, we have chosen a scheme that covers the maximum number of available options. Note that in this example we assume that all criteria are equally important. Therefore, we do not add weights to any criteria.

After performing the search process, we obtain the following initial set of alternatives:

$$RedX = \{x_{50}, x_{77}, x_{25}, x_{17}, x_{57}\}$$

With the initial set of alternatives calculated, the experts start the discussion. They then provide preferences to the system using linguistic labels from the set S^7 .

They are aggregated in order to generate C^1 . The resulting matrix is shown below:

$$C^1 = \begin{pmatrix} - & 2 & 2.75 & 2.25 & 3.5 \\ 5.25 & - & 4.5 & 3.5 & 4.75 \\ 1.5 & 2 & - & 1.25 & 2.25 \\ 3 & 2.25 & 3.5 & - & 4 \\ 2 & 1.5 & 2.75 & 2.75 & - \end{pmatrix}$$

By applying expression (2), the temporary alternatives ranking used for the stagnation analysis is obtained. GDD values are shown below:

$$GDD = \{0.27, 0.58, 0.125, 0.364, 0.21\}$$

	x_{77}	x_{17}	mean	query
cr_1	s_3	s_2	2.5	5
cr_2	s_1	s_6	3.5	4
cr_3	s_1	s_7	3	5

Table 2. Labels associated to the relations of x_{77} and x_{17} for the first three criteria values.

This generates the following temporary ranking: $R^1 = \{x_{77}, x_{17}, x_{50}, x_{57}, x_{25}\}$.

Once the time ranking has been calculated, rounds 2 and 3 of the debate are carried out. Once round 3 has been reached, we apply the stagnation analysis to determine whether we should modify the set of alternatives or whether, on the contrary, the experts should continue debating about it. During the 3 rounds that have taken place, expert e_1 has provided the following information to the system:

$$P^{11} = \begin{pmatrix} - & s_4 & s_4 & s_2 & s_3 \\ s_6 & - & s_4 & s_4 & s_5 \\ s_1 & s_3 & - & s_2 & s_1 \\ s_1 & s_2 & s_4 & - & s_2 \\ s_2 & s_2 & s_4 & s_3 & - \end{pmatrix} P^{12} = \begin{pmatrix} - & s_6 & s_4 & s_2 & s_3 \\ s_6 & - & s_4 & s_4 & s_5 \\ s_2 & s_2 & - & s_3 & s_1 \\ s_1 & s_2 & s_3 & - & s_2 \\ s_2 & s_2 & s_4 & s_3 & - \end{pmatrix} P^{13} = \begin{pmatrix} - & s_5 & s_4 & s_2 & s_3 \\ s_6 & - & s_5 & s_4 & s_5 \\ s_2 & s_3 & - & s_3 & s_1 \\ s_1 & s_2 & s_3 & - & s_2 \\ s_1 & s_2 & s_4 & s_3 & - \end{pmatrix}$$

It is possible to obtain the stagnation value associated to e_1 as follows:

$$Stag_{e_1} = \phi(sim(P^{12}, P^{11}), sim(P^{13}, P^{12})) = 0.2.$$

For e_2 we obtain the value of 0.3. Moreover, for e_3 the value is 0.34. To calculate the global stagnation value, we apply the arithmetic mean. The result obtained is 0.28 within the range [0,6]. Therefore, as the value obtained is less than 2, then the debate can be considered to be stagnant. Therefore, we replace the 3 alternatives that are furthest back in the ranking, x_{50} , x_{57} and x_{25} . As this is the first round, we apply the exploration method. To perform the search for the new alternatives, we use the 2 best alternatives: x_{77} and x_{17} . We can see this search in the FO in more detail in the table 2.

After the search process, we determine that the alternatives that best meet the search criteria are x_{35} , x_2 and x_{120} . Therefore, the set of alternatives that the experts will discuss in the fourth round is: $RedX = x_{77}, x_{17}, x_{35}, x_2, x_{120}$. Preferences provided are shown below:

$$P^{14} = \begin{pmatrix} - & s_3 & s_5 & s_3 & s_2 \\ s_6 & - & s_2 & s_1 & s_4 \\ s_2 & s_3 & - & s_4 & s_2 \\ s_3 & s_2 & s_3 & - & s_2 \\ s_6 & s_6 & s_5 & s_7 & - \end{pmatrix} P^{24} = \begin{pmatrix} - & s_4 & s_3 & s_2 & s_4 \\ s_3 & - & s_2 & s_2 & s_1 \\ s_2 & s_3 & - & s_3 & s_5 \\ s_2 & s_3 & s_1 & - & s_5 \\ s_6 & s_7 & s_6 & s_6 & - \end{pmatrix}$$

$$P^{34} = \begin{pmatrix} - & s_3 & s_1 & s_2 & s_2 \\ s_2 & - & s_3 & s_5 & s_5 \\ s_1 & s_1 & - & s_3 & s_4 \\ s_2 & s_1 & s_4 & - & s_2 \\ s_7 & s_7 & s_7 & s_6 & - \end{pmatrix} P^{44} = \begin{pmatrix} - & s_1 & s_1 & s_1 & s_2 \\ s_1 & - & s_2 & s_1 & s_3 \\ s_2 & s_3 & - & s_2 & s_1 \\ s_1 & s_2 & s_3 & - & s_2 \\ s_6 & s_6 & s_5 & s_6 & - \end{pmatrix}$$

The collective preference matrix, C^4 , is as: The GDD vector in the fourth round can be seen below:

$$GDD = \{0.24, 0.281, 0.26, 0.23, 0.864\}$$

Finally, the ranking on round 4 is:

$$R^4 = \{x_{120}, x_{17}, x_{35}, x_{77}, x_2\}$$

4. Conclusions

This article proposes a new GDM system that works in environments with large numbers of alternatives and criteria. The experts focus on discussing alternatives extracted from the FO and, once the worst rated alternatives

have been discussed in depth, they are replaced by others following a series of characteristics related to the criteria of the alternatives that have been best rated by the experts. In total we propose 3 different replacement methods, each with its own advantages and disadvantages.

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