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Continuum Model of an Avalanche-Like Spread of Information on Twitter

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Abstract. The research results devoted to the construction and analysis of the stochastic nonlinear dynamic system of equations that simulate the self-organization of Twitter into the critical state are presented. A nonlinear dynamic system links three dynamic variables. The first variable corresponds to the order parameter determined by the average size of avalanches of microposts (tweets, retweets). The second variable is the conjugate field to the sizes of micropost avalanches is Tsallis entropy. The third variable is the control parameter determined by the generalized velocity of micropost distribution. Fluctuations of synergistic parameters considered in the form of additive white noise. The three-parameter scheme of selforganization allows to describe the basic properties of both a second-order phase transition and self-organization into a critical state. To describe a phase transition of the second order in a social network, the standard system of Lorenz equations in the adiabatic approximation is sufficient. To describe the transit of a social network into the self-organized criticality state, it is necessary to weaken the feedbacks by raising the order parameter (the size of the avalanches of microposts) to a fractional power and consider the fluctuations of the control parameter (the speed of microposts distribution in the network).

INTRODUCTION

The presented research is motivated by numerous empirical evidences of an avalanche-like information distribution in microblogging social networks, in particular, in Twitter (see the papers [1][2][3][4][5][6][7][8][9] and references to them).

It is known from [10] that an avalanche-like behavior of systems is due to their self-organization into the critical state. If a social network is in the critical state, probability distribution density of occurring information avalanches of size η is determined by the following relationship:

$$p(\eta) \propto \eta^{-\alpha} \mathfrak{M}\left(\frac{\eta}{\eta_s}\right), \tag{1}$$

where $\alpha \in (1,3)$, η_s is a scaling constant.

Indicators α of distribution of avalanches of microposts corresponding to some topics are presented in table 1.

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TABLE 1. Estimates of the values of the α for the time series of microposts

Topic	Point estimation	p-value
2016 United States Presidential Election	1.23	0.0121
Women's March	2.11	0.0234
End of Term 2016 US government	3.24	0.6743
Hurricanes Harvey	2.12	0.0312
Hurricanes Irma	2.23	0.0234
Immigration and Travel Ban	2.18	0.0401
Charlottesville	2.18	0.0313
Winter Olympics 2018	3.59	0.7239
US Government	3.28	0.6361
News Outlet	3.36	0.4275
2018 US Congressional Election	1.47	0.0281
115th US Congress	3.99	0.3189
Ireland 8th	2.18	0.0311

Despite the numerous empirical evidences that the social network can be in a self-organized critical state, the analyzed literature does not present synergistic schemes for the self-organization of social networks into a critical state. This scheme is a generalization of one-parameter and two-parameter schemes of critical transitions widely presented in the scientific literature (see the papers [11]).

The one-parameter scheme is usually based on the diffusion equation, for example, diffusion in ultrametric space. The two-parameter scheme gives a more meaningful picture of critical transitions. So, in the well-known sandpile model, the order parameter corresponds to the density of active nodes, the control parameter corresponds to the conserved energy. Self-organization into a critical state is the result of a competition between energy pumping and energy dissipation. The one-parameter and two-parameter approaches do not take into account the relationship between the open system and the environment, and, therefore, does not provide an opportunity to describe the self-consistent behavior of the avalanche's dynamics. To obtain a general picture of self-organization into the critical state, it is necessary to consider the coordinated dynamics of at least three parameters.

SINERGETIC MODEL OF CRITICAL TRANSITION

A generalization of the two-parameter model (order parameter versus control parameter) of critical and phase transitions is the three-parameter scheme of self-organization of an open nonequilibrium system into a critical state. Such a scheme is based on the consistent nonlinear dynamics of the order parameter, conjugate field, and control parameter. The three-parameter scheme describes the self-organization of open systems, the significant distance of which from equilibrium leads to a decrease in the system's entropy. As a result of such self-organization, a metastable ordered state arises, which corresponds to the minimum of the synergistic potential. The system can be in such a state indefinitely, at least as long as the system is under the external influence. This state is stationary and nonequilibrium. The justification for using the three-parameter scheme is the Ruelle–Takkens theorem, according to which a non-trivial picture of self-organization is achieved only if the number of degrees of freedom (parameters) parameterizing the evolution of the system is at least three.

Let's define the parameters (dynamic variables), the consistent evolution of which describes the self-organization of Twitter.

