Introduction

There is a paradoxical situation in modern physics. On the one hand, it has achieved amazing success and the technical devices based on modern physics make it possible what recently seemed like a miracle. Unique microelectronics, Internet, mobile telecommunication systems and many other inventions made at the end of 20th century look like something fantastic. On the other hand, by now there are a large number of natural phenomena that cannot be explained with the help of conventional physical science, which indicates its serious incompleteness. There are also many fundamental questions of physics that remain unsolved despite numerous attempts to find clues to them. All this indicates that theoretical physics needs new approaches and new theories that could shed light on the secrets of nature.

One of the ways to renew physical science is to study nonlinear generalizations of classical theories. This topic is central to the present monograph. It begins with an analysis of the famous Yang–Mills theory proposed in 1954. At first, this theory did not arouse much interest, since it was focused on the description of hypothetical particles with isospin about which the existing experimental data were silent. But after it played an important role in creating a model of electroweak interactions and correctly predicting intermediate Z and W bosons involved in them, interest in the Yang–Mills theory became very large.

However, this model of electroweak interactions was based not only on the Yang–Mills field but also on other concepts. In particular, on the Higgs field. As a result, there was no great clarity in the question of what the Yang–Mills field itself is. It should be noted that this field has always been treated as a purely quantum phenomenon, applicable only in the microcosm. However, little attention has been paid to an important feature of the Yang–Mills field equations: they are a reasonable nonlinear generalization of Maxwell's equations for the electromagnetic field. At the same time, in relation to Maxwell's theory, which is undoubtedly a reliable foundation for describing a very wide range of electromagnetic phenomena, the question sometimes arose as follows: Does it cover the entire set of electromagnetic phenomena? The reason for the emerging doubts is the linearity of Maxwell's equations at arbitrarily high intensities of electromagnetic fields. That is why such outstanding scientists as Einstein, Born, Infeld and some others believed that the true theory of the electromagnetic field should be nonlinear.

One of the most famous attempts to give a nonlinear generalization of Maxwell's equations was the Born–Infeld approach proposed in 1934. There is still a continuing interest in these nonlinear equations. But for almost nine decades of their existence, they have not received any experimental confirmation. At the same time, in 1954 a worthy competitor appeared for the Born–Infeld theory: the Yang–Mills theory. However, no special attention was paid to this, since the Yang–Mills theory was firmly assigned the label of a purely quantum theory, applicably to the description of only physical processes on a microscopic scale.

It should be said that the nonlinear equations proposed by C. N. Yang and R. Mills in 1954 are undoubtedly beautiful and have a three-dimensional Lie group of internal symmetry. They describe three vector fields, while Maxwell's equations describe only one such field. As for the energy–momentum tensors of the Yang–Mills fields, they are expressed by their field strengths in the same way as in Maxwell's theory. Besides, if the second and third Yang–Mills fields are absent, the first field is described by Maxwell's linear equations. Therefore, it makes sense to regard the Yang–Mills theory as a reasonable nonlinear generalization of Maxwell's electromagnetic theory.

For this reason, the present monograph pays great attention to the equations of nonlinear electrodynamics based on the Yang–Mills theory. To them, a number of exact solutions in the cases of spherically symmetric and axially symmetric field sources and a number of classes of exact wave solutions are found in the monograph. The results obtained are used to explain a variety of phenomena that are mysterious within the framework of Maxwell's electrodynamics. This is the mysterious properties of ball lightning, the unusual features of streak lightning, the striking phenomenon of reversals of the magnetic poles of the Sun and the Earth, the unrevealed puzzle of amazing circles on cereal-planted fields and a number of other intriguing phenomena.

