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Erratum to: "On surfaces with zero vanishing cycles"

Published online: 4 July 2022

Erratum to: manuscripta math. 145, 235–242 (2014)
https://doi.org/10.1007/s00229-014-0672-z

In the proof of Proposition 2.3 of the paper [1] I referred to a result form the paper [2] which is valid only in characteristic zero, while later in the paper I use Proposition 2.3 for varieties over an arbitrary algebraically closed field. What follows is a proof of this proposition that is valid in any characteristic.

Proof. Suppose that X is a smooth projective surface and $C \subset X$ is a curve such that $C \cong \mathbb{P}^1$, the self-intersection index (C, C) equals 1, and $C \subset X$ is an ample divisor. We are to prove that $X \cong \mathbb{P}^2$.

Observe that any flat deformation $C' \subset X$ of the curve $C \subset X$ is isomorphic to $\mathbb{P}^1$. Indeed, $\chi(\mathcal{O}_{C'}) = 1$ and C' is irreducible since $(C', C) = 1$ and C is ample. Hence, $h^0(N_{X|C'}) = 2$, $h^i(N_{X|C'}) = 0$ for $i > 0$, so if B is the connected component of the Hilbert scheme of curves on X which (the component) contains the point corresponding to C, then B is a (smooth) projective surface. If

$$\begin{tikzcd} T \arrow[r, "q"] \arrow[d, "p"'] & X \\ B & \end{tikzcd}$$

is the standard diagram representing the family of curves on X parameterized by B, then, for a general (closed) point $x \in X$, one has $\dim q^{-1}(x) = 1$.

Let $\sigma : \tilde{X} \to X$ be the blowup of X at x. Proper transforms (with respect to σ) of the curves from the family B passing through x, are isomorphic to $\mathbb{P}^1$ and have zero self-intersection. Arguing as in the proof of Proposition 2.2 in [1], we conclude that $\tilde{X}$ admits a morphism $\pi : \tilde{X} \to C$ onto a smooth curve C such that the fibers of π are the above-mentioned proper transforms, all isomorphic to $\mathbb{P}^1$. Restricting π to the exceptional curve $E = \sigma^{-1}(x) \subset \tilde{X}$, one concludes that there exists a surjective morphism $E \to C$, whence $C \cong \mathbb{P}^1$ by Lüroth's theorem. Besides, E is a section of the morphism π, so $\tilde{X}$ is a $\mathbb{P}^1$-bundle over $\mathbb{P}^1 \cong C$, so

The original article can be found online at https://doi.org/10.1007/s00229-014-0672-z
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https://doi.org/10.1007/s00229-022-01407-w

$X \cong \mathbb{P}(\mathscr{O}_{\mathbb{P}^1} \oplus \mathscr{O}_{\mathbb{P}^1}(d))$ for some $d \geq 0$. Since this $\mathbb{P}^1$-bundle has a section E with self-intersection equal to -1, one concludes that $d = 1$; the blowdown of such a section of $\mathbb{P}(\mathscr{O}_{\mathbb{P}^1} \oplus \mathscr{O}_{\mathbb{P}^1}(1))$ is isomorphic to $\mathbb{P}^2$, and we are done. $\square$

Declarations

Data Availability Statement Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

Conflict of interest The author states that there is no conflict of interest.

References

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