The order parameter (η) is defined as the average size of avalanches of microposts (tweets, retweets) relevant to a certain topic. The parameter $\eta = 0$ if the social network is in a disordered (subcritical) phase, and $\eta \neq 0$ if the network is in an ordered (supercritical) phase.

When determining the conjugate field (h) and the control parameter (ν), we will consider Twitter as a non-additive (non-extensive) system, it means a complex (anomalous) system, which is characterized by the effects of strong long-range action in the exchange of information between users of a social network, non-local (long-distance) correlations between individual elements of the system (individual communities of the network) and the fractal nature of the phase space and graph structure of the social network. The complex space-time structure of anomalous systems leads to a violation of the entropy additivity principle.

Considering the social network as a non-additive system, we define the control parameter in the form of the generalized velocity of micropost distribution, averaged by the parameter $q \neq 1$:

$$v_q \equiv \sum_i v_i p_i^q, \tag{2}$$

where p_i^q is the probability of the *i*-th micropost distribution in the network with the velocity v_i, $q \neq 1$ – is the parameter characterizing the degree of non-additivity of the network. The speed (2) is the velocity of the network user's reflection to the micropost received by him and is not the physical velocity of information distribution.

The parameter of the field conjugate to the sizes of micropost avalanches is Tsallis entropy:

$$h_q \equiv \frac{1-\sum_i p_i^q}{q-1}. \tag{3}$$

Entropy (3) determines the degree of the network's disorder, and as $q \to 1$ entropy (3) converges to Gibbs entropy ($-\sum_i p_i \ln p_i$). The introduction of the entropy (3) allows us to take into account the following important feature of the social network's evolution: the probability of realizing small values of v_i decreases according to a power law.

Phase Transition

In this Subsection, it is shown that external influence on the system can lead to its self-organization, as a result of which a nonequilibrium system passes into an ordered state.

The simplest scheme for describing the self-organization of the system is the Lorenz scheme:

$$\begin{cases} \tau_\eta \dot{\eta} = -\eta + a_\eta h_q \\ \tau_h \dot{h}_q = -h_q + a_h \eta v_q \\ \tau_v \dot{v}_q = \left(v^{(e)} - v_q\right) - a_v \eta h_q \end{cases}. \tag{4}$$

In the system (4) the dot denotes differentiation in time; τ_η, τ_h and τ_v – relaxation times of the parameters η, h_q and v_q to its stationary values 0 and $v^{(e)}$, respectively; a_h and a_v – positive constants of feedbacks between system parameters. A negative sign before $a_h \eta v_q$ in the third equation indicates the presence of negative feedback ηh_q and v_q, a positive sign before $a_h \eta v_q$ in the second equation indicates positive feedback between ηv_q and h_q. Positive feedback leads to self-organization of the system.

System (4) has no analytical solution; therefore, we analyze it in the adiabatic approximation. According to the principle of adiabatic subordination, during the system's evolution, parameters with short relaxation times have time to relax to stationary values. Formally, the principle is expressed by the following dependencies:

$$\tau_\eta \gg \tau_h, \tau_v, \tau_h \dot{h}_q = \tau_v \dot{v}_q = 0. \tag{5}$$

Taking into account (5), we can reduce the dimension of system (4), which describes the self-organization of the social network, to one equation describing the nonlinear dynamics of avalanche sizes:

$$\tau_\eta \dot{\eta} = \frac{a_\eta a_h v^{(e)} \eta}{1 + a_h a_v \eta^2} - \eta. \tag{6}$$

Equation (6), known as the Landau–Khalatnikov equation, describes the dissipative mode of evolution of the nonequilibrium social network into the stationary state.

The zero stationary point is asymptotically stable at $v^{(e)} < v_C$, stationary point $\left(a_h a_v\right)^{-\frac{1}{2}} \left(a_\eta a_h v^{(e)} - 1\right)^{\frac{1}{2}}$ is asymptotically stable at $v^{(e)} > v_C$. Critical value of the pumping parameter: $v_C = \left(a_\eta a_h\right)^{-1}$.

Thus, the v_C value is a critical point of a second order phase transition or the transition point of the social network from a disordered phase to an ordered phase.

Self-Organized Critical Transition

In contrast to a second-order phase transition, which occurs when the control parameter reaches its critical value, the transition of the system into a self-organized critical state does not require an intense external influence. Many systems have been discovered in which self-organization occurs spontaneously. An essential feature of self-organized criticality (SOC) is the discontinuous nature of the process, which corresponds to the intermittency regime. In this regime, dissipation prevents the spontaneous activation of the social network, and the network is in a subcritical state during a long time. Along with this, uncompensated activation of the network can spontaneously occur, which leads to its self-organization. After that, within a short time interval, the network is discharged in the form of microposts spreading in the network, i.e., an avalanche of microposts of size η is formed.