Another group of questions raised in the monograph concerns the old problem of nuclear forces that bind protons and neutrons in atomic nuclei. As is known, the first successful attempt to describe them was the Yukawa meson theory proposed by him in 1935. His idea made it possible to explain the short-range character of nuclear forces and to give a correct prediction of the mass of their carriers — pions and the zero spin of these particles. However, subsequent experiments have shown that this theory, in which the nuclear potential is described by a linear equation of the Klein–Gordon type, can be valid only for relatively small values of this potential. If its value becomes large enough, then there are essentially nonlinear effects that are not described by the linear Yukawa equation. One of them is the interesting effect of saturation of nuclear forces. It manifests itself in changing the sign of nuclear forces at sufficiently large potentials, that is, when passing from the usual property of attraction, like the forces of gravity, to the phenomenon of repulsion inside nuclear matter. Unexplained in Yukawa's theory are the properties of the specific binding energies of atomic nuclei, which for some mysterious reason have a maximum at the iron nucleus, and much more. For these reasons, Yukawa's theory, after a short period of great success and bright hopes, faced a crisis. It became clear that the true equation for the nuclear potential must be essentially nonlinear. Later, a number of nonlinear models were proposed. A very interesting attempt was the now almost forgotten nonlinear meson Schiff model. However, in these models, it was not possible to achieve quantitative agreement with experiments. New hopes for a description of nuclear forces arose with the emergence of quantum chromodynamics, which is now considered the most likely candidate for the role of a correct theory of strong interactions. But it also did not lead to a quantitative theory of nuclear forces due to the impossibility of applying its main approach, perturbation theory, to atomic nuclei.

Thus, the problem of describing nuclear forces remained unresolved. At the same time, it is difficult to completely abandon the Yukawa model, which was successful in the case of relatively small values of nuclear potentials. In view of this, in the monograph, a new attempt to give a nonlinear generalization of Yukawa's theory is undertaken. It is based on the idea of the dependence of the mass of nuclear particles on the potential of nuclear forces, which is just not taken into account in the Yukawa model. As our studies have shown, this dependence can be determined from the general principles of relativistic dynamics. The nonlinear generalization of the Yukawa equation found in this way is applied to theoretically determine the binding energies and radii of medium and heavy atomic nuclei, for which the classical approximation can be valid. The calculations performed for such nuclei showed good agreement between the proposed model and the known experimental data. Using this model, the dynamics of relativistic nucleons and antinucleons near heavy atomic nuclei is studied. The obtained results can be applied to the problem of the formation of quasi-nuclei, in which protons or antiprotons revolve around heavy atomic nuclei. In addition, a detailed analysis of the effect of the saturation of nuclear forces on the equilibrium of cooled massive neutron stars is carried out. It is shown that this effect which manifests itself in the appearance of nuclear repulsive forces can compensate for the gravitational compression of the neutron stars under consideration.

Serious attention in the monograph is also given to the problem of the quantum description of nucleons. As is known, the proton consists of two u-quarks having $+2/3$ of its charge and one d-quark having $-1/3$ of its charge, and the neutron consists of one u-quark and two d-quarks. In addition, nucleons have significant anomalous magnetic moments, positive for the proton and negative for the neutron. In view of these properties, it is impossible to directly apply the famous Dirac equation to the quantum description of nucleons. This equation, which is valid for the relativistic electron, does not give anomalous values of the magnetic moments and quark structure of particles. It should be noted that for the description of nucleons, there is one well-known generalization of the Dirac equation. It assumes an additional, nonminimal interaction of nucleons with an electromagnetic field. But in it, the values of the anomalous magnetic moments of nucleons are not determined theoretically. Their experimental values are simply inserted into the equation as coefficients. Moreover, this equation does not reflect the quark structure of nucleons. That is why the monograph poses the problem of finding a new generalization of the Dirac equation, which would not have these two defects. The idea of such a generalization is based on the multiplication of charge and mass in the Dirac equation by special matrices of the third order, composed of the quark numbers $+2/3$ and $-1/3$. It turns out that these matrices can be chosen so that the resulting generalization of the Dirac equation corresponds to the basic principles of quantum mechanics and to the quark structure of nucleons. Besides, the anomalous magnetic moments of nucleons determined on the basis of this equation agree with experimental data. Further, a more complete generalization of the Dirac equation is considered, which is used to describe the quantum properties of systems of closely spaced nucleons and light atomic nuclei. This is achieved by replacing the Dirac matrices with generalizing matrices of the Clifford algebra. A detailed study of the proposed quantum equation for systems of nucleons and light atomic nuclei is carried out to investigate the properties of their quark currents.