The main difference between SOC and the second order phase transition is that the transition of a social network into the SOC state occurs in the absence of external influence. The evolution of a social network into the SOC state, described by the power law of distribution of the sizes of micropost avalanches (see Eq. 1), is a characteristic feature of this state.

To describe the self-organization of the social network, the following modifications of the Lorenz scheme are required (4):

- Weaken negative and positive feedbacks due to the introduction a fractional power (β) for the parameter η in the second and third equations of the system.
- Take into account the fluctuations of the parameters η, h_q and v_q due to the introduction of an additive noise (ξ) in the equations (4).

As a result, the Lorenz scheme describing the self-similar evolution of a social network will take the following form:

$$\begin{cases} \dot{\eta} = -\eta + h_q + \sqrt{I_\eta}\xi \\ \varepsilon \dot{h}_q = -h_q + \eta^\beta v_q + \sqrt{I_h}\xi \\ \delta \dot{v}_q = \left(v^{(e)} - v_q\right) - \eta^\beta h_q + \sqrt{I_v}\xi \end{cases}, \tag{7}$$

where ξ is the white noise, I is the intensity of noises of each of the parameters, $\beta \in \left(0,1\right)$.

System (7) is written in dimensionless variables obtained as a result of the following renormalizations:

$$\eta \equiv \left(a_h a_v\right)^{-\frac{1}{2\beta}} \eta, h_q \equiv \left(a_h a_v\right)^{-\frac{1}{2\beta}} \left(a_\eta\right)^{-1} h_q, v_q \equiv \left(a_h a_v\right)^{-\frac{1-\beta}{2\beta}} \left(a_\eta\right)^{-1} v_q,$$

$$I_\eta \equiv \left(a_h a_v\right)^{-\frac{1}{\beta}} I_\eta, I_h \equiv \left(a_h a_v\right)^{-\frac{1}{\beta}} \left(a_\eta\right)^{-2} I_h, I_v \equiv a_h^{-\frac{1+\beta}{\beta}} a_v^{-\frac{1-\beta}{\beta}} \left(a_\eta\right)^{-2}, \tag{8}$$

$$\varepsilon \equiv \frac{\tau_h}{\tau_\eta}, \delta \equiv \frac{\tau_v}{\tau_\eta}.$$

System (7) in the adiabatic approximation represents the Langevin equation of the following form:

$$\dot{\eta} = \left[-\eta + \frac{v^{(e)}\eta^\beta}{1+\eta^{2\beta}}\right] + \sqrt{\left(\sqrt{I_\eta} + \frac{\sqrt{I_\eta} + \sqrt{I_v}\eta^{2\beta}}{1+\eta^{2\beta}}\right)^2} \xi. \tag{9}$$

Equation (9) is the stochastic Landau–Khalatnikov equation describing the dissipative mode of evolution of a nonequilibrium social network to a stationary state, considering fluctuations of the order parameter. This equation will make it possible to describe self-organization into a critical state of deterministic and stochastic systems.

It is known that SOC in deterministic systems can be obtained using deterministic cellular automata, for example, BTW model, DR model, and FF model [10]. The Fig. 1 shows the dependences of the stationary values of the size of the microposts avalanches (η_{st}) on the stationary value of the distribution velocity of microposts ($v^{(e)}$) at different β. The value η_{st} is the solution of the equation $f(\eta) = 0$.

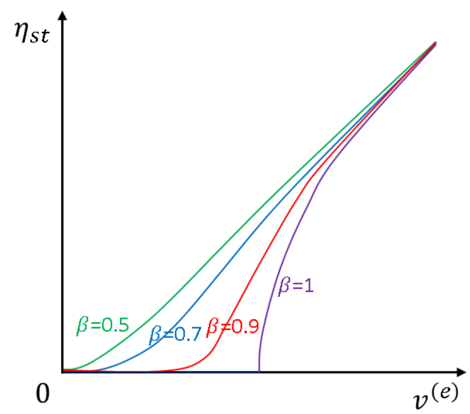

FIGURE 1. η_{st} versus $v^{(e)}$

The dependencies shown in the Fig. 1, clearly demonstrate the possibility of the social network to enter into the SOC state ($\beta = 0.5, 0.7, 0.9$) at $v^{(e)} > 1$ even in the absence of fluctuations of the systems parameters ($I_\eta, I_h, I_v = 0$). Entering the SOC state is solely due to the weakening of negative and positive feedbacks between the parameters of the social network.