The last two chapters of the monograph are devoted to unsolved problems of Einstein's general theory of relativity and the cosmology based on it, as well as to a number of mysterious astronomical phenomena unexplained within its framework.

The first group of these questions is connected with the description within the framework of Einstein's general theory of relativity of a rather wide class of frames of reference: noninertial elastically deformed frames. The problem is the undoubted merit of the equations of the gravitational field of general relativity — their validity in an arbitrary coordinate system also has a negative side. Namely, the solutions of Einstein's equations contain four arbitrary functions of space–time coordinates, due to the arbitrariness of their choice, which makes it impossible to describe specific noninertial frames of reference. This circumstance leads to the emergence of significant difficulties in the study of gravitational waves since their behavior directly depends on the properties of the chosen frames of reference.

Before proceeding to the study of the class of elastically deformed frames of reference, a simpler class of frames is examined in which their own elastic deformations can be neglected. Such frames of reference are called perfect by us. Based on Einstein's principle of equivalence and general requirements for frames of reference, a system of four differential equations for the components of the metric tensor is proposed in the monograph to describe the perfect frames of reference under examination. These systems of equations are analyzed by us and a number of their exact solutions are found which correspond to several specific states of perfect frames. Further, the difference between the metric tensors in an elastically deformed frame and in a perfect frame comoving with it is determined through the strain tensor. The obtained relations make it possible to find metric tensors in elastically deformed frames from known solutions in perfect frames. Of particular interest is the form of solutions found in the monograph which describes gravitational waves relative to extended perfect frames of reference. As it turns out from their analysis, situations of significant amplification of gravitational waves are possible. Such anomalous waves can be regarded by an observer as some others, nongravitational waves. Therefore, the new results concerning gravitational waves may be important for the correct identification of these waves when observing them.

Another circle of issues studied in the monograph is associated with the need to suggest a new cosmological theory since standard cosmology cannot explain the origin of the cosmological singularity, the nature of dark matter, the existence of orderly disk galaxies more than 13.4 billion years ago, as discovered by the James Webb Space Telescope, and a number of other cosmological phenomena.

For this purpose, a generalization of the equations of general relativity is proposed in which the influence of physical vacuum on physical processes is taken into account. Physical vacuum is a special extremely rarefied state of matter, the influence of which on moving bodies is very small. However, over very long cosmological time intervals, this small effect can gradually accumulate and lead to quite noticeable results. This point of view is developed in the monograph. To describe small vacuum corrections to the equations of general relativity, the Weyl conformal geometry is applied, which generalizes the Riemann geometry used in Einstein's theory. A remarkable property of Weyl's geometry is the equivalence of expressions for the space–time intervals which differ in the scale factor. This is achieved by introducing an additional field with four potentials to the Einstein field with the ten gravitational potentials, which are components of the metric tensor. Weyl interpreted his additional field as electromagnetic, trying to create a unified theory of gravitational and electromagnetic forces. But this point of view turned out to be wrong. At the same time, it is possible to give the Weyl field a different interpretation as a very weak field caused by the physical vacuum. This approach turned out to be fruitful, allowing us to explain a number of cosmological phenomena that are difficult to explain in standard cosmology. Its development leads to generalizations of Einstein's gravitational and Maxwell's electromagnetic theories, which differ significantly from them only on cosmological space–time scales. The proposed system of field equations is applied to a homogeneous and isotropic vacuum and a new nonsingular cosmological solution to them is obtained. This solution is used to describe the influence of the Weyl field on propagating electromagnetic waves and moving free particles in vacuum. As a result, we arrive at a new cosmological theory, which is a real alternative to standard cosmology. A detailed analysis shows that it is in agreement with the known observational data, including the latest data obtained from the James Webb Space Telescope. In addition, the proposed cosmology explains the nature of dark matter and gives a simple explanation of the spiral structure of most galaxies and a number of their evolutionary properties, which have not found a satisfactory interpretation within the framework of previous theories.