Stochastic systems are also capable to self-organize into a critical state. Such a mode of self-organization is observed, for example, in stochastic cellular automata (Manna model and PSV model) [10]. In this case, consideration of fluctuations in the parameters of the network ($I_\eta, I_h, I_v \neq 0$) leads to its stochastic behavior, which, even in the absence of external pumping ($v^{(e)} = 0$) can lead the network to self-organization into a critical state.

The stationary distribution density of the probability of occurrence of information avalanches with size η, which is a solution of the corresponding Fokker – Planck equation, has the following form:

$$p(\eta) \propto \frac{1}{I(\eta)} \exp\left[\int_{\eta_0}^{\eta} \frac{f(\eta')}{I(\eta')} d\eta' \right] \tag{10}$$

Power law (1), typical for the SOC state of the network, is realized (1) in the absence of external pumping of the network ($v^{(e)} = 0$) and (2) with $\eta \ll 1$ and $I_\eta, I_h \ll I_v$. In this case, the stationary probability distribution density is determined by the following expression:

$$p(\eta) \propto \frac{\left(1+\eta^{2\beta}\right)^2}{I_v \eta^{2\beta}} \exp\left\{ -\left[\frac{1}{I_v} \left(\frac{\eta^{2-2\beta}}{2-2\beta} + \eta^2 + \frac{\eta^{2+2\beta}}{2+2\beta} \right) \right] \right\} \propto \eta^{-\alpha} \mathfrak{M}\left(\frac{\eta}{\eta_s} \right). \tag{11}$$

with $\alpha = 3/2, \beta = 3/4$ (see the paper [11]) for the SOC state of the social network evolution.

The dependence shown in the Fig. 2, demonstrate the power law distribution (11) at $I_v = 1000, \beta = 0.75, \eta \ll 1$.

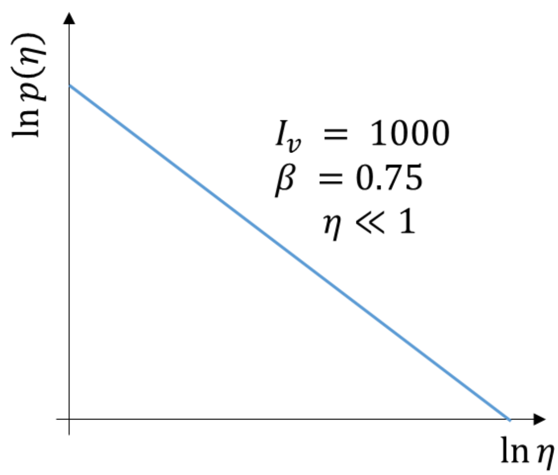

FIGURE 2. Log-log plot of the distribution (11)

CONCLUSION

The three-parameter scheme of self-organization allows to describe the basic properties of both a second-order phase transition and self-organization into a critical state. To describe a phase transition of the second order in a social network, the standard system of Lorenz equations in the adiabatic approximation is sufficient. To describe the transit of a social network into the SOC state, it is necessary to weaken the feedbacks by raising the order parameter (the size of the avalanches of microposts) to a fractional power and consider the fluctuations of the control parameter (the speed of microposts distribution in the network).

It is known [11][12] that the Lorenz system makes it possible to describe a first-order phase transition. For this, in the system of equations (4), it is necessary to consider the increasing nature of the relaxation time in the following form:

$$\frac{1}{\tau_\eta(\eta)} = \frac{1}{\tau_0}\left(1+\frac{\kappa}{1+\left(\frac{\eta}{\tau_\eta}\right)^2}\right). \quad (12)$$

As a result, with a slow increase in the control parameter up to the critical value (v_C), the stationary value η jumps from zero to non-zero, then the value of η gradually increases. When the value of the control parameter decreases to the value v_C, the stationary value of the parameter η smoothly decreases to a non-zero value, then takes zero value.

Despite the possibility of constructing a three-parameter scheme of the first-order phase transition, the problem of constructing a three-parameter Lorenz scheme for describing the transit of a system to a self-organized bistable state (SOB) remains unresolved. This transition corresponds to a first-order phase transition. It is possible that the social network may transit not only in the SOC state, but also in the SOB state. The construction of a three-parameter scheme for the SOB based on the Lorenz system of equations, as well as the detection of the SOB state because of the analysis of the observed time series of microposts, are tasks for further research.

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